Quantum Spectral Curves from Monodromy Bootstrap

Based on work with Dmytro Volin

Motivation

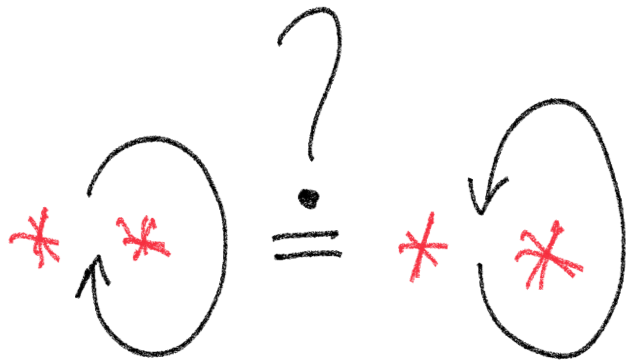
- QSC = <u>analytic Q-system!</u>
- QQ-relations
- Analytic properties (Polynomial = Rational Spin Chain, Branch cuts = AdS/CFT)
- Can we find new QSC's?
- Avoid TBA!
- Use symmetry principles
- Monodromy bootstrap provides a way.

What is Monodomy Bootstrap?

$\hat{S}(Q) \qquad \hat{Q}$

Branch points

Crossing



$$\boxed{\Rightarrow \tilde{Q} = S(Q)}$$

- # The case of (P)SU(2|2)

- Focus exclusively on one or two su(2|2) Q-systems
- Simple, but rich physics!
  - Hubbard Model
  - SU(2|2)c.e Spin Chains
  - AdS3/CFT2
  - ? AdS2/CFT1 ?
- Other symmetry groups? Not today
  - ABJM osp(4|6)?

Overview

Background on SU(2|2) Q-system

Analytical continuation around the branch points

SU(2|2) in detail

Models for SU(2|2) and SU(2|2)^2

AdS3 QSC Conjecture

Conclusions

Algebraic properties of GL(4) Q-systems

The GL(4) Q-system form a flag

$$B_0 \qquad\qquad B_a$$

$$B_{ab} = \frac{1}{B_0} W(B_a, B_b)\,, \qquad B_{abc} = \frac{1}{B_0^+ B_0^-} W(B_a, B_b, B_c),$$

$$B_{1234} = \frac{1}{B_0^{[2]} B_0 B_0^{[-2]}} W(B_1, B_2, B_3, B_4)$$

Implies QQ-relations:

$$W(B_{Aa}, B_{Ab}) = B_{Aab} B_A$$

Flags are preserved under rotations and rescaling of vectors!

$$B_0 \to g_0 B_0\,, \quad B_a \to g_1 {H_a}^b B_b\,.$$

Gauge-transformation!

Supersymmetric GL(2|2) Q-functions can be found from bosonization

$$\mathcal{Q}_{A|I} = \epsilon^{I\bar{I}} B_{A\bar{I}}$$

A GL(2|2) Q-system has 2^4=16 Q-functions $Q_{A|I}$

Define Hodge dual Q-functions $Q^*_{A|I} = Q^{A|I} = (-1)^{|B||I|}\epsilon^{AB}\epsilon^{IJ}Q_{B|J}$

Hasse Diagram

$$\begin{tikzcd}
 & Q_{a|\emptyset} \arrow[dr, dash] & & -Q^{\emptyset|j} \arrow[dr, dash] & \\
Q_{\emptyset|\emptyset} \arrow[ur, dash] \arrow[dr, dash] & & Q_{a|i} = -Q^{b|j} \arrow[ur, dash] \arrow[dr, dash] & & Q^{\emptyset|\emptyset} \\
 & Q_{\emptyset|i} \arrow[ur, dash] & & -Q^{b|\emptyset} \arrow[ur, dash] &
\end{tikzcd},$$

Gauge-transformations preserving QQ-relations:

$$\begin{tikzcd}
 & g_b^+ g_b^- \arrow[dr, dash] & & g_b^+ g_b^- \arrow[dr, dash] & \\
g_b g_f \arrow[ur, dash] \arrow[dr, dash] & & g_b g_f \arrow[ur, dash] \arrow[dr, dash] & & g_b g_f \\
 & g_f^+ g_f^- \arrow[ur, dash] & & g_f^+ g_f^- \arrow[ur, dash] &
\end{tikzcd}, \qquad
\begin{tikzcd}
 & h \arrow[dr, dash] & & h \arrow[dr, dash] & \\
1 \arrow[ur, dash] \arrow[dr, dash] & & 1 \arrow[ur, dash] \arrow[dr, dash] & & 1 \\
 & h^{-1} \arrow[ur, dash] & & h^{-1} \arrow[ur, dash] &
\end{tikzcd}.$$

Scaling

$$Q_{a|0} \to (H_b)_a{}^b Q_{b|0}\,, \quad Q_{0|i} \to (H_f)_i{}^j Q_{0|j}\,.$$

Rotations

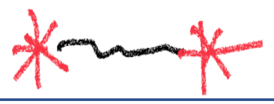
Analytical structure from AdS/CFT

QQ-relations are well defined in the upper halfplane

$\mathcal{Q}_{a|i}$ is an upper half plane analytic Q-system.

Assume the following cut structure inspired by AdS/CFT integrability

$\mathbf{P}_a = \mathcal{Q}_{a|0}$

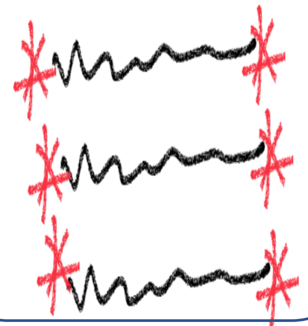


$\mathbf{Q}_i = \mathcal{Q}_{0|i}$

Other Q-functions follows from QQ.

$$\mathcal{Q}^+_{a|i} - \mathcal{Q}^-_{a|i} = \mathbf{P}_a \mathbf{Q}_i \implies \mathcal{Q}_{a|i} = -\sum_{n=0}^{\infty} (\mathbf{P}_a \mathbf{Q}_i)^{[2n+1]}$$

$\mathcal{Q}_{a|i}$



Why upper half plane and not lower? No reason!

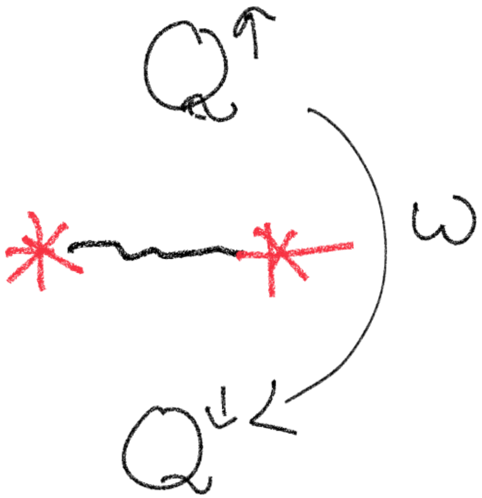
Introduce LHPA Q-system $\mathcal{Q}^{\downarrow}_{A|I}$

Can be killed by definition of LHP system

$\mathcal{Q}^{\downarrow}_{A|I}, \mathcal{Q}_{A|I}$ related by rotation, gauge + potentially Hodge duality

$$\boxed{\mathcal{Q}^{\downarrow} = \omega_{\hat{h}} \cdot \mathcal{Q}} \qquad \omega^{[2]} = \omega$$

Defined in physical kinematics!

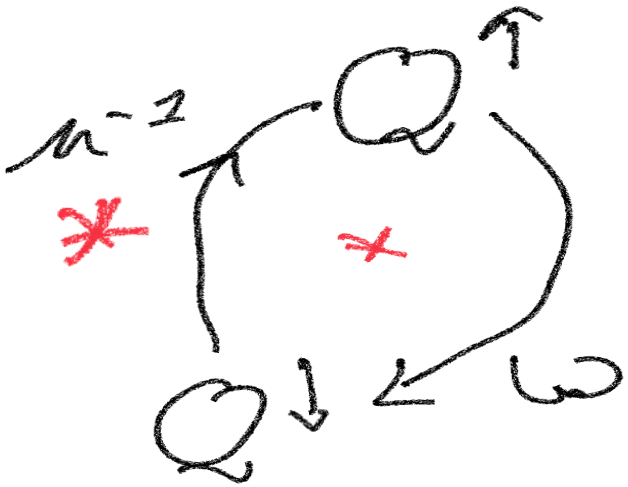


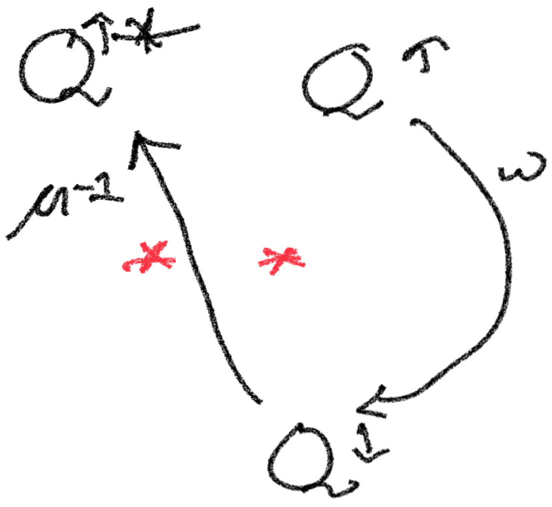
Consider now avoiding long cuts, mirror kinematics

$$\mathcal{Q}^{\downarrow} = \mu_{\check{h}} \cdot \mathcal{Q} \qquad\qquad \mathcal{Q}^{\downarrow} = \mu_{\check{h}} \cdot \mathcal{Q}^{*}$$

following analytical continuation

$$\tilde{\mathcal{Q}} = \mu_{\check{h}}^{-1} \cdot \omega_{\hat{h}} \cdot \mathcal{Q} \qquad\qquad \tilde{\mathcal{Q}} = (\mu_{\check{h}})^{-1} \cdot \omega_{\hat{h}} \cdot \mathcal{Q}^{*}$$





$\tilde{\mathcal{Q}} = (\mu_{\check{h}})^{-1} \cdot \omega_{\hat{h}} \cdot \mathcal{Q}^*$ explicit

Clock-wise	Counterclock-wise
$\tilde{\mathbf{P}}_a = r\,\mu_{ab}\mathbf{P}^b\,,$	$\overset{*}{\tilde{\mathbf{P}}}_a = -\overset{*}{\tilde{r}}\,\mathbf{P}^b \overset{*}{\tilde{\mu}}_{ba} \times (-1)^{\mathrm{p}_\mu + \mathrm{p}_\omega}\,,$
$\tilde{\mathbf{P}}^a = -\frac{1}{r}\mathbf{P}_b\,\mu^{ba} \times (-1)^{\mathrm{p}_\mu + \mathrm{p}_\omega}\,,$	$\overset{*}{\tilde{\mathbf{P}}}{}^a = \frac{1}{\overset{*}{\tilde{r}}}\,\overset{*}{\tilde{\mu}}{}^{ab}\mathbf{P}_b$
$\tilde{\mathbf{Q}}_i = \frac{1}{r}\,\mathbf{Q}^j\tilde{\omega}_{ji}$	$\overset{*}{\tilde{\mathbf{Q}}}_i = -\frac{1}{\overset{*}{\tilde{r}}}\,\omega_{ij}\mathbf{Q}^j \times (-1)^{\mathrm{p}_\mu + \mathrm{p}_\omega}$
$\tilde{\mathbf{Q}}^i = -r\,\tilde{\omega}^{ij}\mathbf{Q}_j \times (-1)^{\mathrm{p}_\mu + \mathrm{p}_\omega}\,,$	$\overset{*}{\tilde{\mathbf{Q}}}{}^i = \overset{*}{\tilde{r}}\,\mathbf{Q}_j\,\omega^{ji}\,.$

$$r = \frac{\check{h}}{\tilde{\hat{h}}}$$

$\mathcal{Q}_i^{\downarrow a} = \mathcal{Q}_{b|i}\mu^{ba}$ does not have a branch cut on the real axis

$$\boxed{\mu_{ab} - \overset{*}{\tilde{\mu}}_{ab} = -\frac{1}{F\,r}\tilde{\mathbf{P}}_a\mathbf{P}_b + \frac{1}{\tilde{F}\overset{*}{\tilde{r}}}\mathbf{P}_a\overset{*}{\tilde{\mathbf{P}}}_b}$$

Familiar structure from AdS5/CFT4!

$$F = \sqrt{\frac{\mathcal{Q}^-_{12|12}}{\mathcal{Q}^+_{12|12}}}$$

Can we pick analytical properties arbitrary? No.

It is general hard to get square root cuts.

$$\tilde{\tilde{\mu}} = \mu \implies (\mathbf{P}^a)^{[2n]} \mu_{ab} (\mathbf{P}^b)^{[2n]} = 0 \implies \mu_{ab} \propto \epsilon_{ab}$$

Are square root possible for SU(2|2)? Yes!

For PSU(2|2)? Yes, but cut disappears.

Even worse for more general models!

[Cavaglia, Cornagliotto,Mattelliano,Tateo 15]

QSC for a single SU(2|2) with square root cuts

$$\tilde{\mathbf{P}}_a = \frac{\mu}{F}\epsilon_{ab}\mathbf{P}^b\,, \quad \tilde{\mathbf{P}}^a = \frac{F}{\mu}\epsilon^{ab}\mathbf{P}_b\,, \quad \mu - \tilde{\mu} = \epsilon^{ab}\mathbf{P}_a\tilde{\mathbf{P}}_b\,, \quad \mathbf{P}^a\mathbf{P}_a = \frac{1}{F} - F\,,$$

From self-consistency $\boxed{\frac{\mu}{\tilde{\mu}} = \frac{F}{\tilde{F}} = F^2\,.}$ ← Think about F(x) as a source term

Can solve $\frac{\mu}{\tilde{\mu}} = \frac{f}{f^{[2]}}, \quad f = \prod_{n=0}^{\infty} F^{[2n]}, \quad \bar{f} = \prod_{n=0}^{\infty} F^{[-2n]} \quad \mu = F\frac{f^{[2]}}{f^{[-2]}}\,.$

We can solve the system for different F's

SU(2|2) spin chain

$$F = \sqrt{\frac{u-u_k-\frac{i}{2}}{u-u_k+\frac{i}{2}}}\,\frac{x-\frac{1}{y_k^-}}{x-\frac{1}{y_k^+}}$$

$$\frac{\mathrm{y}_1}{\mathrm{y}_2}\prod_{k=1}^{M_{1|1}}\frac{u_{1|1}^{(i)}-u_{1|1}^{(k)}+\mathrm{i}}{u_{1|1}^{(i)}-u_{1|1}^{(k)}-\mathrm{i}}\prod_{k=1}^{M_{1|\emptyset}}\frac{u_{1|1}^{(i)}-u_{1|\emptyset}^{(k)}-\frac{\mathrm{i}}{2}}{u_{1|1}^{(i)}-u_{1|\emptyset}^{(k)}+\frac{\mathrm{i}}{2}}\prod_{k=1}^{M_{1|12}}\frac{u_{1|1}^{(i)}-u_{1|12}^{(k)}-\frac{\mathrm{i}}{2}}{u_{1|1}^{(i)}-u_{1|12}^{(k)}+\frac{\mathrm{i}}{2}}=-1,$$

$$\frac{\mathrm{y}_2}{\mathrm{x}_2}\prod_{k=1}^{M_\theta}\frac{x_{1|12}^{(i)}-y_j^+}{x_{1|12}^{(i)}-y^-}\prod_{k=1}^{M_{1|1}}\frac{u_{1|12}^{(i)}-u_{1|1}^{(k)}+\frac{\mathrm{i}}{2}}{u_{1|12}^{(i)}-u_{1|1}^{(k)}-\frac{\mathrm{i}}{2}}=1,$$

Lieb-Wu Equations

$$F = x$$

$$\prod_{k=1}^{N}\frac{\lambda^{(i)}-\lambda^{(k)}+\frac{\mathrm{i}}{2g}}{\lambda^{(i)}-\lambda^{(k)}-\frac{\mathrm{i}}{2g}}\prod_{k=1}^{M}\frac{\lambda^{(i)}-\sin k_i-\frac{\mathrm{i}}{4g}}{\lambda^{(i)}-\sin k_i+\frac{\mathrm{i}}{4g}}=-1, \quad i=1,\dots,N,$$

$$\prod_{k=1}^{M}\frac{\sin k_i-\lambda^{(k)}+\frac{\mathrm{i}}{4h}}{\sin k_i-\lambda^{(k)}-\frac{\mathrm{i}}{4h}}=e^{\mathrm{i}M_\theta k_i}. \quad i=1,\dots,M$$

$x_{1|\emptyset} = i e^{-ik}$, $x_{1|12} = \frac{e^{ik}}{i}$, $\lambda \propto u_{1|1}$

$y^+ \to \infty$, $y^- \to 0$, $y_2/x_2 \to 0$

[Beisert 06]

Generalization

Consider two Q-systems: $(\mathcal{Q}, \bar{\mathcal{Q}})$

We have 2 moves: Swap systems or apply Hodge

Model A	$\tilde{\mathcal{Q}} = \mathcal{Q}$
Model B	$\tilde{\mathcal{Q}} = \mathcal{Q}^*$
Model C	$\tilde{\mathcal{Q}} = \bar{\mathcal{Q}}^*\,, \quad \tilde{\bar{\mathcal{Q}}} = \bar{\mathcal{Q}}^*$
Model D	$\tilde{\mathcal{Q}} = \bar{\mathcal{Q}}^*, \quad \tilde{\bar{\mathcal{Q}}} = \mathcal{Q}\,.$

$\mathbf{P}\mu$ – systems

A

$$\mathbf{P} \otimes \mathbf{P}^* = \mu \, \mathbf{P} \otimes \mathbf{P}^* \, \mu^{-1} \,,$$

$$\mu = \left(1 + \frac{1}{F} \mathbf{P} \otimes \mathbf{P}^* \right) \mu \, (1 - F \, \mathbf{P} \otimes \mathbf{P}^*) \,,$$

$$\mathrm{Tr} \, \mathbf{P} \otimes \mathbf{P}^* = \frac{1}{F} - F \,,$$

$$F \, F = \det \mu = 1 \,.$$

B

$$\mathbf{P} \otimes \mathbf{P}^* = - \, \mu \, \mathbf{P}^* \otimes \mathbf{P} \, \mu^{-1} \,,$$

$$\mu = \left(1 + \frac{1}{F} \mathbf{P} \otimes \mathbf{P}^* \right) \mu \left(1 + \frac{1}{F} \mathbf{P}^* \otimes \mathbf{P} \right) \,,$$

$$\mathrm{Tr} \, \mathbf{P} \otimes \mathbf{P}^* = \frac{1}{F} - F \,,$$

$$F \, F = 1 \,.$$

C

$$\widetilde{\mathbf{P}} \otimes \widetilde{\mathbf{P}}^* = -\bar{\mu} \, \bar{\mathbf{P}}^* \otimes \bar{\mathbf{P}} \, \bar{\mu} \,, \quad \widetilde{\bar{\mathbf{P}}} \otimes \widetilde{\bar{\mathbf{P}}}^* = - \, \mu \, \mathbf{P}^* \otimes \mathbf{P} \, \mu^{-1}$$

$$\tilde{\mu} = \left(1 + \frac{1}{\bar{F}} \bar{\mathbf{P}} \otimes \bar{\mathbf{P}}^* \right) \mu \left(1 + \frac{1}{F} \mathbf{P}^* \otimes \mathbf{P} \right) \,, \quad \tilde{\bar{\mu}} = \left(1 + \frac{1}{F} \mathbf{P} \otimes \mathbf{P}^* \right) \bar{\mu} \left(1 + \frac{1}{\bar{F}} \bar{\mathbf{P}}^* \otimes \bar{\mathbf{P}} \right) \,,$$

$$\mathrm{Tr} \, \mathbf{P} \otimes \mathbf{P}^* = \frac{1}{F} - F \,, \quad \mathrm{Tr} \, \bar{\mathbf{P}} \otimes \bar{\mathbf{P}}^* = \frac{1}{\bar{F}} - \bar{F} \,,$$

$$\widetilde{F} \, \bar{F} = F \, \widetilde{\bar{F}} = 1 \,.$$

D

$$\widetilde{\mathbf{P}} \otimes \widetilde{\mathbf{P}}^* = -\bar{\mu} \, \bar{\mathbf{P}}^* \otimes \bar{\mathbf{P}} \, \bar{\mu} \,, \quad \widetilde{\bar{\mathbf{P}}} \otimes \widetilde{\bar{\mathbf{P}}}^* = - \, \mu \, \mathbf{P}^* \otimes \mathbf{P} \, \mu^{-1}$$

$$\tilde{\mu} = \left(1 + \frac{1}{\bar{F}} \bar{\mathbf{P}} \otimes \bar{\mathbf{P}}^* \right) \mu \left(1 + \frac{1}{F} \mathbf{P}^* \otimes \mathbf{P} \right) \,, \quad \tilde{\bar{\mu}} = \left(1 + \frac{1}{F} \mathbf{P} \otimes \mathbf{P}^* \right) \bar{\mu} \left(1 + \frac{1}{\bar{F}} \bar{\mathbf{P}}^* \otimes \bar{\mathbf{P}} \right) \,,$$

$$\mathrm{Tr} \, \mathbf{P} \otimes \mathbf{P}^* = \frac{1}{F} - F \,, \quad \mathrm{Tr} \, \bar{\mathbf{P}} \otimes \bar{\mathbf{P}}^* = \frac{1}{\bar{F}} - \bar{F} \,,$$

$$\widetilde{F} \, \bar{F} = F \, \widetilde{\bar{F}} = 1 \,.$$

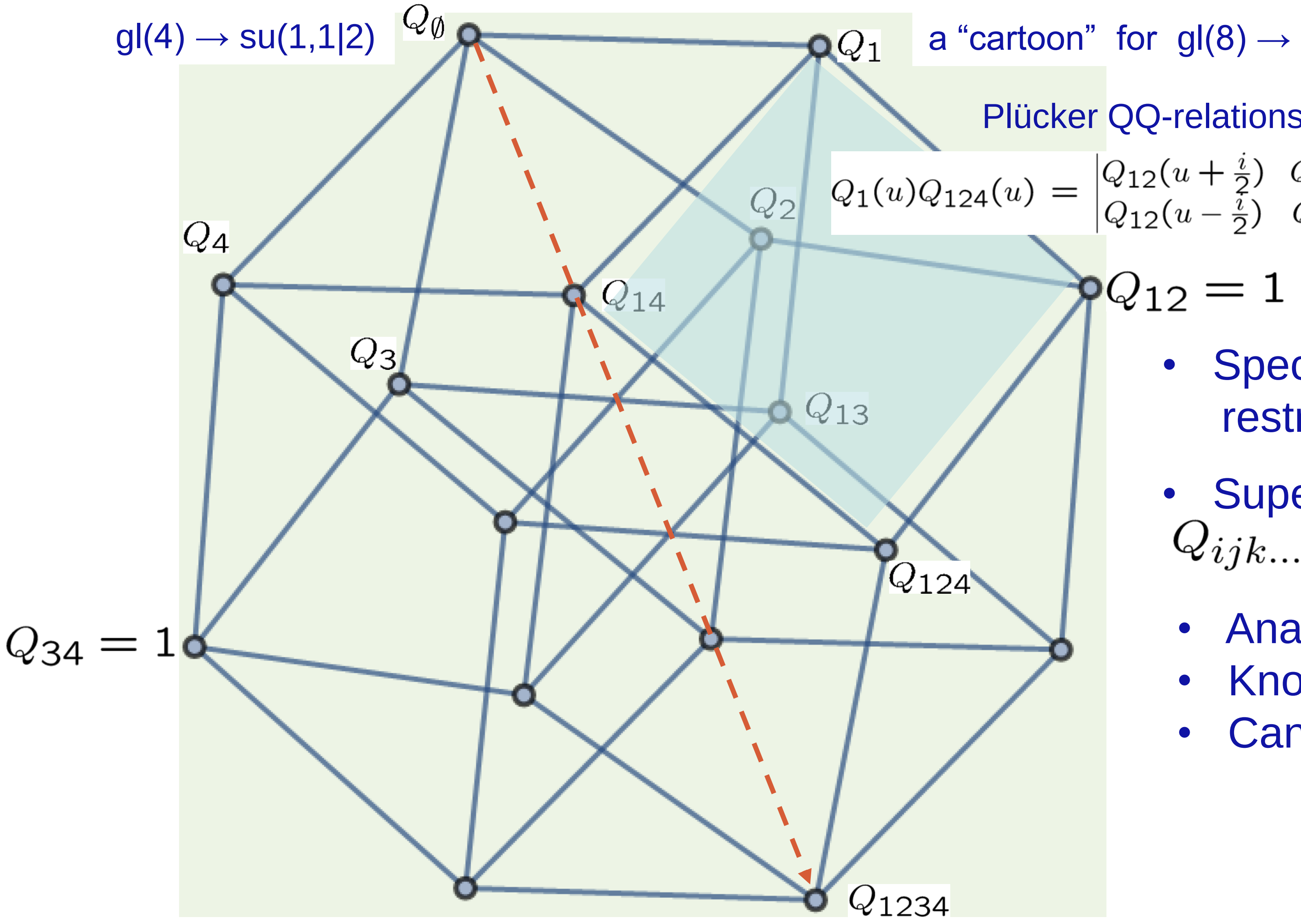
Can we guess what physics all these systems describe?

[Kazakov IGST 2021]

Quantum Spectral Curve and QQ system: algebraic structure

QSC eqs. close on finite number of Baxter functions of spectral parameter $Q_{ijk\ldots}(u)$
each placed at an edge of Hasse diagram - n-hypercube -- QQ system

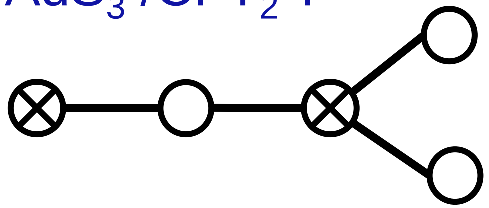
gl(4) → su(1,1|2)   a “cartoon” for gl(8) → su(2,2|4)



Plücker QQ-relations on each face (Grassmanian structure):

$$Q_1(u)Q_{124}(u) = \begin{vmatrix} Q_{12}(u+\frac{i}{2}) & Q_{14}(u+\frac{i}{2}) \\ Q_{12}(u-\frac{i}{2}) & Q_{14}(u-\frac{i}{2}) \end{vmatrix}$$

- Specific (bosonic) model: restrict $Q_\emptyset$ and Q_{1234}
- Supersymmetry: restricting some $Q_{ijk\ldots}$ and its Hodge dual $Q_{\overline{ijk\ldots}}$
- Analyticity and glueing conditions needed!
- Known for AdS_5/CFT_4 and AdS_4/CFT_3
- Can we guess it for AdS_3 /CFT_2 ?



AdS3 should be built on PSU(1,1|2) times PSU(1,1|2)

Model C = D for PSU(1,1|2), Hodge is inessential

1 unique model left! 2 coupled PSU(1,1|2)

Conjecture: Model C in the case of PSU(1,1|2) is the Quantum Spectral Curve for AdS3 * S^3 * T^4

[Cavaglia, Gromov, Stefanski, Torrielli]

[SE,Volin]

Surprising part: Not square root branch cuts!

Reasonable? Check what happens in large volume!

Can study large volume Bethe equations

$$1=\prod_{j}^{M_2}\frac{y_{i,k}-x_j^-}{y_{i,k}-x_j^+}\prod_{j}^{M_2}\frac{1-\frac{1}{y_{i,k}\bar{x}_j^+}}{1-\frac{1}{y_{i,k}\bar{x}_j^-}}\,,$$

$$1=\prod_{j}^{M_2}\frac{y_{\bar{i},k}-\bar{x}_j^-}{y_{\bar{i},k}-\bar{x}_j^+}\prod_{j}^{M_2}\frac{1-\frac{1}{y_{i,k}x_j^+}}{1-\frac{1}{y_{i,k}x_j^-}}\,,$$

$$\left(\frac{x_k^+}{x_k^-}\right)^L=\prod_{j}^{M_2}\frac{u_k-u_j+i}{u_k-u_j-i}\left(\sigma^{\bullet\bullet}(x_k,x_j)\right)^2\prod_{j}^{M_2}\frac{1-\frac{1}{x_k^+\bar{x}_j^+}}{1-\frac{1}{x_k^-\bar{x}_j^-}}\frac{1-\frac{1}{x_k^+\bar{x}_j^-}}{1-\frac{1}{x_k^-\bar{x}_j^+}}\left(\hat{\sigma}^{\bullet\bullet}(x_k,\bar{x}_j)\right)^2$$
$$\times\prod_{l=1}^{M_1}\frac{x_k^--y_{1,l}}{x_k^+-y_{1,l}}\prod_{k=1}^{M_3}\frac{x_k^--y_{3,l}}{x_k^+-y_{3,l}}\prod_{l=1}^{M_1}\frac{1-\frac{1}{x_k^-y_{1,l}}}{1-\frac{1}{x_k^+y_{1,l}}}\prod_{l=1}^{M_3}\frac{1-\frac{1}{x_k^-y_{3,l}}}{1-\frac{1}{x_k^+y_{3,l}}}$$

$$\left(\frac{\bar{x}_k^+}{\bar{x}_k^-}\right)^L=\prod_{j}^{M_2}\frac{\bar{x}_k^--\bar{x}_j^+}{\bar{x}_k^+-\bar{x}_j^-}\frac{1-\frac{1}{\bar{x}_k^+\bar{x}_j^-}}{1-\frac{1}{\bar{x}_k^-\bar{x}_j^+}}\left(\sigma^{\bullet\bullet}(\bar{x}_k,\bar{x}_j)\right)^2\prod_{j}^{M_2}\frac{1-\frac{1}{\bar{x}_k^-x_j^-}}{1-\frac{1}{\bar{x}_k^-x_j^+}}\frac{1-\frac{1}{\bar{x}_k^+x_j^-}}{1-\frac{1}{\bar{x}_k^+x_j^+}}\left(\hat{\sigma}^{\bullet\bullet}(\bar{x}_k,x_j)\right)^2$$
$$\times\prod_{l=1}^{M_1}\frac{\bar{x}_k^+-y_{\bar{1},l}}{\bar{x}_k^--y_{\bar{1},l}}\prod_{k=1}^{M_3}\frac{\bar{x}_k^+-y_{\bar{3},l}}{\bar{x}_k^--y_{\bar{3},l}}\prod_{l=1}^{M_1}\frac{1-\frac{1}{\bar{x}_k^+y_{1,l}}}{1-\frac{1}{\bar{x}_k^-y_{1,l}}}\prod_{l=1}^{M_3}\frac{1-\frac{1}{\bar{x}_k^+y_{3,l}}}{1-\frac{1}{\bar{x}_k^+y_{3,l}}}\,.$$

Obtained from asymptotic Q-functions

$$\mathtt{Q}_{1|1}\propto\mathbb{Q}\bar{f}^+f^+\,,\qquad \mathtt{Q}_{12|12}=1\,,$$
$$\mathtt{Q}_{1|\emptyset}\propto x^{-L/2}B_-R_{\bar{y}_1}\sigma\hat{\bar{\sigma}}\,,\qquad \mathtt{Q}_{1|12}\propto x^{-L/2}B_+R_{\bar{y}_3}\sigma\hat{\bar{\sigma}}\,,$$
$$\mathtt{Q}_{\emptyset|1}\propto x^{L/2}\frac{\bar{f}}{\bar{B}_+}fR_{y_1}\frac{1}{\sigma\hat{\bar{\sigma}}}\,,\qquad \mathtt{Q}_{12|1}\propto x^{L/2}\frac{\bar{f}}{\bar{B}_+}f^{[2]}R_{y_3}\frac{1}{\sigma\hat{\bar{\sigma}}}\,,$$
$$\bar{\mathtt{Q}}_{\dot{1}|\dot{1}}\propto\bar{\mathbb{Q}}\bar{f}^+f^+\,,$$
$$\bar{\mathtt{Q}}_{\dot{1}|0}\propto x^{-L/2}\bar{B}_-B_{y_1}\hat{\sigma}\bar{\sigma}\,,\qquad \bar{\mathtt{Q}}_{\dot{1}|\dot{1}\dot{2}}\propto x^{-L/2}\bar{B}_+B_{y_3}\hat{\sigma}\bar{\sigma}\,,$$
$$\bar{\mathtt{Q}}_{\emptyset|\dot{1}}\propto x^{L/2}\frac{f}{B_+}\bar{f}B_{\tilde{y}_1}\frac{1}{\hat{\sigma}\bar{\sigma}}\,,\qquad \bar{\mathtt{Q}}_{\dot{1}\dot{2}|\dot{1}}\propto x^{L/2}\frac{f}{B_+}\bar{f}^{[2]}B_{\tilde{y}_3}\frac{1}{\hat{\sigma}\bar{\sigma}}\,.$$

$$\widetilde{(\sigma\hat{\bar{\sigma}})}^2(\bar{\sigma}\hat{\sigma})^2=(f^{T[2]}f^{T*[-2]})^2\frac{R_{(-)}}{R_{(+)}}\frac{\bar{B}_{(+)}}{\bar{B}_{(-)}}\qquad \widetilde{(\hat{\bar{\sigma}}\sigma)}^2(\sigma\hat{\bar{\sigma}})^2=(f^{T[2]}f^{T*[-2]})^2\frac{\bar{R}_{(-)}}{\bar{R}_{(+)}}\frac{B_{(+)}}{B_{(-)}}\,,$$
$$\overset{*\sim}{(\sigma\hat{\bar{\sigma}})}{}^2(\bar{\sigma}\hat{\sigma})^2=(f^{T[2]}f^{T*[-2]})^2\frac{R_{(+)}}{R_{(-)}}\frac{\bar{B}_{(-)}}{\bar{B}_{(+)}}\qquad \overset{*\sim}{(\hat{\bar{\sigma}}\sigma)}{}^2(\sigma\hat{\bar{\sigma}})^2=(f^{T[2]}f^{T*[-2]})^2\frac{\bar{R}_{(+)}}{\bar{R}_{(-)}}\frac{B_{(+)}}{B_{(-)}}\,.$$

Non-square root cut dressing phases!

[Cavaglia,Gromov,Stefanski,Torrielli]

Conclusions

Monodromy Bootstrap provides a way to generate new QSC's.

Successfully found 4 models, reproducing the Hubbard model

Non-square root type models (AdS2 for PSU(1,1|2) non-sq-root?)

Conjectured QSC for AdS3

Outlook

Extend to Q-systems with other symmetry groups? (ABJM types)

There could be more models, something to consider?

$$\begin{matrix}\tilde{P}_a \\ \tilde{\bar{P}}_{\dot a}\end{matrix} = \begin{pmatrix} \epsilon_{ab} & \bar\mu_{a\dot b} \\ -\mu_{\dot a b} & \epsilon_{\dot a \dot b}\end{pmatrix}\begin{pmatrix} P^b \\ \bar P^b\end{pmatrix}$$