



**How to study the location of the critical point
in the phase diagram of nuclear matter
with the event generator EPOS 4 ?**

Johannès JAHAN (Ph.D. student) - Subatech / CNRS / Nantes University

JRJC 2021 (22th October 2021)

Under the supervision of :

Klaus WERNER - Subatech / Nantes University

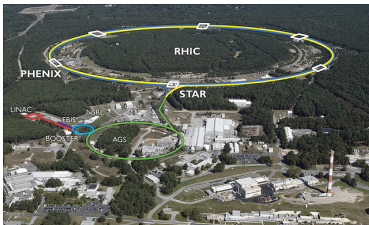
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- 2 EPOS, an event generator
 - Event generators
 - Generation of an event in EPOS
 - EPOS 4
 - What is RIVET ?
- 3 Physical context
 - What are we looking for ?
 - How can we find it ?
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Heavy-ion collisions

My work takes place in the context of **high energy particle collisions**.

In particular, I use an **event generator (EPOS)** to **simulate heavy-ion collisions (HIC)** and compare my results mainly to :

Au + Au collisions with center-of-mass energy $\sqrt{s_{NN}} = 7.7 - 200 \text{ GeV/A}$



RHIC facility

Pb + Pb collisions with center-of-mass energy $\sqrt{s_{NN}} = 2.76 - 5.02 \text{ TeV/A}$



LHC facility

1 collision

≡

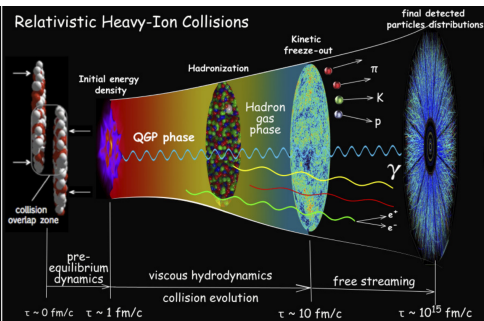
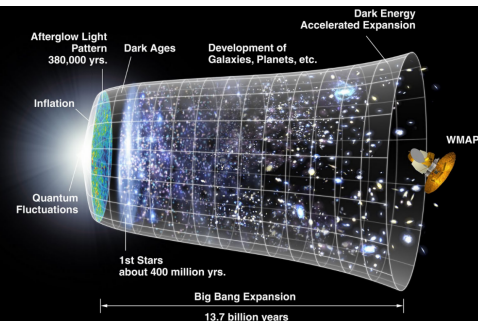
kinetic energy of 3 to 2000 flying mosquitoes in a 10^{-15} m size object !



Little Big-Bangs

But *why* do we perform such heavy-ion collisions ?

...to **recreate the Big-Bang** !



Analogy between the Big-Bang and HIC (U. W. Heinz, 2013)

We want to understand how the fundamental interactions work at very high energy scales.

We will focus in particular on the **strong interaction**, and more especially on the state of matter once created at the early stages of the Universe : the **Quark-Gluon Plasma (QGP)**.

This boiling nuclear matter made of **deconfined quarks and gluons** that can move freely (while **usually bounded into hadrons**) can be recreated in HIC !

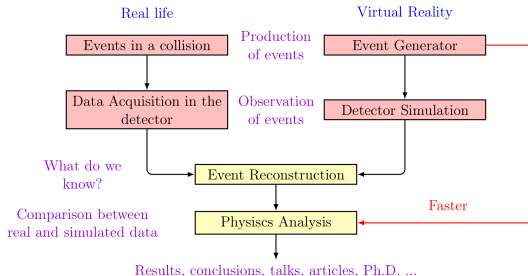
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What is EPOS ?

Event generators are programs made to **compute models** in order to **simulate every step** of a **collision** (e.g. **PYTHIA**, **HIJING++**...).

Advantages : - **perfect detector**, as final-state particles are all listed (no uncertainties)
- **dynamical approach**

*(indeed, there's always a **shadow in the picture** : one has to be careful on the applicability, and phenomenological approaches generally requires parametrisation)*



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Energy conserving quantum mechanical approach, based on
Partons, parton ladders, strings,
Off-shell remnants, and
Saturation of parton ladders

Event generator based on **parton-based Gribov-Regge Theory** (PBGRT), unifying **Parton model** and **Gribov-Regge theory** by **solving inconsistencies** of both models.

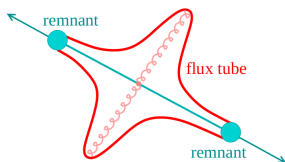
Can simulate with the same formalism **any type of collision** consistently :

$e^{+/-} + e^{+/-}$ $e^{+/-} + p$ $p + p$ $p + A$ $A + A$

Initial conditions & core-corona procedure

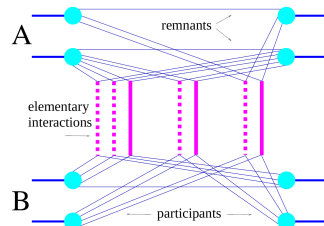
Primary interactions treated with PBGRT

Exchange of multiple Pomerons in parallel



A simple interaction within the PBGRT

(K. Werner, 2018)



Schematic representation of a collision

(K. Werner et al., 2000)

Core-corona separation

Those ladders are formed by strings, or color flux tubes
 $(q - g - \dots - g - \bar{q} \text{ chains})$
 with "kinks" due to transverse gluons.

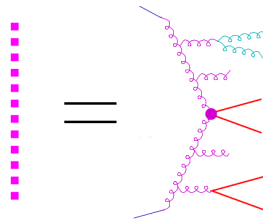
Initial conditions & core-corona procedure

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⇒ can be seen as parton ladders which are cut (particle production) or uncut (σ calculation)

(= *Multiple Parton Interaction*)



Diagrammatic view of a cut ladder

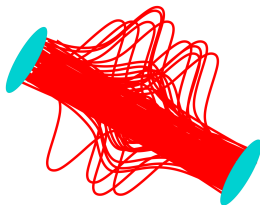
(K. Werner et al., 2016)

Core-corona separation

Those ladders are formed by strings, or color flux tubes ($q - g - \dots - g - \bar{q}$ chains) with "kinks" due to transverse gluons.

In HIC (but not only !), many strings may overlap, so we can separate :

- **core** = high string density region ($> \epsilon_c$)
- **corona** = escaping segments (with high p_T) ($< \epsilon_c$)



Multiple interactions within the PBGRT

(K. Werner, 2018)

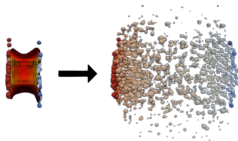
Medium evolution, hadronisation and re-scattering

Core evolution

Viscous 3D+1 relativistic
hydrodynamics expansion

+

Hadronisation of the medium via
Cooper-Frye procedure



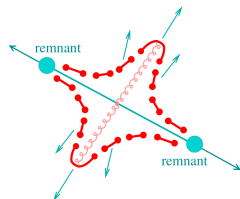
(MADAI collaboration)

Corona evolution

Strings evolution following dynamics of
gauge invariant Lagrangian

+

String fragmentation to produce hadrons



Re-scatterings between formed hadrons with the UrQMD model until
chemical freeze-out (no more inelastic scatterings)
kinetic freeze-out (no more elastic scatterings)



Final state particle

Toward the next public release : EPOS 4

As an important part of my Ph.D., I am involved in the **development of EPOS 4**, a **new version** planed to be **released publicly in late 2021 / early 2022**.

In order to **help** and **improve** the **validation process** of this new version before its release, I've been working on :

- ① **searching for experimental data** of basic observables (like p_T spectra, production yields of particles...) and **writing** the **corresponding analyses**
⇒ **mandatory for validation of the new EPOS version**
- ② adding the **HepMC output format** to enable **EPOS usage with RIVET**
⇒ **makes it more user-friendly**
- + **integrating RIVET** to the **online EPOS analysis framework**
⇒ **provides huge and constantly growing library of data and analyses**
+ **fastens the validation process**

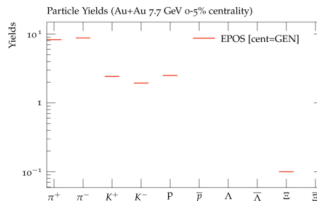
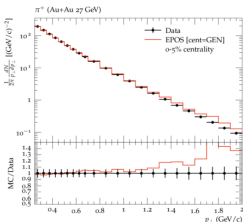
Robust Independant Validation of Experiment and Theory

Software based on C++ libraries, installed with different packages :

- YODA : Python libraries and classes used for analyses and histogramming
- HepMC : simulations recording and reading for analyses
- Fastjet : recombination algorithms, mainly used for jet analyses

Purpose : offer a **simple and standardised tool** to **automatise comparison** between **event generators simulations** and **experimental data**

```
rivet EG_DATA.hepmc -a ANALYSIS_NAME -p CALIB_NAME.yoda -o OUTPUT_FILE.yoda
```



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RIVET contains many analyses based on publications from many different experiments (experimental results included), and develops thanks to contributions from the users community (experimentalists & theoreticians).

Advantages :

- provides huge and constantly growing library of data and analyses
- easy to handle (a lot of documentation + helpful reactive developers)
- don't have to "think about" the analysis details anymore

⇒ **RIVET is a very useful tool for us !**

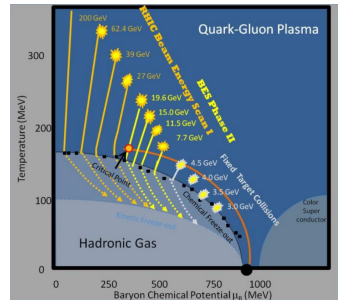
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What are we looking for ?

Quantum Chromodynamics phase diagram and critical point

Since the QGP has been observed (indirectly), efforts has been made to learn about its properties, and to map the QCD phase diagram.

- **Theoretically** : use models & theories to make predictions (T_C , μ_{B_C}) or to extract information from measurements (T & μ_B of a collision, viscosity of the QGP...)
- **Experimentally** : exploration of QCD phase diagram thanks to the Beam Energy Scan (BES) program, measurements of observables of interest (jet quenching, collective flow...)



Phase diagram of nuclear matter

(D. Cebra, 2013)

Question(s) of interest : is there a 1st order phase transition and a critical endpoint (CEP) between QGP and hadronic gas phases ? If yes, where ?

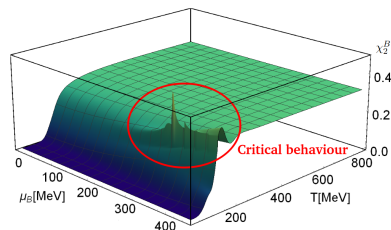
Susceptibilities

To answer this question, many tools can be used, among which are the **susceptibilities**, which **quantify** how an **extensive property** of a system **changes** under the **variation** of an **intensive property**.

In a **grand-canonical ensemble (GCE)**, a formalism often used to describe HIC, they are **theoretically defined** as derivatives of the partition function $Z(T, V, \mu)$:

$$\chi_{i,j}^{X,Y} = \frac{1}{VT^3} \cdot \left[\frac{\partial^{i+j} Z(T, V, \mu)}{(\partial \hat{\mu}_X)^i (\partial \hat{\mu}_Y)^j} \right]_{\mu_X, \mu_Y} \quad (\hat{\mu} = \frac{\mu}{T})$$

As we are searching for **radical changes in the state of nuclear matter**, i.e. phase transition, these derivatives of Z should reveal them.



2nd order baryonic susceptibility as a function of T and μ_B

(P. Parotto et al., 2020)

Susceptibilities

In a more convenient and understandable way, susceptibilities can be written as a function of the **net-charge cumulants**
 $(N_{B,Q,S} = n_{B,Q,S} - n_{\bar{B},\bar{Q},\bar{S}}).$

They represent in fact **event-by-event fluctuations** of the considered net charges, and can be linked to the statistical moments of their distributions.

Also, in order to **get rid of volume and temperature factors**, as they cannot be measured directly in experiments, **ratios** are often used.

2nd order susceptibilities for $X/Y = B, Q, S$

Linked to the **(co)variances** of the considered charges :

$$\chi_{11}^{XY} = \frac{1}{VT^3} \sigma_{XY}^{11} = \frac{\langle N_X N_Y \rangle - \langle N_X \rangle \langle N_Y \rangle}{VT^3}$$

$$\chi_2^X = \frac{1}{VT^3} \sigma_X^2 = \frac{\langle N_X^2 \rangle - \langle N_X \rangle^2}{VT^3}$$

Ratios

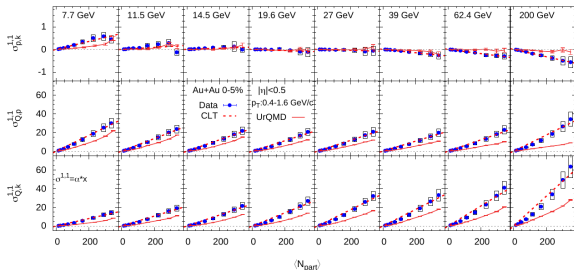
$$C_{BS} = \frac{\sigma_{BS}^{11}}{\sigma_S^2} \quad C_{QB} = \frac{\sigma_{QB}^{11}}{\sigma_B^2} \quad C_{QS} = \frac{\sigma_{QS}^{11}}{\sigma_S^2}$$

What has been done recently ?

Experimental results

STAR collaboration measured, for N_Q , $N_{protons}$ and N_{kaons} (proxies for N_B and N_S) in a restrained phase space ($|\eta| < 0.5 + 0.4 < p_T < 1.6 \text{ GeV/c}$) :

$$\bullet \begin{pmatrix} \sigma_Q^2 & \sigma_{Q,p}^{11} & \sigma_{Q,k}^{11} \\ " & \sigma_p^2 & \sigma_{p,k}^{11} \\ " & " & \sigma_k^2 \end{pmatrix} \text{ vs } \langle N_{part} \rangle (\chi_{11,2}^{B,Q,S} \text{ proxies})$$



What has been done recently ?

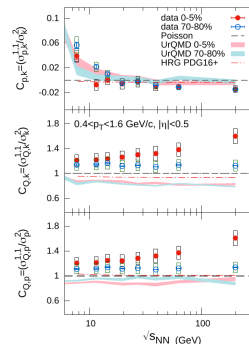
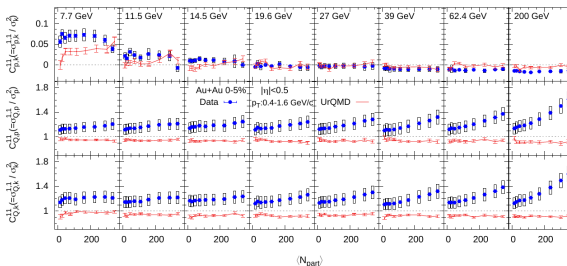
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- $\begin{pmatrix} \sigma_Q^2 & \sigma_{Q,p}^{11} & \sigma_{Q,k}^{11} \\ " & \sigma_p^{22} & \sigma_{p,k}^{11} \\ " & " & \sigma_k^{22} \end{pmatrix}$ vs $\langle N_{part} \rangle$ ($\chi_{11,2}^{B,Q,S}$ proxies)

- Koch ratios $C_{Qp,Qk,pk}$ (proxies for $C_{QB,QS,BS}$)

- as a function of $\langle N_{part} \rangle$
- as a function of $\sqrt{s_{NN}}$



What we can(not) study with EPOS

Recent feature : inclusion of a **new EoS** containing **CEP + 1st order phase transition**.

However, the **hydrodynamic evolution** of the core in EPOS (macroscopic quantities) **does not include fluctuations** : susceptibilities are **NOT expected to be sensitive** to any possible **CEP** within the hydro phase

⇒ search for signatures of CEP **impossible with EPOS** by construction ?

Recent work with EPOS (see [M. Stefaniak's thesis](#)) showed almost no differences between new and old EoS

In fact, in EPOS, we expect that most of the **fluctuations** come from **initial conditions**, **hadronisation process** and/or **hadronic cascades**.
(may even dominate the fluctuations of phase transition we are seeking...)

Then, what we plan to do is

1. comparing cumulants before & after UrQMD (+ with STAR results), to see the impact of hadronic cascades on the susceptibilities

What we can(not) study with EPOS

Furthermore, the **choice** of **grand-canonical ensemble** to describe heavy-ion collisions is **questionable** (taken from *M. Nahrgang's talk*) :

in a GCE, the system is :

- in thermal equilibrium (=long-lived)
- in equilibrium with a particle heat bath
- static

the system created in a HIC is :

- short-lived
- inhomogeneous
- highly dynamical

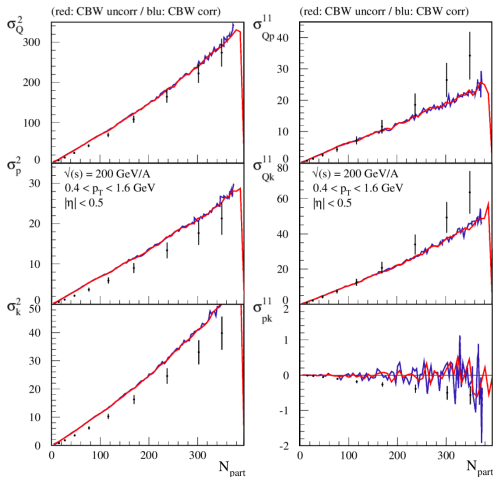
Hence, we also include in our plan

2. comparing cumulants after decays for micro (new standard in EPOS 4) & grand canonical (= classical Cooper-Frye procedure) with STAR results, to see the impact of hadronisation on the susceptibilities

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Au+Au @ $\sqrt{s_{NN}} = 200$ GeV/A

Results from **recent EPOS 4 version** (3 months-old) compared with **STAR** data

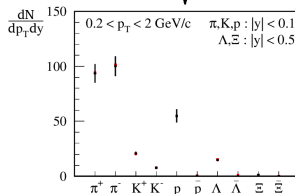
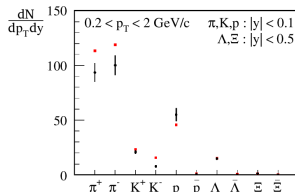


⇒ As expected for $\sigma^{11,2}$ ([Sahar et al.](#)),
 no difference w/o CBWE correction

- EPOS reproduces qualitatively well the N_{part} dependence of variances
- pretty good estimation of σ_Q^2
 + σ_{Qp}^{11} & σ_{Qk}^{11} for peripheral collisions

- EPOS fails to describe quantitatively σ_p^2 and σ_k^2
 → particle production
- fails to reproduce properly the N_{part} dependence of covariances, especially σ_{pk}^{11} (no dependence ?)
 → check the feed-down

Hadronic species multiplicity



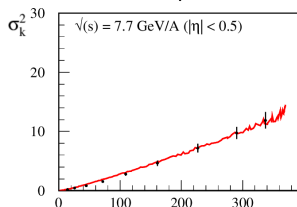
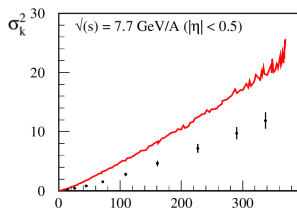
Why does particle production should be checked ?

Simply because **amplitudes** of these 2nd order cumulants are **directly linked to the net-multiplicities** of the **considered species**.

Indeed, if we multiply net particle numbers N_X and N_Y by factors c_X and c_Y , we get :

$$\begin{aligned}\sigma_{XY}'^{11} &= \langle N_X' \cdot N_Y' \rangle - \langle N_X' \rangle \langle N_Y' \rangle \\ &= \langle c_X N_X \cdot c_Y N_Y \rangle - \langle c_X N_X \rangle \langle c_Y N_Y \rangle \\ &= c_X c_Y \langle N_X \cdot N_Y \rangle - c_X \langle N_X \rangle c_Y \langle N_Y \rangle\end{aligned}$$

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Then

$$\sigma_{XY}'^{11} = (c_X \cdot c_Y) \times \sigma_{XY}^{11}$$

and

$$\sigma_X'^2 = (c_X)^2 \times \sigma_X^{11}$$

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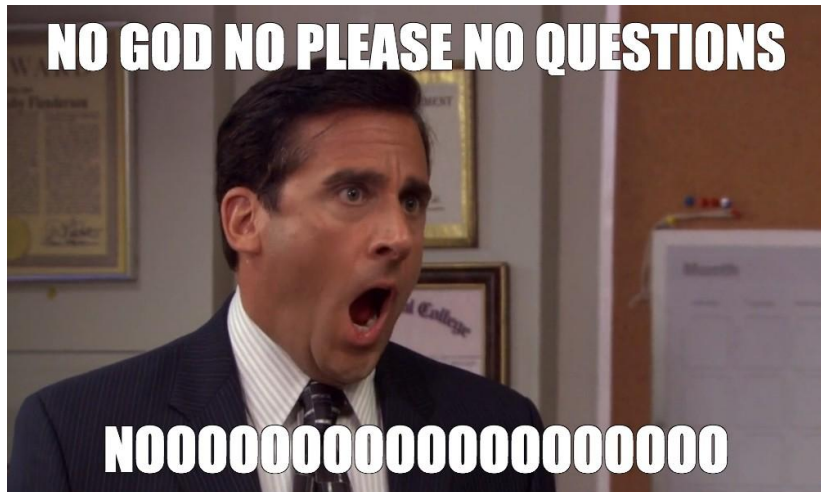
Summary & Outlook

Main research goal : use last version of **EPOS 4** study the **impact** of **hadronisation** and **hadronic cascades** on **2nd order susceptibilities** of **B, Q, S** , using **STAR proxies** and best proxies proposed by **C. Ratti *et al.*** through BES

Status :

1. compare EPOS results with STAR measured proxies :
 - $\sqrt{s_{NN}} = 200 \text{ GeV/A} :$
OK qualitatively for variances, even almost quantitatively
covariances fall for central collisions
 - ⇒ finish EPOS 4 validation (\approx OK @ 200 GeV/A → go to lower energies)
→ check results for other energies in order to check the energy dependence
2. implement the best proxies from C. Ratti *et al.* (see backup slides)
3. compare results from different hadronisation processes
4. compare results before and after hadronic cascades
5. take a look at higher order cumulants and ratios (skewness, kurtosis...) ?

Thanks for your attention !



... just kidding of course 😊

A bit more about EPOS...

More references about EPOS :

- primary interactions & hydrodynamics in EPOS
- hydrodynamics in EPOS
- heavy flavors in EPOS
- jet-fluid interaction in EPOS

Recent developments for EPOS 4 :

- [parton saturation](#) (see also [here](#))
- microcanonical decay of the core

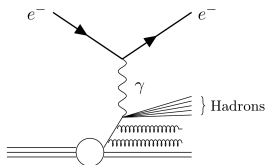
+ development of **EPOS-HQ** for heavy flavour observables

Stay tuned ! More papers to come...

PBGR - The motivations

Parton model

Mainly used for inclusive cross-section calculations



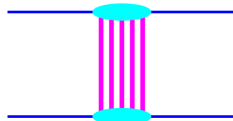
Deep Inelastic Scattering

Problems :

- can only calculate cross-section for hard processes \rightarrow not suitable alone for HIC

Gribov-Regge theory

EFT for Multiple *Pomeron* Interaction



(K. Werner et al., 2000)

Inconsistencies :

- energy conserved for particle production but NOT for cross-section calculations
- although multiple scattering approach, all interactions are not treated equally

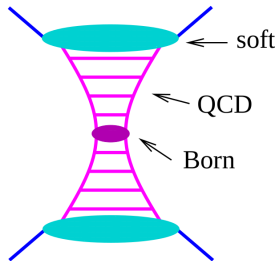
Solution : merge both into a formalism treating consistently hard and soft scattering

\Rightarrow **Parton-based Gribov-Regge Theory !**

Main principle of PBGRT

In the PBGRT, an **elementary interaction** is modeled as a *Pomeron*.

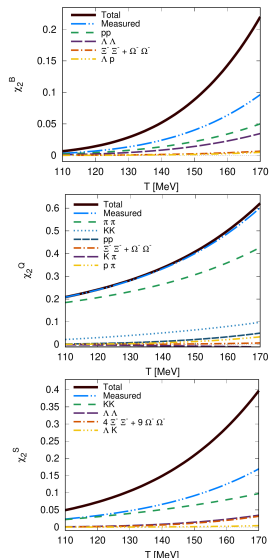
- **Soft process** ($Q^2 < 1 \text{ GeV}$) : mainly elastic scatterings, parametrised T-matrix (Regge poles)
- **Hard process** ($Q^2 > 1 \text{ GeV}$) : pQCD applicable, computed T-matrix (DGLAP equation)
- **Semi-hard process** ($Q^2 > 1 \text{ GeV}$ $q_{sea}/\bar{q}_{sea}/g$) : using both previous formalisms



Lattice QCD + Hadron Resonance Gas model

C. Ratti *et al.* :

- breakdown of hadronic species contributions to susceptibilities, studied from IQCD + HRG model calculations (*gas of non-interacting hadrons and resonances in a box*)



Lattice QCD + Hadron Resonance Gas model

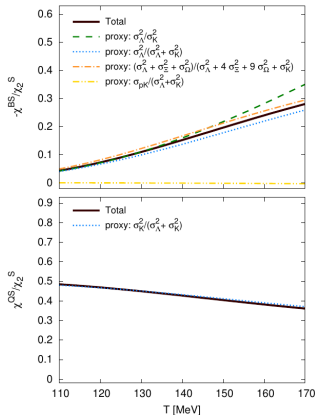
C. Ratti *et al.* :

- breakdown of hadronic species contributions to susceptibilities, studied from IQCD
+ HRG model calculations (*gas of non-interacting hadrons and resonances in a box*)
⇒ **best proxies for ratios**
(so potentially the most sensitive ones)

$$C_{BS} = \frac{\chi_{11}^{BS}}{\chi_2^S} = \frac{\sigma_\Lambda^2 + 2\sigma_\Xi^2 + 3\sigma_\Omega^2}{\sigma_\Lambda^2 + 4\sigma_\Xi^2 + 9\sigma_\Omega^2 + \sigma_k^2} \quad \left(= \frac{\sigma_{pk}^{11}}{\sigma_k^2} \right)_{STAR}$$

or $= \frac{\sigma_\Lambda^2}{\sigma_k^2 + \sigma_\Lambda^2}$ (easier to measure experimentally !)

$$C_{QS} = \frac{\chi_{11}^{QS}}{\chi_2^S} = \frac{1}{2} \cdot \frac{\sigma_k^2}{\sigma_k^2 + \sigma_\Lambda^2} \quad \left(= \frac{\sigma_{Qk}^{11}}{\sigma_k^2} \right)_{STAR}$$



Lattice QCD + Hadron Resonance Gas model

C. Ratti *et al.* :

- breakdown of hadronic species contributions to susceptibilities, studied from IQCD + HRG model calculations (*gas of non-interacting hadrons and resonances in a box*)

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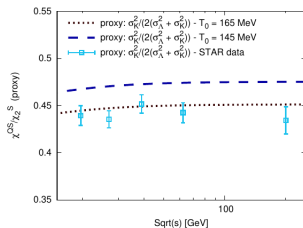
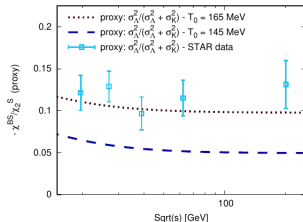
⇒ **results depending on \sqrt{s} + kinematic cuts compared with STAR data**

$$C_{BS} = \frac{\chi_{11}^{BS}}{\chi_2^S} = \frac{\sigma_\Lambda^2 + 2\sigma_\Xi^2 + 3\sigma_\Omega^2}{\sigma_\Lambda^2 + 4\sigma_\Xi^2 + 9\sigma_\Omega^2 + \sigma_k^2} \quad \left(= \frac{\sigma_{pk}^{11}}{\sigma_k^2} \right)_{STAR}$$

or
$$= \frac{\sigma_\Lambda^2}{\sigma_k^2 + \sigma_\Lambda^2} \quad (\text{easier to measure experimentally !})$$

$$C_{QS} = \frac{\chi_{11}^{QS}}{\chi_2^S} = \frac{1}{2} \cdot \frac{\sigma_k^2}{\sigma_k^2 + \sigma_\Lambda^2} \quad \left(= \frac{\sigma_{Qk}^{11}}{\sigma_k^2} \right)_{STAR}$$

... and what about event generators ?



Hadron Resonance Gas Model (summarised from **C. Ratti *et al.***)

It assumes that a gas of interacting hadrons in ground states can be described by a gas of non-interacting hadrons and resonances.

One can then re-write partition function, allowing to consider kinematic cuts simply by changing the phase space integration :

$$\ln(\mathcal{Z}_R) = \eta_R \frac{V \cdot d_R}{2\pi^2 T^3} \int_0^\infty p^2 \cdot dp \cdot \ln \left(1 - \eta_R \cdot z_R \cdot e^{-\varepsilon_R/T} \right)$$

Hence, with such assumption, one can decompose susceptibilities as a function of hadronic species :

$$\chi_{ijk}^{BQS}(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S) = \sum_R \sum_{i \in \text{stable}} (P_{R \rightarrow p})^i \times B_p^i Q_p^j S_p^k \times I_l^R(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)$$

with :

- $l = i + j + k$
- $P_{R \rightarrow p} = \sum_\alpha N_{R \rightarrow p}^\alpha \times n_{p,\alpha}^R : \quad \langle n_p \rangle$ produced in process α by each resonance R
- $B_p^i, Q_p^j, S_p^k : \quad$ quantum numbers of particle specie p
- $I_l^R(T, \hat{\mu}_{B,Q,S}) = \frac{\partial^l}{\partial \hat{\mu}_R^l} \left[\frac{1}{VT^3} \sum_R \ln(\mathcal{Z}_R) \right] \quad (\hat{\mu}_R = \hat{\mu}_B \cdot B_R + \hat{\mu}_Q \cdot Q_R + \hat{\mu}_S \cdot S_R)$

Centrality bin width effect (CBWE)

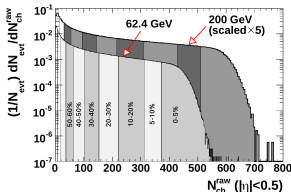
When plotting whatever moment $\sigma^{i,j}$ vs N_{part} , one induces **trivial fluctuations** due to the **volume variation** of the system : this is the **CBWE**.

In fact, for a certain centrality bin considered (*and even for a single N_{part} value*), there will be volume variations in the collisions (\leftrightarrow *different final-state multiplicities*) that will contribute to $\sigma_{p,Q,k}^{11,2}$ without being "real fluctuations" (the one we are seeking).

To **minimise this effect**, STAR collaboration measure $\sigma_{p,Q,k}^{11,2}$ vs N_{ch} for each centrality bin considered, and calculate the corresponding **weighted mean value** :

$$\sigma_c = \sum_i \frac{n_i \times \sigma_i}{n_c}$$

n_i the number of events for the multiplicity bin i
 n_c the number of events in the centrality bin c



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\Rightarrow **Our method** (faster & easier) : **calculate $\sigma_{p,Q,k}^{11,2}$ vs N_{ch}** , and then **convert $N_{ch} \rightarrow N_{part}$** from the **$< N_{part} >$ vs N_{ch}** distribution

