ID de Contribution: **9** Type: **Non spécifié**

A universal theory of enumerative invariants and wall-crossing formulae.

I outline a (very long and complicated, sorry) programme which gives a common universal structure to many theories of enumerative invariants counting semistable objects in abelian or derived categories in Algebraic Geometry, for example, counting coherent sheaves on curves, surfaces, Fano 3-folds, Calabi-Yau 3- or 4-folds, or representations of quivers (with relations). Write A for your abelian or derived category, K(A) for its numerical Grothendieck group, t for the stability condition, M for the usual moduli stack of objects in A, and M^{pl} for the 'projective linear'moduli stack of objects modulo "projective linear"isomorphisms (quotient by multiples of identity morphisms). Then (oversimplifying a bit):
(i) the homology H_*(M,Q) has the structure of a graded vertex algebra (or a graded vertex Lie algebra in the 3-Calabi-Yau case).*
(ii) We have H_*(M^{pl},Q) = H_*(M,Q) / D(H_*(M,Q)), where D is the translation operator in the vertex algebra. Therefore H_*(M^{pl},Q) has the structure of a graded Lie algebra. It seems very difficult to understand this Lie bracket without going via the vertex algebra.
(iii) For each class a in K(A) we have a moduli stack M_a^{ss}(t) of t-semistable object in A in class K(A). We can define invariants [M_a^{ss}(t)]{inv} in H(M^{pl}*a,Q). If there are no semistables in class a, this is just the virtual class of M_a^{ss}(t), and is defined over Z. If there are semistables, it is defined over Q, and has a complicated definition involving auxiliary pair invariants.*
*(iv) If t, t*are two stability conditions, there is a universal wall-crossing formula which writes [M_a^{ss}(t*)]*{inv} as a Q-linear combination of repeated Lie brackets of invariants [M_b^{ss}(t)]*{inv}, using the Lie bracket on H**(M^{pl},Q) from (ii).
The programme above is proved for invariants defined using Behrend-Fantechi perfect obstruction theories and virtual classes. I expect to extend it to Calabi-Yau 4-fold obstruction theories and virtual classes, a la Borisov-Joyce / Oh-Thomas. The programme includes "reduced"invariants (for example, counting coherent sheaves on surfaces with p_g > 0), for which the wall-crossing formula is modified.The wall-crossing formulae are effective computational tools in examples. I am currently using them to compute invariants counting semistable sheaves on projective surfaces (algebraic Donaldson invariants) from Seiberg-Witten invariants.The appearance of the vertex algebras in (i), in relation to enumerative invariants, is a complete mystery (at least to me). I invite String Theorists to explain it. Based on: arXiv:2005.05637 (joint with Jacob Gross and Yuuji Tanaka), arXiv:2111.04694, and work in progress.

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