Study of surface properties of heavy atomic nuclei

Da Costa Philippe
PhD supervisor : K. Bennaceur
Working group : K. Bennaceur, M. Bender, J. Meyer

Institut de physique des deux infinis (IP2I)

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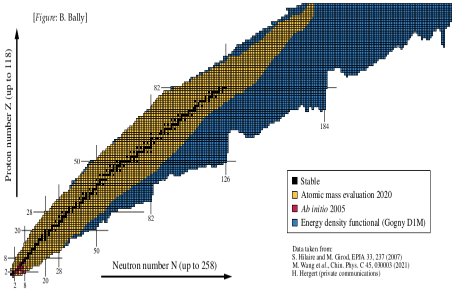
1.1) Problematic

Nuclear structure is the study of system with N quantum body in interaction. Thus its treatment is non trivial and we will face some difficulties such as :



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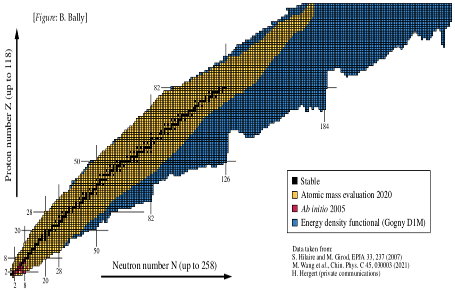

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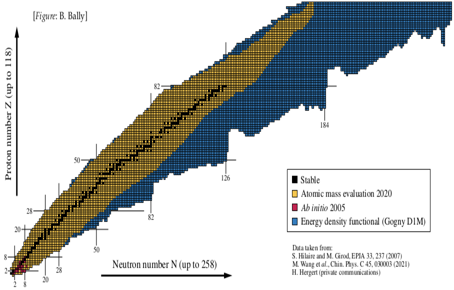

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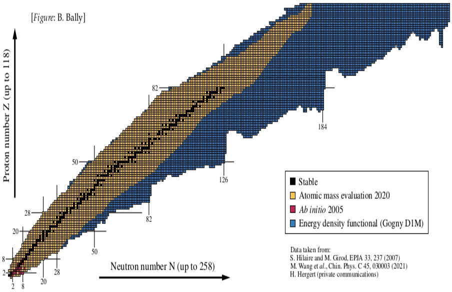

- Effective potentiel unknown a priori ;
- Ambiguity for the choice of experimental data for constraining the interaction ;
- Numerical resolution cost non negligible.

1.2) Variational principle in a nutshell

In the variational approach (HF or HFB) we want to minimize the energy which is a function of the wave-function.

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We can also perform this variation with constraint as a quadrupole deformation for example :

$$\delta\langle\hat{H} - \lambda(\langle\hat{Q}_{20}\rangle - Q_{20})^2\rangle = 0$$

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Effective interaction

- Finite-range interaction (Gogny) $\rightarrow \sum_{k=1,2} e^{-\frac{(r_1-r_2)^2}{\mu_k^2}}$

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- Contact interaction (Skyrme) $\rightarrow \delta(r_1 - r_2)$

$$
\begin{aligned}
V_{\mathrm{Sky}} = & \; t_0 \left(1 + x_0 P^\sigma\right) \delta(\vec{r}) \\
& + \tfrac{1}{2}\, t_1 \left(1 + x_1 P^\sigma\right) \left[\overleftarrow{k}^2 \delta(\vec{r}) + \delta(\vec{r})\, \overrightarrow{k}^2 \right] \\
& + t_2 \left(1 + x_2 P^\sigma\right) \overleftarrow{k} \cdot \delta(\vec{r}) \overrightarrow{k} \\
& + \tfrac{1}{6}\, t_3 \left(1 + x_3 P^\sigma\right) \rho_0^\alpha \delta(\vec{r}) \\
& + \mathrm{i}\, W_0 \overrightarrow{\sigma_{12}} \cdot \overleftarrow{k} \times \delta(\vec{r}) \overrightarrow{k}
\end{aligned}
$$

1.2) Variational principle in a nutshell
Penalty function

The parameters of a functional has to be adjusted by minimizing a penalty function built from a series of constraints :

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{n_{\mathrm{obs}}} \frac{(\mathcal{O}_i(\mathbf{p}) - \mathcal{O}_i^{\mathrm{target}})^2}{\Delta\mathcal{O}_i^2}$$

with

- $\mathbf{p}$: Parameters of the model ;
- $\mathcal{O}_i(\mathbf{p})$: Calculated observable (pseudo-obersvable) ;
- $\mathcal{O}_i^{\mathrm{target}}$: Targeted value for the observable (pseudo-obersvable) ;
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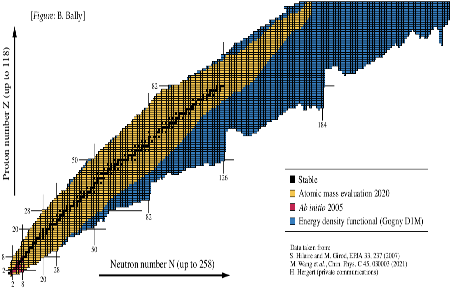
We can then build the penarlty function as a sum of different compronents :

$$\chi^2 = \chi^2_{\text{inm}} + \chi^2_{\text{pol}} + \chi^2_{\text{BE}} + ... + \chi^2_{\text{rad}},$$

2.1) Why surface properties so important ?



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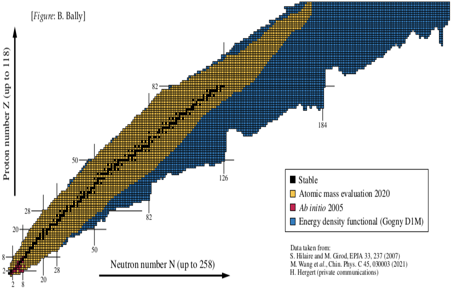

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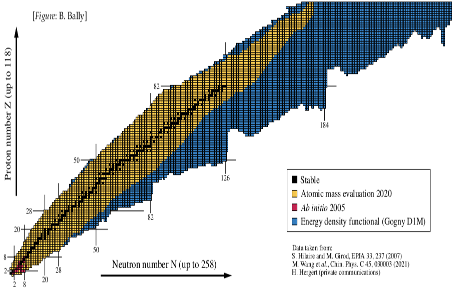

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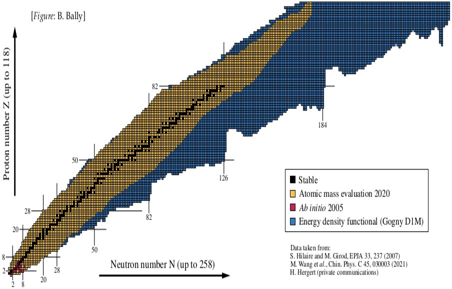

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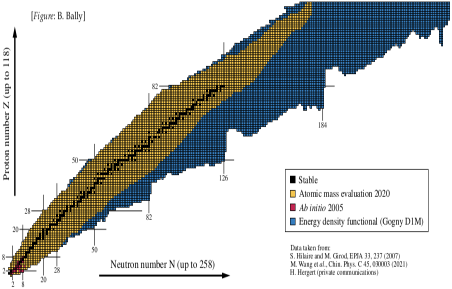

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- Rotationnal bands.

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Until now ?

Several attempts to reproduce nuclear surface properties : SkM* [1], D1S [2] and UNEDEF2 [3].

1. J. Bartel *et al*. Nuclear Physics A, 386(1), 79-100 (1982).
2. J.F. Berger, M. Girod and D.Gogny, Comp. Phys. Comm., 63 (1991) 365.
3. M. Kortelainen et al., Phys. Rev. C 89, 054314 (2014).
4. R. Jodon, M. Bender, K. Bennaceur, and J. Meyer, Phys. Rev. C 94, 024335 (2016).

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SLy5s1 [4] {
- Able to reproduce a lot of heavy nuclei properties.
- Not really good for mass residuals...

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- Having a good description of ^{240}Pu isomer ;
- Being able to describe ^{180}Hg fission barrier with an oblate groud state ;
- Improving binding energies predictions of nuclei compared to SLy5s1 interaction.

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- Different recipies of center of mass correction 1F2F, 1T2F, 1T2T ;
- Different values for the effective mass $m_0^*/m = 0.70, 0.80, 0.85$.

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$$E_{\mathrm{ph}} = E_{\mathrm{kin}} + E_{\mathrm{Sky}} + E_{\mathrm{Coulomb}} + E_{\mathrm{corr}}$$

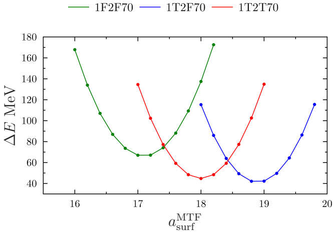
E_{corr} is limited to the correction of the cm :

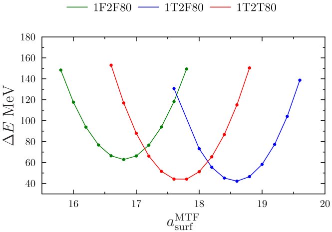
$$E_{\mathrm{cm}} = -\frac{\langle \mathbf{P}^2 \rangle}{2Am} = -\left(\sum_i \frac{\langle \mathbf{p_i}^2 \rangle}{2Am} + \sum_{i<j} \frac{\langle \mathbf{p_i} \cdot \mathbf{p_j} \rangle}{Am} \right)$$

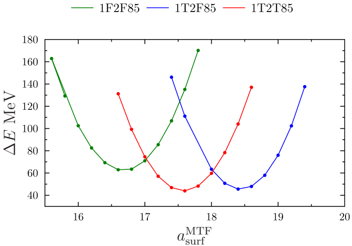
With **P** the sum of the impulsions of the A nucleons in the nucleus.

$$\mathrm{E}_{\mathrm{cm}}^{(1)} = \sum_i \frac{\langle \mathbf{p_i}^2 \rangle}{2Am} \qquad \mathrm{E}_{\mathrm{cm2}}^{(2)} = \sum_{i<j} \frac{\langle \mathbf{p_i} \cdot \mathbf{p_j} \rangle}{Am}$$

3.1) Penalty function



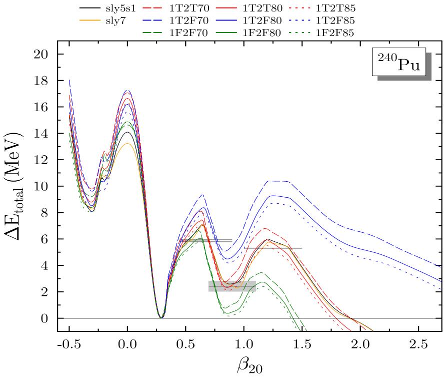




χ^2 as a function of $a_{\mathrm{surf}}^{\mathrm{MTF}}$

3.2) Fission barriers

Calculation are compared with SLy5s1 and SLy7 [5] interaction.



Fission barrier of ^{240}Pu

- Grey areas indicate experimental excitation energies and barriers height.

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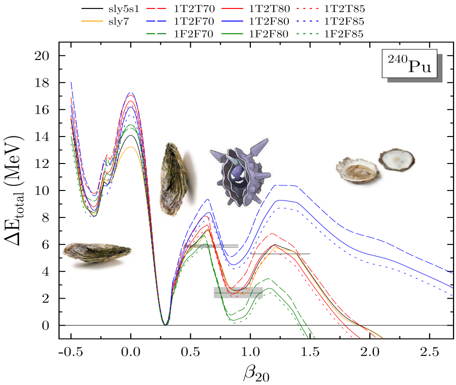

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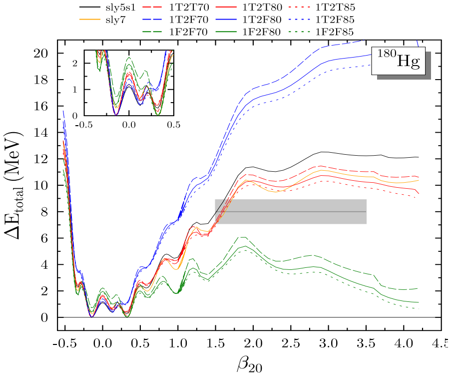
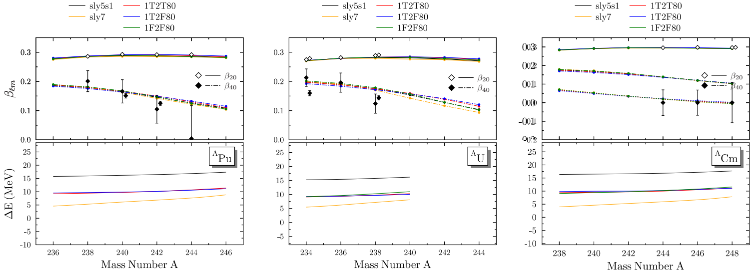
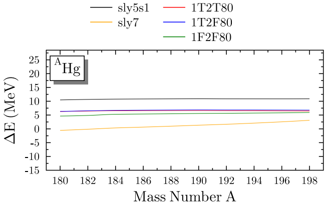


Fission barrier of ^{180}Hg

- Grey areas indicate experimental excitation energies and barriers height ;
- Experimentally, the ground state of is ^{180}Hg oblate.

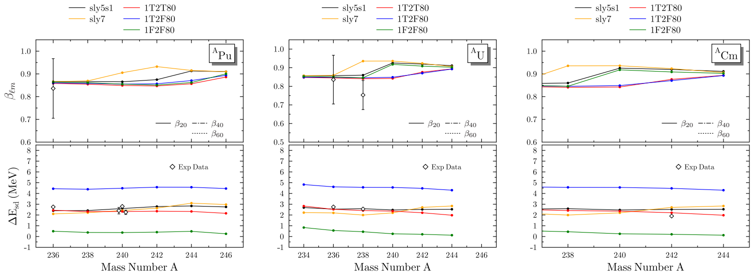
3.3) Normal deformation





Lower panels : mass residuals of the calculated ground states

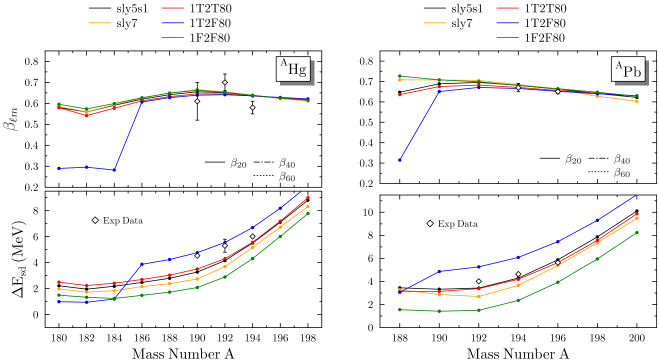
3.4) Super deformation for actinide



• Lower panel : excitation energy of the 0^+ fission isomers of even-even Pu, U and Cm isotopes.

• Upper panel : calculated β_{20}, values, and experimental data for charge quadrupole deformation β_{20} for comparison.

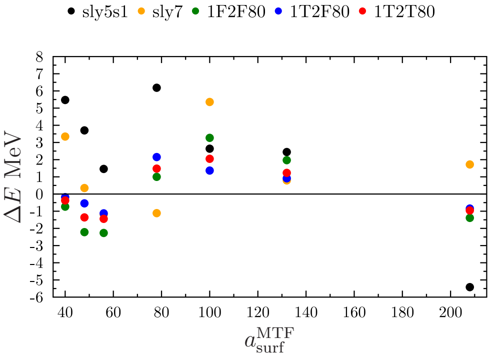
3.4) Super deformation Hg and Pb



- Lower panel : excitation energy of the 0^+ band head of the superdeformed rotational bands of Hg and Pb isotopes.

- Upper panel : Deformation parameter $\beta_{lm} = \beta_{20}$, β_{40},and β_{60}.

3.5) Mass of doubly magic nuclei



Mass rediduals of doubly magic nuclei : $\Delta E = E_{calc} - E_{exp}$

Used nuclei :

- ^{40}Ca, ^{56}Ni, ^{78}Ni, ^{100}Sn, ^{132}Sn, ^{208}Pb.

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- Covariant analysis.

Thanks for your attention.