Constraining higher-dimensional cosmology with gravitational wave standard sirens

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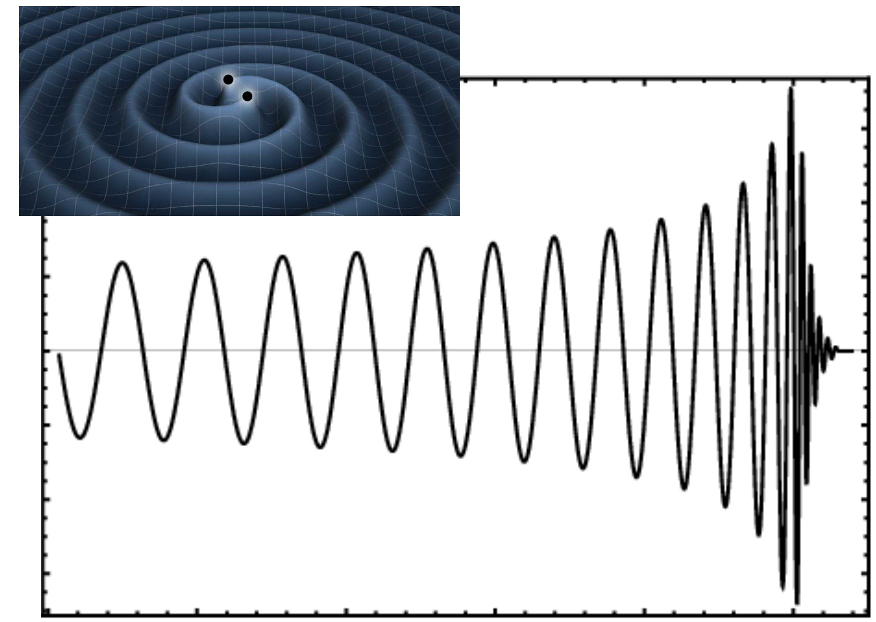
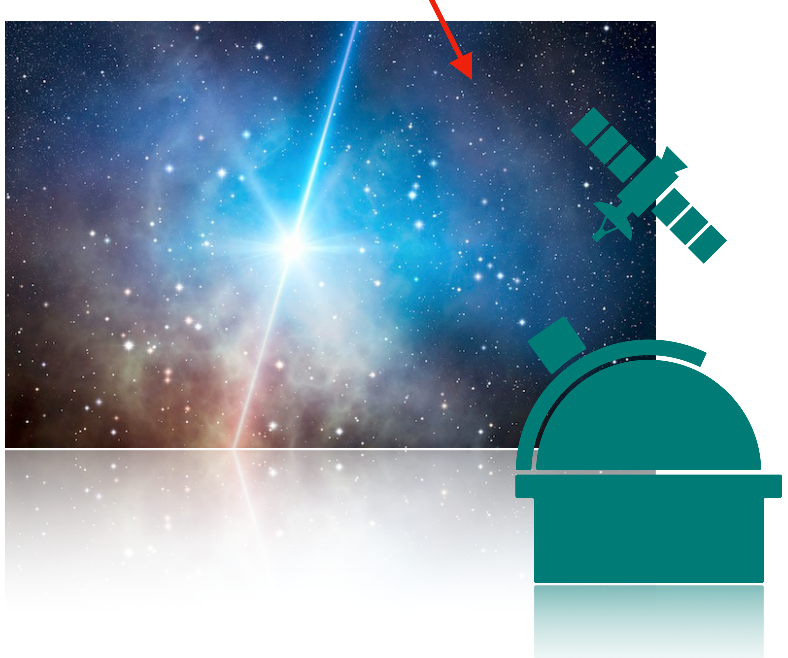
What are standard sirens?

<u>Standard sirens</u> are GW events that can be used as absolute cosmological distance indicators

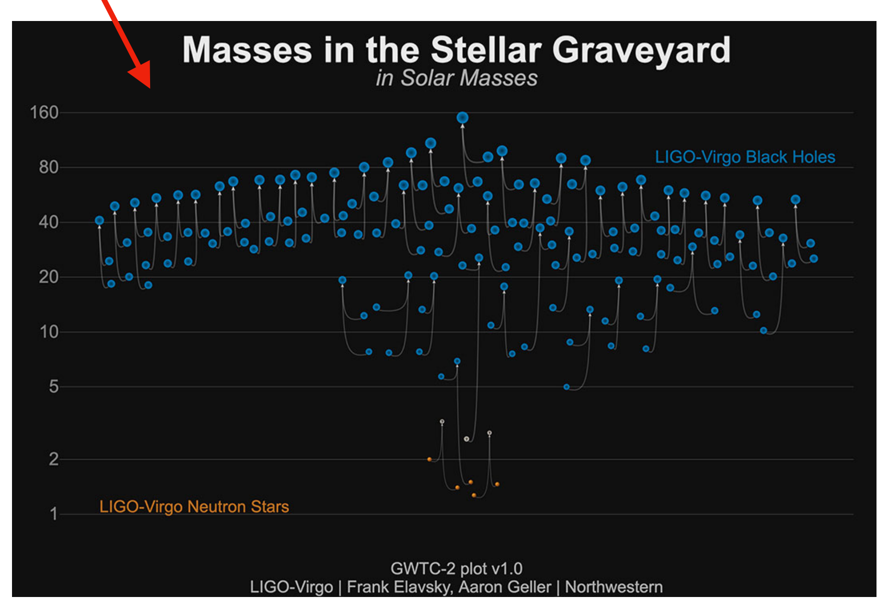
- **Luminosity distance** estimated from GW signal
- **Redshift** obtained from EM observations (or statistical information on sources' properties)



$$h_\times = \frac{4}{d_L}\left(\frac{GM_c}{c^2}\right)^{\frac{5}{3}}\left(\frac{\pi f}{c}\right)^{\frac{2}{3}} \cos\iota \sin\Phi(t)$$



[Schutz, *Nature* (1986)]



[Mastrogiovanni+, arXiv:2103.14663]

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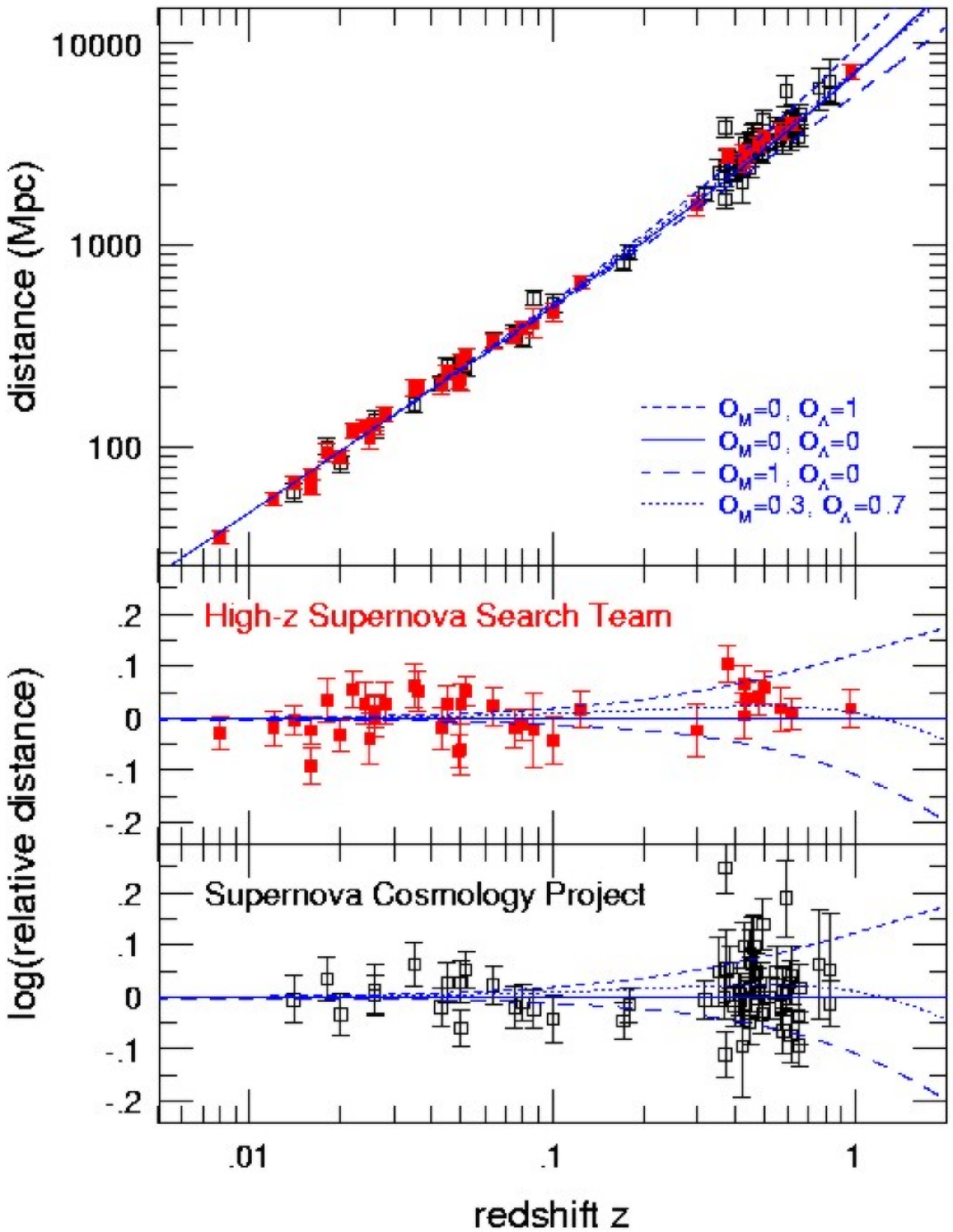
With these two measurements one can then fit the **distance-redshift relation** and obtain constraints on the **cosmological parameters** (similarly to *standard candles* ⇒ type-Ia SNs)

$$d_L(z) = \frac{c}{H_0}\frac{1+z}{\sqrt{\Omega_k}}\sinh\left[\sqrt{\Omega_k}\int_0^z \frac{H_0}{H(z')}dz'\right]$$



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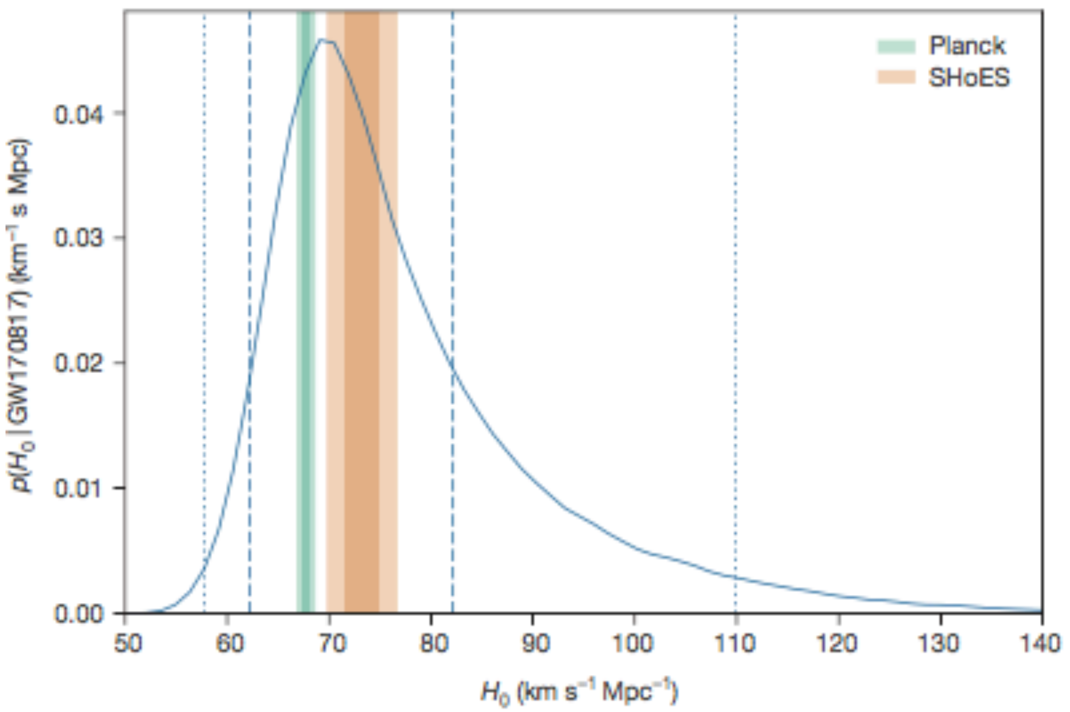

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GW170817

$$H_0 = 69^{+17}_{-8}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

[LVC+, *Nature* (2017)]
[LVC, *PRX* (2019)]

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Standard sirens beyond general relativity

By assuming GR and an homogeneous and isotropic universe, GW propagates according to the equation:

$$h''_{ij} + 2Hh'_{ij} + c^2k^2h_{ij} = 0$$

Which implies: $h \propto \frac{1}{d_L}$ and $v_{gw} = c$

- h_{ij} = GW amplitude
- H = Hubble rate
- c = speed of light
- k = wave number
- $'$ = derivative w.r.t. (conformal) time

- What if GR is no longer valid at cosmological distances?
- What if we consider cosmic inhomogeneities?

Standard sirens beyond general relativity

If homogeneity holds (very large scale) but GR is modified, the most general GW propagation equation reads:

$$h''_{ij} + 2H(1+\nu)h'_{ij} + (c_T{}^2k^2 + a^2\mu^2)h_{ij} = a^2\Gamma\gamma_{ij}$$

$h \neq \frac{1}{d_L}$ $\quad v_{gw} \neq c$ $\quad m_{gw} \neq 0$ $\quad h_{\times,+}$ mixing, birefringence, oscillations, …

- h_{ij} = GW amplitude
- H = Hubble rate
- c = speed of light
- k = wave number
- $'$ = derivative w.r.t. (conformal) time
- ν, c_T, μ, Γ are all spacetime functions
- γ_{ij} = source of anisotropic stresses

Different modifications of the GW propagation equation correspond to different physical effects

[Nishizawa, *PRD* (2018)]

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Can be tested with GW+EM multi-messenger events (or population analyses)

Can be tested with GW-only matched-filtered searches

Can be tested with GW polarisation measurements

[Nishizawa, *PRD* (2018)]

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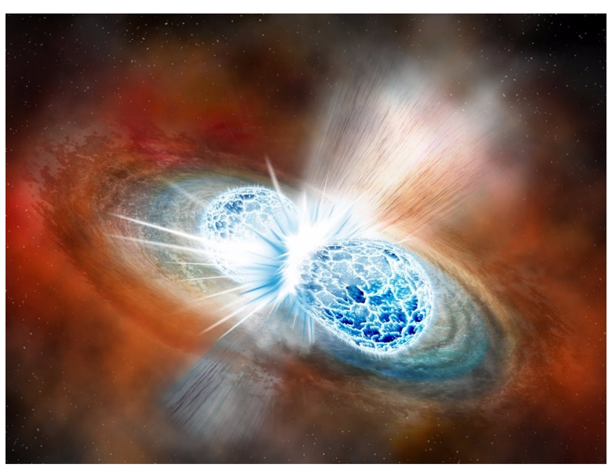
An additional amplitude damping of GWs translates into a different (luminosity) distance inferred by EM and GW



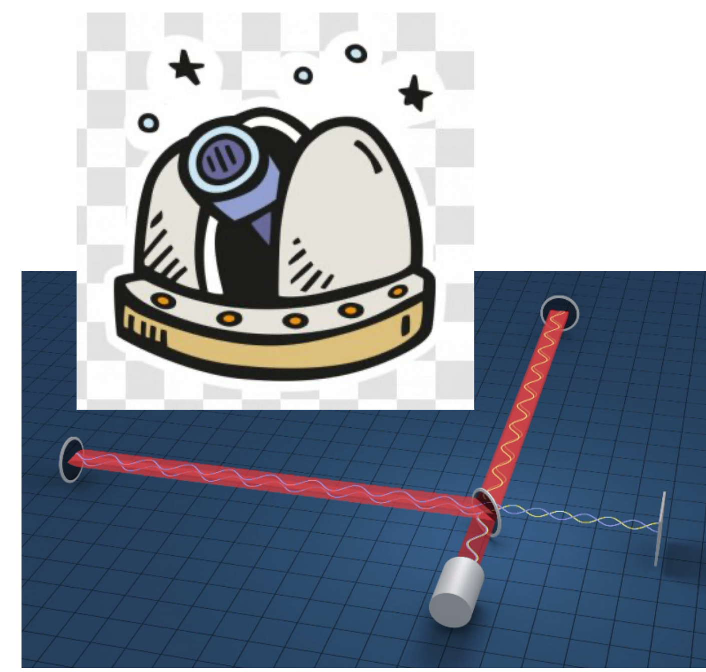
Source emitting both GW and EM radiation

EM radiation (photons)

Gravitational waves

d_L^{EM}

d_L^{GW}



GW and EM detectors

Standard sirens beyond general relativity

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$$h \neq \frac{1}{d_L}$$

$$h \propto d_L^{-(D-2)/2}$$

[Deffayet&Menou, *ApJ* (2007)]

Gravity theory	ν	$c_T^2 - 1$	μ	Γ
General relativity	0	0	0	0
Extra-dimensional theory	$(D-4)(1+\frac{1+z}{\mathcal{H}d_L})$	0	0	0
Horndeski theory	α_M	α_T	0	0
f(R) gravity	$F'/\mathcal{H}F$	0	0	0
Einstein-aether theory	0	$c_\sigma/(1+c_\sigma)$	0	0
Modified dispersion relation	0	$(n_{\rm mdr}-1)\mathbb{A}E^{n_{\rm mdr}-2}$	when $n_{\rm mdr}=0$	0
Bimetric massive gravity theory	0	0	$m^2 f_1$	$m^2 f_1$

[Nishizawa, *PRD* (2018)]

Standard sirens in a higher-dimensional universe

If GWs propagate in a *D*-dimensional FRW universe, flux/energy conservation dictates that

$$h \propto (d_L^{\mathrm{EM}})^{-(D-2)/2}$$

where d_L^{EM} is the standard 4-dimensional luminosity distance as measured by EM observations*.

If we define d_L^{GW} as the distance inferred by GW observations assuming a standard GR templates

$$h \propto (d_L^{\mathrm{GW}})^{-1}$$

we find that

$$d_L^{\mathrm{GW}} \propto (d_L^{\mathrm{EM}})^{(D-2)/2}$$

Measured by GW observations

Measured by EM observations

***EM (photons) radiation is strongly confined to 4 dimensions by observations**

Standard sirens in a higher-dimensional universe

At small distances however we must recover 4 dimensions, in agreement with observations (e.g. Solar System tests).

One can phenomenologically introduce a transition scale R_c and steepness factor n such that [Deffayet&Menou, *ApJ* (2007)]

$$d_L^{\rm GW} = d_L^{\rm EM} \left[1 + \left(\frac{d_L^{\rm EM}}{R_c} \right)^n \right]^{\frac{D-4}{2n}}$$

- For $d_L^{EM} \ll R_c$ and for $D = 4$ one recovers 4-dimensions: $d_L^{\rm GW} = d_L^{\rm EM}$
- For $d_L^{EM} \gg R_c$ one recovers *D*-dimensions: $d_L^{\rm GW} \propto (d_L^{\rm EM})^{(D-2)/2}$

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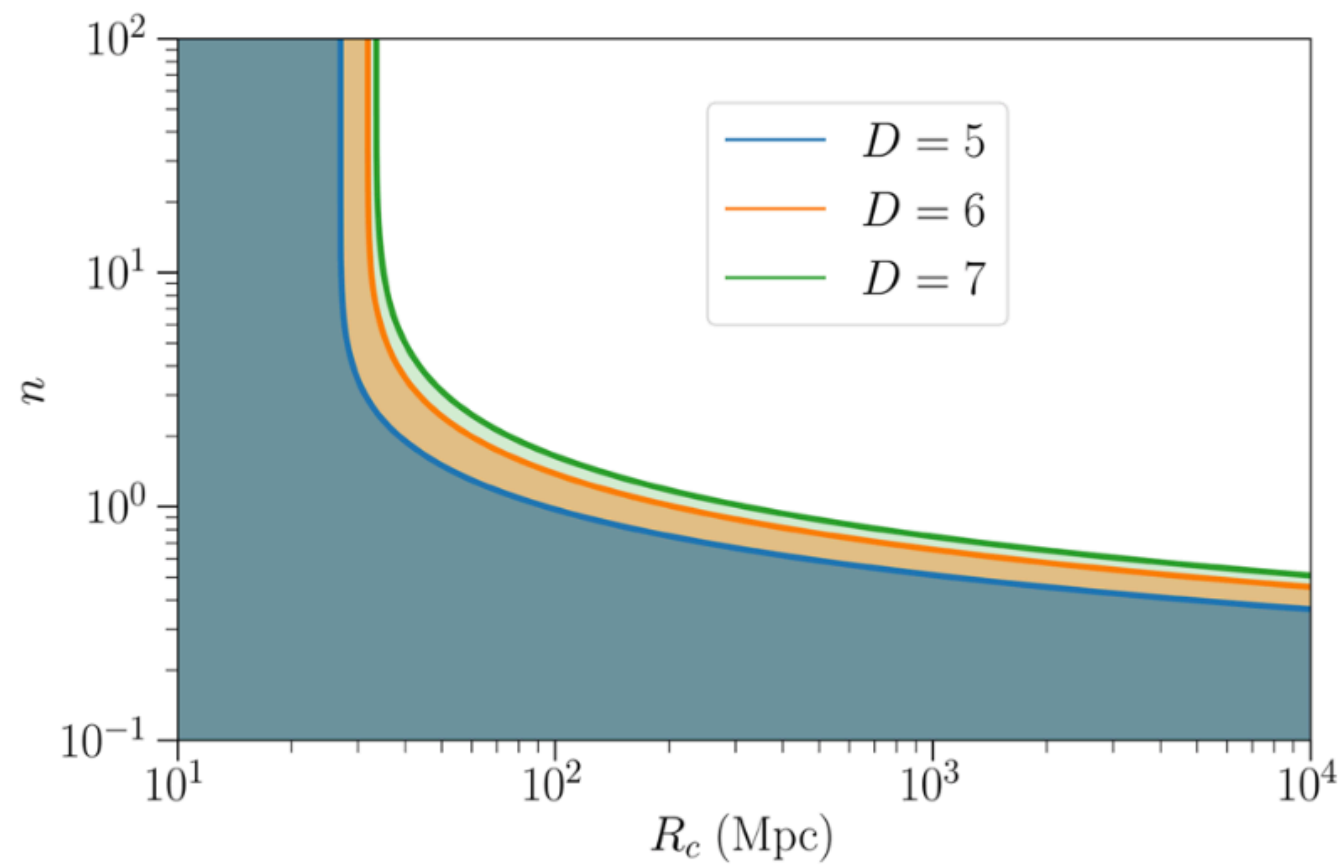

This equations has recently been used in different analyses to constraints GW propagation in higher-dimensions

[Pardo+, *JCAP* (2018)] [LIGO/Virgo, *PRL* (2019)]
[Mastrogiovanni+, *JCAP* (2021)] [Corman+, *JCAP* (2021)]

Standard sirens in a higher-dimensional universe

Recently we re-derived from first principles the d_L^{GW}-d_L^{EM} relation, and found missing redshift factor

$$d_L^{\mathrm{GW}} = d_L^{\mathrm{EM}} \left[1 + \left(\frac{d_L^{\mathrm{EM}}}{(1+z)R_c} \right)^n \right]^{\frac{D-4}{2n}}$$

Ignored in all previous analyses

The new factor redshifts the transition scale R_c, making it more difficult to constrain at high-redshift

[Corman, Ghosh, Escamilla-Rivera, Hendry, Marsat, Tamanini, *ArXiv* (2021)]

Standard sirens in a higher-dimensional universe

We then re-derived the constraints from current results (GW170817) and for future observations (LISA)

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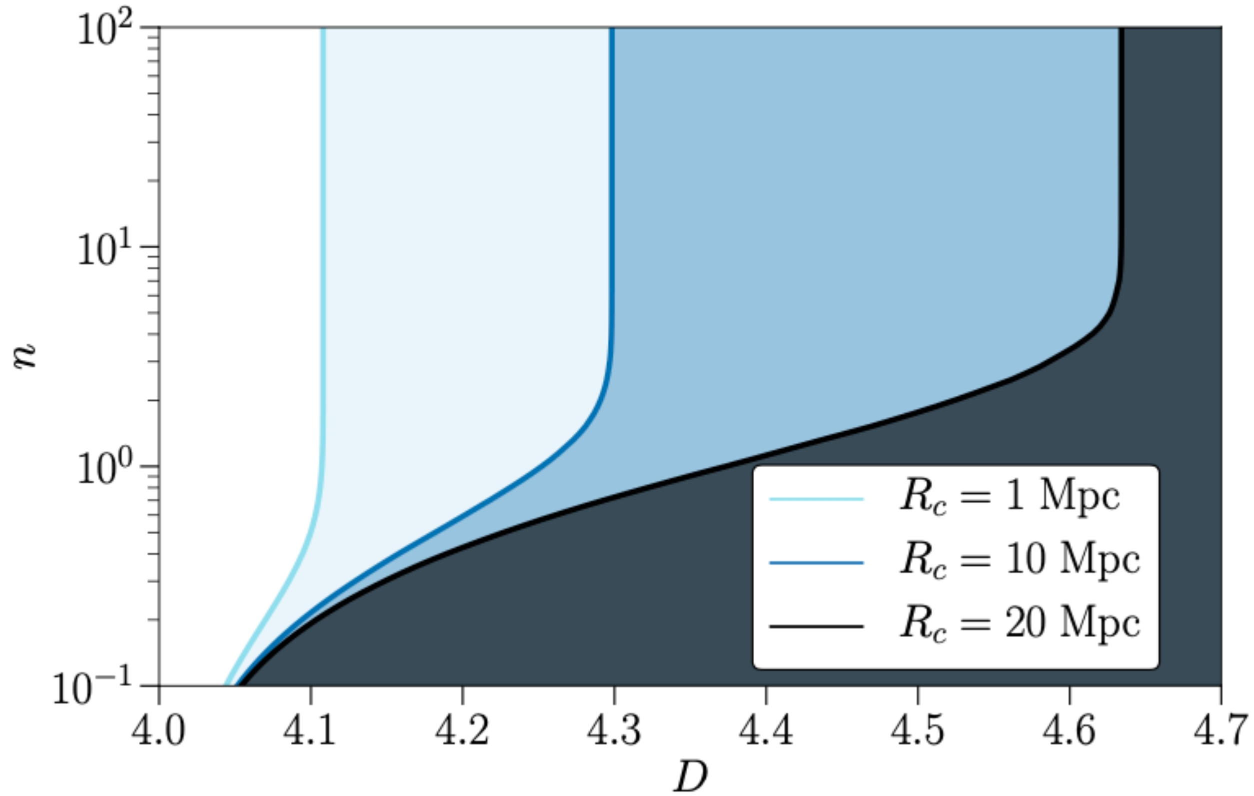

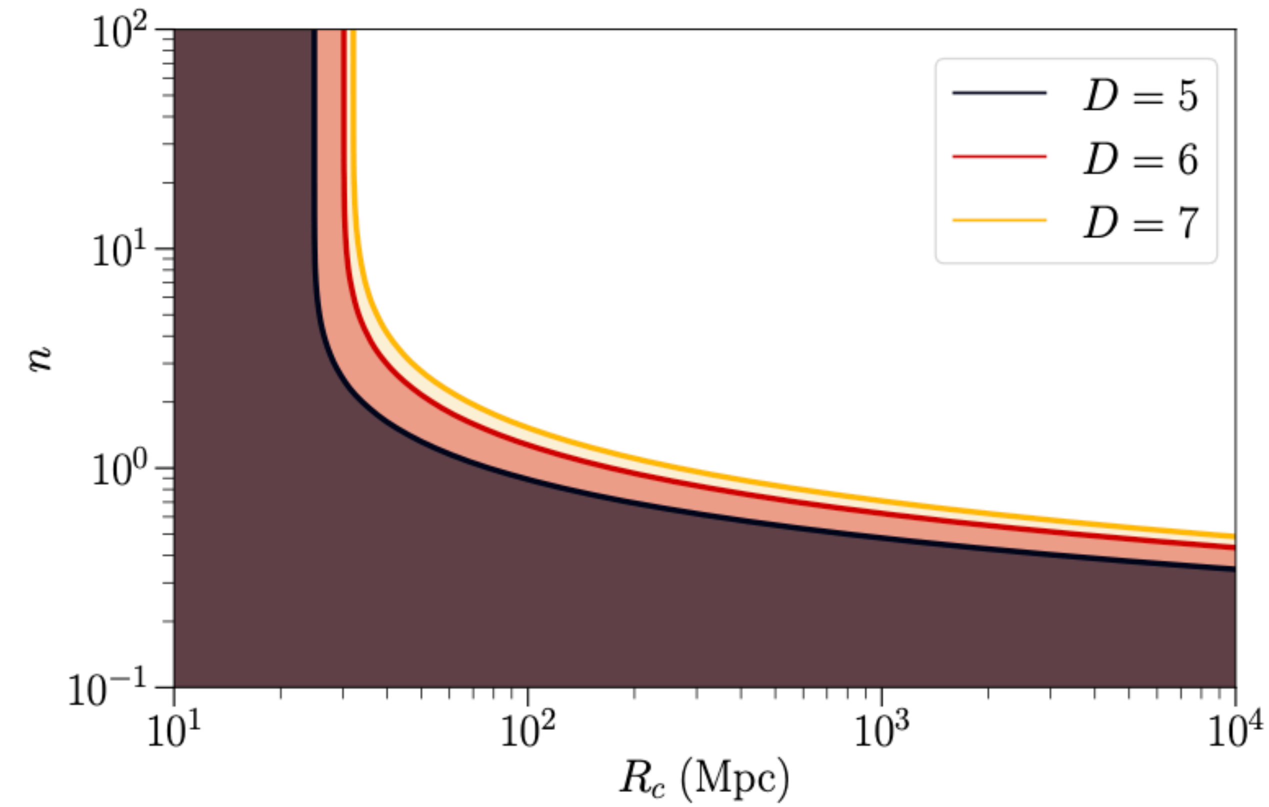

Constraints from GW170817 are basically unchanged since the event is at $z \sim 0.01$ and thus the redshift correction is negligible

[Corman, Ghosh, Escamilla-Rivera, Hendry, Marsat, Tamanini, *ArXiv* (2021)]

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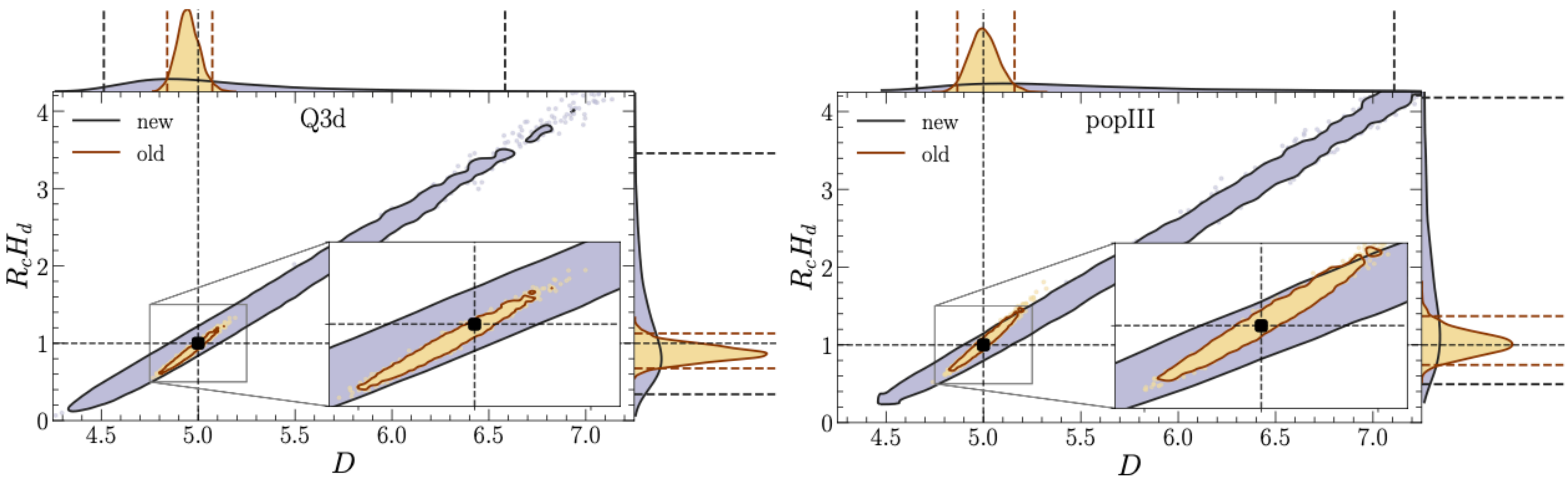

Forecasts for LISA MBHBs are instead drastically affected since these standard sirens will have large redshift $(1 \lesssim z \lesssim 8)$

[Corman, Ghosh, Escamilla-Rivera, Hendry, Marsat, Tamanini, *ArXiv* (2021)]

Conclusions and perspectives

Standard sirens can be used to:

- Map the cosmic expansion independently of EM observations
- Test deviations from GR otherwise unobservable in EM
- Probe higher spacetime dimensions at cosmological scales

Testing large extra-dimensions is useful to:

- To constrain alternative models of dark energy
  [e.g. Dvali, Gadabadze, Porrati, *PLB* (2000)]
- To understand fundamental properties of spacetime at large scales
- As a toy-model to show how GWs and multi-messenger observations can test the Universe beyond what EM telescopes can offer