Fondúireacht Eolaíochta Éireann
Dá bhfuil romhainn

Science Foundation Ireland
For what's next

European Research Council
Established by the European Commission

Integrable deformations of AdS/CFT

Based on work with M. de Leeuw, C. Paletta, A. Pribytok and A. Retore

Motivation

Deformations allow us to embed integrable models in multi-parameter families of integrable models

Why study deformations?

Motivation

New integrable systems

New interesting mathematical structures

$$\text{XXX } Y(\mathfrak{g}) \rightarrow \text{XXZ } U_q(\widehat{\mathfrak{g}})$$

Deformations can be used as regulators

Access to limits with interesting physics

Where to introduce deformations?

Hamiltonian! XXX $\longrightarrow$ XXZ $\longrightarrow$ XYZ, Long-range Haldane-Shastry

Sigma model action! Yang-Baxter deformations

4d field theory action! Deform field product, dipole, fishnets

Worldsheet S-matrix!

Where to introduce deformations?

Hamiltonian! XXX $\longrightarrow$ XXZ $\longrightarrow$ XYZ, Long-range Haldane-Shastry

What is an “integrable” Hamiltonian?

Sigma model action! Yang-Baxter deformations

4d field theory action! Deform field product, dipole, fishnets

Worldsheet S-matrix!

Where to introduce deformations?

Hamiltonian! XXX $\longrightarrow$ XXZ $\longrightarrow$ XYZ, Long-range Haldane-Shastry

What is an “integrable” Hamiltonian?

Sigma model action! Yang-Baxter deformations

Will it survive quantisation?

4d field theory action! Deform field product, dipole, fishnets

Worldsheet S-matrix!

Where to introduce deformations?

Hamiltonian! XXX $\longrightarrow$ XXZ $\longrightarrow$ XYZ, Long-range Haldane-Shastry

What is an “integrable” Hamiltonian?

Sigma model action! Yang-Baxter deformations

Will it survive quantisation?

4d field theory action! Deform field product, dipole, fishnets

Beyond one-loop? Non-perturbatively?

Worldsheet S-matrix!

Where to introduce deformations?

Hamiltonian! XXX $\longrightarrow$ XXZ $\longrightarrow$ XYZ, Long-range Haldane-Shastry

What is an "integrable" Hamiltonian?

Sigma model action! Yang-Baxter deformations

Will it survive quantisation?

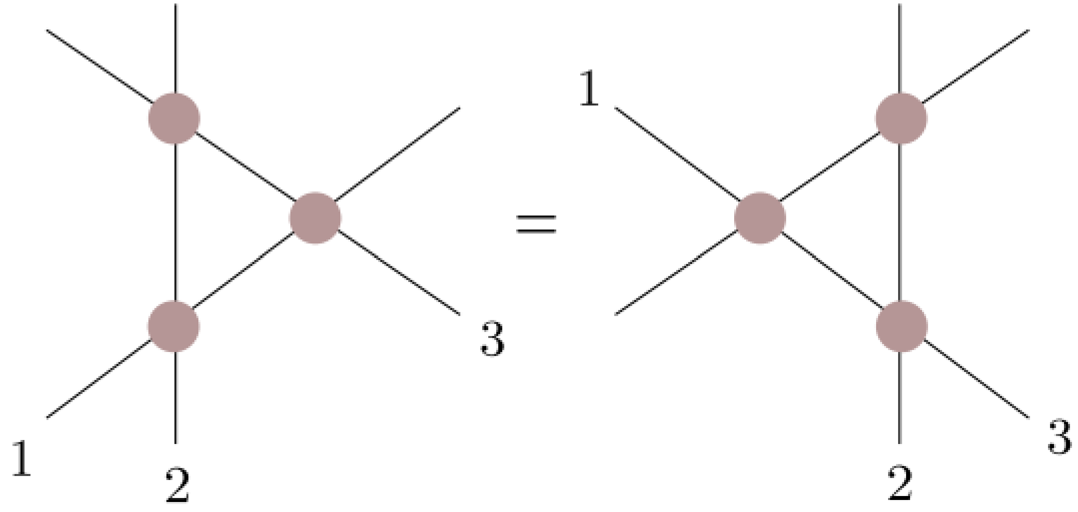
4d field theory action! Deform field product, dipole, fishnets

Beyond one-loop? Non-perturbatively?

Worldsheet S-matrix!

Clear notion of quantum, non-perturbative, integrable deformation!

Deforming the S-matrix



Integrable S-matrix in 1+1 dim QFT

2 body S-matrix satisfies Yang-Baxter equation!

$$R_{12}(u-v)R_{13}(u)R_{23}(v) = R_{23}(v)R_{13}(u)R_{12}(u-v)$$

Classify solutions → Classify integrable S-matrices

Classification leads to deformations

$\mathfrak{su}(2)$ spin $\frac{1}{2}$ XXX $\qquad R(u) = u\,1 + P$

Identify solutions for which XXX can be embedded

$$R(u) = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{pmatrix}$$

Solutions contain free functions / parameter
Tuning them fixes XXX

Flowing away preserves integrability, deforms XXX

XXZ $\longrightarrow$ XXX
as q $\longrightarrow$ 1

We need to solve the YBE!

$n \times n$ matrix

$$R_{12}(u-v)R_{13}(u)R_{23}(v) = R_{23}(v)R_{13}(u)R_{12}(u-v)$$

Functional equation in 2 variables, up to n^2 unknowns

Situation is even worse without worldsheet Lorentz invariance!

$$R(u-v) \to R(u,v)$$

YBE becomes a functional equation in 3 variables

Most relevant for AdS/CFT applications!

Techniques on the market

[Kulish, Reshetikhin, Sklyanin]

Algebraic

[Beisert]

- Exploit symmetry (Yangian, quantum affine…)
- Baxterisation of Temperley-Lieb
- Based on physical input from a model

$$\Delta^{\mathrm{op}}(a)R(u,v) = R(u,v)\Delta(a)$$

Direct approach

[Vieira]

- Functional equations reduce to diff. eqns
- Guaranteed to find all solutions

Techniques on the market

[Kulish, Reshetikhin, Sklyanin]

[Beisert]

Algebraic

- Exploit symmetry (Yangian, quantum affine…)
- Baxterisation of Temperley-Lieb
- Based on physical input from a model

$$\Delta^{\mathrm{op}}(a)R(u,v) = R(u,v)\Delta(a)$$

- Assumes we know symmetry in the first place… not good for deformations
- Only works for a limited subclass of models

Direct approach

[Vieira]

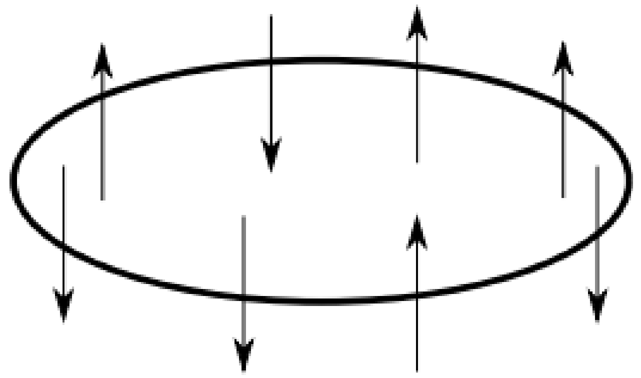
- Functional equations reduce to diff. eqns
- Guaranteed to find all solutions

- Very difficult to solve many coupled ODEs / PDEs
- What even are good initial conditions?

New approach!

Based on associated integrable spin chain

Key tool: Spin chain boost operator $B[H]$



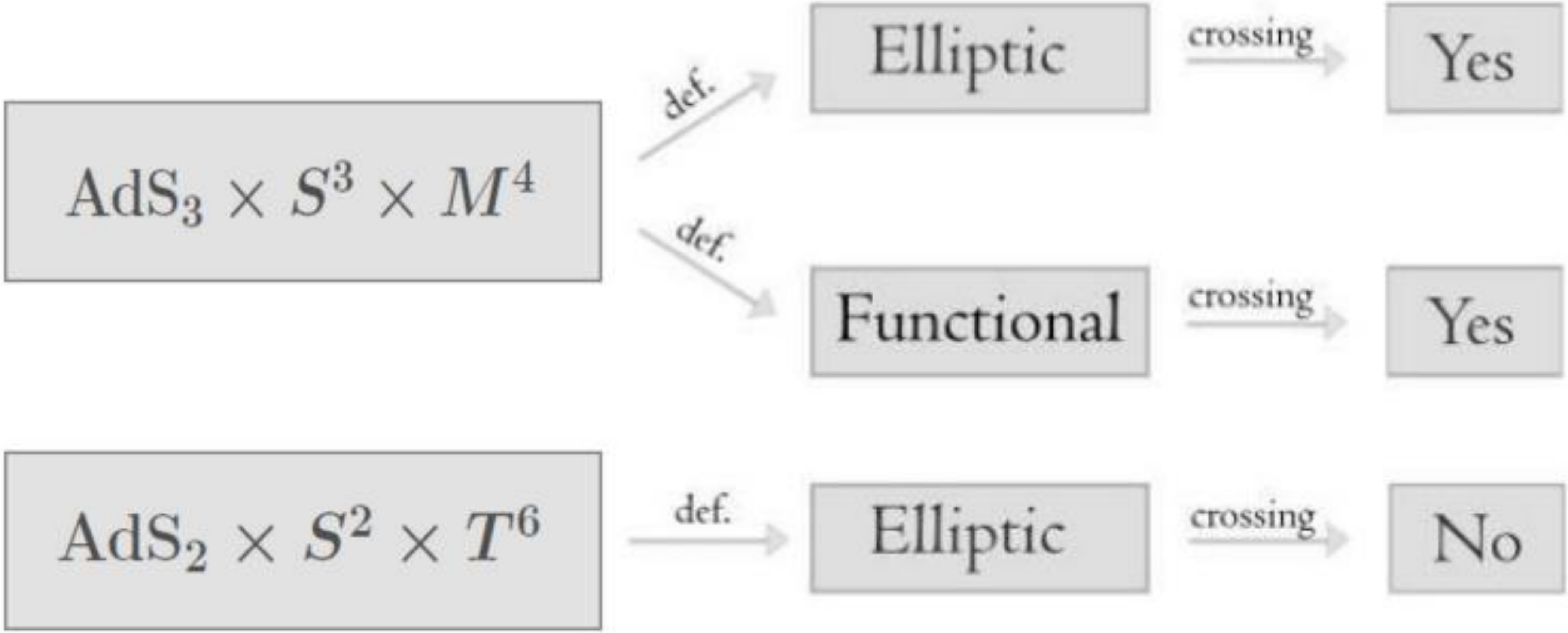
Recursively generates higher conserved charges

$$\mathbb{Q}_{r+1} = [B[H], \mathbb{Q}_r]$$

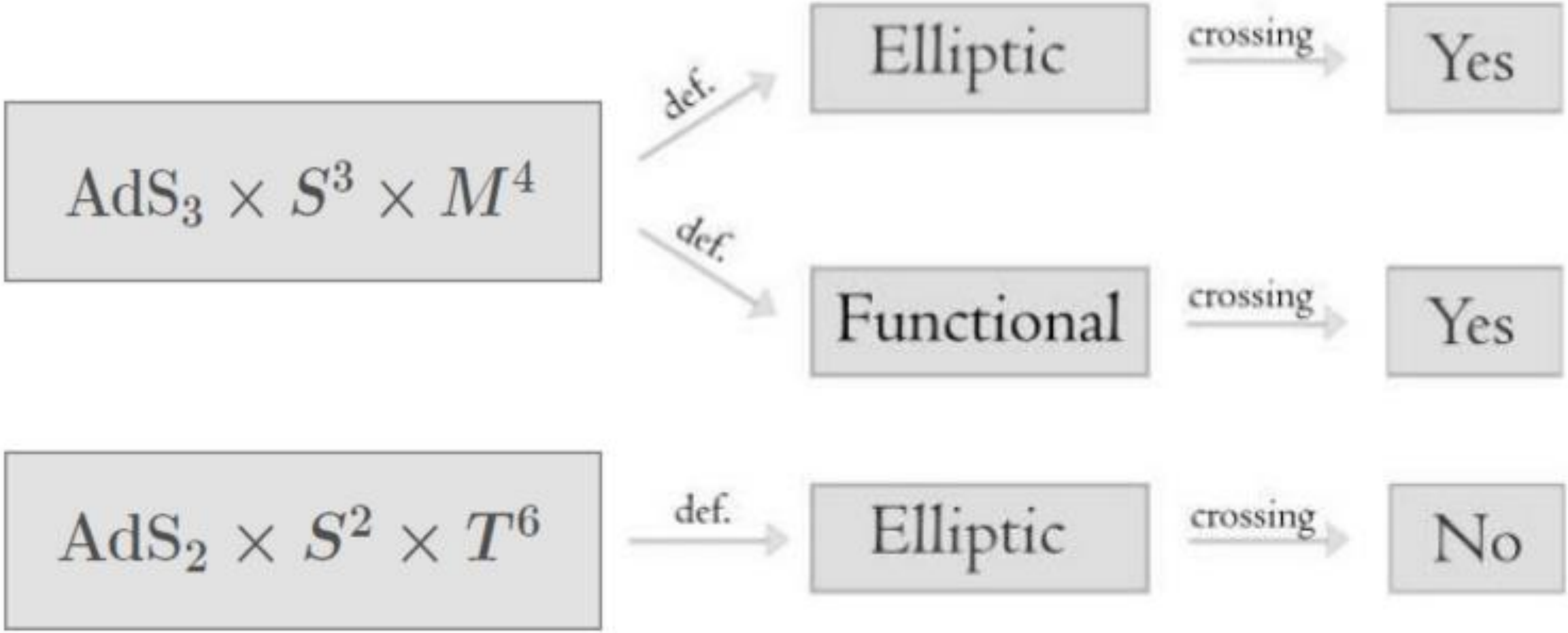
- Approach based on consistency of conserved charges
- Generates all regular solutions

- Highly efficient!
- Lorentz invariant: cubic ODEs ⟶ cubic polynomials
- Non-Lorentz invariant: cubic PDEs ⟶ cubic ODEs

Deformations of low-dimensional AdS models



Deformations of low-dimensional AdS models



Everything applies to $AdS_5 \times S^5$

Outline

1. Boost automorphism

2. Boost as a tool for solving the YBE

3. Classification of solutions

4. Applications to AdS/CFT

5. Symmetry algebra + crossing symmetry

Boost automorphism

Every R-matrix defines an integrable spin chain

$$R_{ab}(u) : V \otimes V \to V \otimes V$$

Transfer matrix $\quad T(u) = \mathrm{tr}_a \left(R_{aL}(u) \ldots R_{a2}(u) R_{a1}(u) \right)$

$[T(u), T(v)] = 0 \quad$ Family of commuting charges

Integrability!

Regular R-matrix $\quad R(0) = P \quad$ Permutation operator

Conserved charges $\quad \log T(u) = 1 + u\, \mathbb{Q}_2 + u^2\, \mathbb{Q}_3 + \ldots \qquad [\mathbb{Q}_r, \mathbb{Q}_s] = 0$

Nearest-neighbour local Hamiltonian

$$\log T(u) = 1 + u\,\mathbb{Q}_2 + u^2\,\mathbb{Q}_3 + \dots$$

$\mathbb{Q}_2 := H$ sum of local densities $H = \mathcal{H}_{12} + \mathcal{H}_{23} + \cdots + \mathcal{H}_{L1}$

Intimately related to R-matrix

$$R'_{12}(0) = P_{12}\,\mathcal{H}_{12}$$

$$\log T(u) = 1 + u\,\mathbb{Q}_2 + u^2\,\mathbb{Q}_3 + \ldots$$

$\mathbb{Q}_2 := H$ sum of local densities $H = \mathcal{H}_{12} + \mathcal{H}_{23} + \cdots + \mathcal{H}_{L1}$

Intimately related to R-matrix

$$R'_{12}(0) = P_{12}\,\mathcal{H}_{12}$$

R-matrix ⟶ Hamiltonian + higher charges

Crucial fact: higher conserved charges completely determined by Hamiltonian density!

$$\mathbb{Q}_2 = H = \mathcal{H}_{12} + \mathcal{H}_{23} + \cdots + \mathcal{H}_{L1}$$

$$\mathbb{Q}_3 = \sum_j \mathcal{Q}^{(3)}_{j,j+1,j+2}$$

$$\mathcal{Q}^{(3)}_{j,j+1,j+2} = [\mathcal{H}_{j,j+1}, \mathcal{H}_{j+1,j+2}]$$

Similar story for higher charges

Boost operator: a ladder operator for conserved charges

$$B[H] = \sum_{n=-\infty}^{\infty} n\mathcal{H}_{n,n+1}$$

$$\mathbb{Q}_{r+1} = [B[H], \mathbb{Q}_r]$$

Works just as well for models without Lorentz invariance

$$R(u-v) \to R(u,v)$$

$$R(u,v) = P(1 + (u-v)\mathcal{H}(v) + \dots)$$

Regular

1-parameter family of Hamiltonians

$$\mathbb{Q}_r \to \mathbb{Q}_r(v)$$

Modification to B[H] is very simple! [Links, Zhou, McKenzie, Gould]

$$B[H] = \frac{\partial}{\partial v} + \sum_{n=-\infty}^{\infty} n\,\mathcal{H}_{n,n+1}(v) \qquad \mathbb{Q}_{r+1}(v) = [B[H], \mathbb{Q}_r(v)]$$

Boost as a tool for solving the YBE

The boost operator allows us to turn the problem of solving the YBE on its head

Let $\mathcal{H}_{12} \in \mathrm{End}(V \otimes V)$ be a generic $n \times n$ matrix

$$\mathbb{Q}_2 = H = \mathcal{H}_{12} + \mathcal{H}_{23} + \cdots + \mathcal{H}_{L1}$$

Define operator $\mathbb{Q}_3$ by property $\mathbb{Q}_3 = [B[H], \mathbb{Q}_2]$

Impose $[\mathbb{Q}_2, \mathbb{Q}_3] = 0$

Solve for entries of $\mathcal{H}$

Is there an R-matrix with $R'(v,v) = P\mathcal{H}(v)$?

Yes!

[de Leeuw, Paletta, Pribytok, Retore, PR]

Let $R(u,v)$ satisfy $R(v,v) = P$ and $R'(v,v) = P\mathcal{H}(v)$ and let $\mathcal{H}$ be such that $[\mathbb{Q}_2(v), \mathbb{Q}_3(v)] = 0$.

Then R extends uniquely to a solution of the YBE.

Not (yet) a theorem

- have considerable experimental evidence
- No counterexamples yet found

Yes!

[de Leeuw, Paletta, Pribytok, Retore, PR]

Let $R(u,v)$ satisfy $R(v,v) = P$ and $R'(v,v) = P\mathcal{H}(v)$ and let $\mathcal{H}$ be such that $[\mathbb{Q}_2(v), \mathbb{Q}_3(v)] = 0$.
Then R extends uniquely to a solution of the YBE.

Not (yet) a theorem

- have considerable experimental evidence
- No counterexamples yet found

Modulo symmetries of the YBE
eg rescaling R(u) → f(u) R(u)

Our claim is related to an old conjecture of Reshetikhin

[Grabowski, Mathieu]

A Hamiltonian $\mathcal{H}$ is integrable if there exists an operator G such that $G_{12} - G_{23} = [\mathcal{H}_{12} + \mathcal{H}_{23}, [\mathcal{H}_{12}, \mathcal{H}_{23}]]$.

Remarkably, G exists $\iff$ $[\mathbb{Q}_2, \mathbb{Q}_3] = 0$!

Seems possible to express R in terms of H and G

$$R(u) = P(1 + u\,\mathcal{H} + \frac{u^2}{2}\mathcal{H}^2 + \frac{u^3}{3!}(\mathcal{H}^3 + G) + \frac{u^4}{4!}(\mathcal{H}^4 + G\mathcal{H} + \mathcal{H}G) + \dots$$

Higher terms unknown – if a closed form found our method would produce an R-matrix immediately after inputting a Hamiltonian!

Our method at work

Sutherland equations

YBE implies two sets of differential eqns

$$[R_{13}R_{23}, \mathcal{H}_{12}(u)] = \dot{R}_{13}R_{23} - R_{13}\dot{R}_{23}$$
$$[R_{13}R_{12}, \mathcal{H}_{23}(v)] = R_{13}R'_{12} - R'_{13}R_{12}$$

Initial conditions:

$$R(v,v) = P$$
$$R'(v,v) = P\mathcal{H}(v)$$

→ Found from $[\mathbb{Q}_2, \mathbb{Q}_3] = 0$

Make suitable ansatz for R using the expansion

$$R(u) = P(1 + u\,\mathcal{H} + \tfrac{u^2}{2}\,\mathcal{H}^2 + \dots)$$

to determine which entries of R should be 0

Our method at work

$$\mathcal{H}_{12}(\theta) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_1(\theta) & h_3(\theta) & 0 \\ 0 & h_4(\theta) & h_2(\theta) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$[\mathbb{Q}_2, \mathbb{Q}_3] = 0$ Implies H must have the form

$$\mathcal{H}_{12}(\theta) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_1 & \frac{c_3}{2}(h_1 + h_2) & 0 \\ 0 & \frac{c_4}{2}(h_1 + h_2) & h_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Ansatz $R = \begin{pmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & r_3 & 0 \\ 0 & r_4 & r_5 & 0 \\ 0 & 0 & 0 & r_6 \end{pmatrix}$

Solve Sutherland

Conclusion: XXZ with a twist!

Classification of solutions

[de Leeuw, Pribytok, PR]

Proof of concept

$$R(u,v) = R(u-v)$$

$$R(u) = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{pmatrix}$$

We classified all solutions

In all cases our conjecture works perfectly, i.e. $[\mathbb{Q}_2, \mathbb{Q}_3] = 0$

$\longrightarrow$ YBE

We recover well-known solutions, XXX, XXZ, XYZ etc

New solutions!

Many peculiar features eg nilpotent or non-diagonalisable H

(related to fishnet CFT?!)

[Ipsen, Staudacher, Zippelius]

We classified

[de Leeuw, Paletta, Pribytok, Retore, PR]

All 4 x 4 solutions with $R(u,v) = R(u-v)$

All 4 x 4 solutions which preserve fermion number

Includes AdS_2 AdS_3

All 9 x 9 solutions with $\mathfrak{u}(1)^3$ symmetry

All 16 x 16 solutions with $\mathfrak{su}(2) \oplus \mathfrak{su}(2)$ symmetry

Includes $\mathrm{AdS}_5 \times S^5$

Solutions preserving fermion number

$$R = \begin{pmatrix} r_1 & 0 & 0 & r_8 \\ 0 & r_2 & r_6 & 0 \\ 0 & r_5 & r_3 & 0 \\ r_7 & 0 & 0 & r_4 \end{pmatrix}$$

no scattering processes with

B x B $\longrightarrow$ F x B etc

6 vertex A, B	$r_7, r_8 = 0$	XXX, XXZ type
8 vertex A, B	$r_7, r_8 \neq 0$	XYZ type

6 vertex B

$$
\begin{aligned}
r_1(p,q) &= \frac{h_2(q)-h_1(p)}{h_2(p)-h_1(p)},\\
r_2(p,q) &= (h_2(p)-h_2(q))X(p)Y(p),\\
r_3(p,q) &= \frac{h_1(p)-h_1(q)}{(h_2(p)-h_1(p))(h_2(q)-h_1(q))}\frac{1}{X(q)Y(q)},\\
r_4(p,q) &= \frac{h_2(p)-h_1(q)}{h_2(q)-h_1(q)}\frac{X(p)Y(p)}{X(q)Y(q)},\\
r_5(p,q) &= \frac{Y(p)}{Y(q)},\\
r_6(p,q) &= \frac{X(p)}{X(q)}.
\end{aligned}
$$

8 vertex B

$$\mathrm{sn} = \mathrm{sn}(u-v,k^2), \qquad \mathrm{cn} = \mathrm{cn}(u-v,k^2), \qquad \mathrm{dn} = \mathrm{dn}(u-v,k^2),$$

$$r_1 = \frac{1}{\sqrt{\sin\eta(u)}\sqrt{\sin\eta(v)}}\left[\sin\eta_+\frac{\mathrm{cn}}{\mathrm{dn}} - \cos\eta_+\mathrm{sn}\right],$$

$$r_2 = \frac{1}{\sqrt{\sin\eta(u)}\sqrt{\sin\eta(v)}}\left[\cos\eta_-\mathrm{sn} + \sin\eta_-\frac{\mathrm{cn}}{\mathrm{dn}}\right],$$

$$r_3 = \frac{1}{\sqrt{\sin\eta(u)}\sqrt{\sin\eta(v)}}\left[\cos\eta_-\mathrm{sn} - \sin\eta_-\frac{\mathrm{cn}}{\mathrm{dn}}\right],$$

$$r_4 = \frac{1}{\sqrt{\sin\eta(u)}\sqrt{\sin\eta(v)}}\left[\sin\eta_+\frac{\mathrm{cn}}{\mathrm{dn}} + \cos\eta_+\mathrm{sn}\right],$$

$$r_5 = r_6 = 1,$$

$$r_7 = r_8 = k\,\mathrm{sn}\frac{\mathrm{cn}}{\mathrm{dn}},$$

where $\eta_\pm = \frac{\eta(u)\pm\eta(v)}{2}$.

Applications to AdS/CFT

Ingredients for AdS/CFT S-matrices

[Borsato, Ohlson Sax, Sfondrini, Stefanski]

Off-shell symmetry algebra

$\mathrm{AdS}_3 \times S^3 \times M^4$	$\mathfrak{su}(1\|1)^2_{\mathrm{ce}}$
$\mathrm{AdS}_2 \times S^2 \times T^6$	$\mathfrak{psu}(1\|1)_{\mathrm{ce}}$

[Hoare, Pittelli, Torrielli]

$$\Delta^{\mathrm{op}}(\mathfrak{Q})R = R\Delta(\mathfrak{Q})$$

$\mathrm{AdS}_2 \times S^2 \times T^6$ 8 vertex B type

Reparameterise $u \to G(u)$ in 8 vertex B

$$x^+(u) = -\frac{\mathrm{Tan}^2(G(u))}{x^-(u)}$$

$$\eta(u) = \mathrm{arccot}(k\,F(u))$$

$$F(u) = -\frac{1}{2}\csc(G(u))\sec(G(u))\,\frac{\cot(G(u))\,x^- + i}{\cot(G(u))\,x^- - i}$$

$$k \to \infty$$

Varying k provides an integrable deformation!

$\mathrm{AdS}_3 \times S^3 \times M^4$

Chiral structure

$$\mathbf{S} = \left(\begin{array}{c|c} \mathbf{S}^{\mathrm{LL}} & \mathbf{S}^{\mathrm{RL}} \\ \hline \mathbf{S}^{\mathrm{LR}} & \mathbf{S}^{\mathrm{RR}} \end{array}\right)$$

16 x 16 matrix satisfies YBE

LL and RR blocks satisfy YBE by themselves and are 4 x 4

Two ways to fit in our classification

6 vertex B

8 vertex B with $k \to 0$

$\mathrm{AdS}_3 \times S^3 \times M^4$

6 vertex B type

Free functions h_1, h_2, X, Y

LL sector

$$h_1^{\mathrm{L}}(p) = \beta\, x_{\mathrm{L}}^-(p)$$

$$h_2^{\mathrm{L}}(p) = \beta\, x_{\mathrm{L}}^+(p)$$

$$X^{\mathrm{L}}(p) = \frac{\rho}{\gamma^{\mathrm{L}}(p)}$$

$$Y^{\mathrm{L}}(p) = \frac{\gamma^{\mathrm{L}}(p)}{\beta\rho\left(x_{\mathrm{L}}^-(p) - x_{\mathrm{L}}^+(p)\right)}\sqrt{\frac{x_{\mathrm{L}}^-(p)}{x_{\mathrm{L}}^+(p)}}$$

RR sector

$$h_1^{\mathrm{R}}(p) = -\frac{x_{\mathrm{R}}^-(p)}{\beta}$$

$$h_2^{\mathrm{R}}(p) = -\frac{x_{\mathrm{R}}^+(p)}{\beta}$$

$$X^{\mathrm{R}}(p) = -i\,\rho\,\frac{x_{\mathrm{R}}^+(p)}{\gamma^{\mathrm{R}}(p)}$$

$$Y^{\mathrm{R}}(p) = \frac{-i\,\gamma^{\mathrm{L}}(p)}{\beta\,\rho}\,\frac{\sqrt{x_{\mathrm{R}}^-(p)x_{\mathrm{R}}^+(p)}}{x_{\mathrm{R}}^-(p) - x_{\mathrm{R}}^+(p)}$$

$\mathrm{AdS}_3 \times S^3 \times M^4$

8 vertex B type

Free functions G, η

LL sector

$$G^L(p) = \pi - \frac{i}{4}\log\left(\sqrt{x_L^-(p)x_L^+(p)}\right) \qquad \eta^L(p) = -\tfrac{i}{2}\log\left(\sqrt{\frac{x_L^+}{x_L^-}}\right) = \frac{p}{4}$$

RR sector

$$G^R(p) = G^L(p)\big|_{L\to R} \qquad \eta^R(p) = \tfrac{\pi}{2} - \eta^L(p)$$

$$\boxed{k=0}$$

$\mathrm{AdS}_3 \times S^3 \times M^4$

Functional deformation

For 6 vertex B we can vary the defining functions in any way and obtain a deformation of LL and RR

[Hoare]

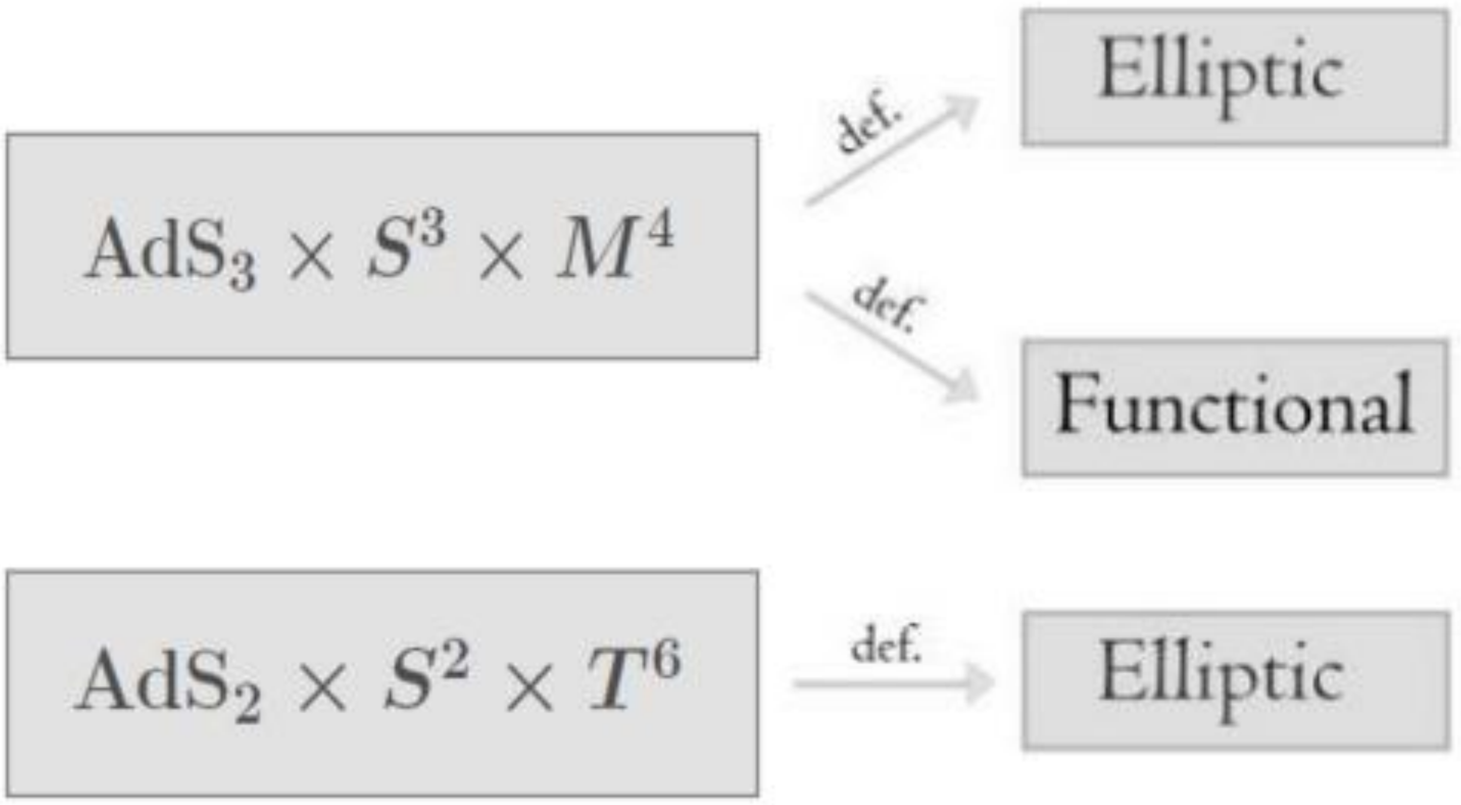
R-matrix equiv to q-def modulo details

For 8 vertex B we can vary k

For 6 vertex B the deformation extends uniquely to LR and RL

The same is true for 8 vertex B $\iff k_R = -k_L$

New elliptic deformation!



$\mathrm{AdS}_3 \times S^3 \times M^4$
def.
Elliptic
def.
Functional
$\mathrm{AdS}_2 \times S^2 \times T^6$
def.
Elliptic

Symmetry algebra + crossing symmetry

Symmetry algebra

$$R_{ab}(u,v)T_a(u)T_b(v) = T_b(v)T_a(u)R_{ab}(u,v)$$

Expand around R diagonal

Lie algebra

$$T_{ab}(u) = T_{ab}^{(-1)}\delta_{ab} + \frac{1}{u}T_{ab}^{(0)} + \frac{1}{u^2}T_{ab}^{(1)} + \dots$$

Quantum algebra (Yangian)

Alternatively can try to solve directly

$$\Delta^{\mathrm{op}}(\mathfrak{Q})R = R\Delta(\mathfrak{Q}) \qquad \Delta(\mathfrak{Q}) = \mathfrak{Q}\otimes 1 + \mathfrak{U}\otimes\mathfrak{Q}$$

$\mathrm{AdS}_3 \times S^3 \times M^4$

6 vertex B

$\mathfrak{su}(1|1)^2_{\mathrm{ce}}$

Deformation equivalent to q-deformed model
+ mass depends on spectral parameter

R-matrices equivalent up to relabelling functions

Exciting things happen for 8 vertex B!

$\mathrm{AdS}_2 \times S^2 \times T^6$ 8 vertex B

$\mathfrak{psu}(1|1)_{\mathrm{ce}}$ $\{\mathfrak{Q}_\pm, \mathfrak{Q}_\pm\} = 2\mathfrak{P}_\pm$ $\{\mathfrak{Q}_+, \mathfrak{Q}_-\} = 2\mathfrak{C}$

For finite k the algebra is the same… trivial redefinition?

$$T_{ab}(u) = T_{ab}^{(-1)}\delta_{ab} + \frac{1}{u}T_{ab}^{(0)} + \frac{1}{u^2}T_{ab}^{(1)} + \ldots$$

Same true for level 1 generators

New k dependence in level 2 generators! New quantum algebra?!

Yangian algebra

$$T_{ab}(u) = T^{(-1)}_{ab}\delta_{ab} + \frac{1}{u}T^{(0)}_{ab} + \frac{1}{u^2}T^{(1)}_{ab} + \dots$$

$T^{(0)}$ and $T^{(1)}$ enough to generate full algebra

$$T^{(m)}T^{(n)} \sim T^{(m+n)}$$

Not true for our algebra!

By appropriate redefinitions can make the first two levels independent of k

Cannot be removed from level 2: independent

Novel quantum algebra structure

$\mathrm{AdS}_3 \times S^3 \times M^4$ 8 vertex B

LL and RR blocks = AdS2, know symmetry!

Naturally expect two copies of $\mathfrak{psu}(1|1)_{\mathrm{ce}}$ $\begin{pmatrix} \mathfrak{Q}^L_\pm & 0 \\ 0 & \mathfrak{Q}^R_\pm \end{pmatrix}$

Not all charges remain a symmetry when extended to full 16 x 16 R-matrix

Symmetry algebra is just one copy of $\mathfrak{psu}(1|1)_{\mathrm{ce}}$

$$\begin{pmatrix} \mathfrak{Q}^L_+ & 0 \\ 0 & \mathfrak{Q}^R_- \end{pmatrix} \qquad \begin{pmatrix} \mathfrak{Q}^L_- & 0 \\ 0 & \mathfrak{Q}^R_+ \end{pmatrix}$$

Crossing symmetry

$$\mathbb{C}_1\mathbb{R}(p+\omega,q)^{t_1}\mathbb{C}_1=\mathbb{R}(p,q)^{-1}$$
$$\mathbb{C}_2\mathbb{R}(p,q-\omega)^{t_2}\mathbb{C}_2=\mathbb{R}(p,q)^{-1}$$

$$\mathbb{C}=\begin{pmatrix}0&0&1&0\\0&0&0&i\\1&0&0&0\\0&i&0&0\end{pmatrix}$$

Charge conjugation for undeformed AdS3

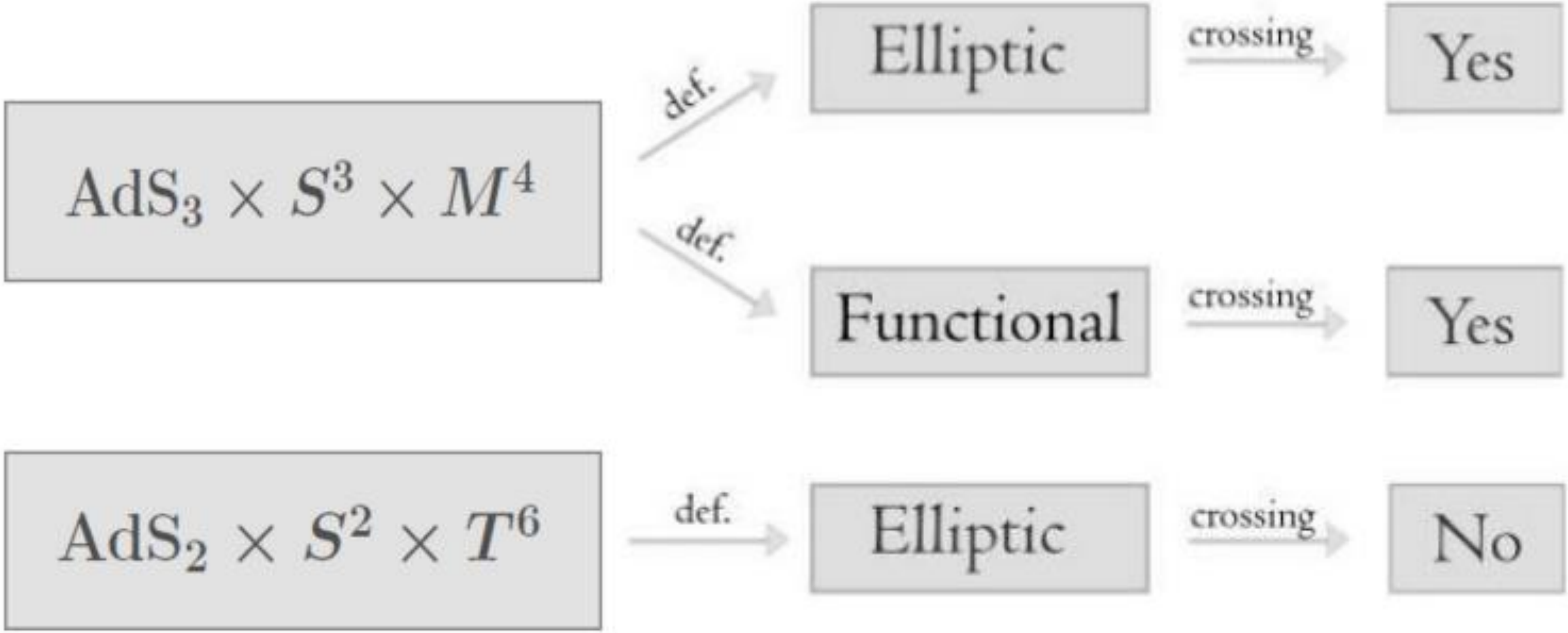
Works for 6 vertex B with simple assumptions on functions eg $h_i^{\mathrm{R}}(p\pm\omega)=-\dfrac{1}{h_i^{\mathrm{L}}(p)}$

8-vertex B

$$\mathbb{C}=\begin{pmatrix}0&0&-\frac{b^2\tau}{c}&0\\0&0&0&b\\c&0&0&0\\0&-b\tau&0&0\end{pmatrix}$$

Requires full 16 x 16 R-matrix to work

8 vertex B AdS2 not crossing symmetric!


$AdS_3 \times S^3 \times M^4$
def.
Elliptic
crossing
Yes
def.
Functional
crossing
Yes
$AdS_2 \times S^2 \times T^6$
def.
Elliptic
crossing
No

Summary & Outlook

Summary

Boost operator as a tool for classifying YBE solutions

4 x 4 solutions which preserve fermion number fully classified

Low dimensional AdS admit deformations based on 6 / 8 vertex B

Elliptic deformation of AdS3 - crossing symmetry

- broken symmetry algebra

Outlook

Closed expression for R-matrix from H?

Elliptic deformation of AdS5?

Physics of elliptic deformation? Bethe Ansatz? XYZ type so need something else (Q-operators, SoV?). TBA? QSC?

Sigma model origins of elliptic deformations?