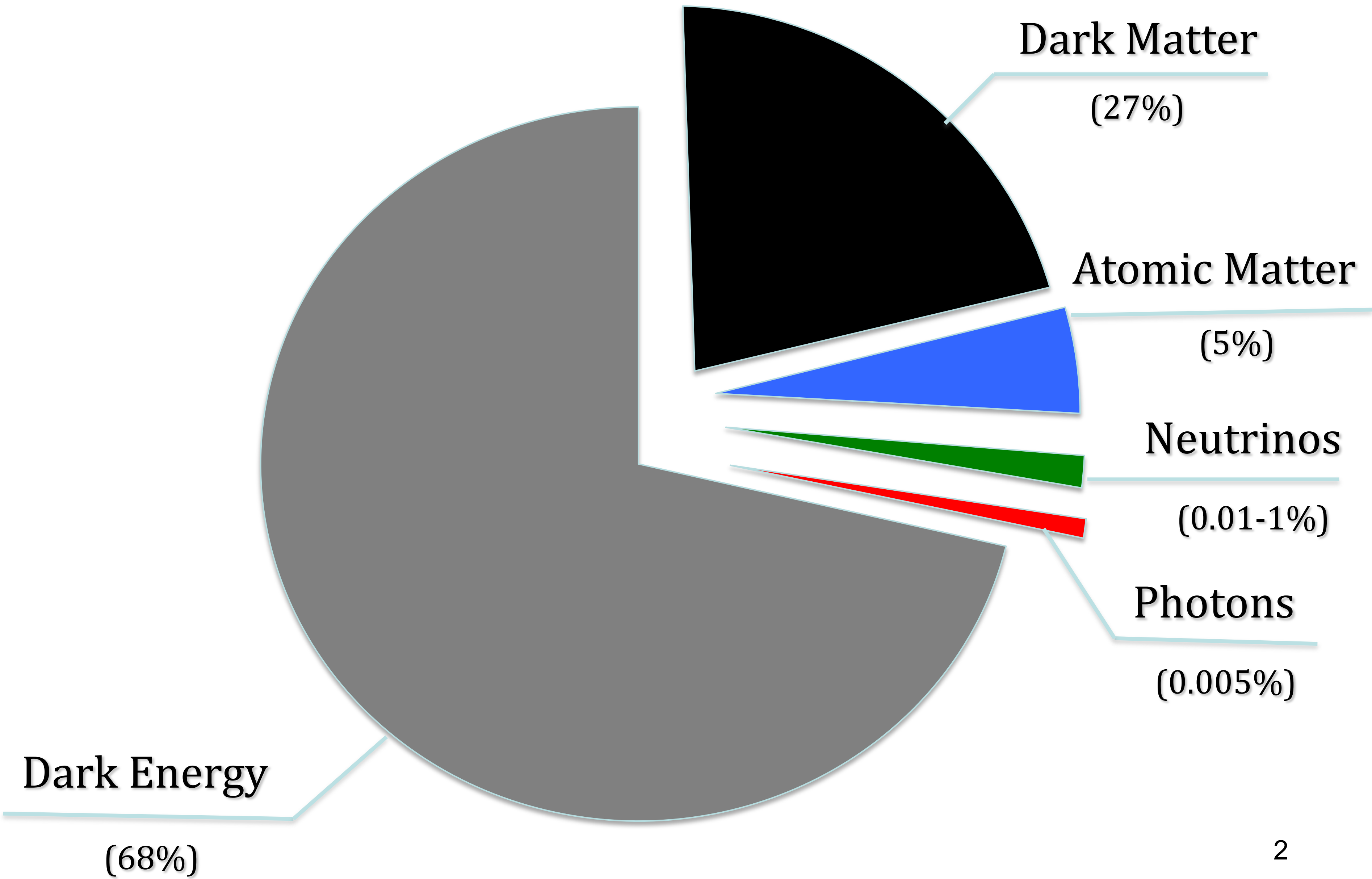
Neutrino IPhU days (June 2021)

The cosmological imprint of massive neutrinos

Julien BEL

Centre de Physique Théorique

The concordance model of cosmology:



Massive neutrinos

Experiments on oscillations of neutrinos of different flavours: Masse difference

WHAT IS THEIR ABSOLUTE MASSE ?

Fermi-Dirac momentum distribution $\rho_\nu \propto T^4 \int_0^\infty \frac{q^2\sqrt{q^2+(m/T)^2}}{1+e^q}$

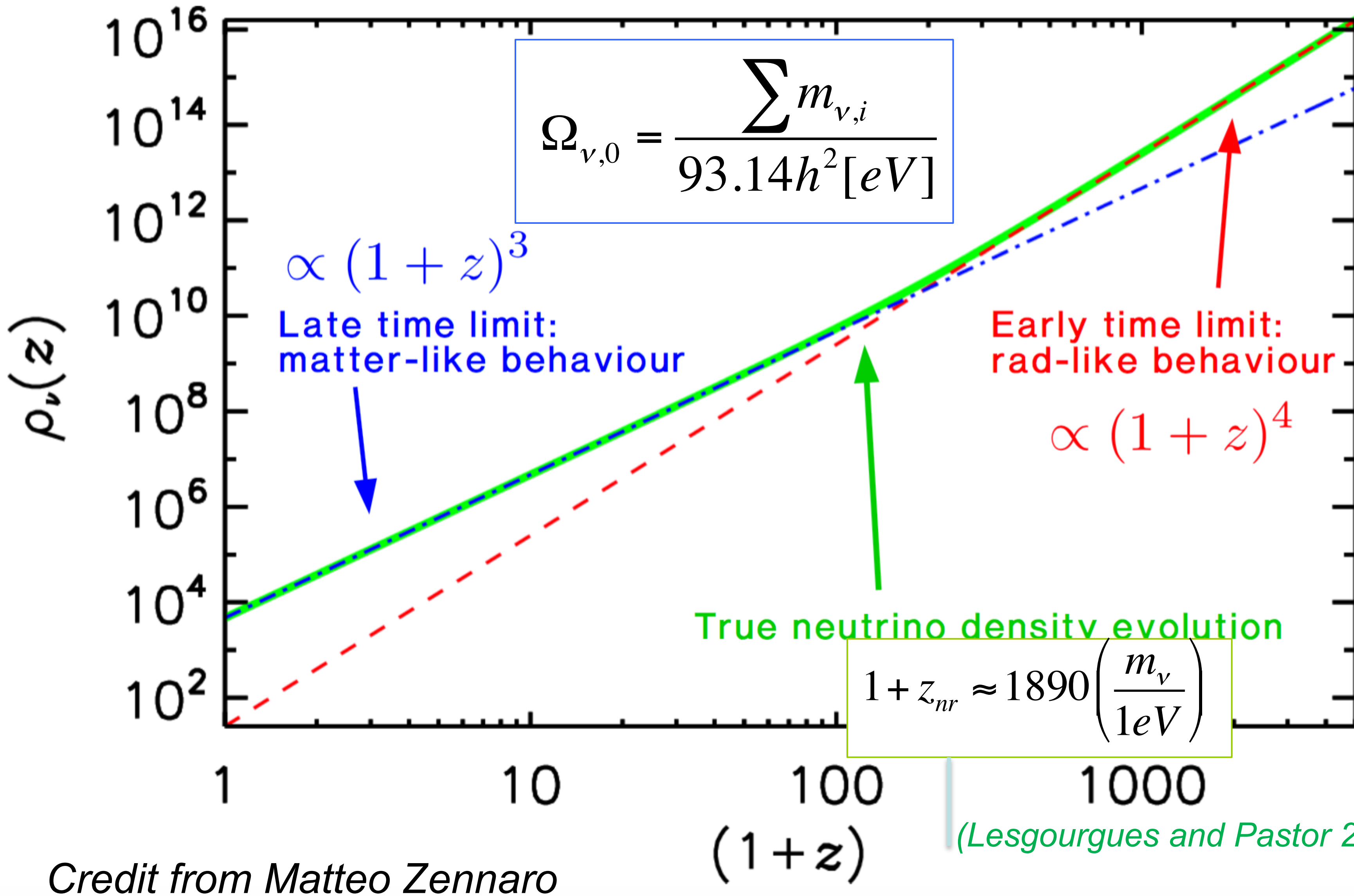
High redshift: $\rho_\nu \propto (1+z)^4$

Low redshift: $\rho_\nu \propto (1+z)^3$

-At high redshift they behave as radiation

-At low redshift they are indistinguishable from matter

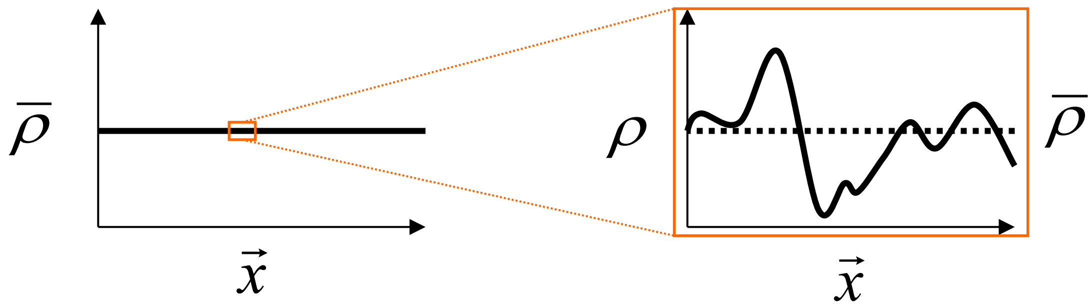
Massive neutrinos



(Lesgourgues and Pastor 2006)

Credit from Matteo Zennaro

Dynamics of the inhomogeneous universe



Homogeneous Background *Deviations on smaller scales*

$$\delta(t,\vec{x}) = \frac{\rho(t,\vec{x}) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

Fluid approximation in the Newtonian limit:

$$d\tau \equiv \frac{dt}{a}$$

$$\frac{\partial \delta}{\partial \tau} + \vec{\nabla}\cdot\vec{v} = -\vec{\nabla}\cdot(\delta\vec{v})$$ *(Continuity)*

$$\frac{\partial \vec{\nabla}\cdot\vec{v}}{\partial \tau} + aH\vec{\nabla}\cdot\vec{v} + \Delta\phi = -\vec{\nabla}\cdot[(\vec{v}\cdot\vec{\nabla})\vec{v}]$$ *(Momentum conservation)*

$$\Delta\phi - \frac{3}{2}\Omega_m (aH)^2 \delta = 0$$ *(Poisson equation)*

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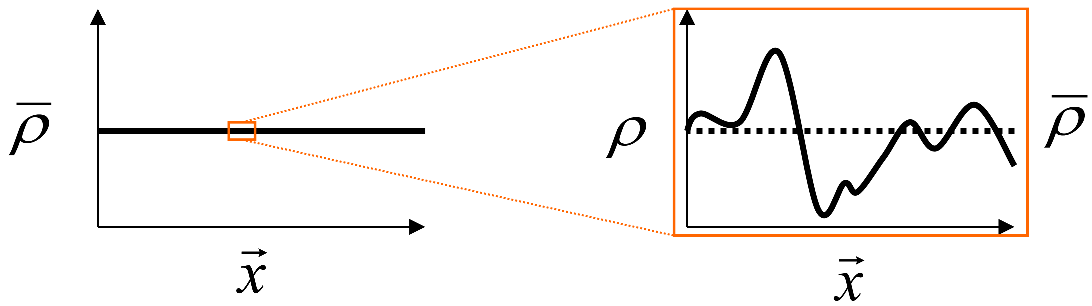

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$$\delta(t,\vec{x}) = \frac{\rho(t,\vec{x}) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

Linear evolution of fluctuations:
(In Newtonian approximation)

$$\frac{d^2\delta}{d\tau^2} + aH\frac{d\delta}{d\tau} - \frac{3}{2}\Omega_m H^2\delta = 0$$

Velocity fluctuations: $\vec{\nabla}\cdot\vec{v} = -aHf\,\delta(t,\vec{x})$

Growth rate: $f \equiv \dfrac{d\ln\delta}{d\ln a}$

$$z_o = z_c + \frac{\vec{v}\cdot\vec{e}_{los}}{c}$$

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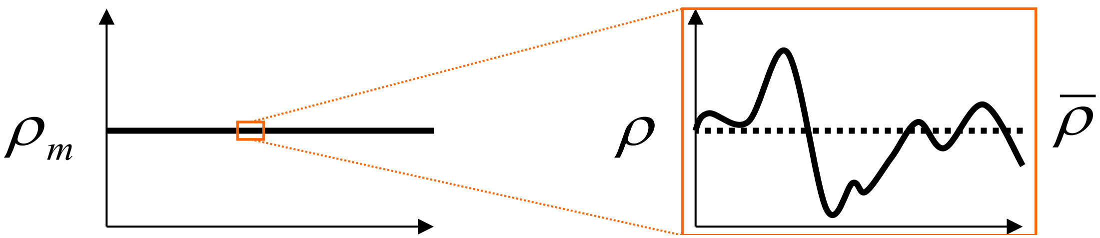

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Densi

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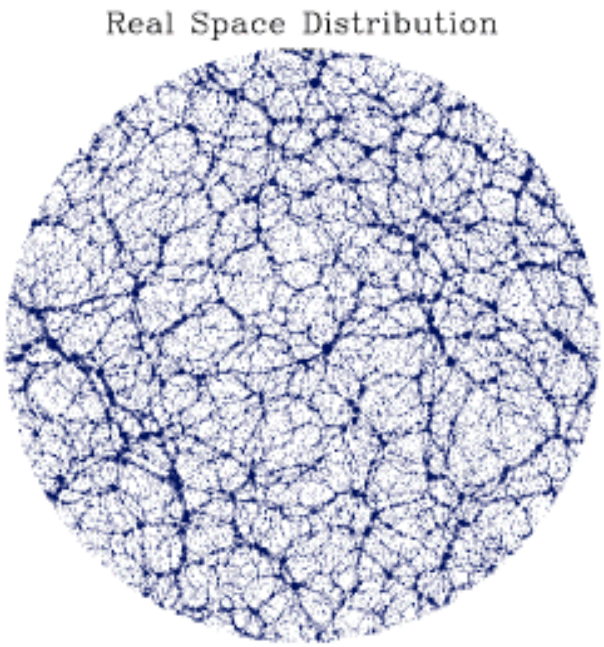

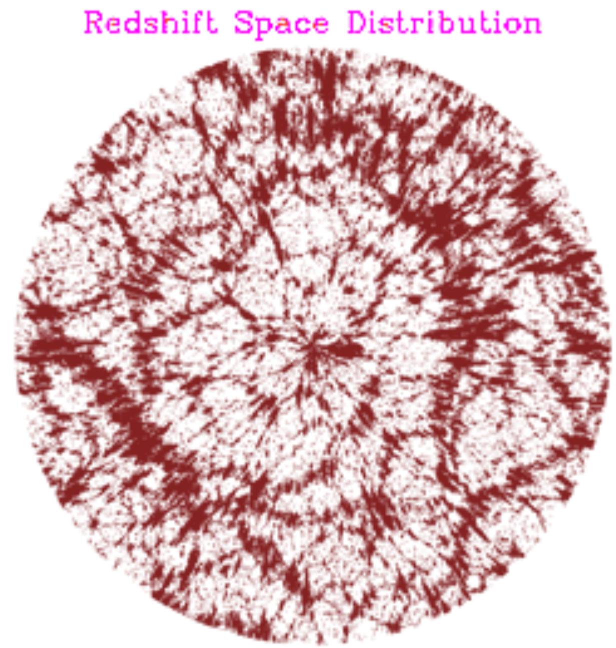

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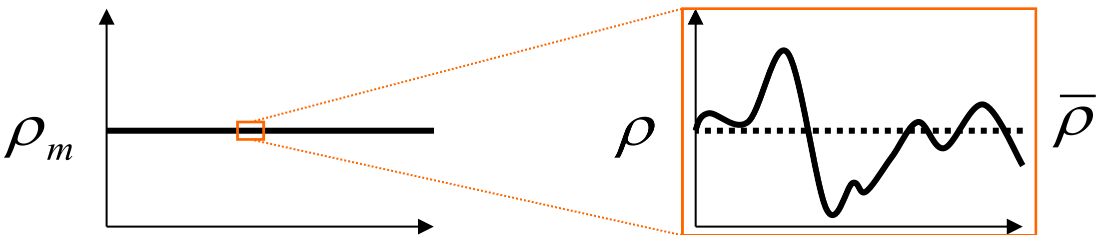

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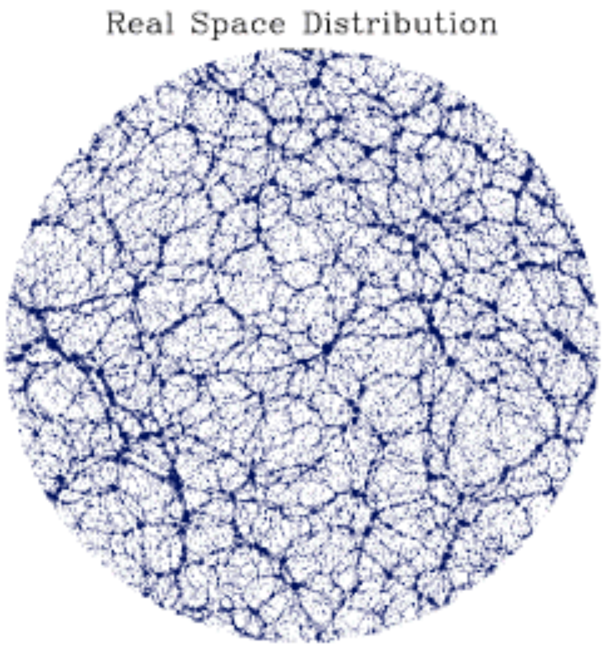

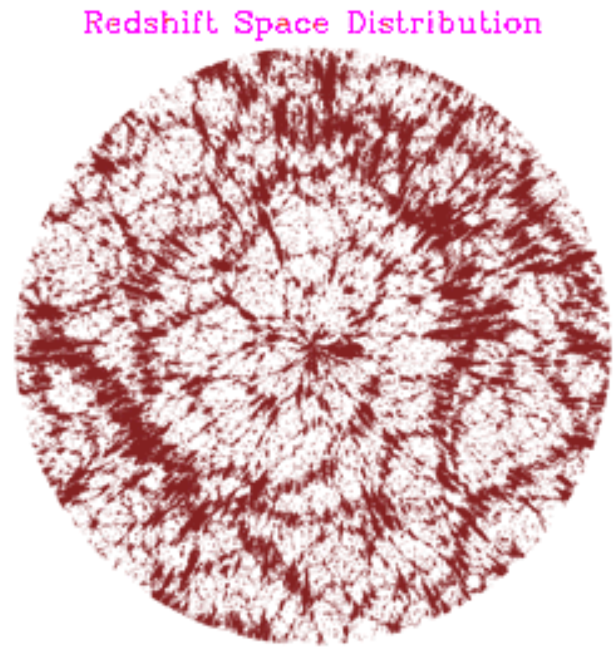

f is directly related to observations

Massive neutrinos

Non-negligible velocity dispersion:

Newtonian approximation and linearized

$$\frac{\partial \delta_\nu}{\partial \tau} + \vec{\nabla} \cdot \vec{v}_\nu = 0$$

(Continuity equation for neutrinos)

$$\frac{\partial \delta_c}{\partial \tau} + \vec{\nabla} \cdot \vec{v}_c = 0$$

(Continuity equation for CDM)

$$\frac{\partial \vec{\nabla} \cdot \vec{v}_\nu}{\partial \tau} + aH\vec{\nabla} \cdot \vec{v}_\nu + \frac{2}{3}\Omega_m (aH)^2[(1-\nu)\delta_c + \nu\delta_\nu] - c_{eff}^2 \Delta\delta_\nu = 0$$

(Eq. of motions)

$$\frac{\partial \vec{\nabla} \cdot \vec{v}_c}{\partial \tau} + aH\vec{\nabla} \cdot \vec{v}_c + \frac{2}{3}\Omega_m (aH)^2[(1-\nu)\delta_c + \nu\delta_\nu] = 0$$

(Blas et al. 2014)

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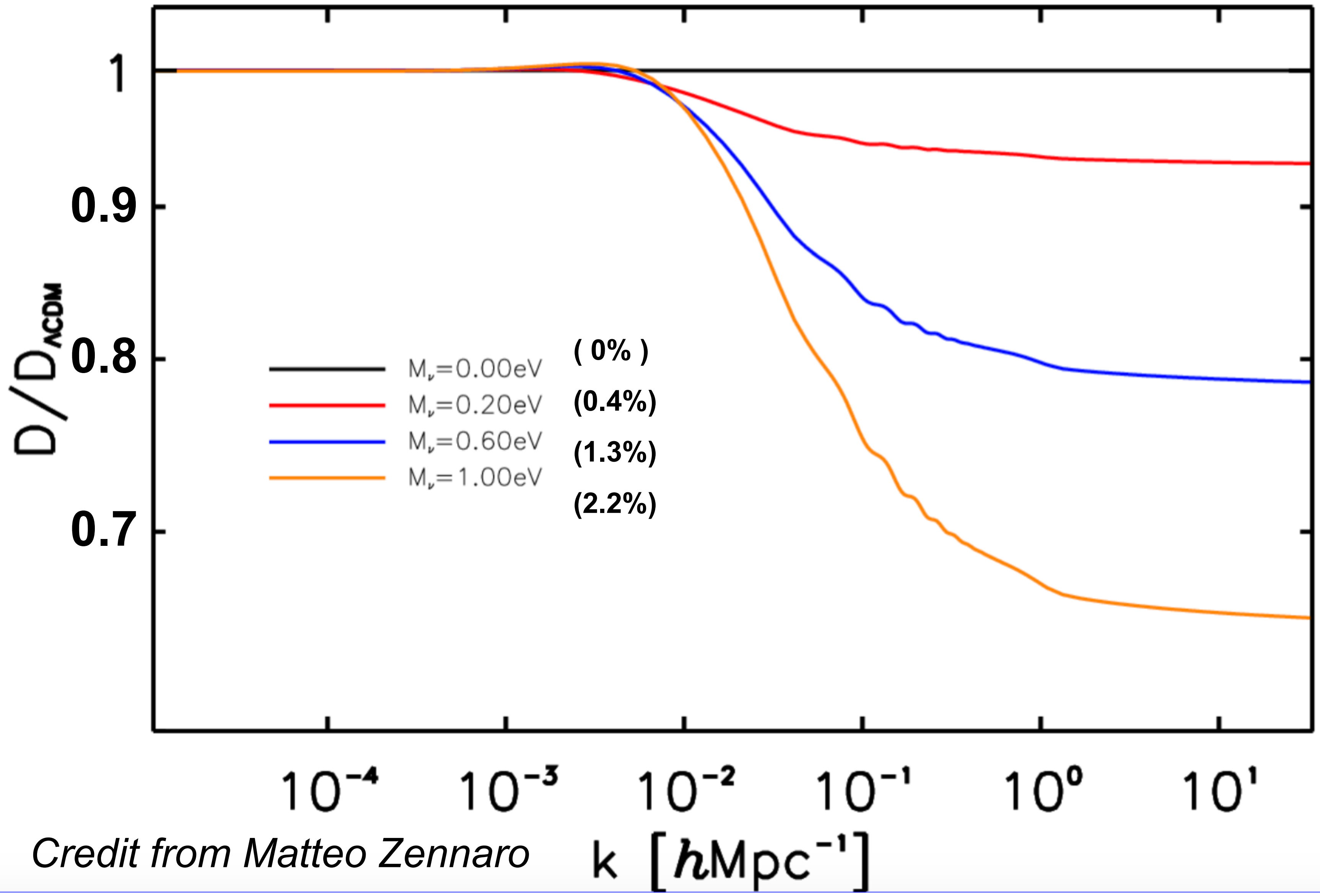

Credit from Matteo Zennaro

Massive neutrinos effects on LSS

Overall: Lower non-linear evolution of the CDM (reduction of the variance σ_8)

-Halo mass function (Castorina et al. 2015) $\frac{dN}{dM}$

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-Integrated Sachs-Wolfe and Rees-Sciama effects (Carbone et al. 2016)

$$\Delta T(\hat{n}) = \frac{2}{c^3} \bar{T}_0 \int_0^{r_L} \dot{\Phi}(r, \hat{n})\, a\, dr$$

DEMNUni simulations with neutrinos

The DEMNUni cosmology:

$h =$	0.67
$\Omega_\Lambda =$	0.68
$\Omega_m =$	0.32
$\Omega_b =$	0.05
$n_s =$	0.96

$$M_\nu = 0, \quad 0.17, \quad 0.3, \quad 0.53 eV$$

$$\sigma_{8,m} = 0.85, \quad 0.80, \quad 0.77, \quad 0.72$$ (total matter)

$$\sigma_{8,c} = 0.85, \quad 0.81, \quad 0.79, \quad 0.74$$ (cold dark matter)

(Carbone, C. et al. 2016)

DEMNUni simulations with neutrinos

CDM velocity spectra (useful for RSD analysis)

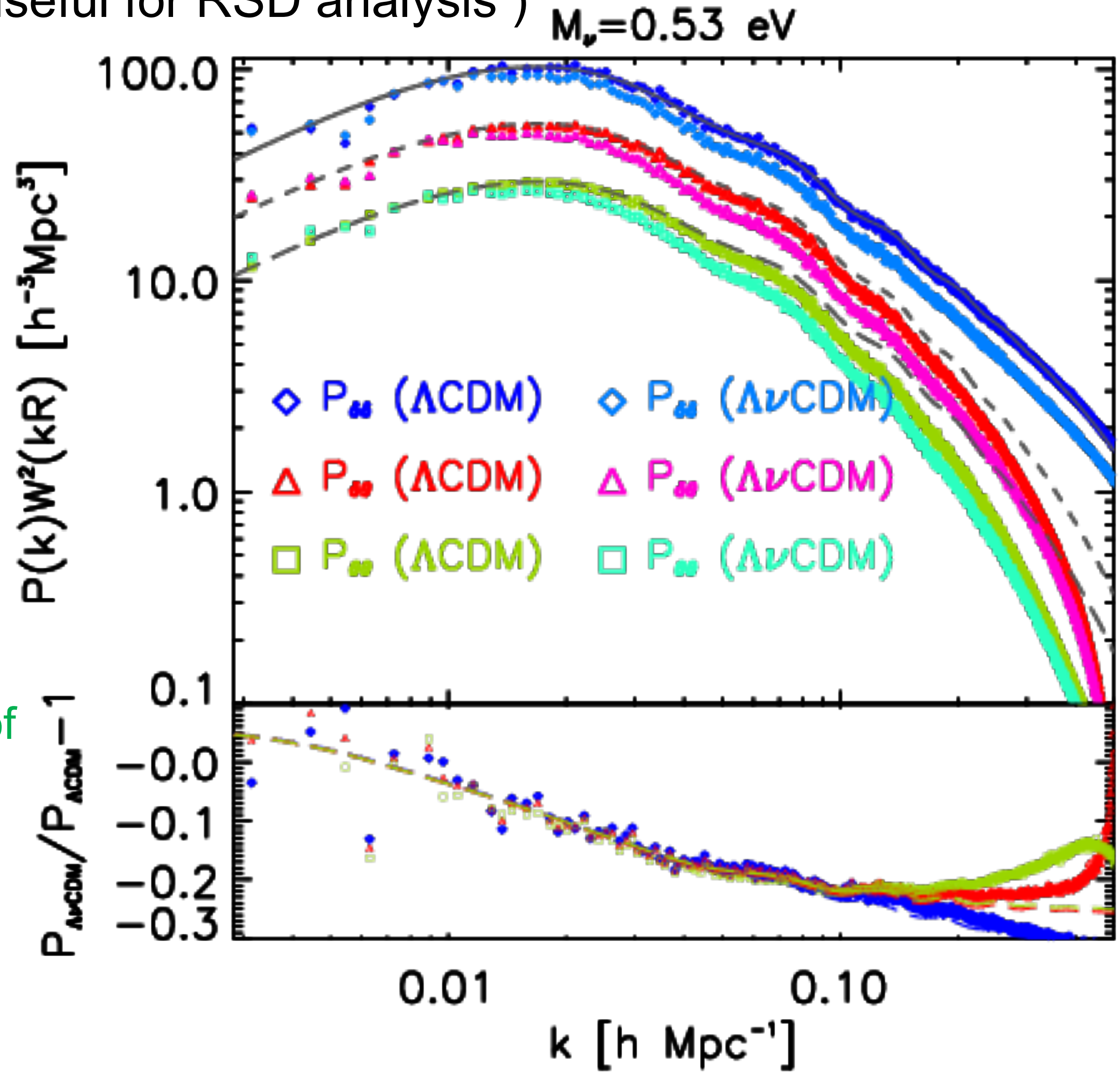
$$\theta \equiv \vec{\nabla} \cdot \vec{v}$$

$$P_{\delta\theta}(k) = k_F^3 \langle \delta_{\vec{k}} \theta^*_{\vec{k}} \rangle$$

$$P_{\theta\theta}(k) = k_F^3 \langle \theta_{\vec{k}} \theta^*_{\vec{k}} \rangle$$

$$k_F = \frac{2\pi}{L}$$

Fundamental frequency of the simulation comoving output (*box*)



Pezzota et al. (2019)

2048^3 particles in a L^3=**2000**^{3} $h^{-3}Mpc^3$ box

DEMNUni simulations with neutrinos

Change the expansion rate with dynamical dark energy:

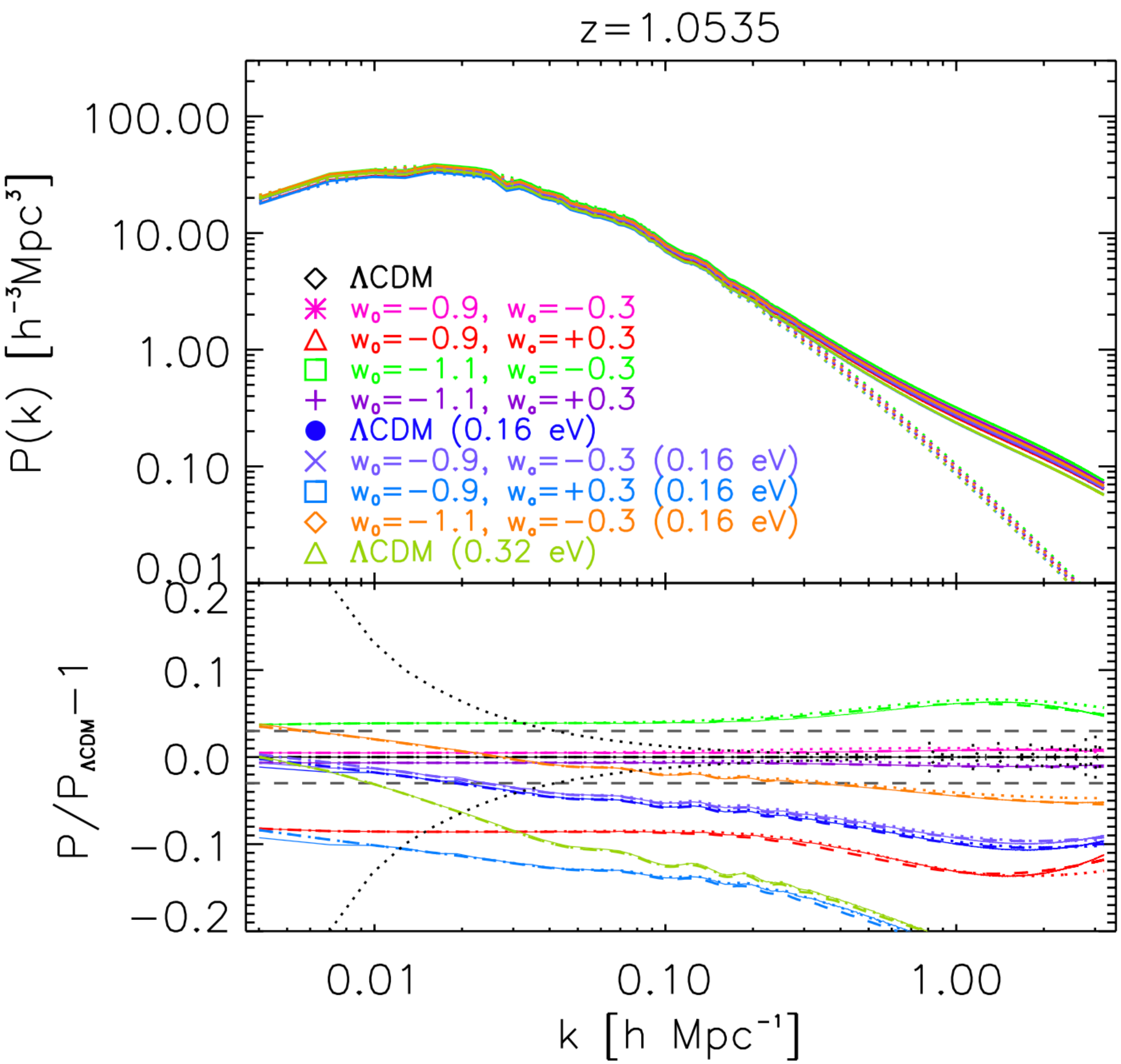
$$w(z) = w_0 + w_a z/(1+z)$$

- ◇ ΛCDM
- ✳ $w_0=-0.9,\ w_a=-0.3$
- △ $w_0=-0.9,\ w_a=+0.3$
- □ $w_0=-1.1,\ w_a=-0.3$
- \+ $w_0=-1.1,\ w_a=+0.3$
- ● ΛCDM (0.16 eV)
- × $w_0=-0.9,\ w_a=-0.3$ (0.16 eV)
- □ $w_0=-0.9,\ w_a=+0.3$ (0.16 eV)
- ◇ $w_0=-1.1,\ w_a=-0.3$ (0.16 eV)
- △ ΛCDM (0.32 eV)

} massive neutrinos

DEMNUni simulations with neutrinos

Power spectra with good spatial resolution ($k_{Nyquist}$=3.14 *h*Mpc^{-1}):



Ongoing project…

Conclusions

1 – Improved LSS simulations with massive neutrinos

(Zennaro et al. 2017)

2 – Study massive neutrino effects on the clustering of dark matter

- Second order statistics (Zennaro et al. 2018)
- Higher order statistics
- Dark matter velocity field for RSD (Pezzota et al. 2019)

2 – We are involved in observational projects

- VIPERS (finished)
- Euclid (ongoing)

Backup

* phase-space distribution or momentum distribution ⇒ neutrinos Fermions ⇒ Fermi-Dirac distribution

$$f_\nu = \frac{1}{1 + e^{\frac{E-\mu}{k_B T}}} \Rightarrow \begin{cases} \rho_\nu = \frac{g_i}{c^2} \int \frac{d^3 p}{(2\pi)^3} f_\nu(\vec{x}, \vec{p})\, E \\ P_\nu = g_i \int \frac{d^3 p}{(2\pi)^3} f_\nu \frac{p^2}{3E} \end{cases}$$

(p^2; p → momentum)

$$n_\nu = g_i \int \frac{d^3 p}{(2\pi)^3 \hbar^3} f_\nu$$

⇒ Relativistic neutrinos

⇒ Decouple very early in time (before photons)

$T_{dec} \simeq 1\ \mathrm{MeV}$ $\left(z_{dec} \simeq 10^9 \right) \sim a^{-9}$

$E_{dec} \simeq pc$

BBN ⇒ $-0{,}05 \leq \frac{\mu_\nu}{T} \leq 0.07 \simeq 0$

$$f_\nu = \frac{1}{1 + e^{\frac{pc}{k_B T}}}$$

$E = \sqrt{p^2 + m^2 c^2}\ c$

Backup

* Decoupling

$\Gamma_\nu \equiv \langle \sigma_\nu n_\nu \rangle \simeq H \Rightarrow T_{dec} = 1\,MeV$

Befor photons decouple $\Rightarrow$ not heated $e^- e^+$ annihilation

$T_\nu < T_\gamma$

Decoupling is instataneously $\Rightarrow \Gamma_\nu \equiv \frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$ (entropy conservation)

$\Rightarrow$ Conservation of the energy momentum tensor

$\Rightarrow \dot{\rho} + 3H(\rho + p) = 0 \Rightarrow d\rho + 3\frac{da}{a}\rho = -3p\frac{da}{a}$

$V = a^3,\ \rho\ ;\ U = a^3\rho \qquad d(a^3\rho) = -p\,d(a^3)$

$\Rightarrow$ $\boxed{dU = -p\,dV}$ $\qquad \underbrace{T\,dS}_{=0} = dU + p\,dV$

Backup

$$\rho_\nu = \frac{2}{c^2}\int \frac{d^3\vec{p}}{(2\pi\hbar)^3} f_\nu(\vec{p})\, E(\vec{p}) = \frac{2}{c^2}\int \frac{d^3\vec{p}}{(2\pi\hbar)^3} \frac{\sqrt{p^2c^2+m_\nu^2c^4}}{1+e^{\frac{pc}{k_BT}}} \rightarrow \text{isotropy} \quad d^3\vec{p} = 4\pi p^2 dp$$

$x \equiv \frac{pc}{k_BT}$

$$= \left(\frac{15}{\pi^4}\right)\Pi_\nu^4\, \rho_\gamma\, F[y] \; ; \quad F(y) \equiv \int dx \frac{x^2\sqrt{x^2+y^2}}{1+e^x} \quad \text{where } y = \frac{mc^2}{k_BT_\nu}$$

$\rho_\gamma \rightarrow \propto T^4$

$\propto T^4$

$y \ll 1 \Rightarrow F \rightarrow$ Riemann-Zeta function

when $y \gg 1 \rightarrow F(y) \propto y \Rightarrow \rho_\nu \propto T^3 \Rightarrow$ matter

$\rho_\nu \propto T^3 \propto a^{-3}$

* Show that $\boxed{\frac{S}{V} = \frac{\partial P}{\partial T}}$ $\quad T \rightarrow a^{-1}$

$\Rightarrow$ first principle of thermodinamics

$dU + PdV = TdS$

$d(PV)$ and $d(TS)$

$$dP = \left(\frac{\partial P}{\partial T}\right) dT + \left(\frac{\partial P}{\partial \mu}\right) d\mu$$

$\left(\frac{\partial P}{\partial \mu}\right) = \frac{1}{V}$

$P_\gamma \propto \rho_\gamma \propto T^4 \Rightarrow T \propto a^{-1}$

Backup

$$y \equiv \frac{m_\nu c^2}{k_B T_{\nu,0}(1+z)} \rightarrow y = 1 \Rightarrow z_{NR}$$

$$1 + z_{NR} = \frac{m_\nu c^2}{k_B T_{\nu,0}}$$

CMB $\rightarrow$ $\sum m_\nu < 1\,eV$

$$\rho_r = \rho_\gamma \left(1 + N_\nu^{rel} \frac{7}{8} \Pi_\nu^4 \right) \quad (y \ll 1)$$

N_{eff} $\left(\frac{4}{11}\right)^{4/3}$

N_{eff} → effective number of relativistic species

→ 3.046 (Non instantaneous decoupling)

→ 3 neutrinos

$$\rho_\nu = \left(\frac{15}{\pi^4}\right) \Pi_\nu^4 \frac{3}{2} \zeta(3)\, y\, \rho_\gamma \propto m_\nu (1+z)^3 \quad (y \gg 1); \quad \rho_{\nu} = \rho_{\nu,0}(1+z)^3$$

$$\boxed{\Omega_{\nu,0} h^2 = \frac{\sum m_\nu}{93{,}14\,eV}} \qquad \boxed{\Omega_\nu(z) = \frac{\Omega_{\nu,0}(1+z)^3}{E^2(z)}}$$

Streaming model

$$\delta^D(\vec{k}) + P_s(\vec{k}) = \int d^3\vec{r} \left\langle e^{ifk\mu_k v_z}[1+\delta(\vec{x})][1+\delta(\vec{x}\,')] \right\rangle e^{-i\vec{k}.\vec{r}}$$

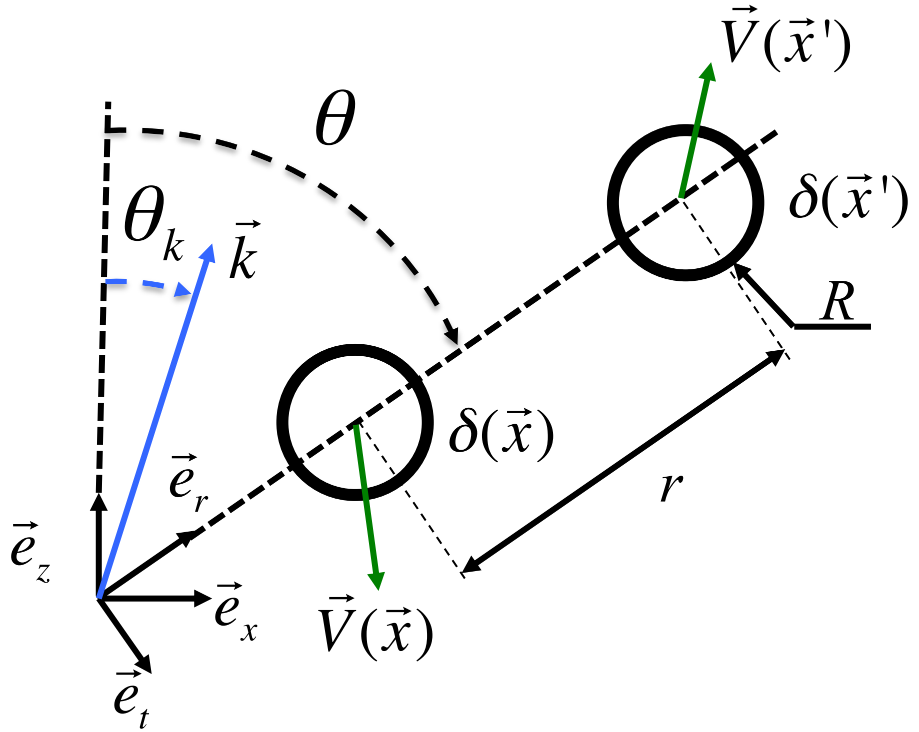
Scoccimarro (2004)

where $\vec{r} \equiv \vec{x}\,' - \vec{x}$ and $v_z \equiv \left[\vec{V}(\vec{x}\,') - \vec{V}(\vec{x})\right] \cdot \vec{e}_z$

$\mu_k \equiv \cos(\theta_k)$

$\mu \equiv \cos(\theta)$

Which can be illustrated by



Growth rate : f

Streaming model

The generating function of the line-of-sight pairwise velocity distribution:

$$[1+\xi(r)]M(\lambda,\vec{r}) \equiv \left\langle e^{\lambda v_z}[1+\delta(\vec{x})][1+\delta(\vec{x}\,')] \right\rangle$$

which can be used to generate the moments

$$[1+\xi(r)]m_n \equiv \left\langle [1+\delta(\vec{x})][1+\delta(\vec{x}\,')]v_z^n \right\rangle = \left.\frac{\partial^n M}{\partial \lambda^n}\right|_{\lambda=0}$$

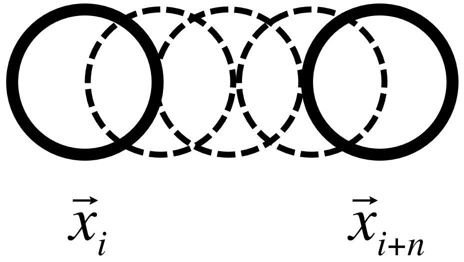
The RSD power spectrum can be obtained from

$$P_s(\vec{k}) = \int d^3\vec{r}\left[(1+\xi)M(ifk\mu_k,\vec{r}) - 1\right]e^{-i\vec{k}.\vec{r}}$$

$$M(\lambda=-it) = 2\pi\tilde{P}(t) = \int P(v_z)e^{-iv_z t}dv_z$$

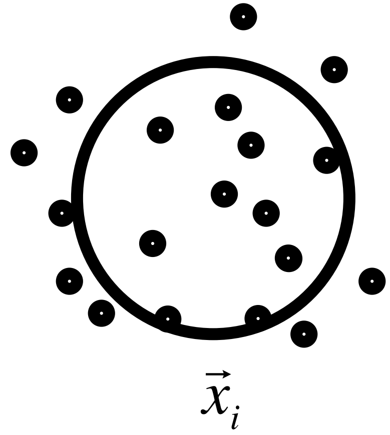
Velocity-density estimator

Estimating the density field: $\delta_R(\vec{x}_i) = \frac{N(\vec{x}_i)}{\bar{N}} - 1$



Estimating the velocity field:

Arithmetic mean (A-M) $\vec{V}_R(\vec{x}_i) \equiv \frac{1}{N_i}\sum_{j=1}^{N_i}\vec{V}(\vec{x}_j)$



Bernardeau & van de Weygaert (1996);

Bernardeau et al. (1997);

van de Weygaert & Bernardeau (1998);

Romano-Díaz & van de Weygaert (2007)

Pueblas & Scoccimarro (2009)

Yu et al. (2015)

Hahn et al. (2015)

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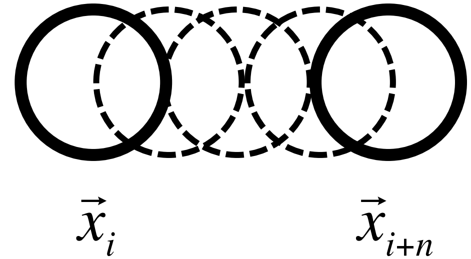

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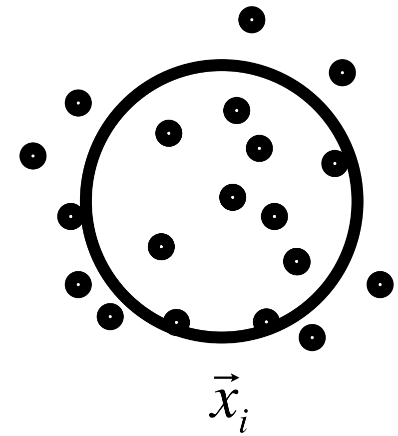

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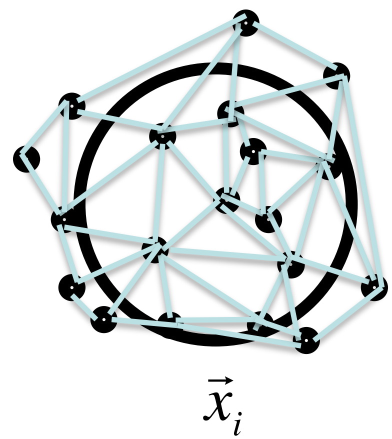
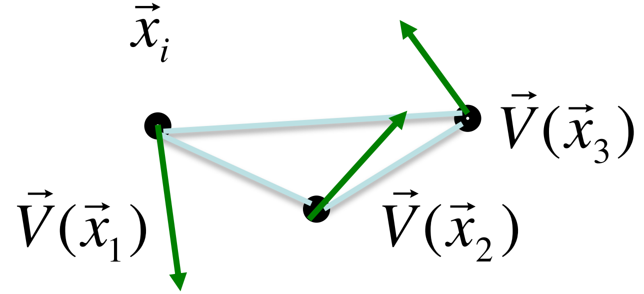
Delaunay tessellation (D-T)



$$\vec{V}_R(\vec{x}_i) \equiv \frac{1}{\vartheta_R}\sum_{k=1}^{N_i} \vartheta_k \vec{V}(\vec{x}_k)$$

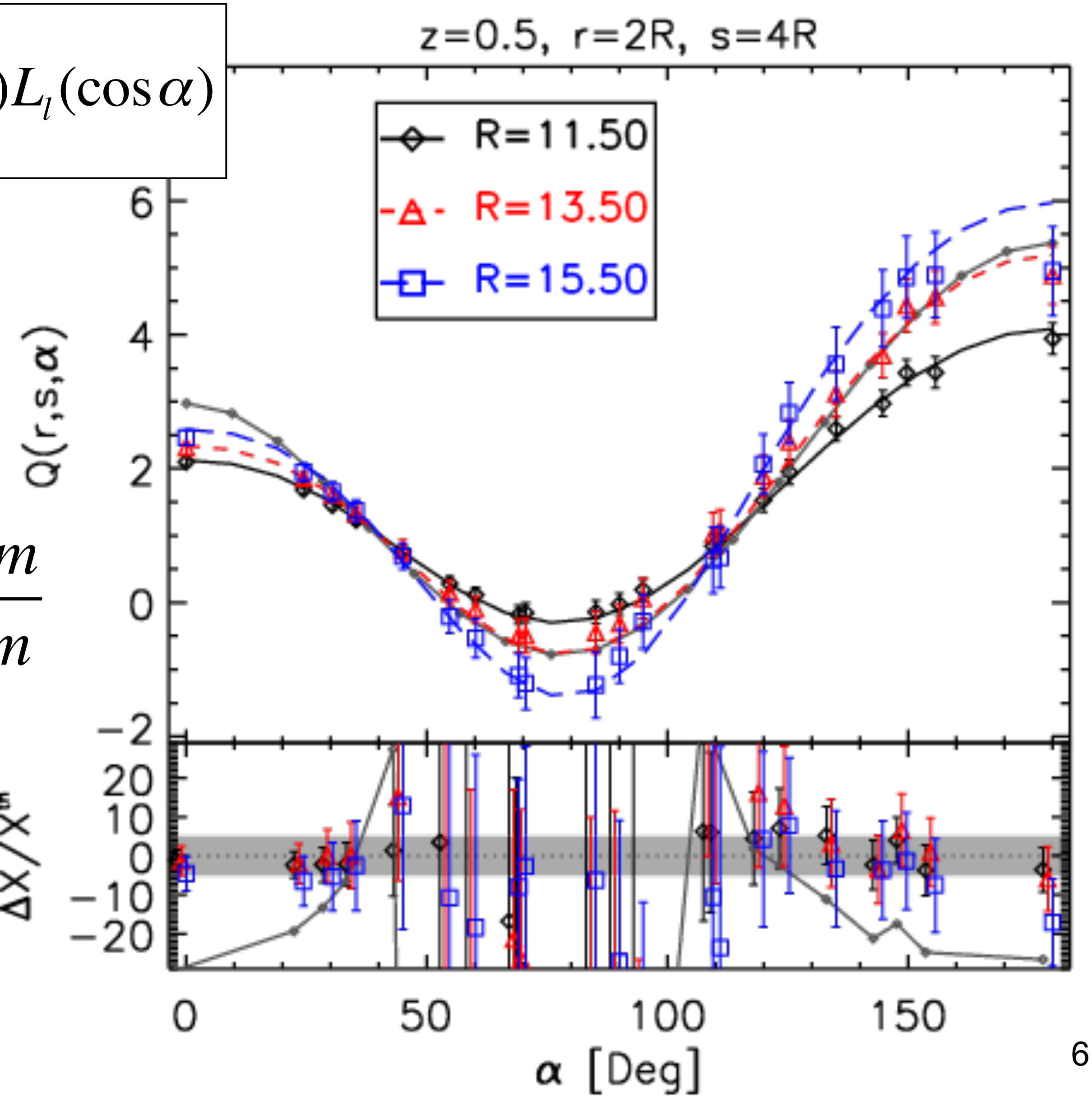
$$\vec{V}(\vec{x}_k) = \frac{1}{D+1}\sum_{q=1}^{D+1} \vec{V}(\vec{x}_q)$$



The small separation limit in the DEMNUni

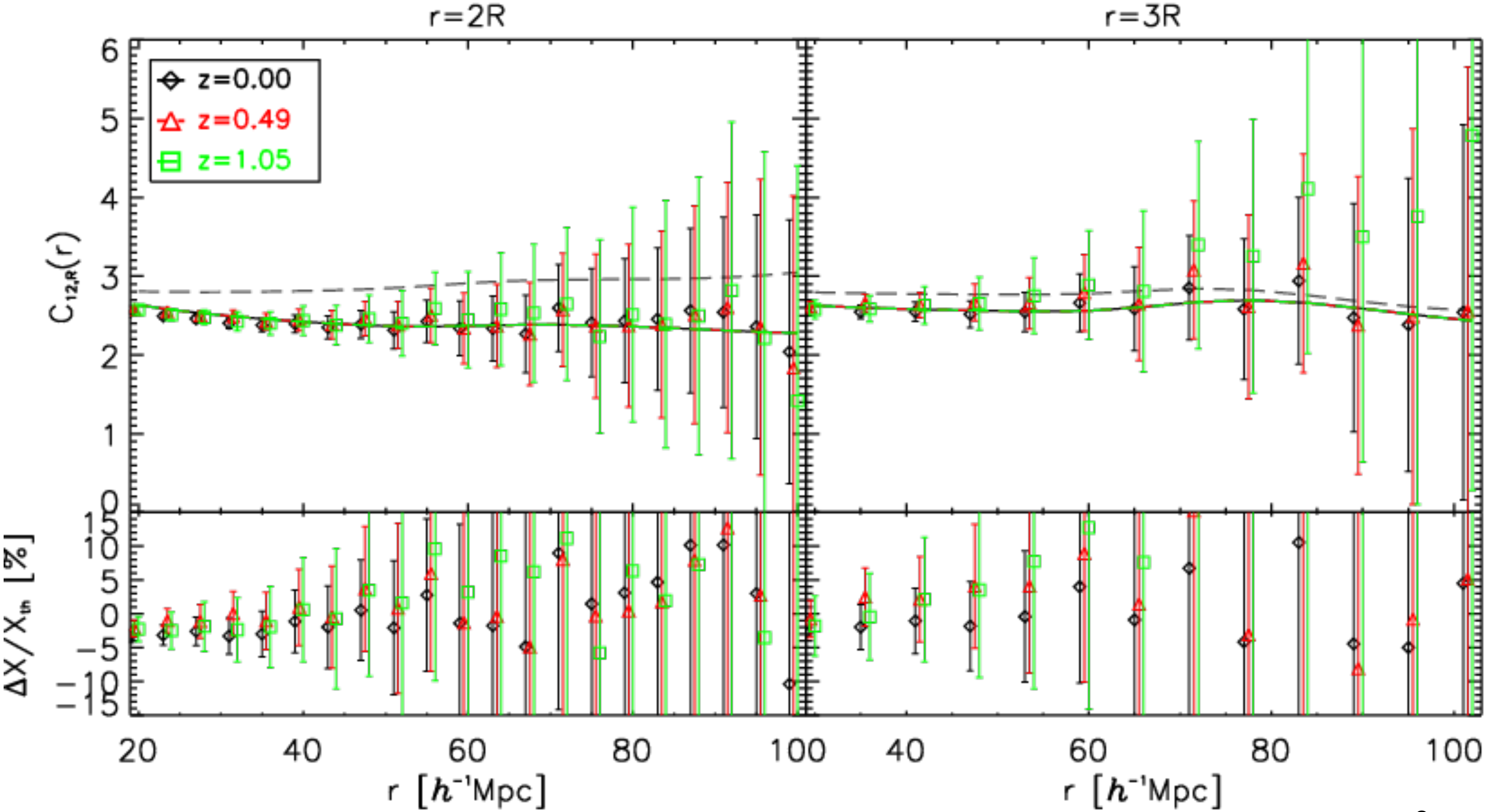
$$\Phi(r,s,\alpha) = \sum_{l=0}^{\infty} w_l(r,s) L_l(\cos\alpha)$$

$$Q \equiv \frac{\Phi(r,s,\alpha) + perm}{\xi(r)\xi(s) + perm}$$



The small separation limit in the DEMNUni

$$C_{12,R}(r) = \frac{68}{21} + \frac{\beta_R + \gamma_R}{3} + 2\left\{1 + \frac{\beta_R}{6} + \frac{\beta'_R}{3}\left(1 + \frac{\bar{\beta}_R}{2}\right)\Theta_R + \frac{1-\Theta_R}{3}\left(\beta_R - \bar{\beta}_R\Theta_R\right) - \frac{4}{21}\Theta_R\left(2-\Theta_R\right)\right\}\frac{\xi_R}{\sigma_R^2}$$



3-point correlators: non-local bias

Chan et al. 2012
Baldauf et al. 2012
Saito et al. 2014

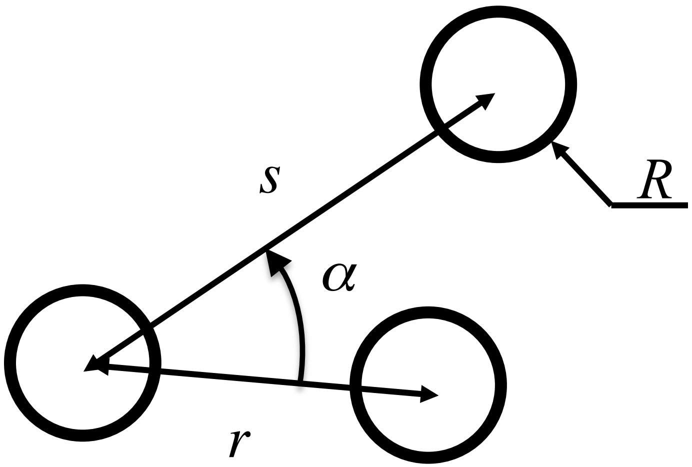
Non-local bias: $$\delta_{g,R} \approx b_1 \left\{ \delta_R + \frac{c_2}{2}\left[\delta_R^2 - \left\langle \delta_R^2 \right\rangle\right] + \frac{g_2}{2}\vartheta_R \right\}$$

Influence of the tidal field on the galaxy distribution. $$\vartheta_R = -\int \beta_{12}\theta_v(\mathbf{q}_1)\theta_v(\mathbf{q}_2)\hat{W}[q_{12}R]e^{i\mathbf{q}_{12}\cdot\mathbf{x}}d^3\mathbf{q}_1 d^3\mathbf{q}_2, \quad (3)$$

where $\beta_{12} \equiv 1 - \left(\frac{\mathbf{q}_1\cdot\mathbf{q}_2}{q_1 q_2}\right)^2$ represents the mode-coupling

3-point correlation function:

$$\varsigma(r,s,\alpha) \equiv \left\langle \delta_R(x)\delta_R(x+r)\delta_R(x+s) \right\rangle$$



Hoffmann, Bel, Gaztanaga et al. (2014)
Hoffmann, Bel & Gaztanaga (2015)
Bel, Hoffmann & Gaztanaga (2015)

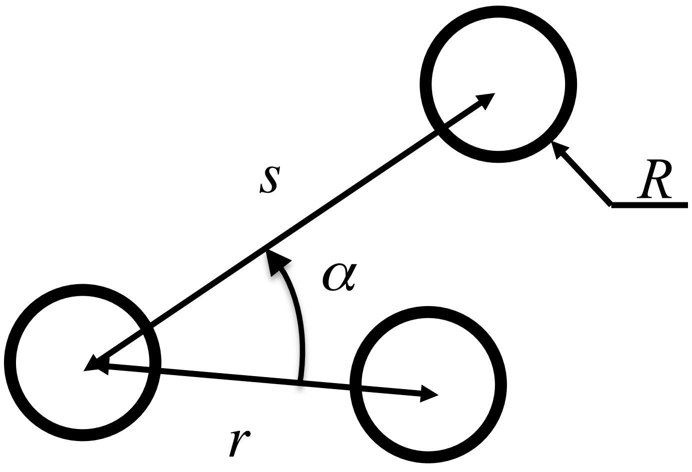
3-point correlators: non-local bias

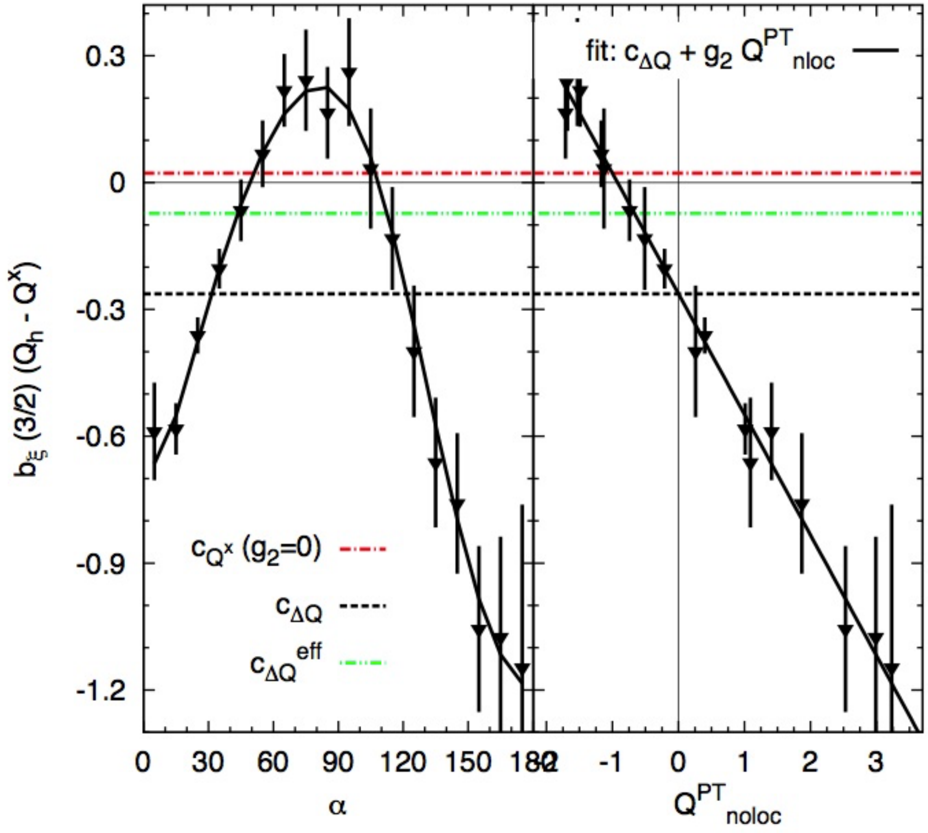
Non-local bias: $$\delta_{g,R} \approx b_1 \left\{ \delta_R + \frac{c_2}{2}\left[\delta_R^2 - \langle \delta_R^2 \rangle\right] + \frac{g_2}{2}\vartheta_R \right\}$$

Influence of the tidal field ϑ_R on the galaxy distribution.

3-point correlation function:

$$\varsigma(r,s,\alpha) \equiv \langle \delta_R(x)\delta_R(x+r)\delta_R(x+s) \rangle$$





Hoffmann, Bel, Gaztanaga et al. (2014)
Hoffmann, Bel & Gaztanaga (2015)
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