Turbulent production of polarized gravitational radiation from primordial helical magnetic fields

Cinquième Assemblée Générale du GdR Ondes Gravitationnelles
Oct. 11–12, 2021
Laboratoire d'Annecy de Physique des Particules (LAPP)

Alberto Roper Pol
Postdoctoral Researcher

Laboratoire Astroparticule et Cosmologie (APC)
Université de Paris, CNRS

Collaborators: A. Brandenburg (Nordita), C. Caprini (APC), G. Gogoberidze (Ilia State U), T. Kahniashvili (CMU), A. Kosowsky (PittU), S. Mandal (SBU), A. Neronov (APC), D. Semikoz (APC).

ARP, S. Mandal, A. Brandenburg, T. Kahniashvili, *submitted to JCAP*, arXiv:2107.05356 (2021).

Overview

Cosmological sources of GWs

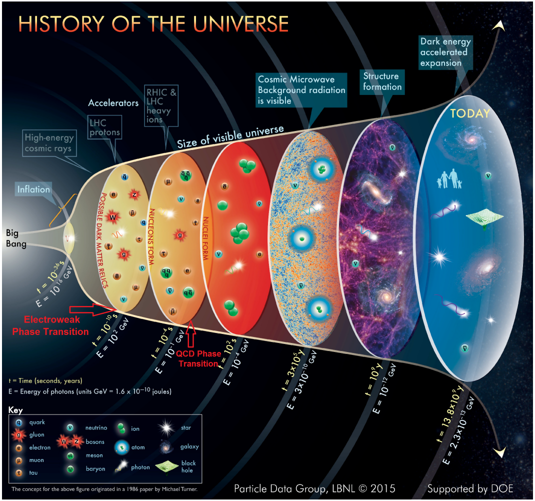
- From phase transitions
  - LIGO-Virgo frequencies are 10–1000 Hz ($T \sim 10^7$ GeV)

    Peccei-Quinn, B-L, left-right symmetries, ...
    (untested physics, SM extensions)
  - **LISA** frequencies are 10^{-5}–10^{-2} Hz

    Electroweak phase transition $\sim$ 100 GeV ($f_c \sim 10^{-5}$ Hz)
  - Pulsar Timing Array **(PTA)** frequencies are 10^{-9}–10^{-7} Hz

    Quantum chromodynamic (QCD) phase transition
    $\sim$ 100 MeV ($f_c \sim 10^{-9}$ Hz)
- B-modes of CMB anisotropies ($f \sim 10^{-18}$ Hz)

  Inflation



The concept for the above figure originated in a 1986 paper by Michael Turner.

Particle Data Group, LBNL © 2015

Supported by DOE

LISA

- Laser Interferometer Space Antenna (LISA) is a space–based GW detector
- LISA is planned for 2034
- LISA was approved in 2017 as one of the main research missions of ESA
- LISA is composed by three spacecrafts in a distance of 2.5M km
- LISA cosmology working group (since 2015, 230 members)

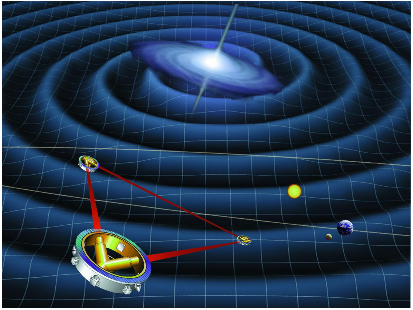
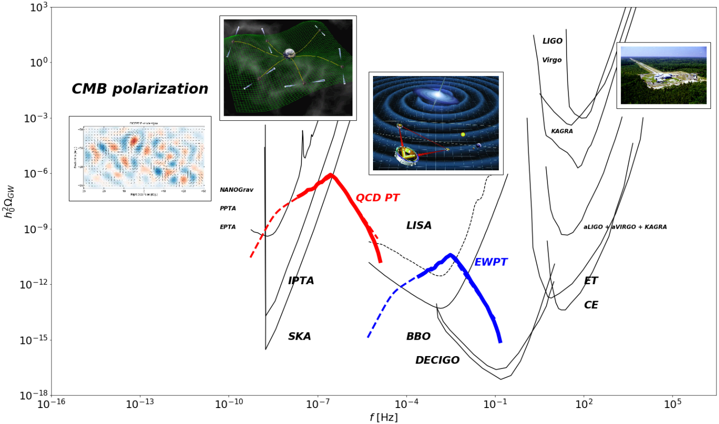


Figure: Artist's impression of LISA from Wikipedia

Gravitational Spectrum



MHD sources

- Magnetohydrodynamic (MHD) sources of GWs
  - Hydrodynamic turbulence from first-order phase transitions
  - **Primordial magnetic fields**

- Other sources of GWs include
  - True vacuum bubble collisions
  - Sound waves
  - Cosmic topological defects (cosmic strings)
  - Primordial black holes

How do we compute this?

- Direct numerical simulations using the PENCIL CODE[1] to solve:
  - Relativistic MHD equations adapted for radiation-dominated era (after electroweak symmetry is broken)
  - Gravitational waves equation

---

[1] Pencil Code Collaboration, JOSS **6**, 2807 (2020), `https://github.com/pencil-code/`

1. Introduction and Motivation

2. **Primordial magnetic fields and MHD**

3. Gravitational waves

Primordial magnetic fields

- Magnetic fields can either be produced at or present during cosmological phase transitions
- The magnetic fields are strongly coupled to the primordial plasma and inevitably lead to MHD turbulence[2]
- Present magnetic fields can be reinforced by primordial turbulence or generated via dynamo[3]
- Primordial magnetic fields would evolve through the history of the universe up to the present time and explain the lower bounds derived by the Fermi collaboration[4]
- Maximum amplitude of primordial magnetic fields is constrained by the big bang nucleosynthesis[5]

---

[2] J. Ahonen and K. Enqvist, *Phys. Lett. B* **382**, 40 (1996).

[3] A. Brandenburg, T. Kahniashvili, S. Mandal, ARP, A. G. Tevzadze and T. Vachaspati *Phys. Rev. Fluids* **4**, 024608 (2019).

[4] A. Neronov and I. Vovk, *Science* **328**, 73 (2010)

[5] V. F. Shvartsman, Pisma Zh. Eksp. Teor. Fiz. 9, 315 (1969).

Helicity

- Magnetic helicity is observed in present astrophysical objects
- Fractional magnetic helicity is required in cosmological seed fields
- Primordial helical magnetic fields require a parity-odd violation (can be used as a test)
- Helicity affects the dynamical evolution of the primordial magnetic fields

Definition (Magnetic Helicity)

$$\mathcal{H} = \left\langle \boldsymbol{B} \cdot (\nabla\times)^{-1} \boldsymbol{B} \right\rangle = \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle$$

Generation of primordial magnetic fields

- Velocity fields induced by first-order phase transitions can generate magnetic fields
- Parity-violating processes during the EWPT can produce helical magnetic fields:
  - Sphaleron decay (non-helical[6] and helical[7])
  - Generation of Chern-Simons number through B+L anomalies[8]
  - Inhomogeneities in the Higgs field in low-scale electroweak hybrid inflation[9]

---

[6] T. Vachaspati, *Phys. Rev. B* **265**, 258 (1991).

[7] T. Vachaspati, *Phys. Rev. Lett.* **87**, 251302 (2001).

[8] J. M. Cornwall, *Phys. Rev. D* **56**, 6146 (1997).

[9] M. Joyce and M. E. Shaposhnikov, *Phys. Rev. Lett.* **79**, 1193 (1997), J. García-Bellido *et al.*, *Phys. Rev. D* **60**, 123504 (1999).

Generation of primordial magnetic fields

- Axion fields can generate helicity in an already existing magnetic field[10]
- Magnetic fields from inflation can be present during phase transitions (non-helical[11] and helical[12])

---

[10] M. M. Forbes and A. R. Zhitnitsky, *Phys. Rev. Lett.* **85**, 5268 (2000).

[11] M. S. Turner and L. M. Widrow, *Phys. Rev. D* **37**, 2743 (1988).

[12] M. Giovannini, *Phys. Rev. D* **58**, 124027 (1998).

MHD description

Right after the electroweak phase transition we can model the plasma using continuum MHD

- Quark-gluon plasma (above QCD phase transition)
- Charge-neutral, electrically conducting fluid
- Relativistic magnetohydrodynamic (MHD) equations
- Ultrarelativistic equation of state

$$p = \rho c^2/3$$

- Friedmann–Lemaître–Robertson–Walker model

$$g_{\mu\nu} = \mathrm{diag}\{-1, a^2, a^2, a^2\}$$

Contributions to the stress-energy tensor

$$T^{\mu\nu} = \left(p/c^2 + \rho\right) U^\mu U^\nu + p g^{\mu\nu} + F^{\mu\gamma} F^{\nu}{}_{\gamma} - \frac{1}{4} g^{\mu\nu} F_{\lambda\gamma} F^{\lambda\gamma},$$

- From fluid motions
  $T_{ij} = \left(p/c^2 + \rho\right) \gamma^2 u_i u_j + p\delta_{ij}$
  Relativistic equation of state: $p = \rho c^2/3$

- From magnetic fields:
  $T_{ij} = -B_i B_j + \delta_{ij} B^2/2$

- 4–velocity $U^\mu = \gamma(c, u^i)$
- 4–potential $A^\mu = (\phi/c, A^i)$

- 4–current $J^\mu = (c\rho_{\mathrm{e}}, J^i)$
- Faraday tensor
  $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

Conservation laws

$$T^{\mu\nu}{}_{;\nu} = 0$$

Relativistic MHD equations are reduced to[13]

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}\left(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho\right) + \frac{1}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2\right],$$

$$\begin{aligned}\frac{D\boldsymbol{u}}{Dt} = {} & \frac{1}{3}\mathrm{u}\left(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho\right) - \frac{\boldsymbol{u}}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta J^2\right] \\ & -\frac{1}{4}\boldsymbol{\nabla} \ln \rho + \frac{3}{4\rho}\boldsymbol{J} \times \boldsymbol{B} + \frac{2}{\rho}\boldsymbol{\nabla} \cdot (\rho \nu \boldsymbol{S}) + \mathcal{F},\end{aligned}$$

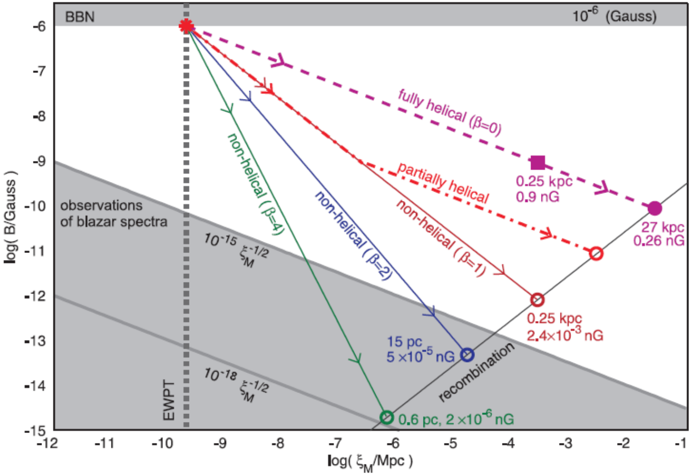
$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \left(\boldsymbol{u} \times \boldsymbol{B} - \eta \boldsymbol{J} + \mathcal{E}\right),$$

for a flat expanding universe with comoving and normalized $p = a^4 p_{\rm phys}, \rho = a^4 \rho_{\rm phys}, B_i = a^2 B_{i,_{\rm phys}}, u_i$, and conformal time t.

---

[13] A. Brandenburg, *et al.*, *Phys. Rev. D* **54**, 1291 (1996)

Evolution of magnetic strength and correlation length[14]



---

[14] A. Brandenburg, T. Kahniashvili, S. Mandal, ARP, A. Tevzadze and T. Vachaspati, *Phys. Rev. D* **96**, 123528 (2017)

1. Introduction and Motivation

2. Primordial magnetic fields and MHD

3. Gravitational waves

GWs equation for an expanding flat Universe

- Assumptions: isotropic and homogeneous Universe
- Friedmann–Lemaître–Robertson–Walker (FLRW) metric $\gamma_{ij} = a^2 \delta_{ij}$
- Tensor-mode perturbations above the FLRW model:

$$g_{ij} = a^2 \left(\delta_{ij} + h_{ij}^{\mathrm{phys}} \right)$$

- GWs equation is[15]

$$\left(\partial_t^2 - \cancel{\frac{a''}{a}} - c^2 \nabla^2 \right) h_{ij} = \frac{16 \pi G}{a c^2} T_{ij}^{\mathrm{TT}}$$

  - h_{ij} are rescaled $h_{ij} = a h_{ij}^{\mathrm{phys}}$
  - Comoving spatial coordinates $\nabla = a \nabla^{\mathrm{phys}}$
  - Conformal time $\mathrm{d}t = a \, \mathrm{d}t^{\mathrm{phys}}$
  - Comoving stress-energy tensor components $T_{ij} = a^4 T_{ij}^{\mathrm{phys}}$
  - Radiation-dominated epoch such that $a'' = 0$

---

[15] L. P. Grishchuk, *Sov. Phys. JETP* **40**, 409 (1974).

Normalized GW equation[16]

$$\left(\partial_t^2 - \nabla^2\right) h_{ij} = 6\, T_{ij}^{\mathrm{TT}}/t$$

Properties

- All variables are normalized and non-dimensional
- Conformal time is normalized with t_*
- Comoving coordinates are normalized with c/H_*
- Stress-energy tensor is normalized with $\mathcal{E}_{\mathrm{rad}}^* = 3H_*^2c^2/(8\pi G)$
- Scale factor is $a_* = 1$, such that $a = t$

---

[16] ARP *et al.*, *Geophys. Astrophys. Fluid Dyn.* **114**, 130 (2020). arXiv:1807.05479.

Numerical results for decaying MHD turbulence[17]

Initial conditions

- Initial stochastic magnetic field with fractional helicity $\mathcal{P}_{\mathrm{M}} = 2\sigma(1+\sigma^2)$
- Batchelor spectrum, i.e., $E_{\mathrm{M}} \propto k^4$ for small k
- Kolmogorov spectrum in the inertial range, i.e., $E_{\mathrm{M}} \propto k^{-5/3}$

$$kB_i = \left(P_{ij} - i\sigma\varepsilon_{ijl}\ \hat{k}_l\right) g_j \sqrt{2E_{\mathrm{M}}(k)},$$

$$E_{\mathrm{M}}(k) = \tfrac{1}{2} B_0^2 k_*^{-1/2} \frac{(k/k_*)^4}{\left(1+(k/k_*)^{34/3}\right)^{1/2}}$$

---

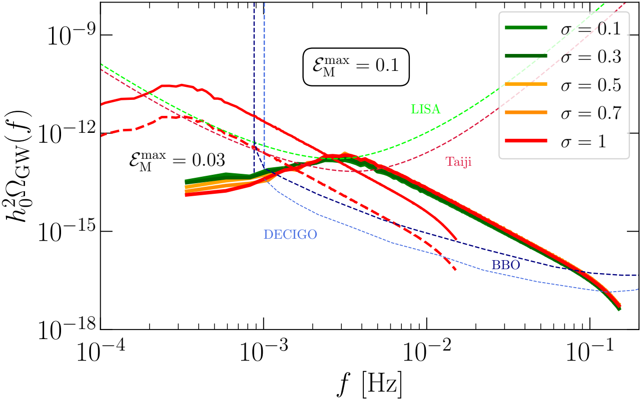
[17] A. Brandenburg *et al.*, *Phys. Rev. D* **96**, 123528 (2017)
ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020)
ARP *et al.*, arXiv:2107.05356

Numerical results for decaying MHD turbulence

Initial conditions

- Magnetic energy density at t_* is a fraction of the radiation energy density, $\mathcal{E}_{\rm M}/\mathcal{E}^*_{\rm rad} = \frac{1}{2}B_0^2 \leq 0.1$ (BBN limit).
- Spectral peak $k_* = N_* \times 2\pi$, normalized by H_*/c is given by the characteristic scale of the sourcing turbulence (as a fraction of the Hubble radius).

Detectability of the SGWB from the EWPT with LISA (for decaying MHD turbulence with initial magnetic field)[18]



---

[18] ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020)

ARP *et al.*, arXiv:2107.05356

Numerical results for decaying MHD turbulence[19]

Driven magnetic field

- Initial magnetic and velocity are zero
- Magnetic field is built-up for a short duration ($\sim 0.1 H_*^{-1}$) via the induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \left(\boldsymbol{u} \times \boldsymbol{B} - \eta \boldsymbol{J} + \mathcal{F} \right).$$

- The forcing term is quasi-monochromatic with fractional magnetic helicity

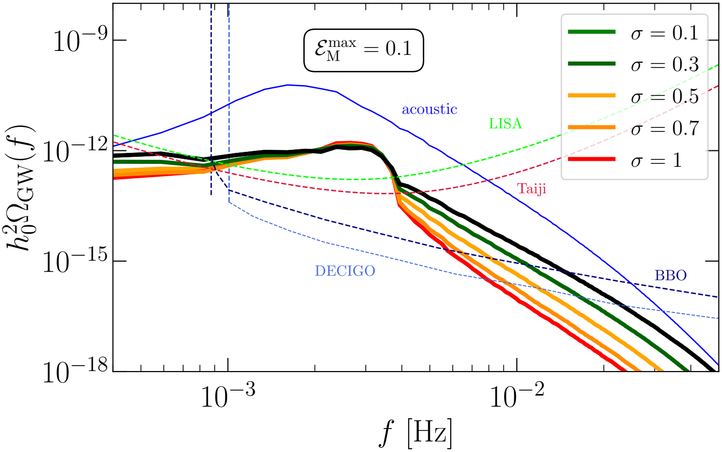
$$\mathcal{F} \; = \mathrm{Re}(\mathcal{A} \boldsymbol{f}) \exp\left[i \boldsymbol{k} \cdot \boldsymbol{x} + i\phi \right], \quad k_* - \tfrac{1}{2}\delta k \le |\boldsymbol{k}| \; \le k_* + \tfrac{1}{2}\delta k$$

$$f_i = \left(\delta_{ij} - i \sigma \varepsilon_{ijl} \hat{k}_l \right) f_j^{(0)} / \sqrt{1 + \sigma^2}$$

---

[19] ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020)
ARP *et al.*, arXiv:2107.05356

Detectability of the SGWB from the EWPT with LISA (for decaying MHD turbulence with an initially forced magnetic field)[20]



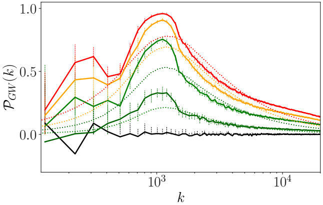
---

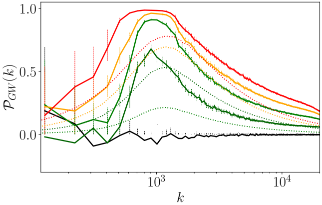
[20] ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020)
ARP *et al.*, arXiv:2107.05356

Polarization degree from stationary turbulence (long-time forcing)

- Helical magnetic fields induce circularly polarized GWs[21]

- Kinetic turbulence
- Magnetic turbulence





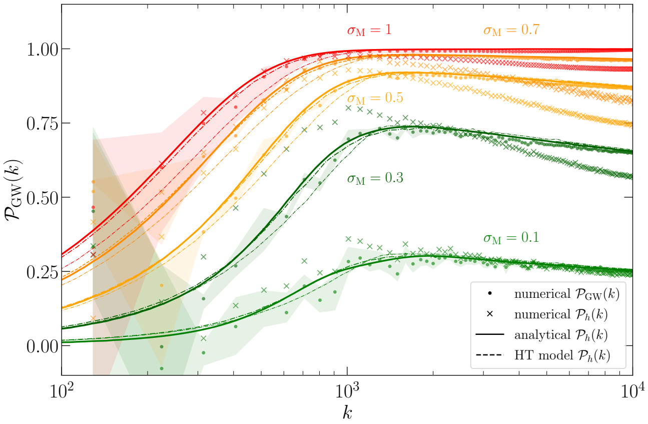
- Degree of circular polarization

$$\mathcal{P}_{\rm GW}(k) = \frac{\Xi_{\rm GW}(k)}{\Omega_{\rm GW}(k)} = \frac{\left\langle \dot{\tilde{h}}_\times \dot{\tilde{h}}_+^* - \dot{\tilde{h}}_+ \dot{\tilde{h}}_\times^* \right\rangle}{\left\langle \dot{\tilde{h}}_+ \dot{\tilde{h}}_+^* + \dot{\tilde{h}}_\times \dot{\tilde{h}}_\times^* \right\rangle}$$

---

[21] L. Kisslinger and T. Kahniashvili, Phys. Rev. D **92**, (2015)
T. Kahniashvili, G. Gogoberidze and B. Ratra, Phys. Rev. Lett. **95**, 151301 (2005)
T. Kahniashvili, A. Brandenburg, A. Kosowsky, S. Mandal, ARP, *Phys. Rev. Res.* **3**, 013193 (2021)

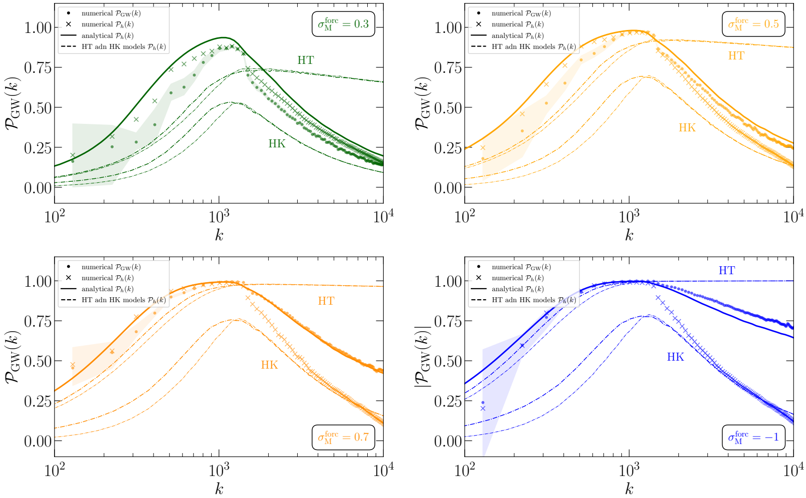
Polarization degree from decaying turbulence (initially given field)[22]



---

[22] ARP *et al.*, arXiv:2107.05356

Polarization degree from decaying turbulence (initially driven field)[23]

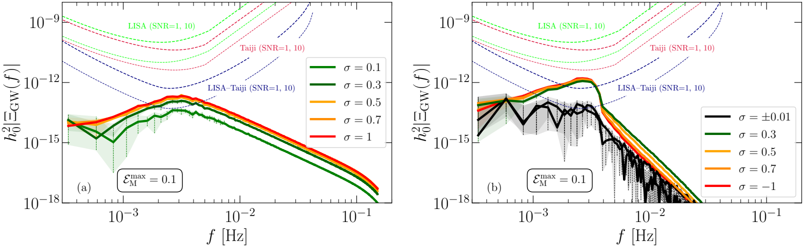


---

[23] ARP *et al.*, arXiv:2107.05356

Detectability of the polarized SGWB from the EWPT with LISA and Taiji[26]

- LISA's dipole response function can provide us with a polarized gravitational wave background due to our proper motion[24]
- Cross-correlation of LISA and an additional space-based GW detector can improve the detectability of a polarized GW background[25]



---

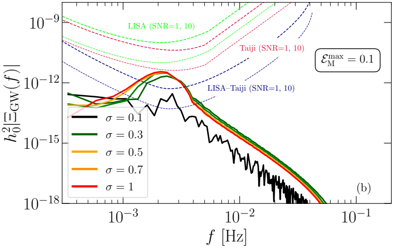
[24] V. Domcke, et al., JCAP **05**, 028 (2020)

[25] G. Orlando, M. Pieroni and A. Ricciardone, JCAP **03**, 069 (2021)

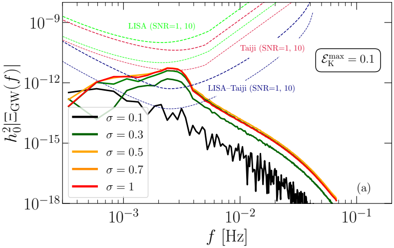
[26] ARP *et al.*, arXiv:2107.05356

Detectability of the polarized SGWB from the EWPT with LISA and Taiji[27]

- Magnetic turbulence



- Kinetic turbulence



---

[27] T. Kahniashvili, A. Brandenburg, A. Kosowsky, S. Mandal, ARP, *Phys. Rev. Res.* **3**, 013193 (2021)
ARP *et al.* arXiv:2107.05356

Conclusions

- Detection of GW spectrum can provide *clean* information from the epoch of generation and the turbulence characteristics
- Polarization degree can provide information on magnetic helicity of the seed field, about its nature (kinetically or magnetically dominant), and formation process
- Production of helical magnetic fields can be related to Chern-Simons violations and to production of particles, shedding light on the baryon-asymmetry problem
- Depending on the mechanism of turbulence generation and/or the initial energy density and characteristic scale, the GW signal from primordial magnetic fields produced/present at the EWPT is detectable by LISA
- The circular polarization of GWs produced by helical magnetic fields might not be detectably by LISA but it will be detectable by correlating LISA and one additional space-based GW detectors (e.g., Taiji)
- Probe of the origin of magnetic fields in the largest scales of our Universe, which is still an open question in cosmology

The End
Thank You!

roperpol@apc.in2p3.fr

შოთა რუსთაველის ეროვნული სამეცნიერო ფონდი
SHOTA RUSTAVELI NATIONAL SCIENCE FOUNDATION

APC, CNRS, FRANCE

LASP
Laboratory for Atmospheric and Space Physics
University of Colorado **Boulder**

NORDITA
The Nordic Institute for Theoretical Physics