Quantum dynamics beyond the independent particle picture

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Motivations

Objective of the thesis: give an accurate description of the dynamical properties of mesoscopic systems ($A = 1$ to ~ 300)

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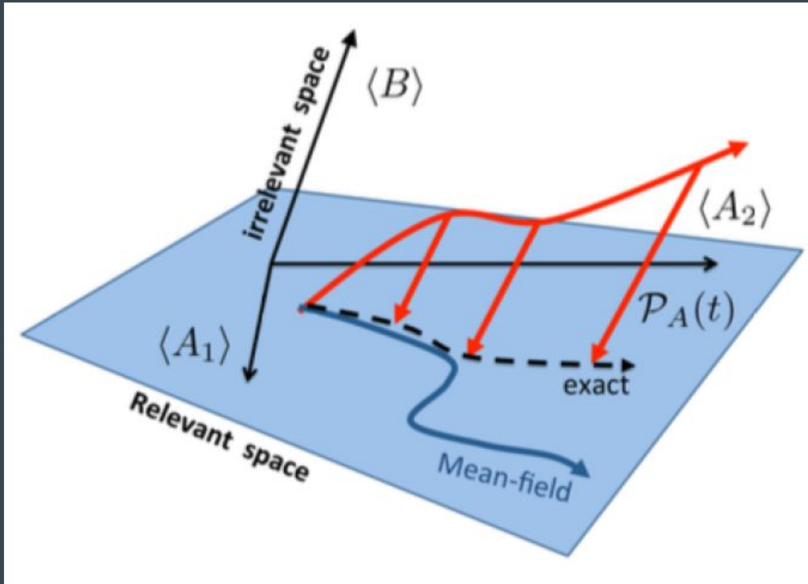
Simplification: only take the relevant degrees of freedom (DoFs), then add correlations



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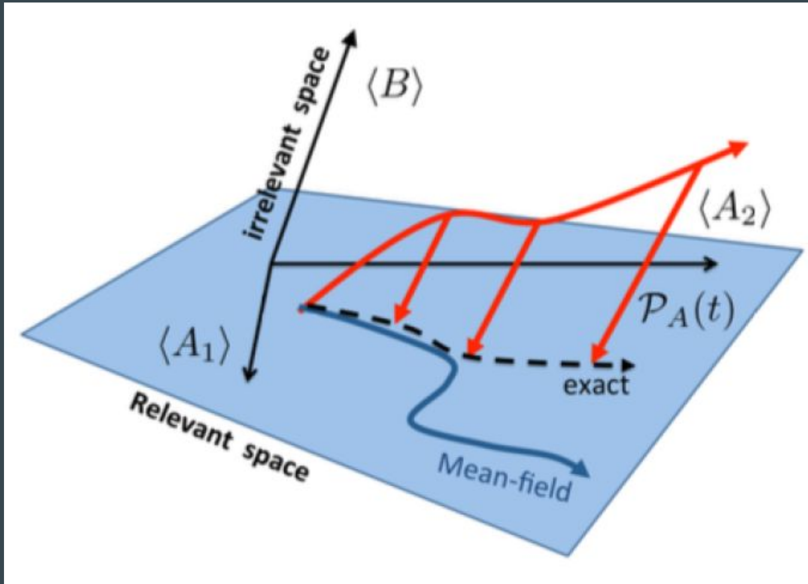
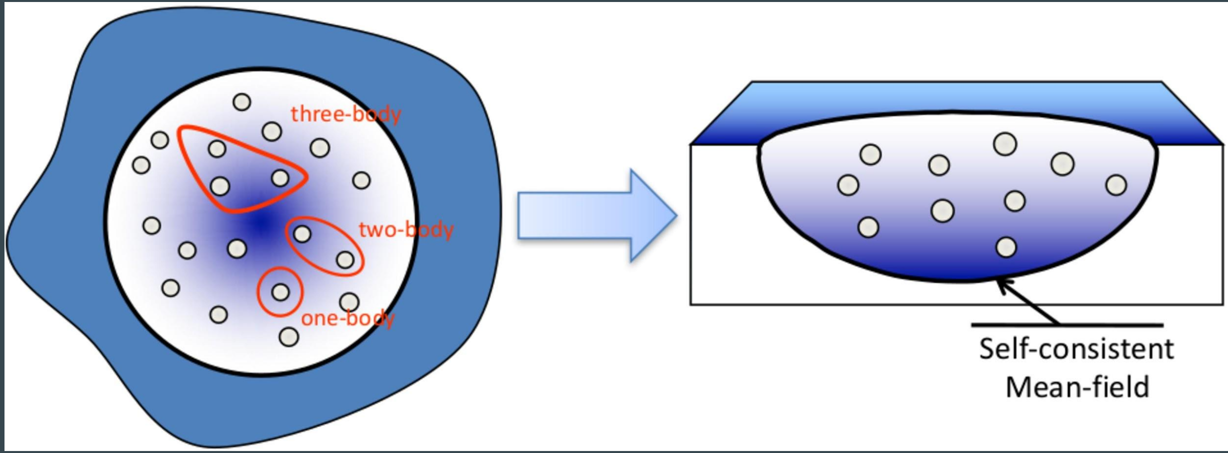
Nuclear Physics: strongly correlated dynamics => free particles in a new (simplified) potential



The Mean-Field ?

Free movement of particles + some effects of inter-particle interaction



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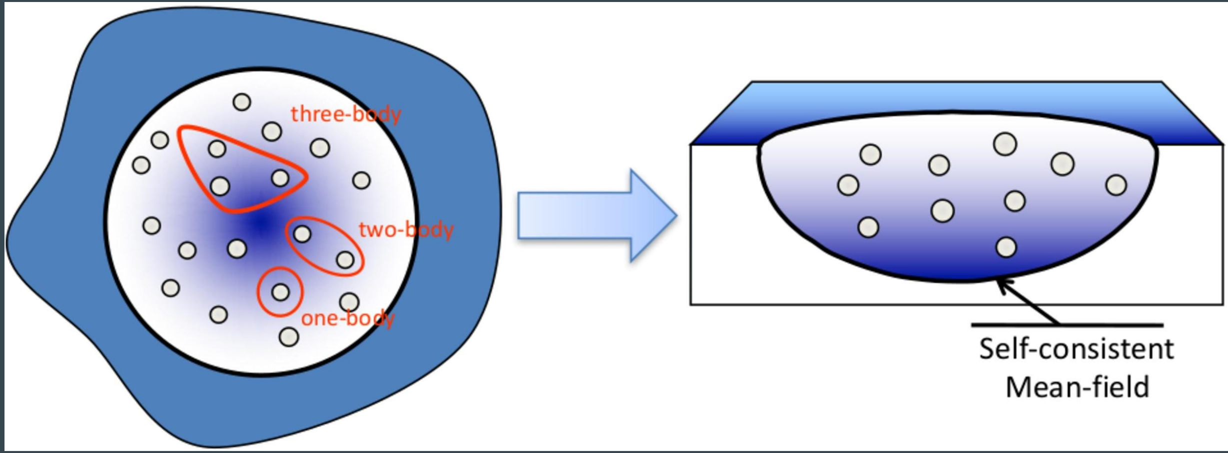
Dependence on the 1-body density matrix: $h = h[\rho]$

Contains all info on 1-body DoFs (e.g. occupation numbers)



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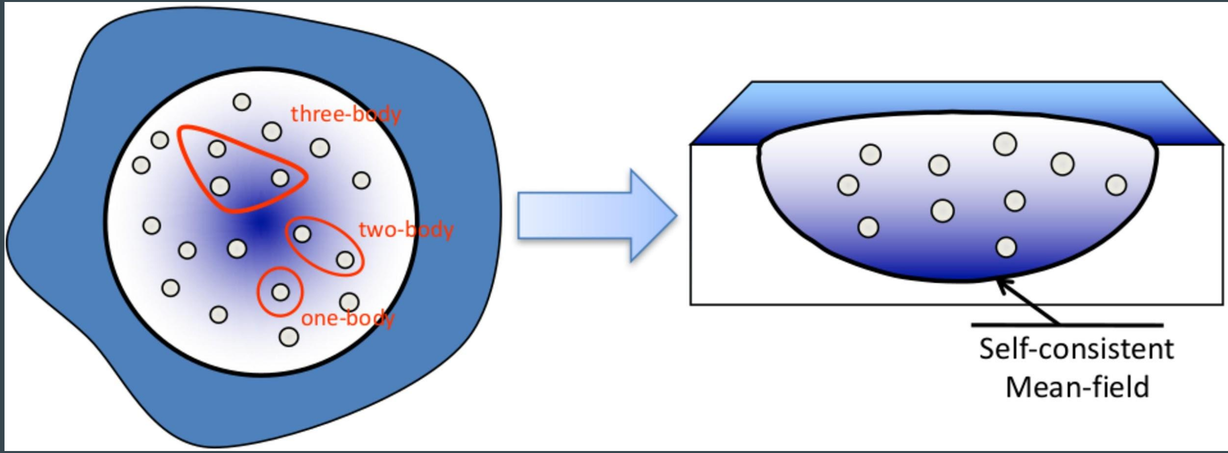

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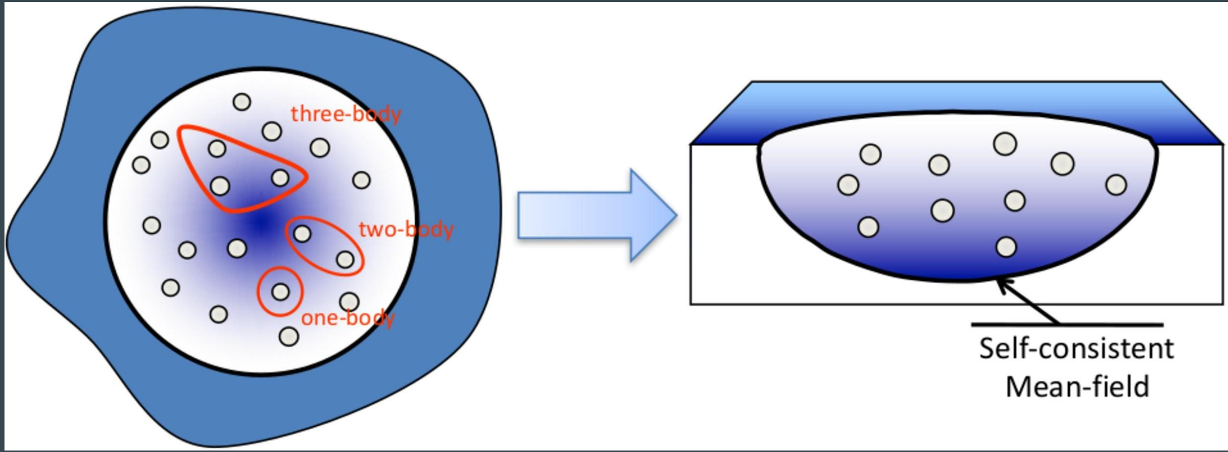
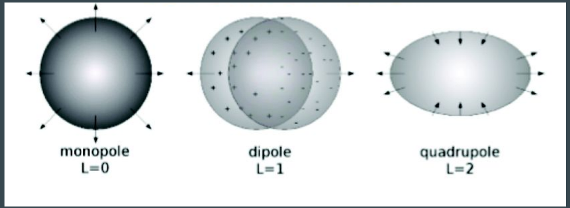


Non-linear self-consistent equations of motion: $i\hbar\,\partial_t \rho = [h[\rho], \rho]$

Many applications (e.g. giant resonances, fission, etc)

Misses important correlations (e.g. lifetimes)



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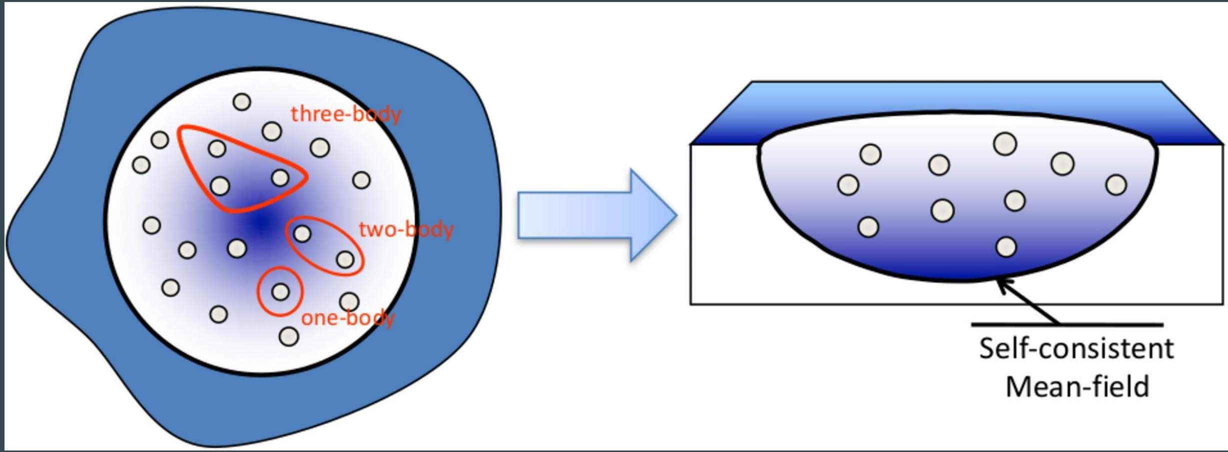


Non-linear self-consistent equations of motion: $i\hbar\,\partial_t \rho = [h[\rho], \rho]$

Add correlations beyond the mean-field ?

1) Stochastic path
2) Deterministic path

Beyond Mean-Field: stochastic path

Random initial fluctuations around the Mean-Field:

Beyond Mean-Field: stochastic path

Random <u>initial</u> fluctuations around the Mean-Field:

$$\rho \implies \rho^{(n)} = \rho + \delta\rho^{(n)}$$

$$\rho(t_0) \rightsquigarrow \rho(t) \quad \boxed{\text{Mean-Field}}$$

Beyond Mean-Field: stochastic path

Random initial fluctuations around the Mean-Field: complex problem => N multiple simple ones

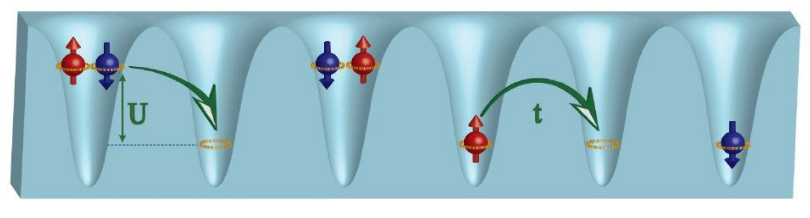
$$\rho \implies \rho^{(n)} = \rho + \delta\rho^{(n)}$$

$\rho(t_0)$ $\rho(t)$ Mean-Field

$\rho^{(n)}(t_0)$ $\overline{\rho^{(n)}(t)}$

Stochastic Mean-Field: (Initially) gaussian statistics of Mean-Field trajectories

Beyond Mean-Field: stochastic path



Hubbard model: electrons hopping from sites to sites

a) 2 spin up, 2 spin down
b) 4 spin up, 4 spin down

$\rho(t_0)$

$\rho(t)$

Mean-Field

$\rho^{(n)}(t_0)$

$\overline{\rho^{(n)}(t)}$

Stochastic Mean-Field: (Initially) gaussian statistics of Mean-Field trajectories

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Hubbard model: electrons hopping from sites to sites

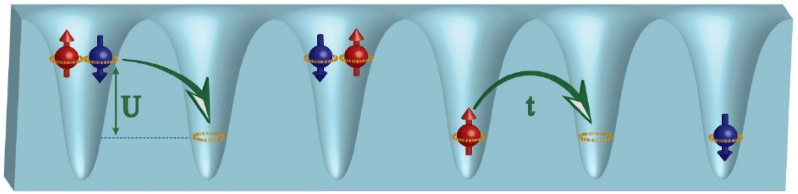
a) 2 spin up, 2 spin down
b) 4 spin up, 4 spin down

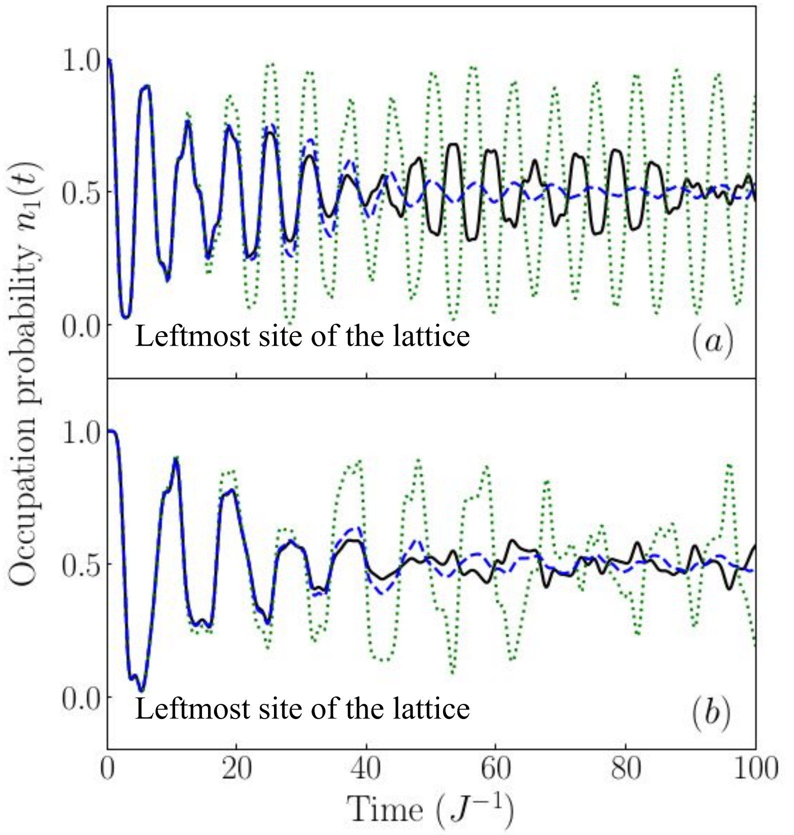
Exact (Schrödinger)

Mean-Field

Stochastic Mean-Field: (Initially) gaussian statistics of Mean-Field trajectories

Perturbative regime U = 0.1 J





Beyond Mean-Field: deterministic path

Adding many-body corrections: the deterministic path

D. Lacroix, P. Chomaz, and S. Ayik, Nucl. Phys. A **651**, 369 (1999).

Beyond Mean-Field: deterministic path

Adding many-body corrections: the deterministic path

Systematic expansion in powers of the two-body interaction: BBGKY hierarchy

2-body correlations

$$i\hbar\partial_t\rho = [h[\rho],\rho] + F(v_{12}, C_{12}),$$
$$i\hbar\partial_t C_{12} = \dots \; + \; G(v_{12}, v_{13}, v_{23}, C_{123})$$
$$\vdots$$

3-body correlations

2 body collisions + pairing + …

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Dissipator: in medium inter-particle collision

Non-Markovian effects, particle-hole excitations, thermalization...

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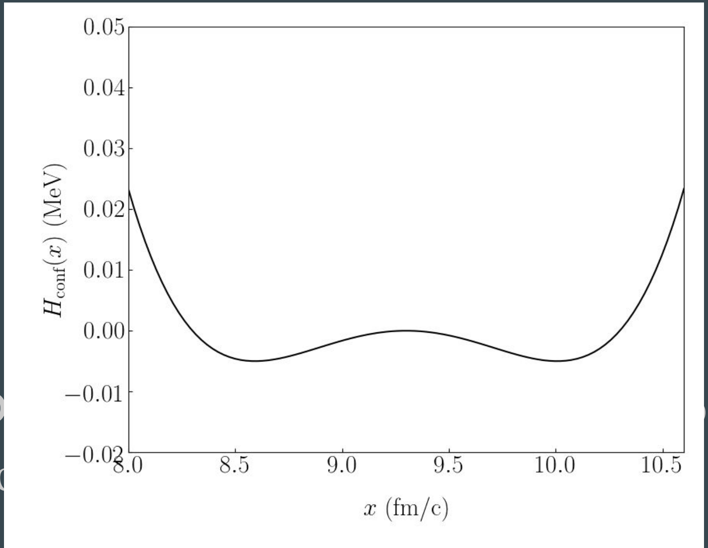
Beyond Mean-Field: deterministic path

2 particles in a double well + gaussian interaction



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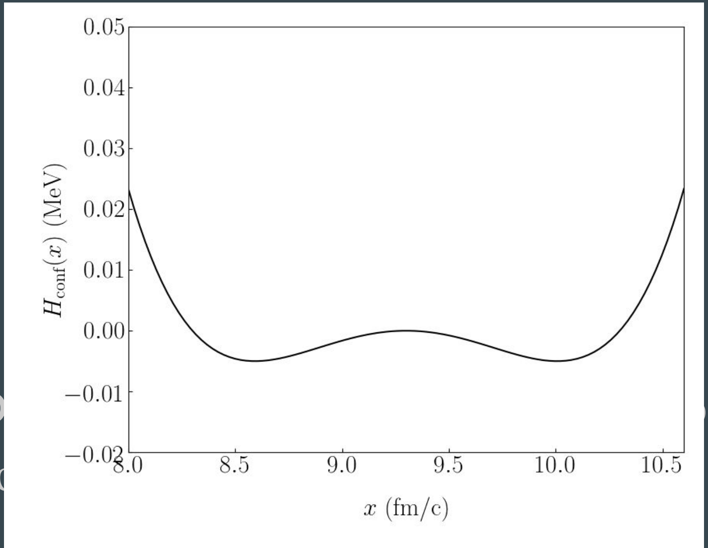

D [illegible] particle
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Collision integral: difficult treatment

Search for approximations: catching the essentials of non-markovian dynamics

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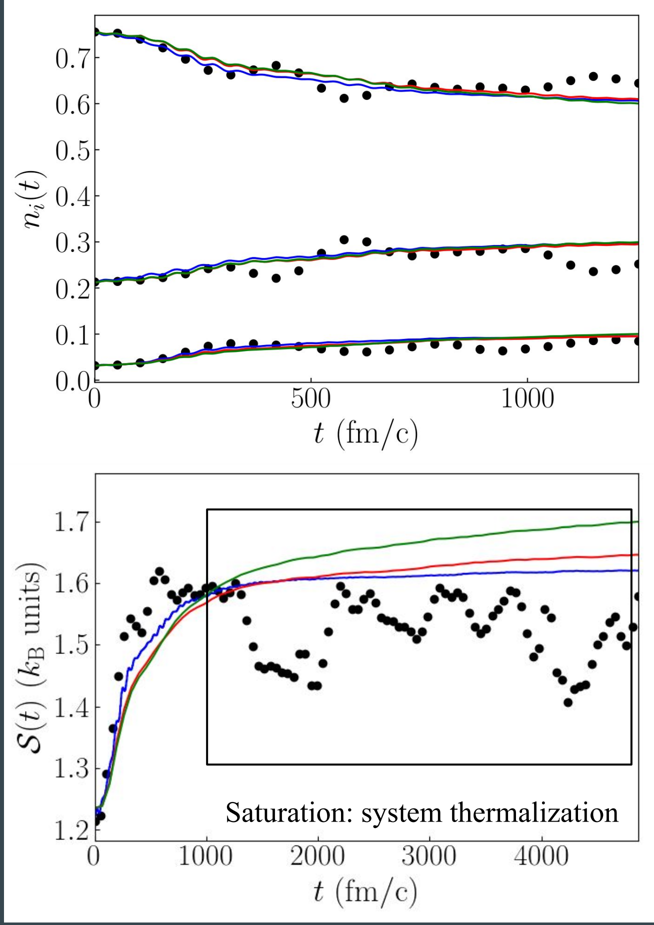
Search for approximations: catching the essentials of non-markovian dynamics

1) Perturbation theory: master equation

$$i\hbar\partial_t n_\alpha = (1 - n_\alpha(t))G_\alpha(t) - n_\alpha(t)L_\alpha(t)$$

T = 5 MeV



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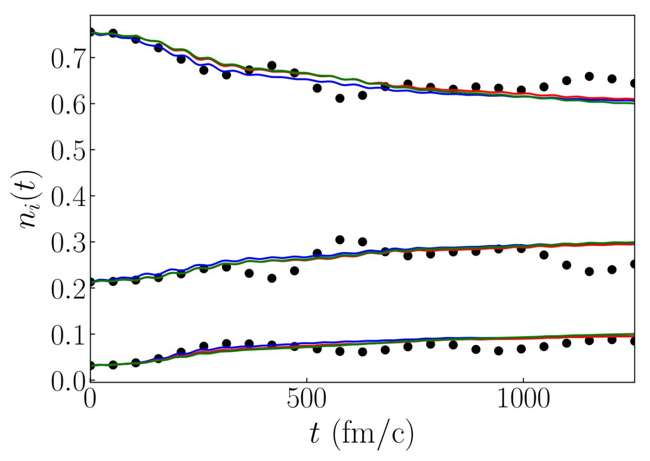
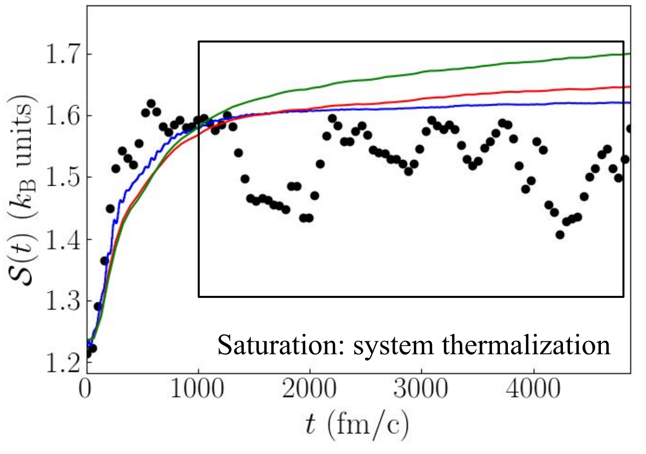
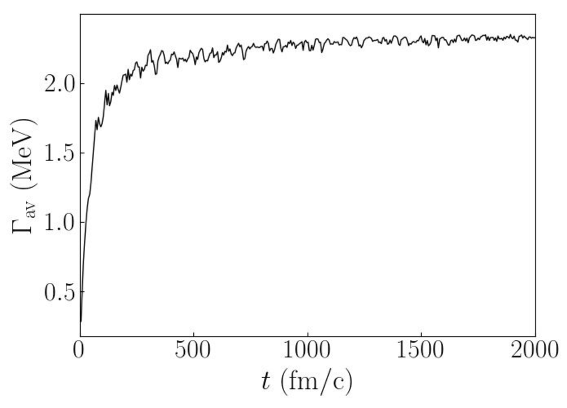
2) Analytical formulation
3) Particles lifetimes (secular approximation)

Average lifetime ~ 80 fm/c

T = 5 MeV







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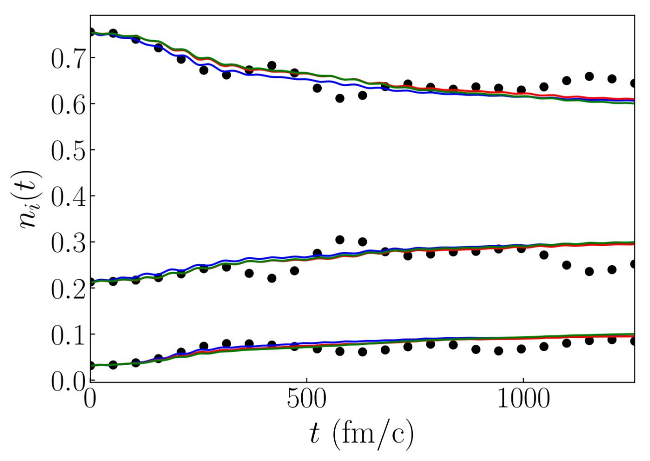

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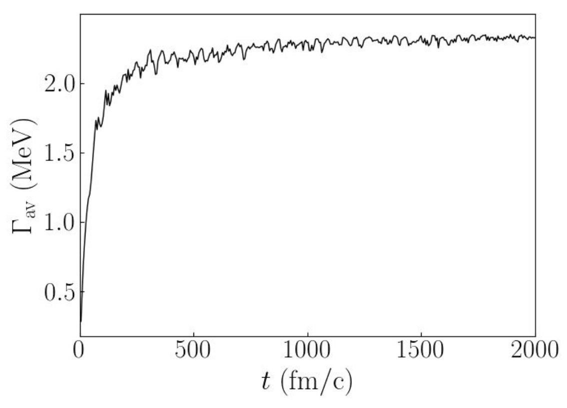

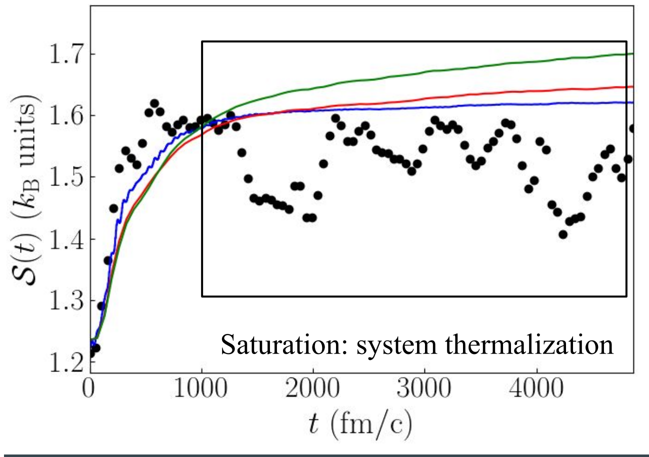

Average lifetime ~ 80 fm/c

Problems: why can’t we do some perturbation theory ?

Erratic effects of truncation: unphysical behaviors (e.g. occupation numbers > 1)

Non-trivial instabilities at long times

Exponential number of complex terms...

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Get a maximum of correlations while not expanding that much the BBGKY hierarchy

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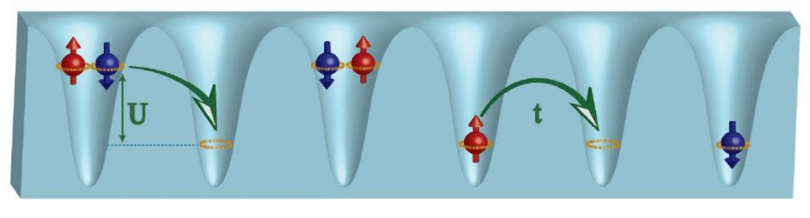
WHY NOT BOTH?

A hybrid method ?

Fluctuations around MF + supplementary correlation terms => 2 x N equations

T. Czuba, D. Lacroix, D. Regnier, I. Ulgen, and B. Yilmaz, Eur. Phys. J. A 56, (2020).

A hybrid method ?



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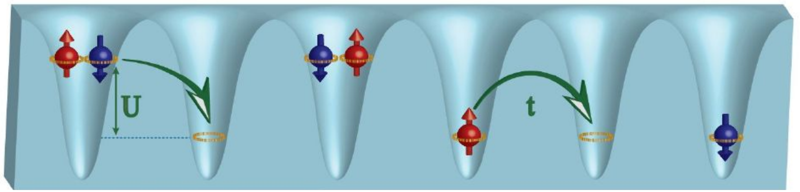

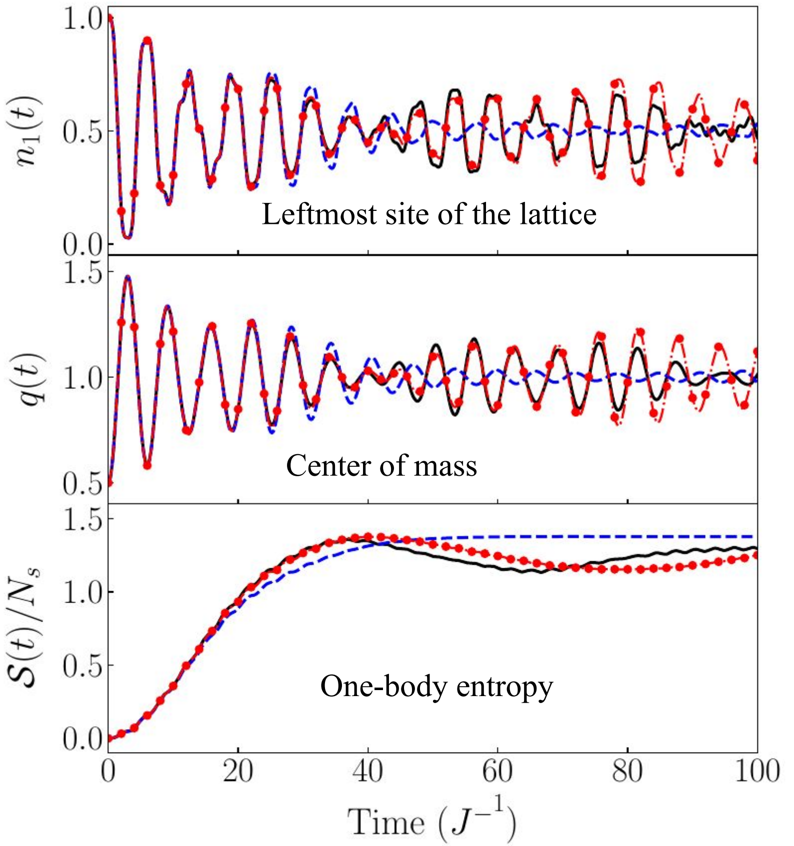

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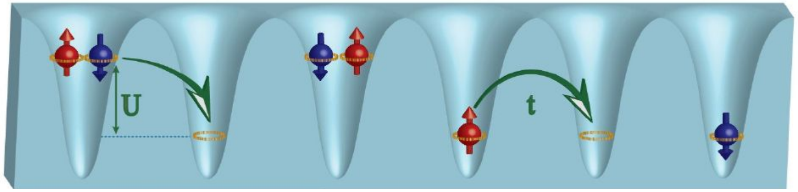
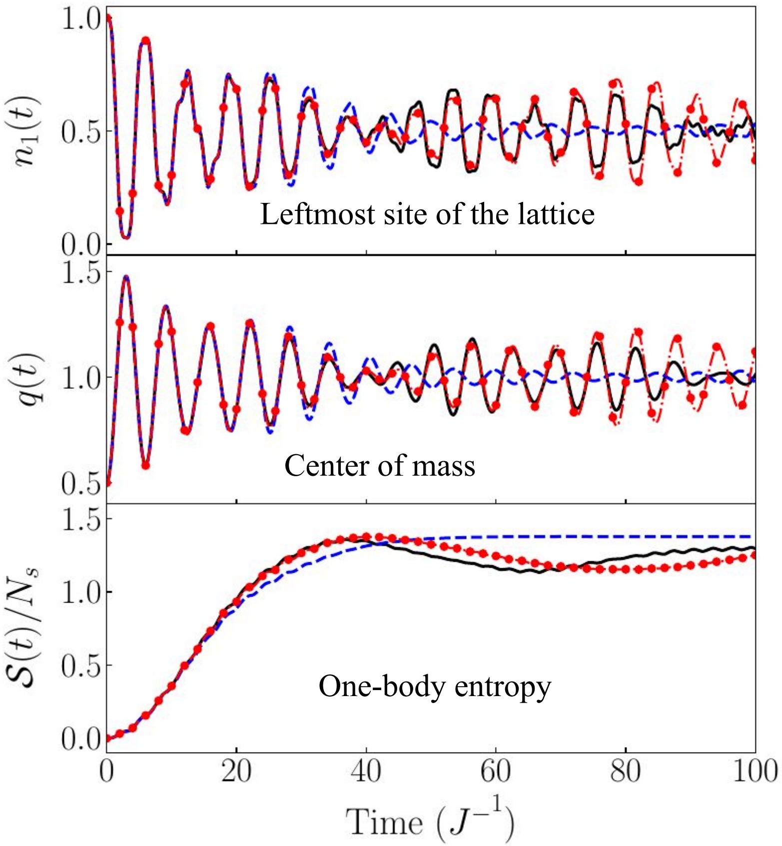
Success ! On a simple system





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The next step

Fluctuations around MF + supplementary correlation terms => 2 x N equations

1) Add pairing + simplify correlations integrals

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2) Application to real nuclei

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2) Application to real nuclei
3) Realistic applications of the new method ?

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Thank you for listening