Correlation functions and wave functions in solvable models
13-16 September 2021, Paris

Finite volume form factors

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In collaboration with: J. Balog, R. Janik, M. Lájer, F. Smirnov, B. Szépfalvi, I. Vona, Ch. Wu

based on 1707.08027, 1802.04021, 1903.06990, 1904.00492, 1911.08525

Ongoing work in collaboration with I. Szécsényi, G. Benas, I. Vona

Plan

Overview of last 5 years work

- Motivation: why finite volume form factors, AdS/CFT 3pt functions
- Finite volume corrections

$$x \equiv x + L$$

Small volume → Large volume

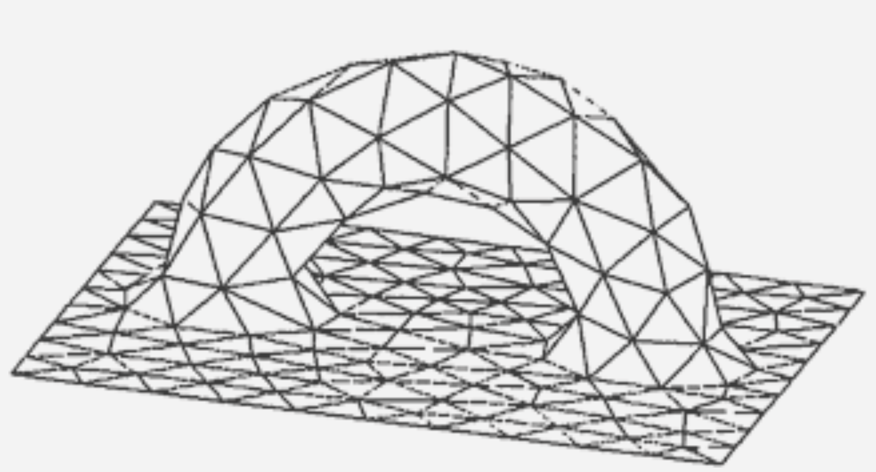
Energy spectrum	Exact description, Thermodynamic Bethe Ansatz	Lüscher correction of masses	Momentum quantisation, Bethe Yang equation	Masses, scatterings
Form Factors Correlators	Linear integral equations based on the TBA	Leading exponential volume corrections	Change in the normalisation of states	Form factors

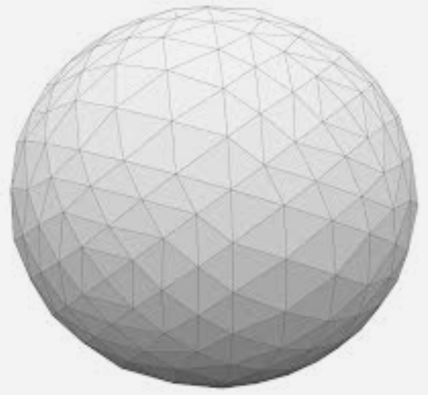
Motivation

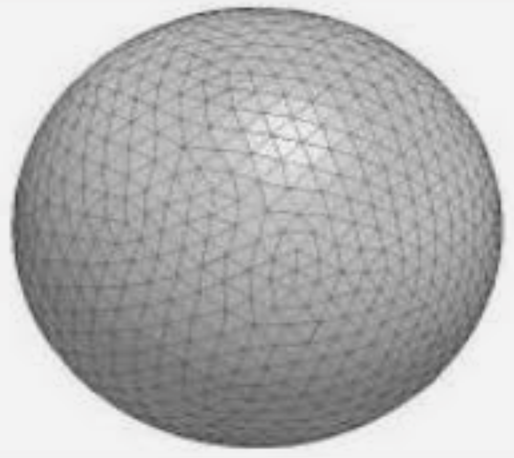
AdS/CFT

[Beisert et al 2012]

Dense Feynman graphs

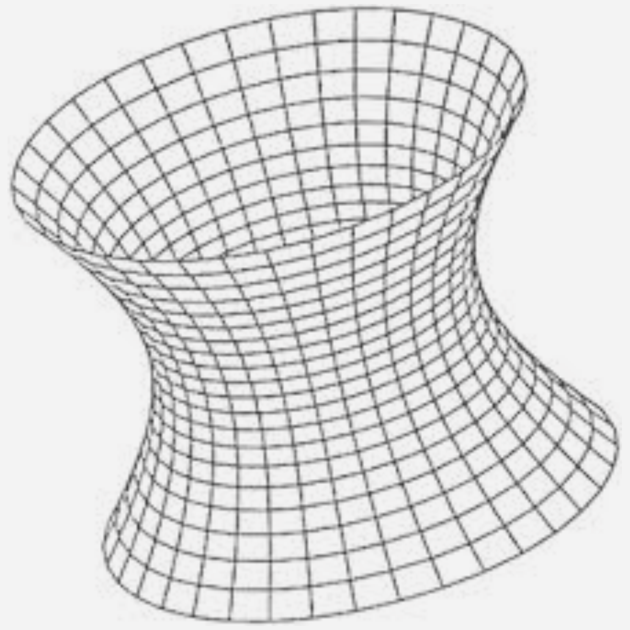






$X_1^2 + X_2^2 + \cdots + X_6^2 = R^2$

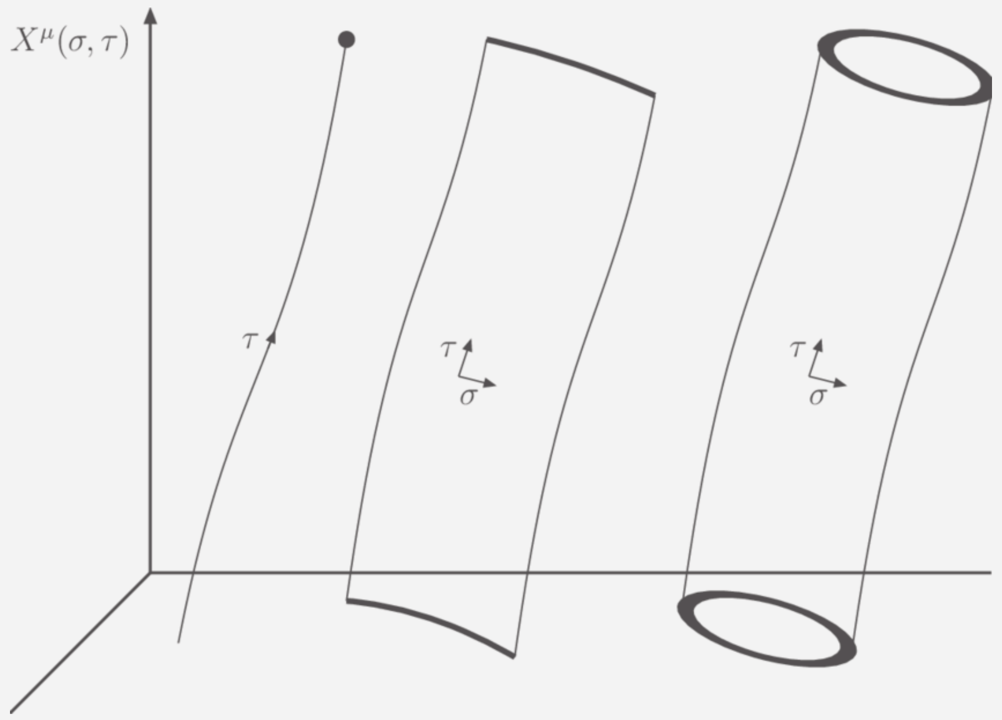
$-Y_1^2 - Y_2^2 + \cdots + Y_5^2 = -R^2$

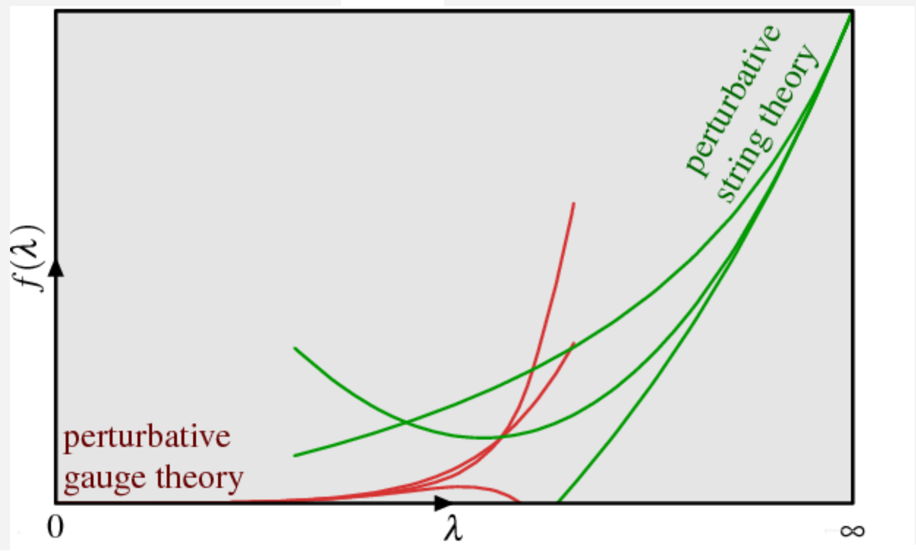


String action

$$\sqrt{\lambda} \int d\tau d\sigma \, \mathrm{Area}(\tau, \sigma)$$

$$E = E_\infty + \frac{1}{\sqrt{\lambda}} E_{\frac{1}{2}} + \dots$$





Maximally supersymmetric 4D SU(N) gauge theory

$$\frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \overline{\Psi} \not{D} \Psi + V \right]$$

1 gauge boson, 8 fermion
6 scalar

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \overline{\Psi} [\Phi, \Psi]$$

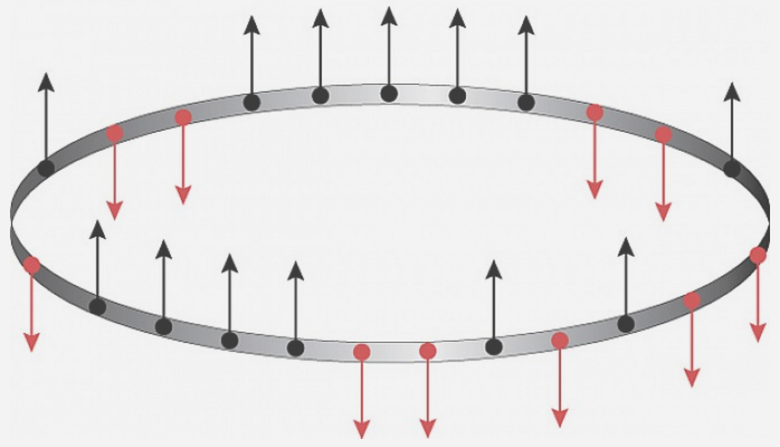
$$\langle \mathcal{O}_i(x) \mathcal{O}_j(0) \rangle = \frac{\delta_{ij}}{x^{2\Delta(\lambda)}}$$

$$\mathcal{O} = \mathrm{Tr}(Z^J)$$

$$Z = \Phi_1 + i\Phi_2$$
$$X = \Phi_3 + i\Phi_4$$

$$\mathcal{O} = \mathrm{Tr}(Z^{J-M} X^M) \quad |\uparrow\uparrow \cdot \downarrow\downarrow\rangle$$

Integrable spin chain

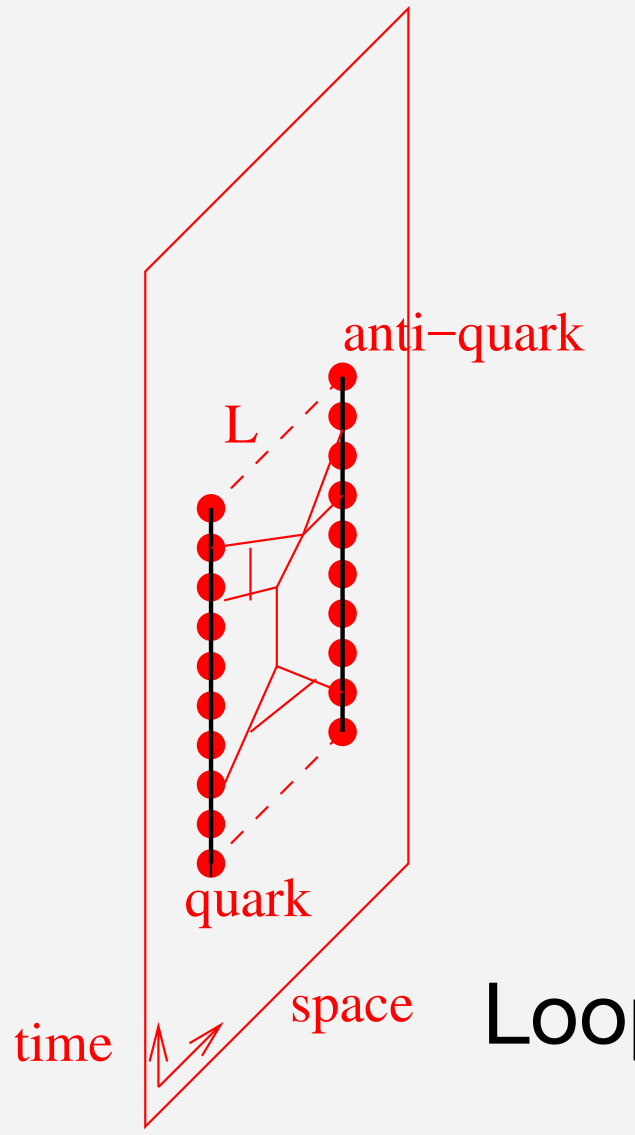


$$\lambda = g_{YM}^2 N$$

$$E = E_0 + \lambda E_1 + \dots$$

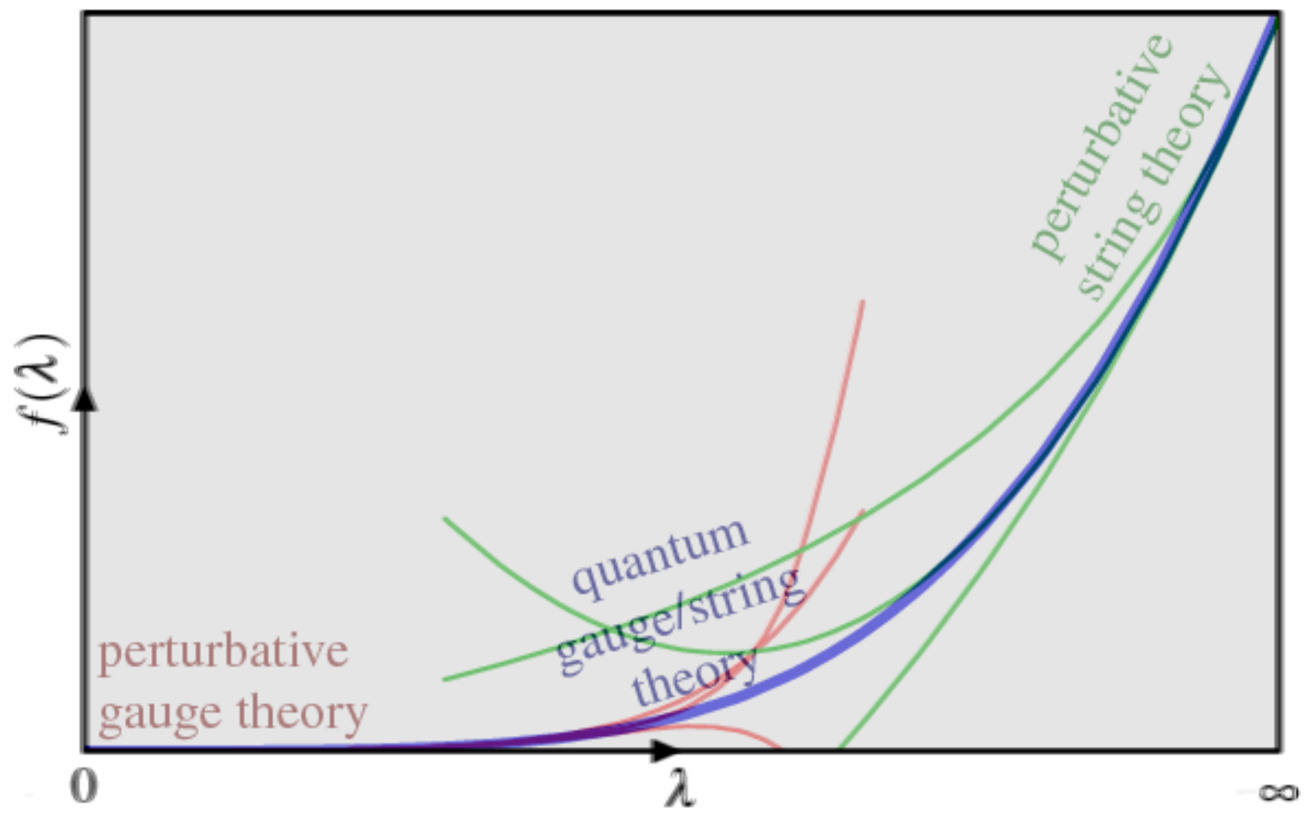
Triality

Gauge theory

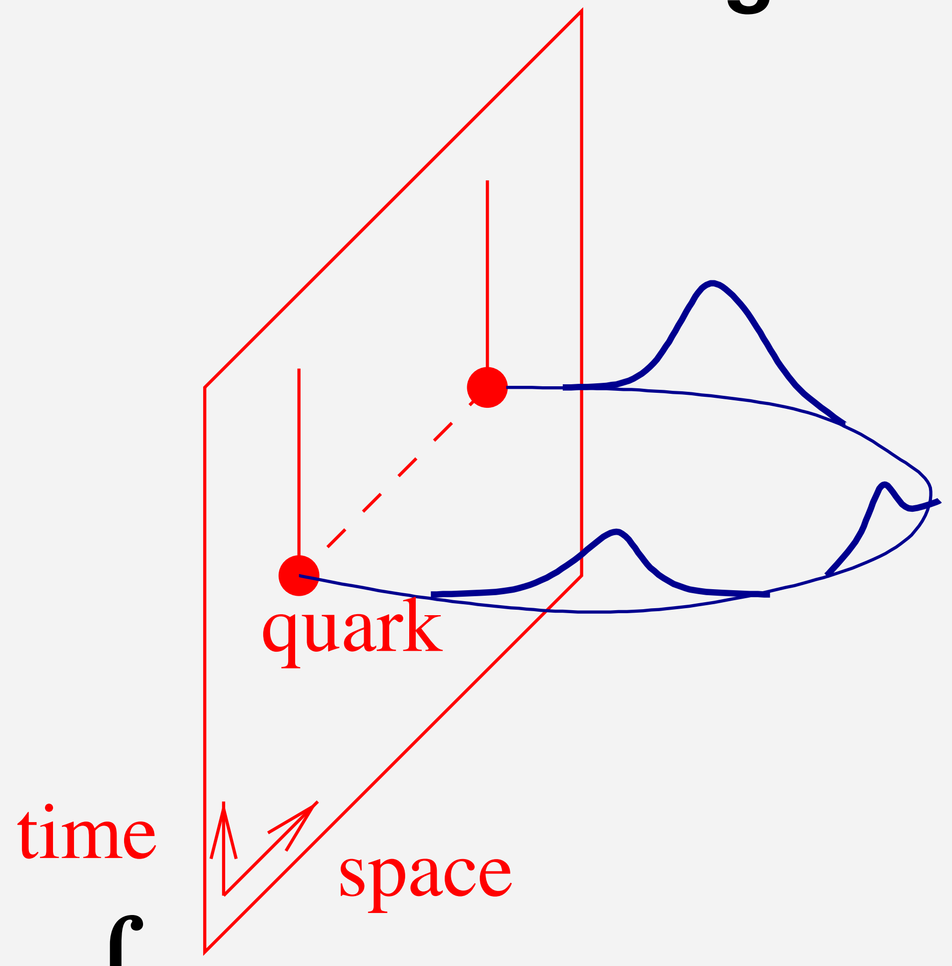


Loop expansion

$$V(L) = \frac{-\lambda}{4\pi L} + \dots$$



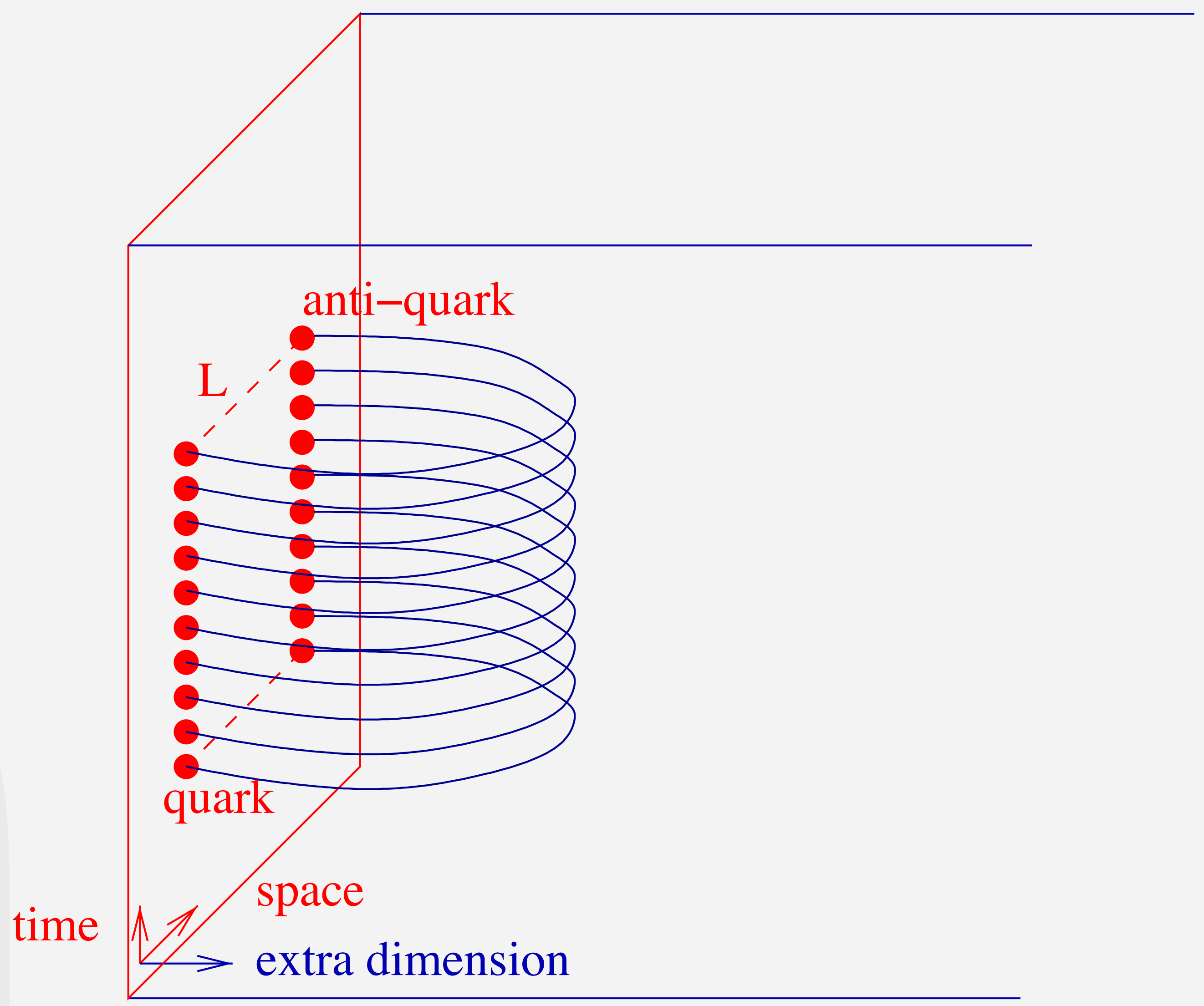
1+1 dimensional integrable theory



$$V(L) = L^{-1}\int dp\,\log(1 - R(p)\bar{R}(p)e^{-\epsilon(p)})$$

Exact ground state energy

String theory

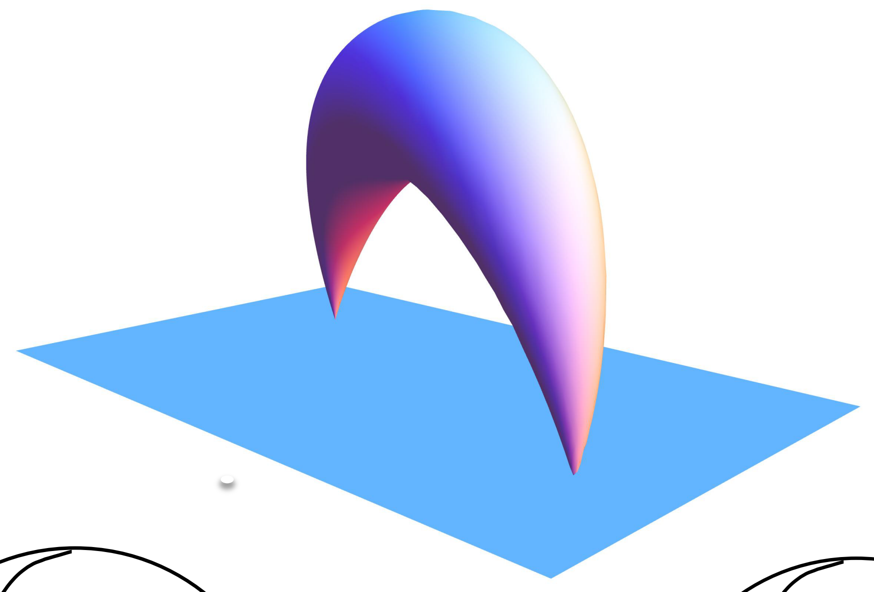


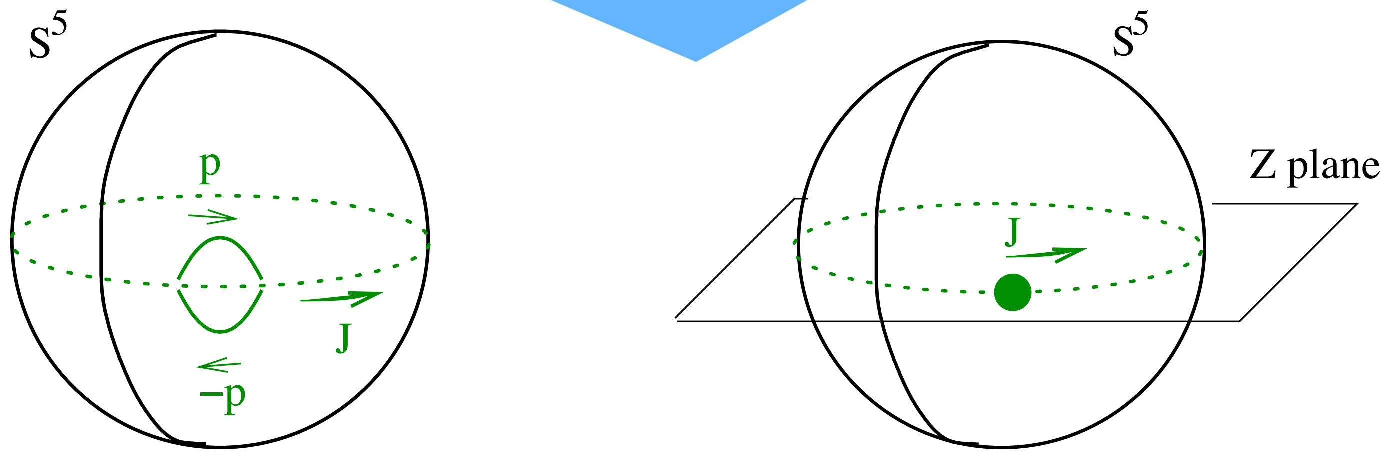
$$V(L) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{L}\left(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots\right)$$

Minimal surface | Fluctuation

[Correa et al 2012]
[Drukker 2012]
[Bajnok et al 2014]

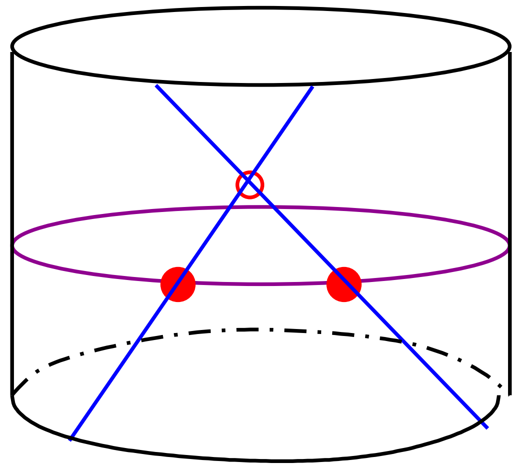
AdS/IM/CFT





Finite volume energy
sevels of multi particle states

$$e^{ipJ}S(p,-p)=1$$

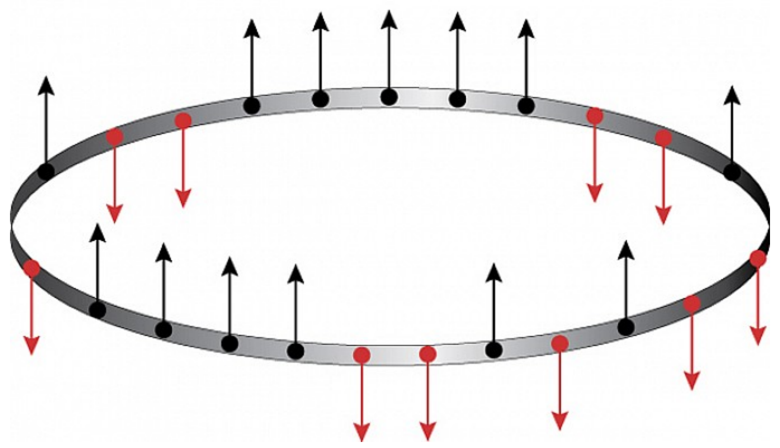


$$E(J)=2E(p)+\delta E(p)$$

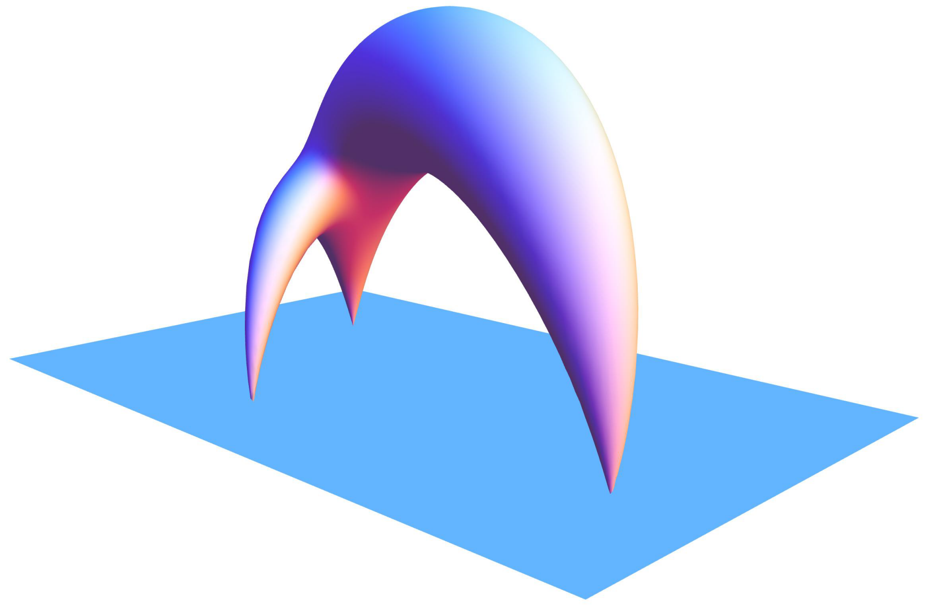
$$\delta E(p)=\int dqS(q,p)S(q,-p)e^{-\epsilon(q)}$$

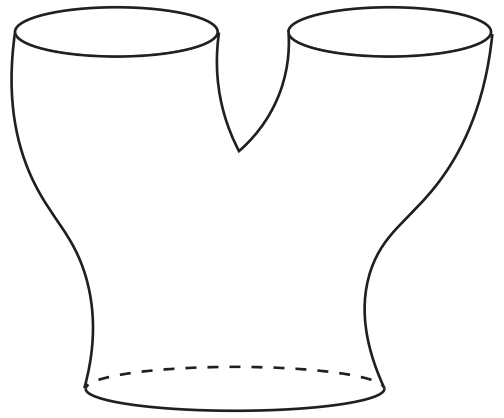
2pt functions
spin chain spectrum

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle=\frac{\delta_{ij}}{x^{2\Delta(\lambda)}}$$



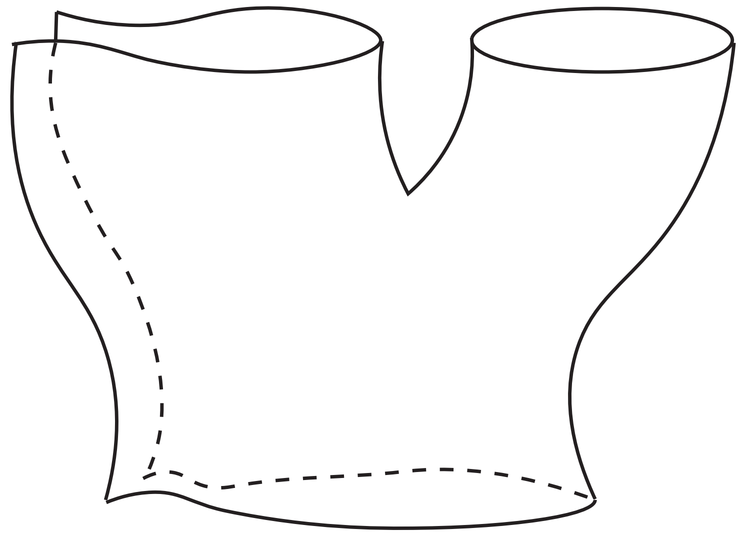
$$\mathcal{O}=\mathrm{Tr}(Z^{J-M}X^M)$$

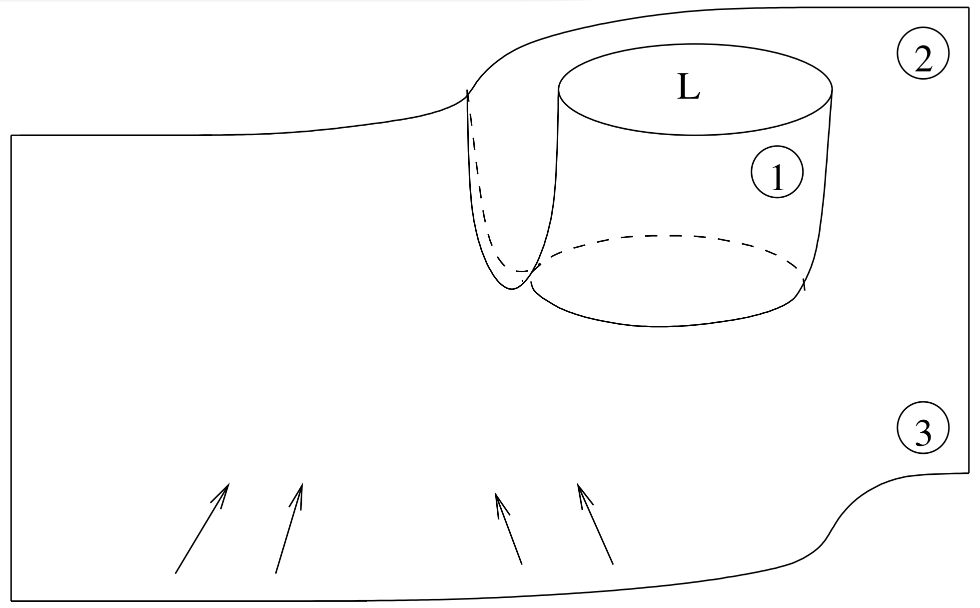




Finite volume formfactor
of a nonlocal operators

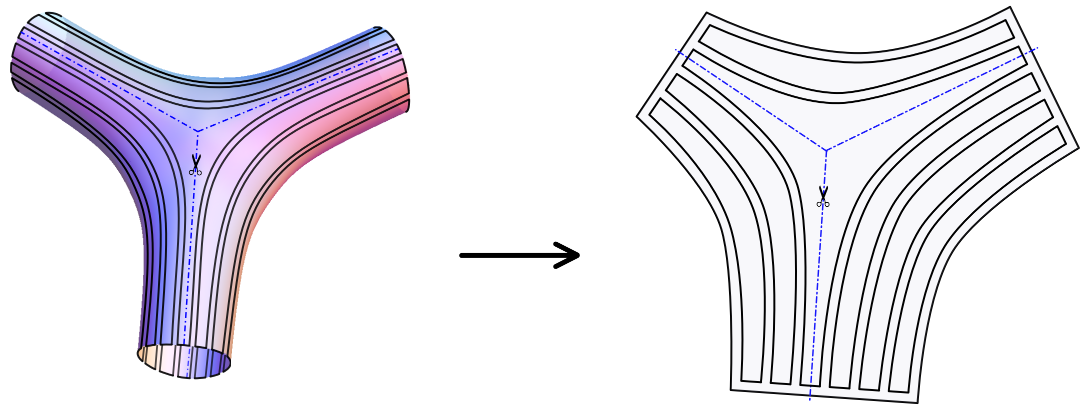
[Bajnok, Janik 2015]





3pt functions
spin chain overlaps

$$\langle \mathcal{O}_i\mathcal{O}_j\mathcal{O}_k\rangle=C_{ijk}(\lambda)$$

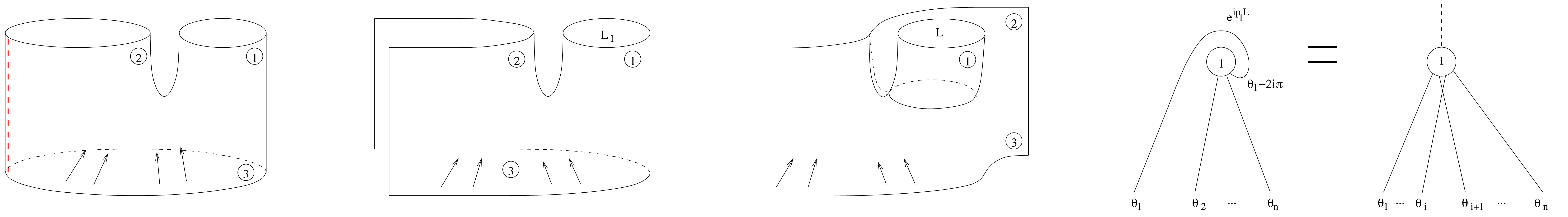


Finite volume form factor from hexagons

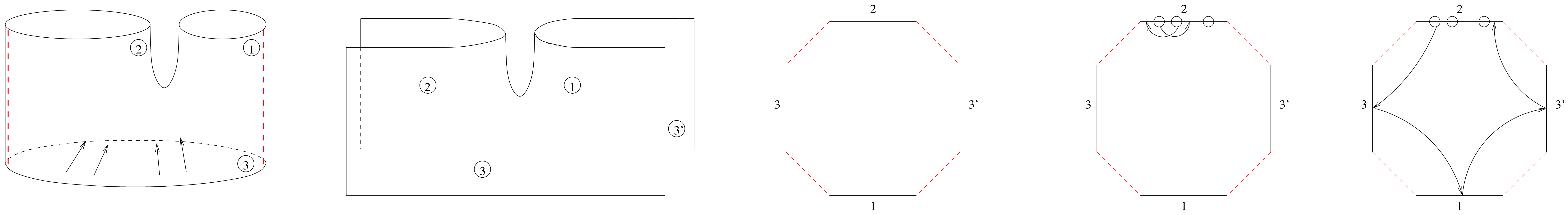
[Basso, Komatsu, Vieira 2015]

[Bajnok, Janik 2015]

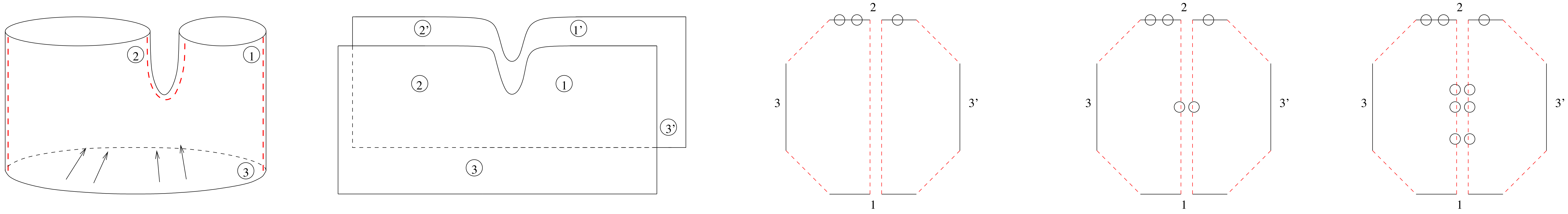
one cut:
nonlocal
form factor



two cuts:
octagon
form factor



Three cuts:
hexagon
form factor



Gluing and wrapping: insert a complete system of mirror basis, but kinematical singularities!

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S-matrix bootstrap

Description at infinite volume

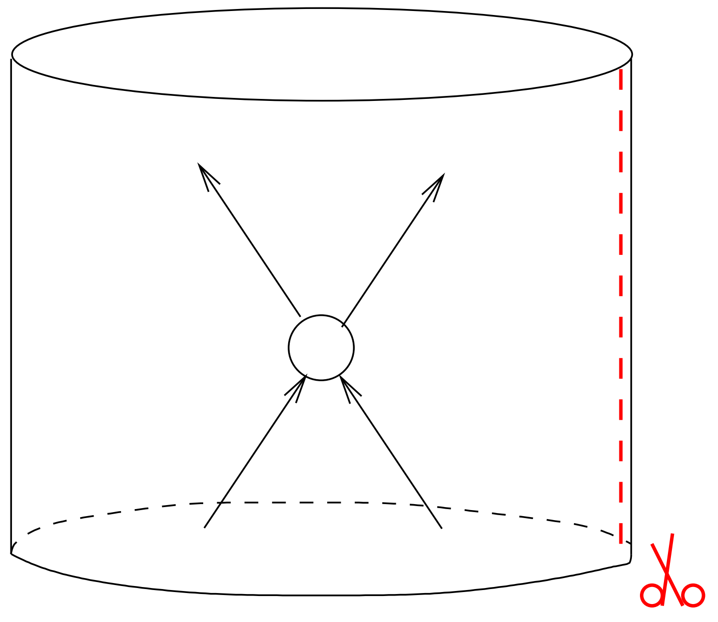
[Zamolochikovs1979]

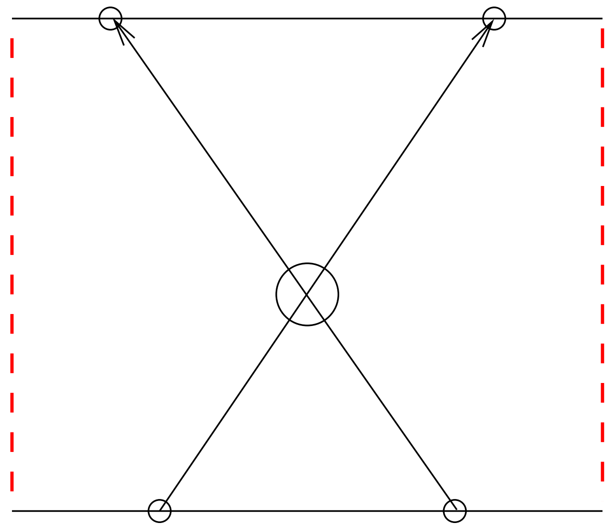
Sinh-Gordon theory

1+1dimensional scalar theory $\mathscr{L} = \frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2}(\partial_x \varphi)^2 - V(\varphi)$

$$V(\varphi) = \frac{m^2}{b^2}\left(\cosh b\varphi - 1\right)$$

Scattering matrix from Decompactification





$$\mathscr{D}_j = i \int d^2 x_j e^{i p_j x - i\omega_j t} \left\{ \partial_t^2 - \partial_x^2 + m^2 \right\}$$

Relativistic invariance

$$p = m \sinh \theta$$

LSZ reduction formula

$$\langle p_1', p_2' \,|\, \mathcal{O} \,|\, p_1, p_2 \rangle = \bar{\mathscr{D}}_1' \bar{\mathscr{D}}_2' \mathscr{D}_1 \mathscr{D}_2 \langle 0 \,|\, T(\mathcal{O}\varphi(1)\varphi(2)\varphi(3)\varphi(4)) \,|\, 0 \rangle = S(\theta_1 - \theta_2)$$

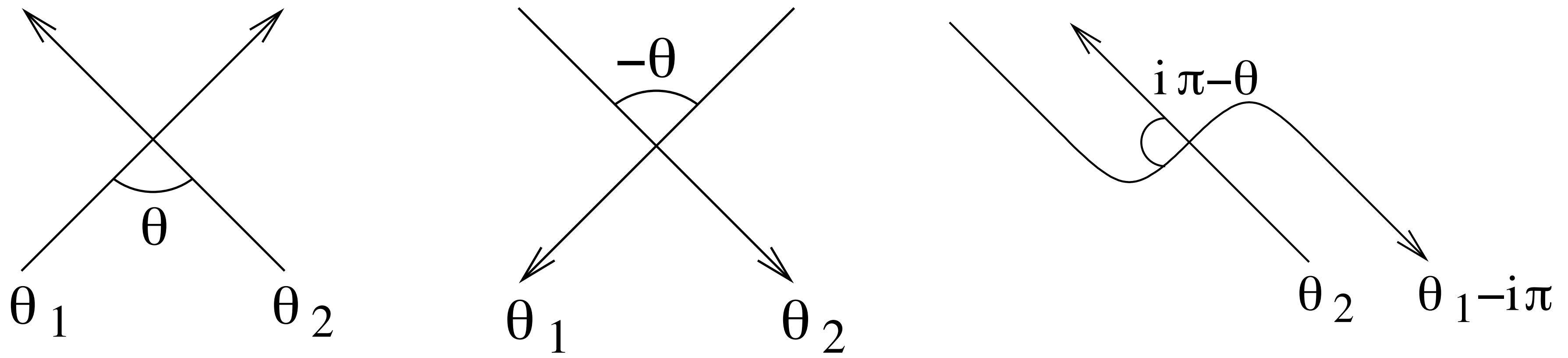
for $\mathcal{O} = \mathbb{1}$

Perturbative results

$$S(\theta) = 1 - \frac{i b^2}{4 \sinh \theta} - \frac{b^4(\theta(\pi/\sinh\theta - i))}{32\pi \sinh \theta} + \frac{i b^6 (\pi/\sinh\theta - i)^2}{256 \pi^2 \sinh\theta} + O\left(b^8\right)$$

[Arefyeva et al 1974]

Analytical properties



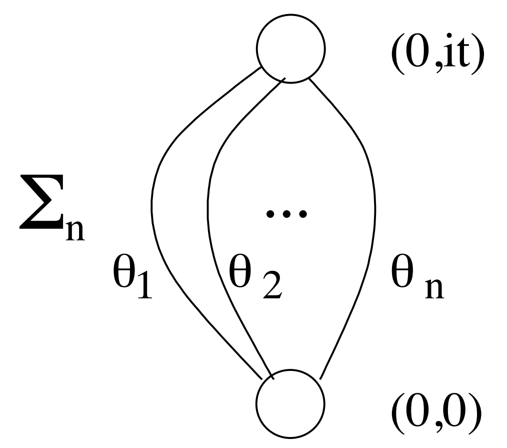
$$S(\theta) = S(-\theta)^{-1} = S(i\pi - \theta)$$

Solutions

$$S(\theta) = \frac{\sinh\theta - i \sin \pi a}{\sinh\theta + i \sin \pi a} \qquad a = \frac{b^2}{8\pi + b^2}$$

Form factor bootstrap

[Smirnov 1992, Karowski et al 1979]

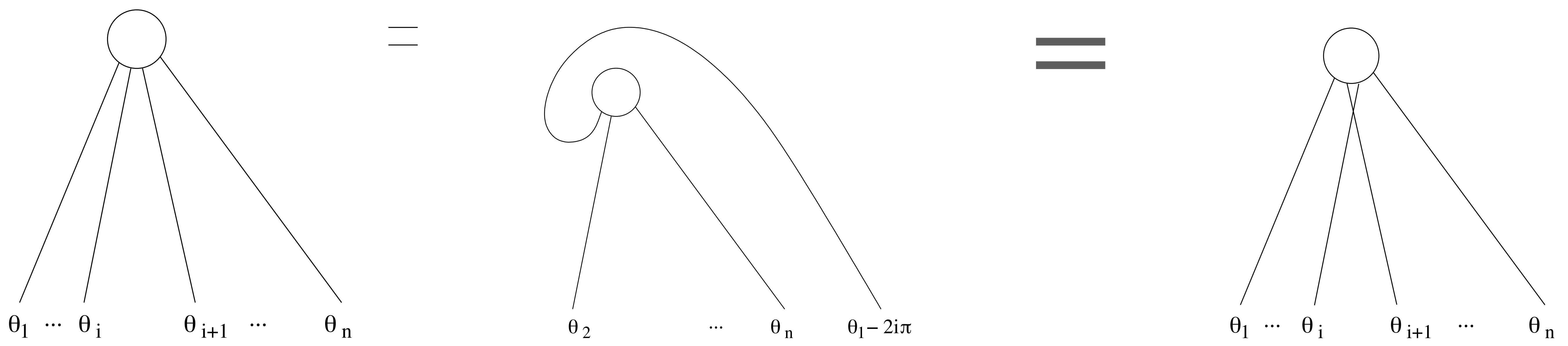


Two point function

$$\langle 0|\mathcal{O}(it)\mathcal{O}(0)|0\rangle = \sum_n \frac{1}{n!}\int\frac{d\theta_1}{2\pi}\cdots\int\frac{d\theta_n}{2\pi}\,|\langle 0|\mathcal{O}(0)|\theta_1,\ldots,\theta_n\rangle|^2 e^{-m(\sum_i \cosh\theta_i)t}$$

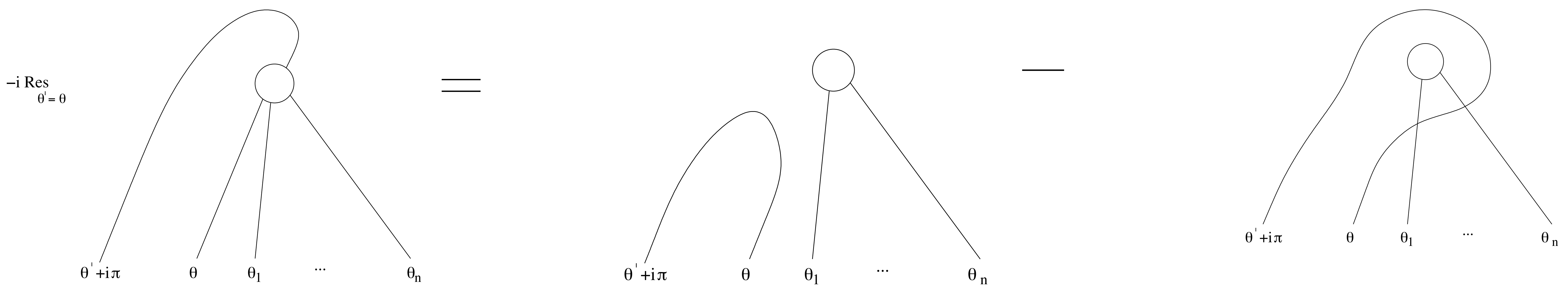
Form factors

From LSZ reduction formula



$$\langle 0|\mathcal{O}|\theta_1,\ldots,\theta_n\rangle = \langle 0|\mathcal{O}|\theta_2,\ldots,\theta_n,\theta_1-2i\pi\rangle = S(\theta_i-\theta_{i+1})\langle 0|\mathcal{O}|\ldots,\theta_{i+1},\theta_i,\ldots\rangle$$

Analytical structure



$$-i\mathrm{Res}_{\theta'=\theta}\langle 0|\mathcal{O}|\theta'+i\pi,\theta,\theta_1,\ldots,\theta_n\rangle = (1-\prod_i S(\theta-\theta_i))\langle 0|\mathcal{O}|\theta_1,\ldots,\theta_n\rangle$$

Form factor solutions

minimal solutions $f(\theta_1, \theta_2) = \langle 0 | \mathcal{O} | \theta_1, \theta_2 \rangle = e^{(D+D^{-1})^{-1} \log S}$ $Df(\theta) = f(\theta + i\pi)$

Generic solution

$$\langle 0 | \mathcal{O} | \theta_1, \ldots, \theta_n \rangle = H_n \prod_{i<j} \frac{f(\theta_i - \theta_j)}{e^{\theta_i} + e^{\theta_j}} Q^{\mathcal{O}}(e^{\theta_1}, \ldots, e^{\theta_n})$$

[Fring et al 1992]

Diagonal limit

$$\langle \theta | \mathcal{O} | \theta_1, \ldots, \theta_n \rangle = \sum_k \delta(\theta - \theta_k) \prod_{i=1}^{k} S(\theta_i - \theta_k) \rangle \langle 0 | \mathcal{O} | \theta_1, \ldots, \hat{\theta}_k, \ldots, \theta_n \rangle$$
$$+ \langle 0 | \mathcal{O} | \theta + i\pi - i\epsilon, \theta_1, \ldots, \theta_n \rangle$$

Connected form factor

$$\langle \theta_1 + \epsilon_1, \ldots, \theta_n + \epsilon_n | \mathcal{O} | \theta_n, \ldots, \theta_1 \rangle = \text{function of } \epsilon_i / \epsilon_j + F_c(\theta_1, \ldots, \theta_n) + O(\epsilon)$$

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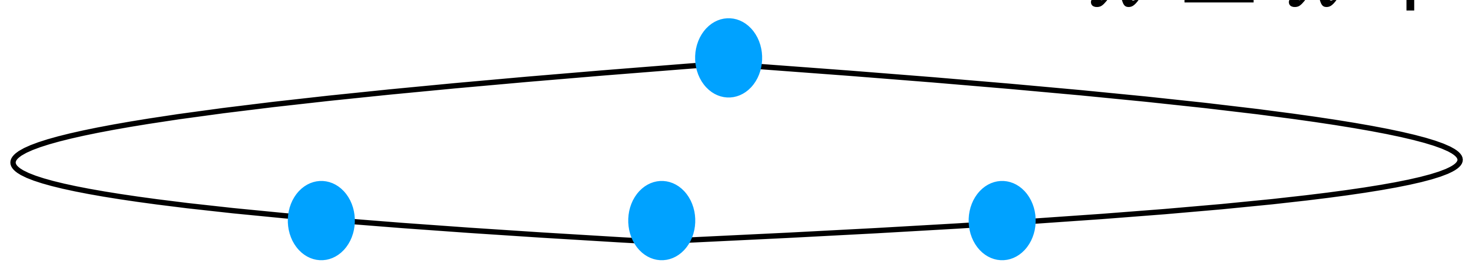
Large volume spectrum

Bethe Yang equations

multiparticle state on the circle
momentum quantization

$$p = m \sinh \theta$$

$$x \equiv x + L$$

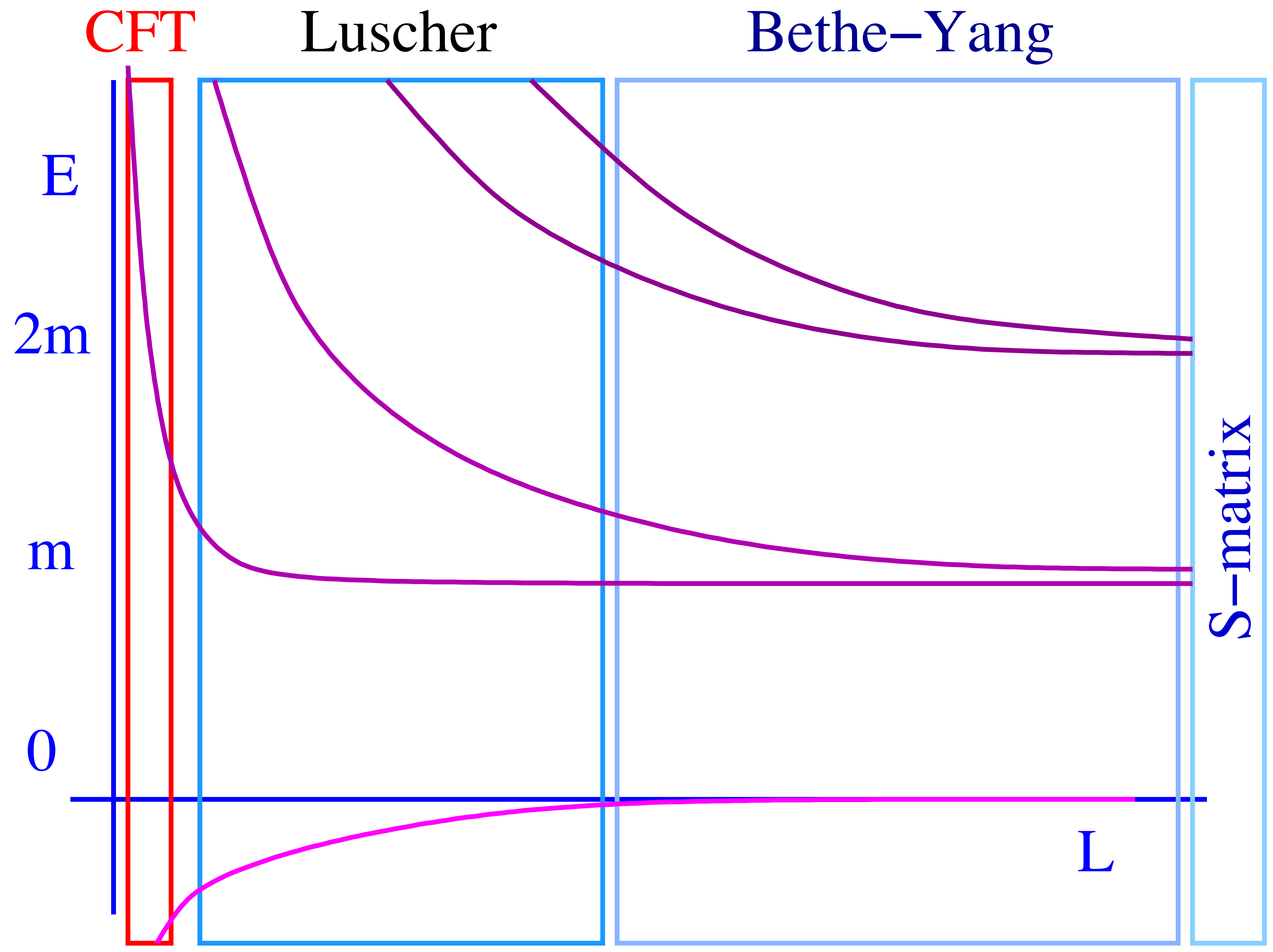


Periodicity of wave function

$$e^{imL \sinh \theta_j} \prod_k S(\theta_j - \theta_k) = 1$$

$$\Phi_j = mL \sinh \theta_j - i \sum_{k:k\neq j} \log S(\theta_j - \theta_k) = 2\pi n_j$$

Finite volume state $\quad |\theta_1, \ldots, \theta_N\rangle_L \equiv |n_1, \ldots, n_N\rangle_L$



Total energy

$$E(\{\theta\}) = \sum_k m \cosh \theta_k$$

Large volume form factors

Polynomial correction

normalisation of infinite volume states $\langle \theta | \theta_k \rangle$

Crossing for form factors

$$\langle \theta | \mathcal{O} | \theta_1, \ldots, \theta_n \rangle = \sum_k \delta(\theta - \theta_k) \rangle \langle 0 | \mathcal{O} | \theta_1, \ldots, \hat{\theta}_k, \ldots, \theta_n \rangle + \langle 0 | \mathcal{O} | \theta + i\pi - i\epsilon, \theta_1, \ldots, \theta_n \rangle$$

Normalization of finite volume states $\langle n_i | n_k \rangle = \delta_{i,k}$

$$\rho = \det[\partial \Phi_i / \partial \theta_j]$$

identity $$1 + \sum_n | n \rangle \langle n | + \ldots = 1 + \int d\theta | \theta \rangle \langle \theta | + \ldots$$

Nondiagonal form factors $$\langle 0 | \mathcal{O} | \theta_1, \ldots, \theta_n \rangle_L = \frac{\langle 0 | \mathcal{O} | \theta_1, \ldots, \theta_n \rangle}{\sqrt{\rho(\theta_1, \ldots, \theta_n)}}$$ [Pozsgay, Takács 2008]

Diagonal form factors $$\langle \theta_n, \ldots, \theta_1 | \mathcal{O} | \theta_1, \ldots, \theta_n \rangle_L = \frac{\sum_{\alpha \cup \bar{\alpha} = \{1, \ldots, n\}} \rho(\bar{\alpha}) F_c^{\mathcal{O}}(\alpha)}{\rho(\theta_1, \ldots, \theta_n)}$$

[Saleur 2000] [Pozsgay, Takács 2008] [Bajnok, Wu 2017]

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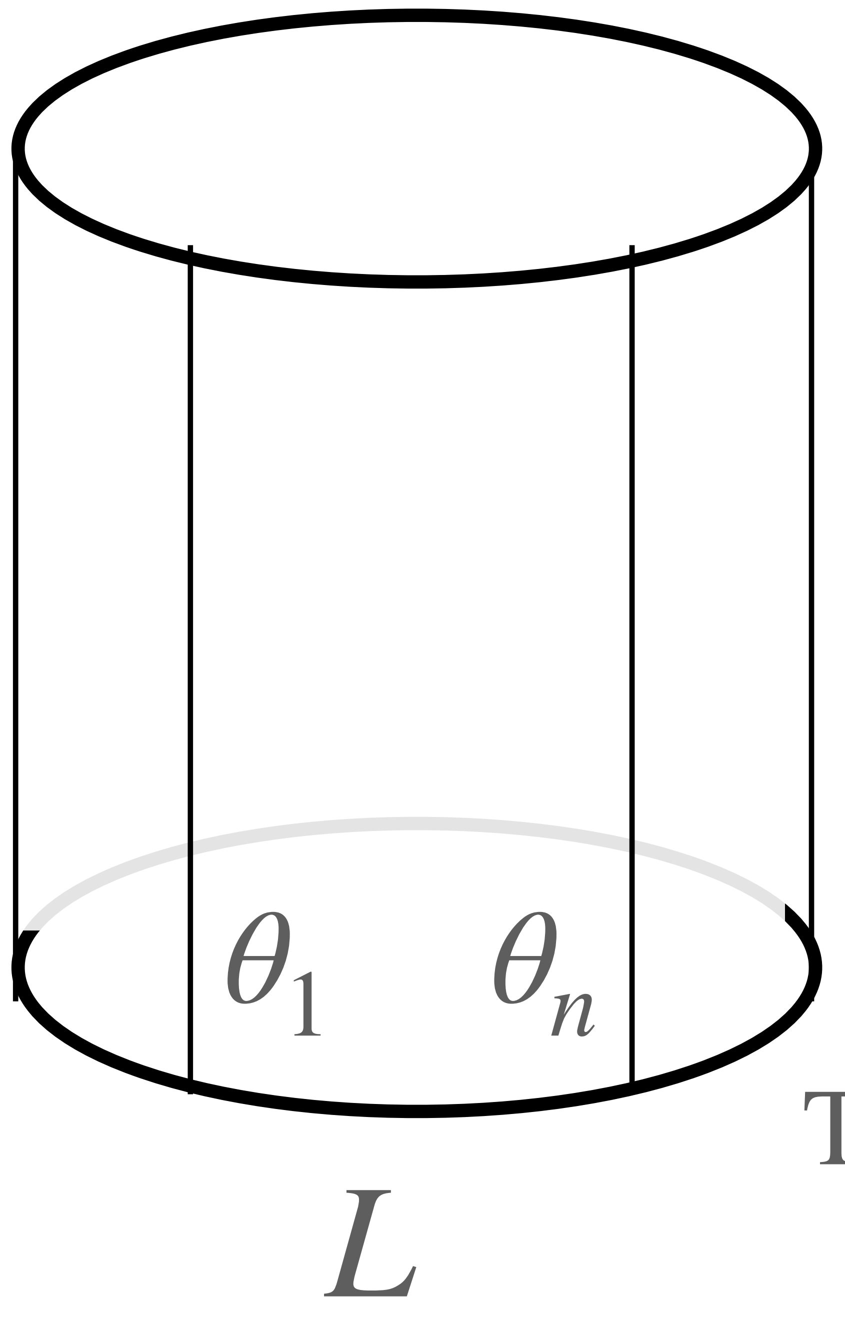
Exponential volume corrections: spectrum

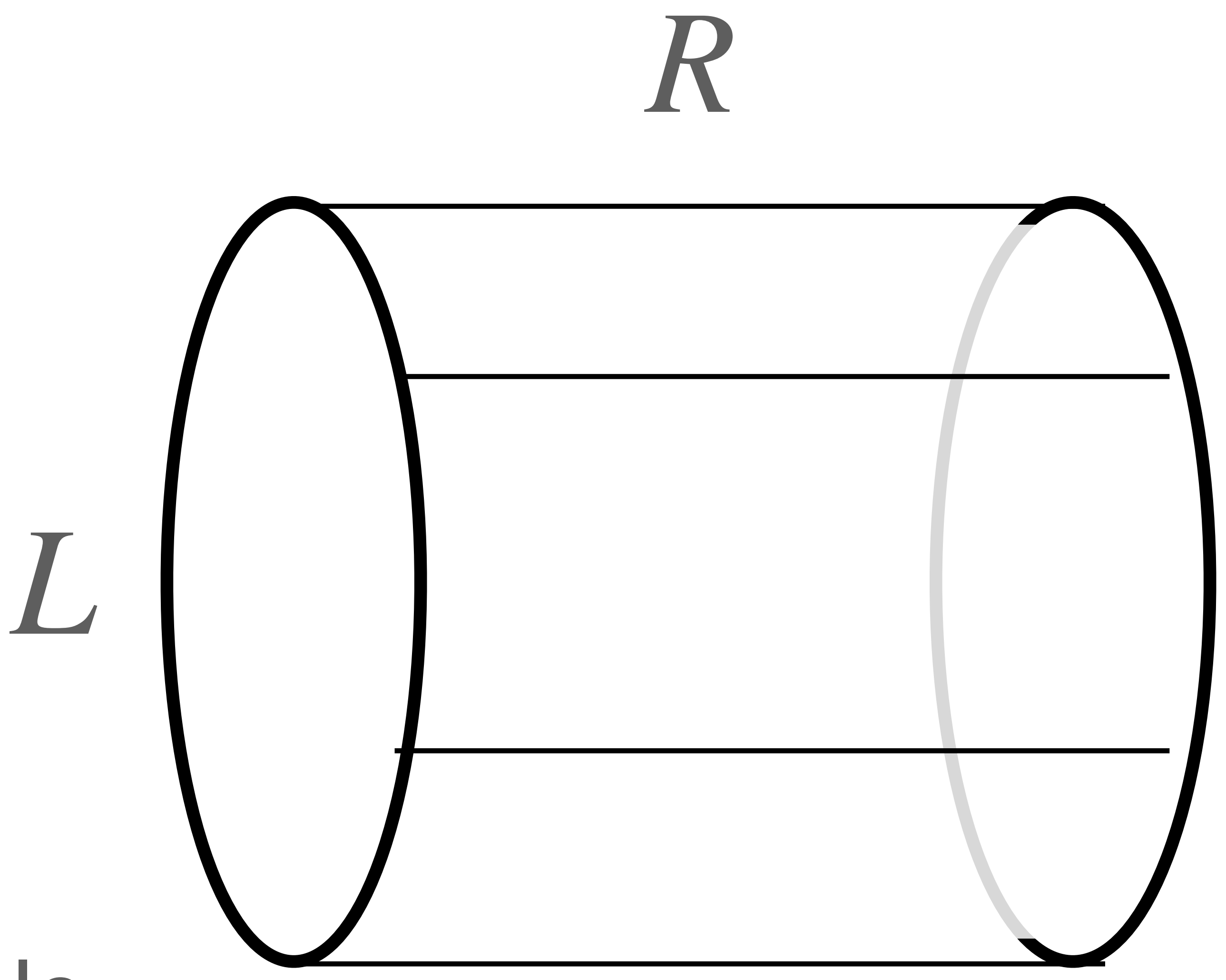
[Lüscher 1982]



$R \to \infty$

Lowest state survives

$$\mathrm{Tr}(e^{-H(L)R}) \to e^{-E_n(L)R}$$



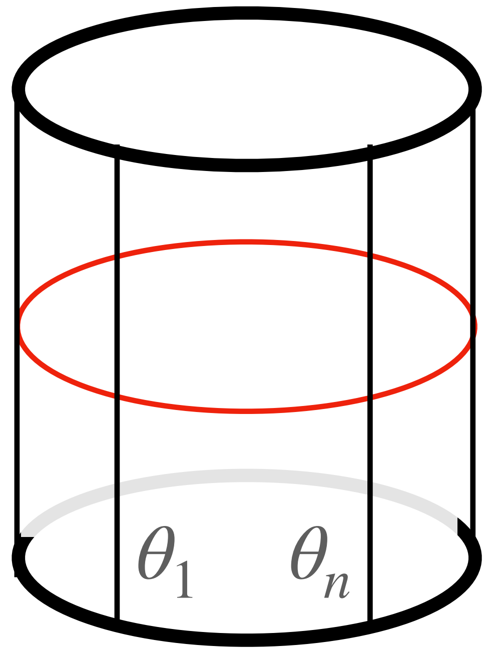
Bethe Yang reliable

$$\mathrm{Tr}(e^{-H(R)L}D_n) = \sum_{|n\rangle \in \mathcal{H}} e^{E_{|n\rangle}(R)L}\langle n | D_n | n \rangle = 1 + \sum_{\theta} e^{-m\cosh\theta(R)L}\prod_{i=1}^{n} S(\theta - \theta_i + i\pi/2)$$

Virtual particle's contribution

$$E_n(L) = \sum_k m\cosh\theta_k - m\int \frac{d\theta}{2\pi}\cosh\theta \prod_{i=1}^{n} S(\theta - \theta_i + i\pi/2)e^{-mL\cosh\theta}$$

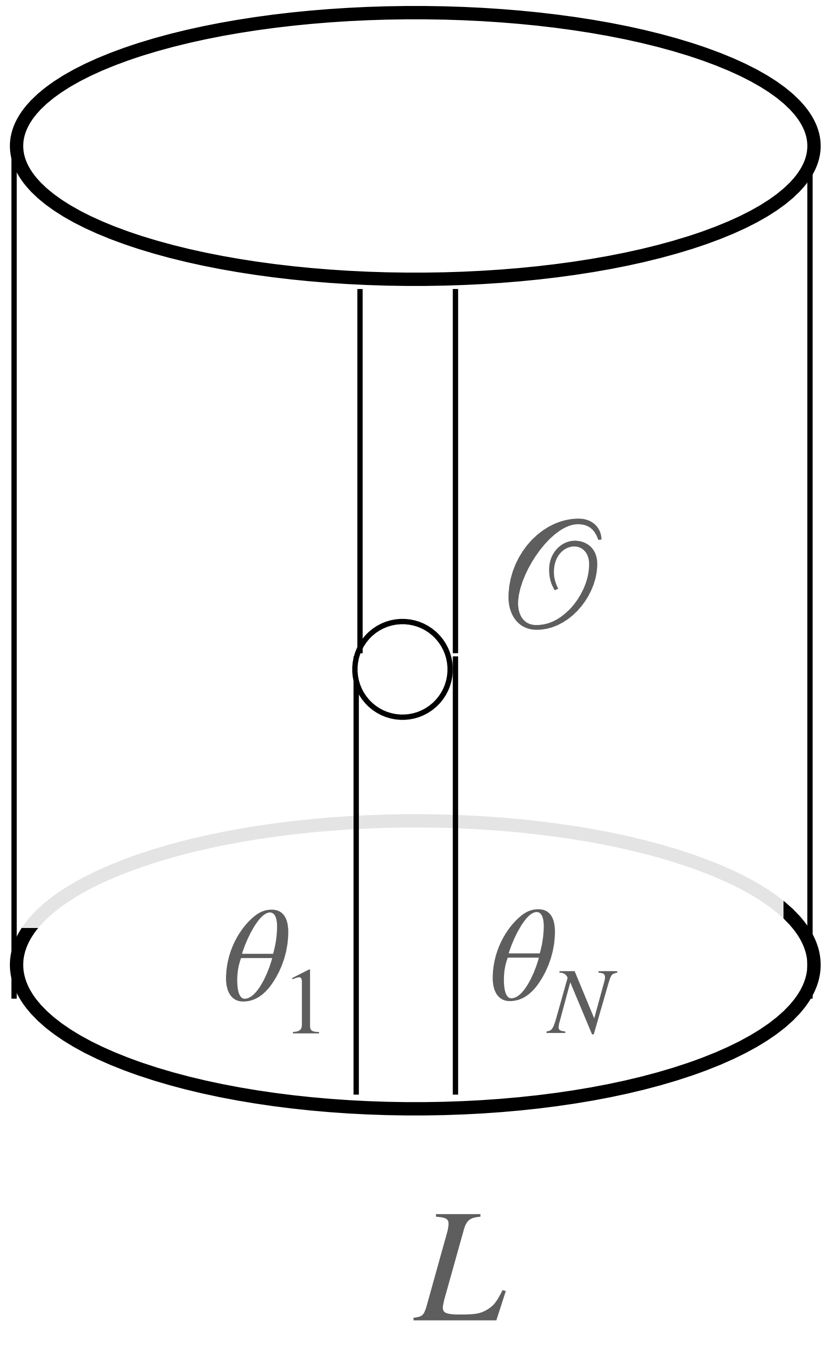
[Bajnok, Janik 2008]



$$\Phi_j = mL\sinh\theta_j - i\sum_{k:k\neq j}\log S(\theta_j - \theta_k) - \int \frac{d\theta}{2\pi}K(\theta_j + i\pi/2 - \theta)\prod_{i=1}^{n} S(\theta - \theta_i + i\pi/2)e^{-mL\cosh\theta} = 2\pi n_j$$

$$K(\theta) = -i\partial_\theta \log S(\theta)$$

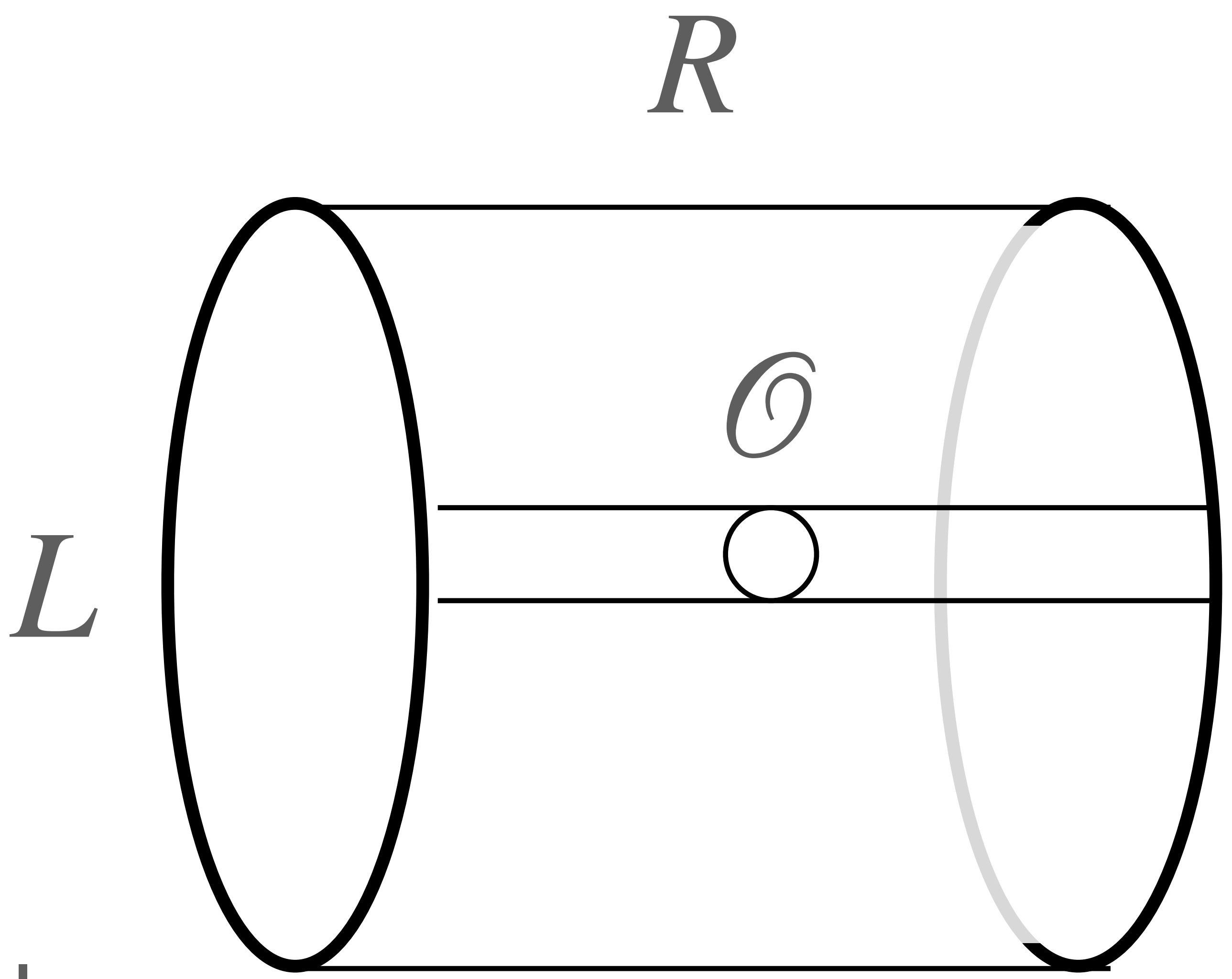
Exponential corrections: diagonal form factors



$R \to \infty$

Lowest state survives

$$\langle \theta_1, \ldots, \theta_n | \mathcal{O} | \theta_n, \ldots, \theta_1 \rangle_L e^{-E_n(L)R}$$

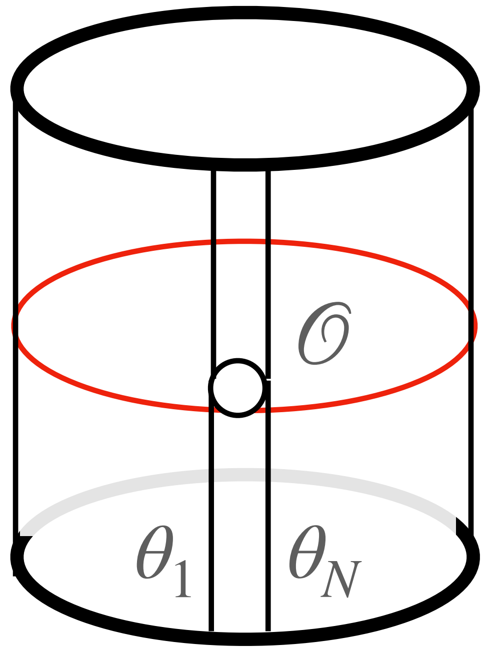


Bethe Yang reliable

$$\sum_{|n\rangle \in \mathcal{H}} e^{E_{|n\rangle}(R)L} \langle n | \mathcal{O}_n | n \rangle_R = 1 + \sum_{\theta} e^{-m\cosh\theta(R)L} \langle \theta | \mathcal{O}_n | \theta \rangle_R + \ldots$$

Virtual particle's contribution

$$\langle \theta_n, \ldots, \theta_1 | \mathcal{O} | \theta_1, \ldots, \theta_n \rangle_L = \frac{\sum_{\alpha \cup \bar{\alpha} = \{1,\ldots,n\}} \rho(\bar{\alpha}) \mathcal{D}_c^{\mathcal{O}}(\alpha)}{\rho(\theta_1, \ldots, \theta_n)}$$



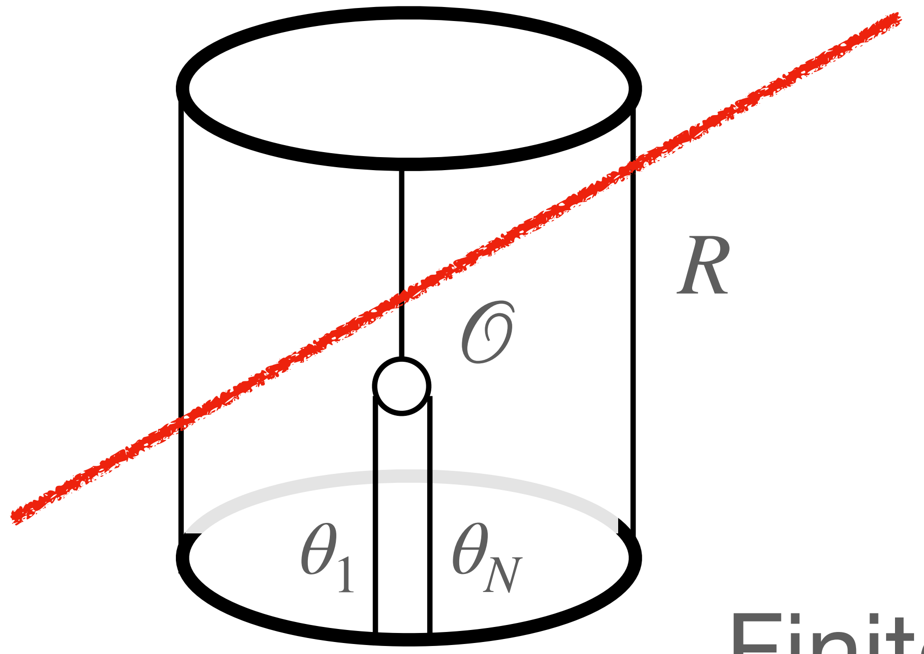
$$\mathcal{D}_c^{\mathcal{O}}(\alpha) = F_c^{\mathcal{O}}(\alpha) - \int d\theta F_c^{\mathcal{O}}(\alpha + i\pi/2, \theta) \prod_{i=1}^{n} S(\theta - \theta_i + i\pi/2) e^{-mL\cosh\theta}$$

[Pozsgay 2013]

Exponential corrections: non-diagonal case

[Bajnok et al 2018]

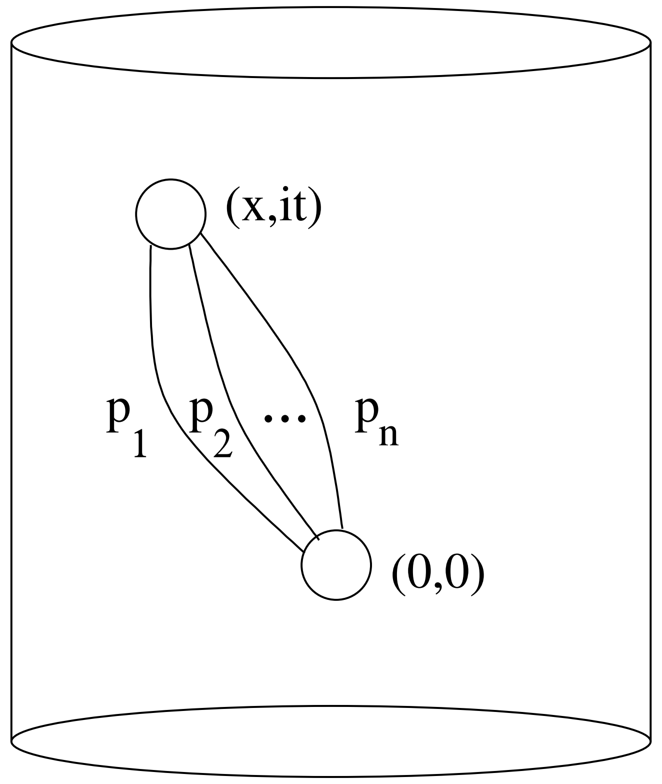
Main problem



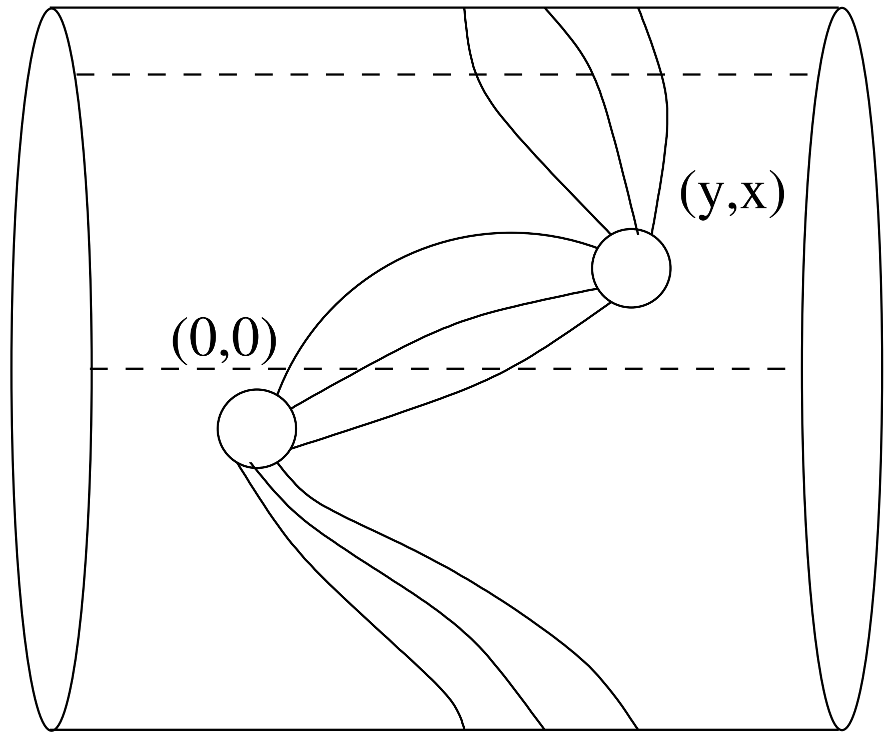
Two point function

$$\frac{1}{L}\int_{-L/2}^{L/2} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{i(\omega t+qx)}\langle \mathcal{O}(x,t)\mathcal{O}\rangle_L =$$

$$\sum_N |\langle 0|\mathcal{O}|\theta_1,\ldots,\theta_N\rangle_L|^2 \left\{ \frac{\delta_{q-P_N(L)}}{E_N(L)-i\omega} + \frac{\delta_{q+P_N(L)}}{E_N(L)+i\omega} \right\}$$



Finite volume LSZ $\lim_{\omega\to iE_N(L)} (E_N(L)+i\omega)\Gamma(\omega,P_N(L)) = |\langle 0|\mathcal{O}|\theta_1,\ldots,\theta_N\rangle_L|^2$



Temperature channel and continue $\omega \to iE_N(L)$

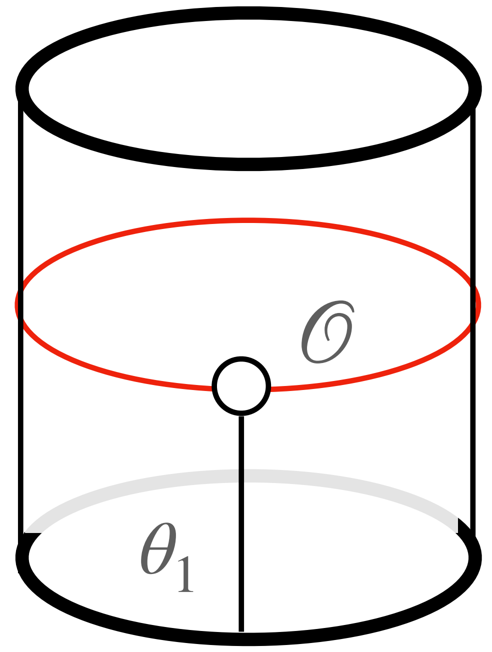
$$Z\Gamma(\omega,q) = \frac{2\pi}{L}\sum_{\mu,\nu} |\langle \nu|\mathcal{O}|\mu\rangle|^2 e^{-E_\nu L}\delta(P_\mu - P_\nu + \omega)\left\{ \frac{1}{E_\mu - E_\nu - iq} + \frac{1}{E_\mu - E_\nu + iq} \right\}$$

Energy corrections are correctly obtained $F_1 = \langle 0|\mathcal{O}|\theta\rangle$

Form factor corrections

$$\langle 0|\mathcal{O}|\theta_1\rangle_L = \sqrt{2\pi}/\sqrt{\rho_1^{(1)}}\left(F_1 + \int_{-\infty}^{\infty} d\theta\, F_3^{\mathrm{reg}}(\theta+i\pi,\theta,\theta_1 - i\frac{\pi}{2})e^{-mL\cosh\theta} + \ldots\right)$$

$$F_3^{\mathrm{reg}}(\theta,\theta_1,\theta_2) = \langle 0|\mathcal{O}|\theta,\theta_1,\theta_2\rangle - \frac{iF_1}{\theta-\theta_1-i\pi}\left[1 - S(\theta_1-\theta_2)\right] + i\frac{F_1}{2}S'(\theta_1-\theta_2)$$



Plan

Overview of last 5 years work

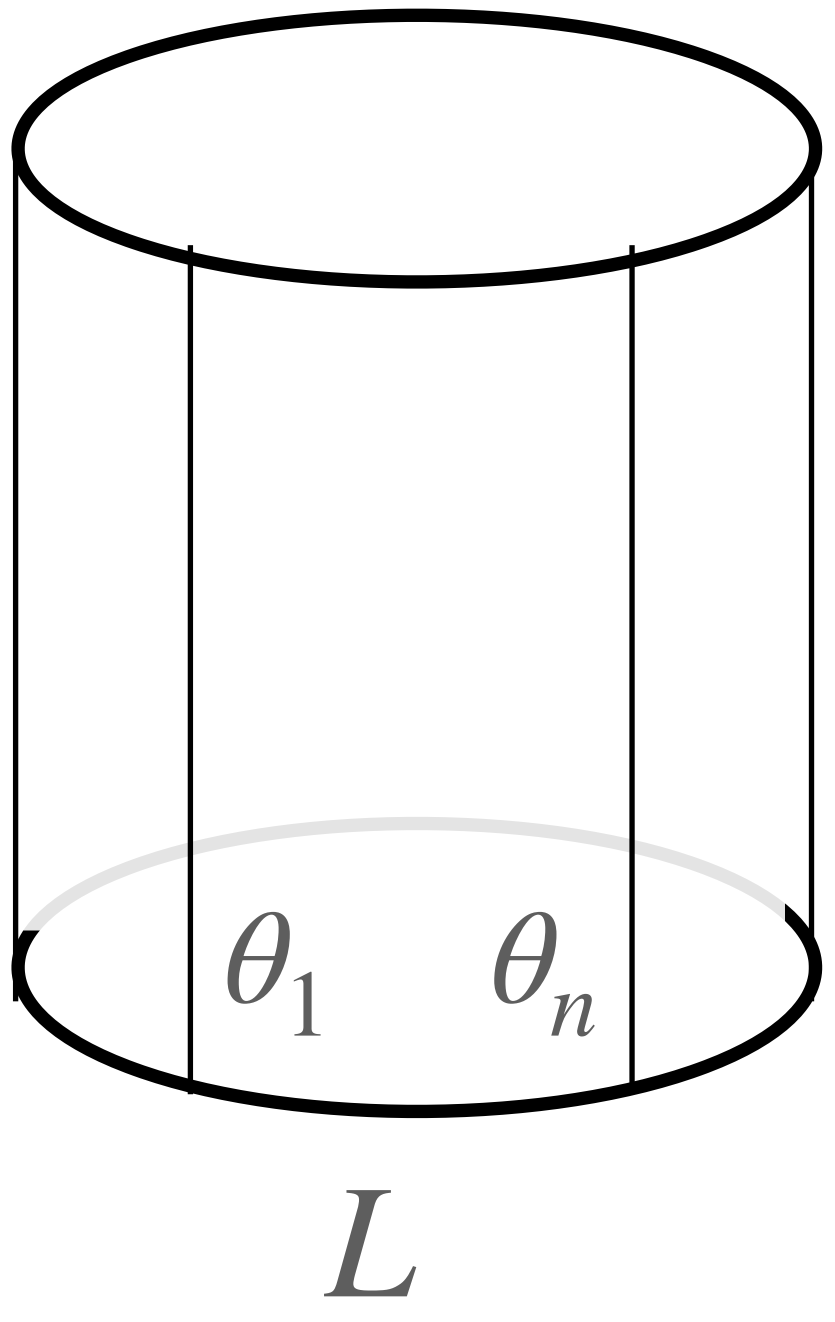
- Motivation: why finite volume form factors, AdS/CFT 3pt functions
- Finite volume corrections

$$x \equiv x + L$$

Small volume → Large volume

Energy spectrum	Exact description, Thermodynamic Bethe Ansatz	Lüscher correction of masses	Momentum quantisation, Bethe Yang equation	Masses, scatterings
Form Factors Correlators	Linear integral equations based on the TBA	Leading exponential volume corrections	Change in the normalisation of states	Form factors

Exact finite volume description: spectrum



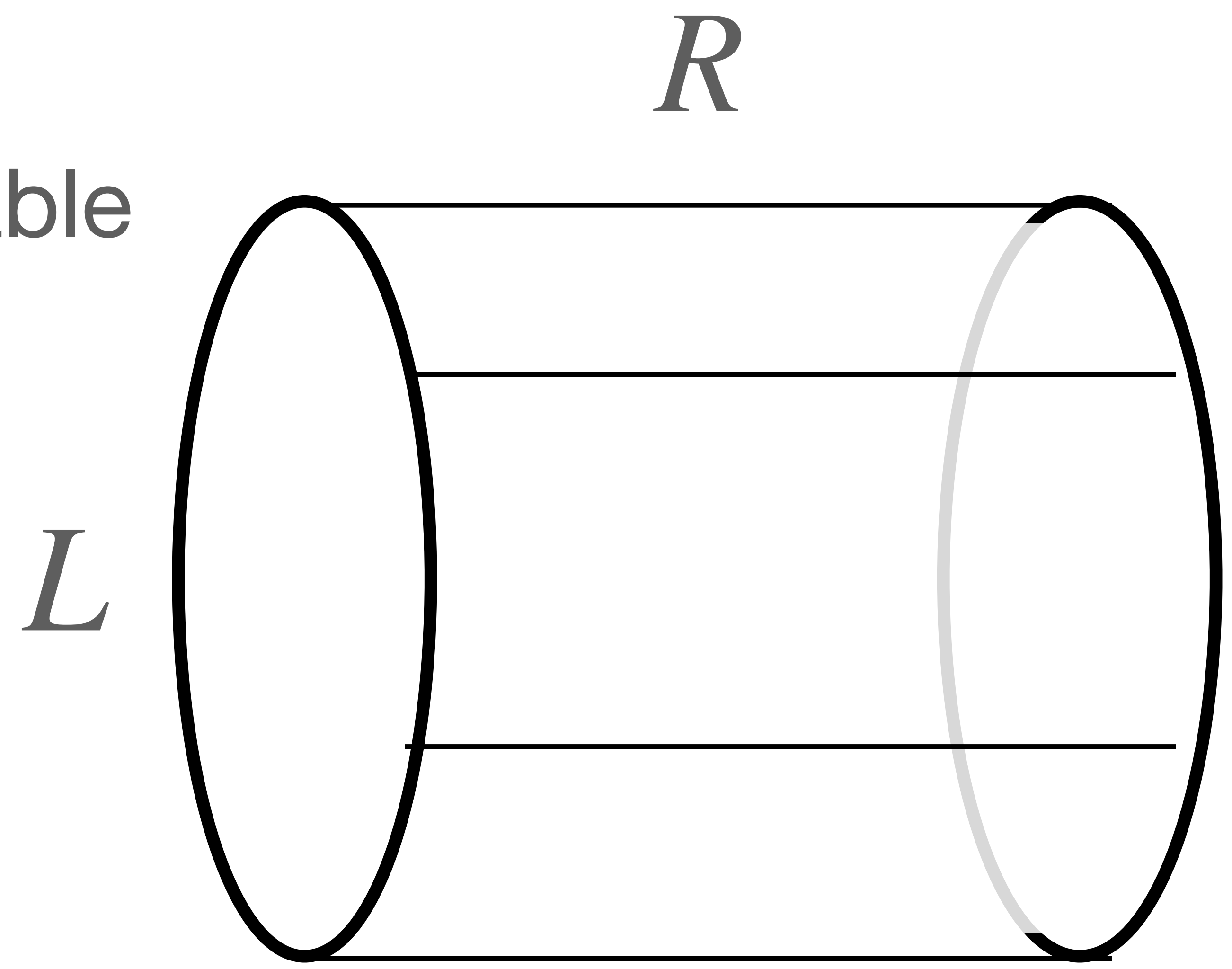
$R \to \infty$

Lowest state survives

$$\mathrm{Tr}(e^{-H(L)R}) \to e^{-E_n(L)R}$$

$$\mathrm{Tr}(e^{-H(R)L}D_n) = \sum_{|n\rangle \in \mathscr{H}} e^{E_{|n\rangle}(R)L}\langle n \,|\, D_n \,|\, n\rangle$$

Bethe Yang reliable



Sum up all terms

[Kostov et al 2017]

Calculate the dominant finite density state

[Zamolodchikov 1990]

Saddle point equation

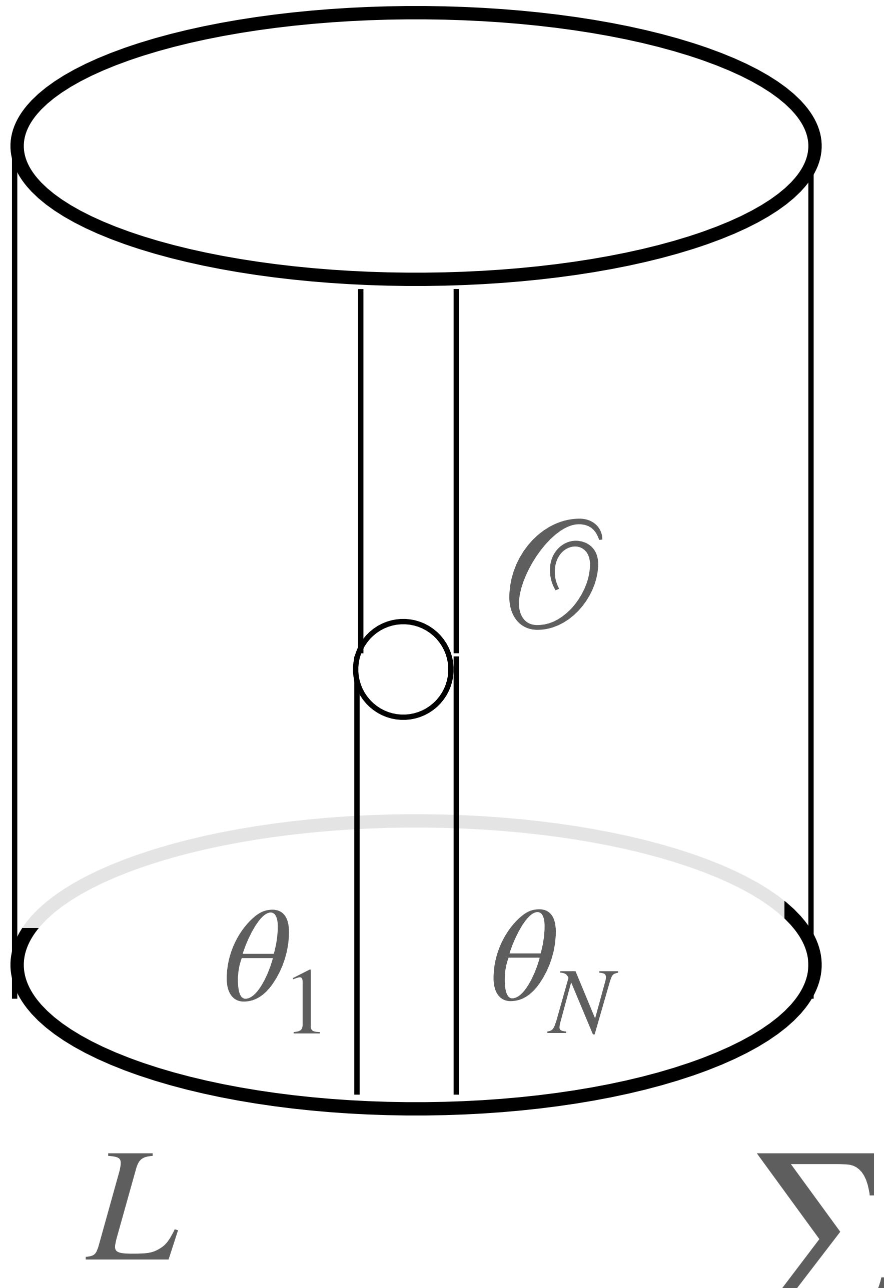
$$\epsilon(\theta) = mL\cosh\theta + \sum_k \log S(\theta - \theta_k - i\pi/2) - \int \frac{d\theta}{2\pi} K(\theta - \theta')\log(1 + e^{-\epsilon(\theta')})$$

[Dorey et al 1996]

$$E_n(L) = \sum_k m\cosh\theta_k - m\int \frac{d\theta}{2\pi}\cosh\theta\log(1 + e^{-\epsilon(\theta)})$$

$$-i\epsilon(\theta_k + i\pi/2) = \Phi_k = \pi(2n_k + 1)$$

Exact finite volume description: diagonal case

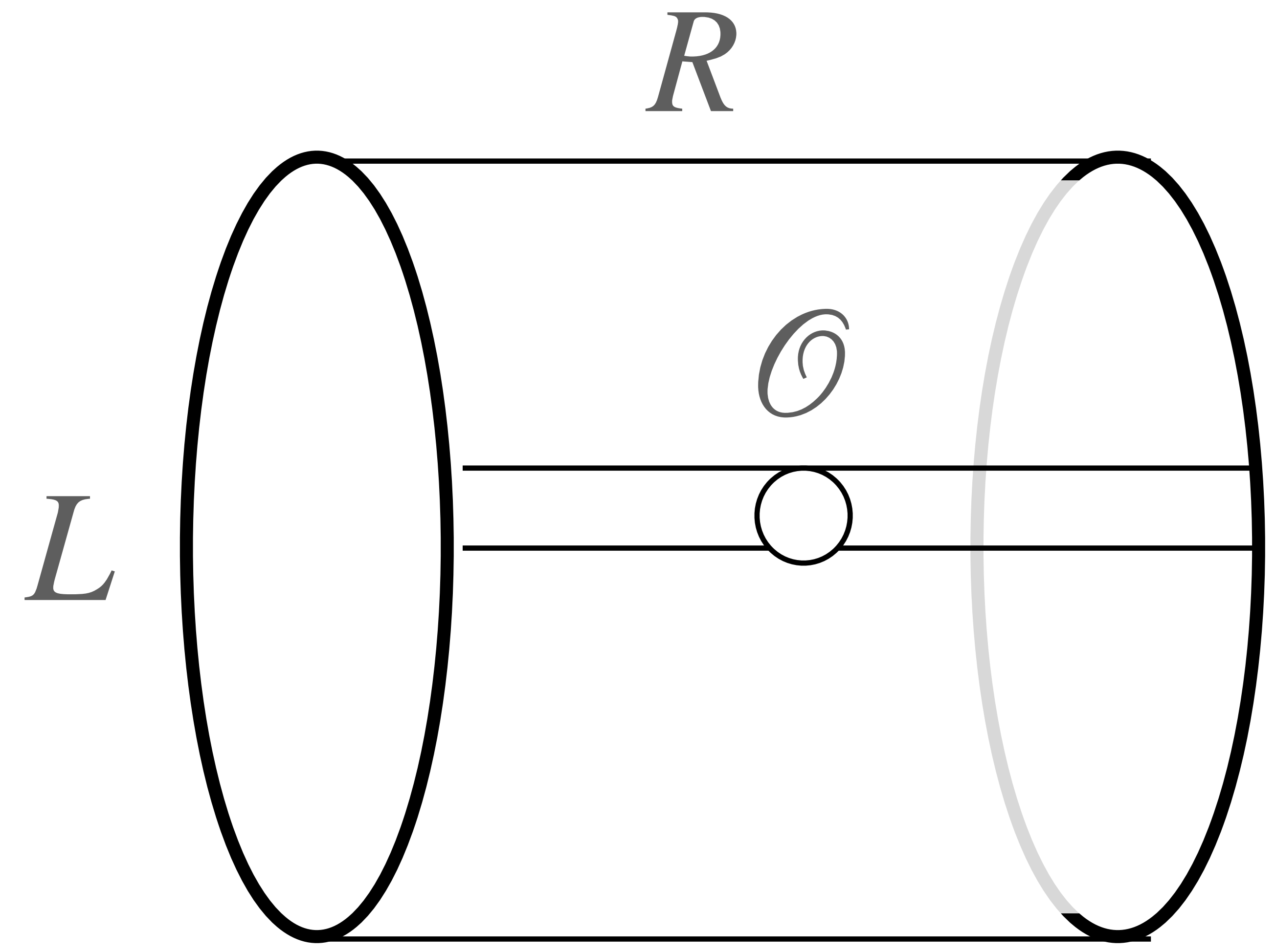


$R \to \infty$

Lowest state survives

$$\langle \theta_1, \ldots, \theta_n | \mathcal{O} | \theta_n, \ldots, \theta_1 \rangle_L e^{-E_n(L)R}$$

Bethe Yang reliable

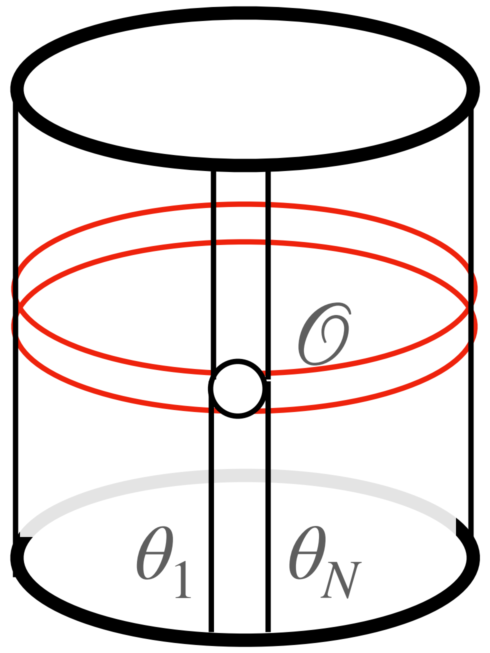


Sum up all terms

$$\sum_{|n\rangle \in \mathcal{H}} e^{E_{|n\rangle}(R)L} \langle n | \mathcal{O}_n | n \rangle_R = 1 + \sum_{\theta} e^{-m\cosh\theta(R)L} \langle \theta | \mathcal{O}_n | \theta \rangle_R + \ldots$$

Expectation values in a highly excited Bethe state

$$\langle \theta_n, \ldots, \theta_1 | \mathcal{O} | \theta_1, \ldots, \theta_n \rangle_L = \frac{\sum_{\alpha \cup \bar{\alpha} = \{1,\ldots,n\}} \rho(\bar{\alpha}) \mathcal{D}_c^{\mathcal{O}}(\alpha)}{\rho(\theta_1, \ldots, \theta_n)}$$



LeClair-Mussardo and generalisation [LeClair, Mussardo 1990] [Pozsgay 2013]

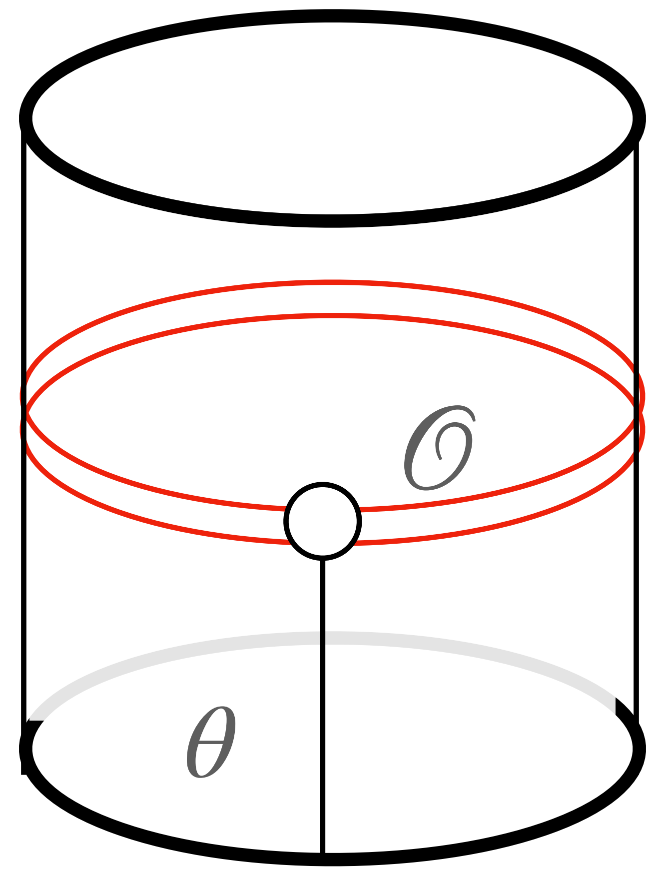
$$\mathcal{D}_c^{\mathcal{O}}(\alpha) = \sum_n \frac{1}{n!} \prod_{j=1}^{n} \int \frac{d\theta_i}{2\pi} \frac{e^{-\epsilon(\theta_j)}}{1 + e^{-\epsilon(\theta_j)}} F_c^{\mathcal{O}}(\alpha + i\pi/2, \theta_1, \ldots, \theta_n)$$

Exact finite volume description: non-diagonal case

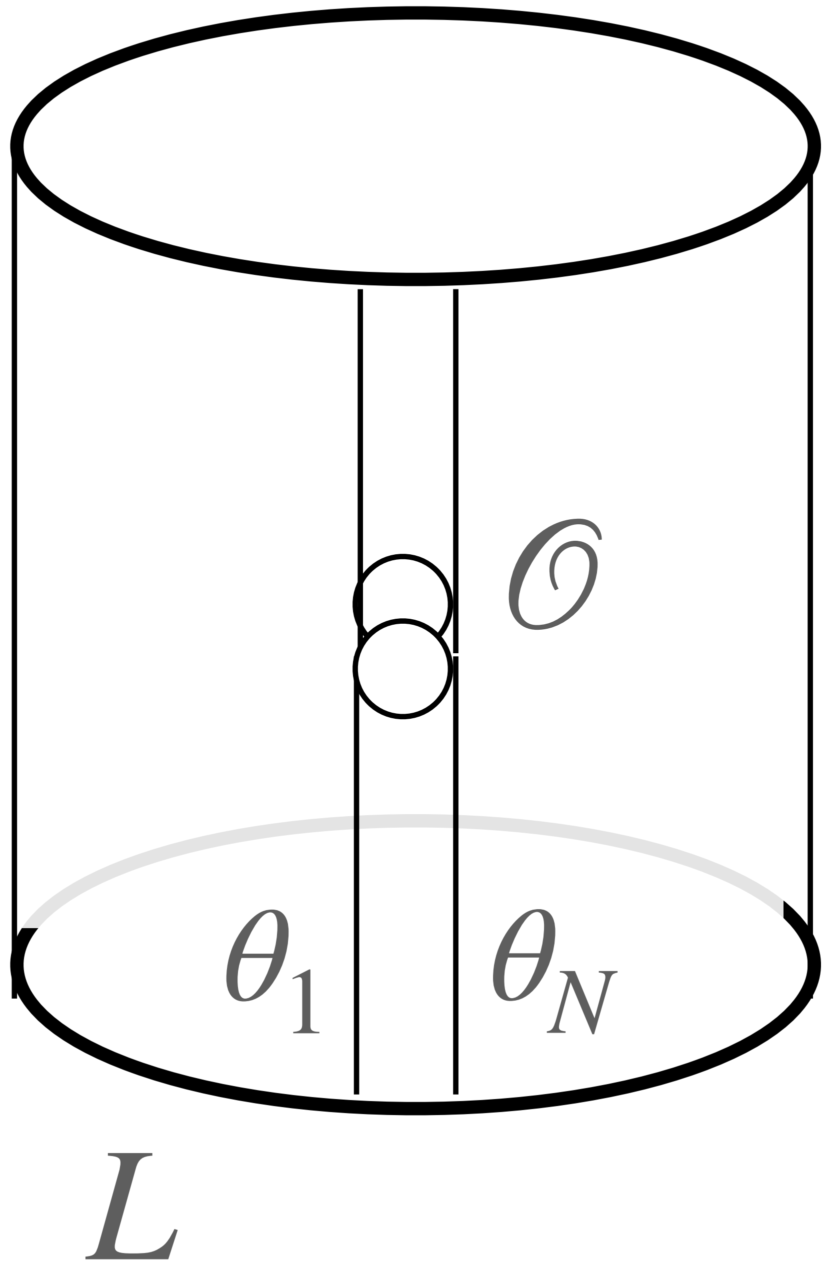
Work in progress

Expected answer

$$\langle 0|\mathcal{O}|\theta\rangle_L = \sum_n \frac{1}{n!}\prod_{j=1}^{n}\int \frac{d\theta_i}{2\pi} f(\theta_j) F_c^{\mathcal{O}}(\theta + i\pi/2, \theta_1, \ldots, \theta_n)$$



Excited state LM for bilocal operators



$R \to \infty$

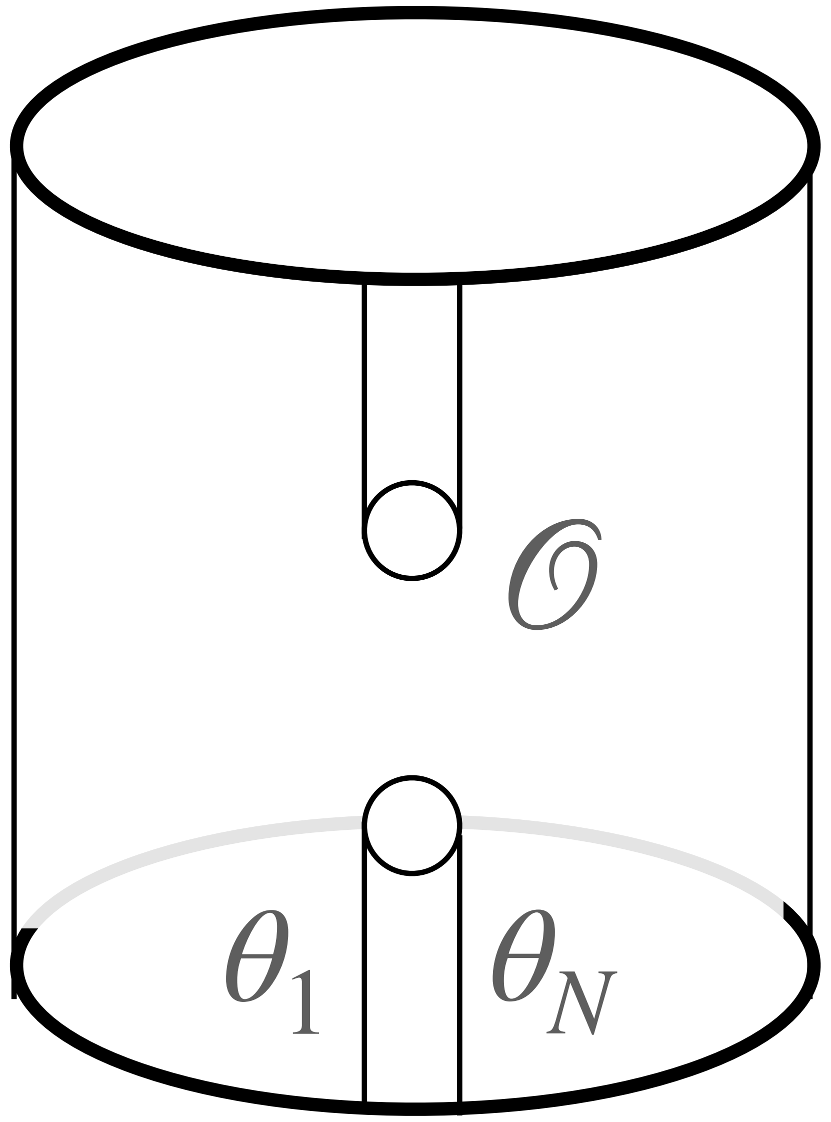
$$\langle \theta_1, \ldots, \theta_n | \mathcal{O}(x,t)\mathcal{O}(0,0) | \theta_n, \ldots, \theta_1 \rangle_L e^{-E_n(L)R}$$

Lowest state survives

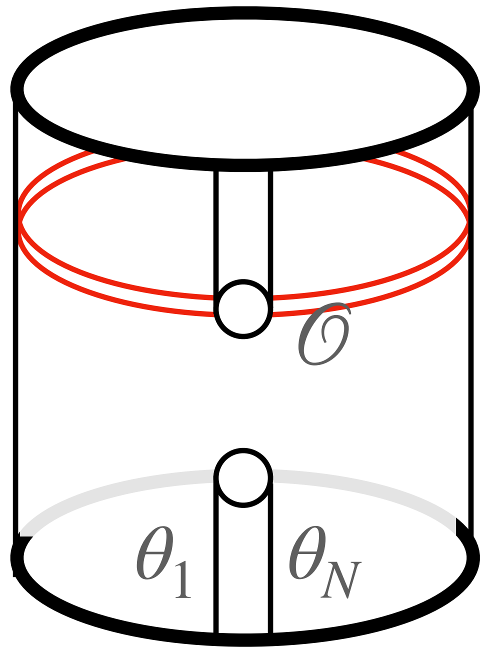
$$\langle \theta_n, \ldots, \theta_1 | \mathcal{O}(x,t)\mathcal{O}(0,0) | \theta_1, \ldots, \theta_n \rangle_L = \frac{\sum_{\alpha \cup \bar{\alpha} = \{1,\ldots,n\}} \rho(\bar{\alpha}) \mathcal{D}_c^{\mathcal{O}(x,t)\mathcal{O}}(\alpha)}{\rho(\theta_1, \ldots, \theta_n)}$$

$$\mathcal{D}_c^{\mathcal{O}(x,t)\mathcal{O}}(\alpha) = \sum_n \frac{1}{n!} \prod_{j=1}^{n} \int \frac{d\theta_i}{2\pi} \frac{e^{-\epsilon(\theta_j)}}{1 + e^{-\epsilon(\theta_j)}} F_c^{\mathcal{O}(x,t)\mathcal{O}}(\alpha + i\pi/2, \theta_1, \ldots, \theta_n)$$

[Pozsgay, Szécsényi 2018]



$$\langle \theta_1, \ldots, \theta_n | \mathcal{O}(0, i\tau) | 0 \rangle \langle 0 |)\mathcal{O} | \theta_n, \ldots, \theta_1 \rangle_L e^{-\tau(E_n(L) - E_0(L))} + \ldots$$



Exact finite volume expectation values of special operators

Conserved charge $\partial_x j(x,t) + \partial_t q(x,t) = 0$ $\quad Q = \int_0^L dx q(x,t) \quad Q\,|\,\theta\rangle = q(\theta)\,|\,\theta\rangle$

$$F_c^q(\theta_1,\ldots,\theta_n) = m\cosh\theta_1 K(\theta_1-\theta_2)\ldots K(\theta_{n-1}-\theta_n)q(\theta_n) + \text{permutations}$$

$$\langle\theta_1,\ldots,\theta_n\,|\,Lq(0,0)\,|\,\theta_n,\ldots,\theta_n\rangle_L = \sum_k q(\theta_k) + \int\frac{d\theta}{2\pi}q(\theta)\log(1+e^{-\epsilon(\theta)})$$

Conserved current

$$F_c^j(\theta_1,\ldots,\theta_n) = m\sinh\theta_1 K(\theta_1-\theta_2)\ldots K(\theta_{n-1}-\theta_n)q(\theta_n) + \text{permutations}$$

$$\langle\theta_1,\ldots,\theta_n\,|\,j\,|\,\theta_n,\ldots,\theta_n\rangle_L = \sum_{m,k}(m\cosh\theta_m)^{dr}(\partial\Phi)^{-1}_{mk}q(\theta_k)^{dr} + m\cosh\theta\circ q(\theta)^{dr}$$

[Bajnok, Vona 2019]

[Borsi et al 2019]

Dressing

$$f(\theta)^{dr} = f(\theta) + K(\theta-\theta')\circ f(\theta')^{dr} \qquad f(\theta)\circ g(\theta) = \int\frac{d\theta}{2\pi}\frac{f(\theta)g(\theta)}{1+e^{\epsilon(\theta)})}$$

Exact finite volume expectation values in the sinh-Gordon theory

Conserved current with a new convolution $\langle \theta_1, \ldots, \theta_n | j | \theta_n, \ldots, \theta_n \rangle_L = (m \cosh \theta)^{dr} \bullet q(\theta)$

$$f(\theta) \bullet g(\theta) = \sum_j \frac{if(\theta_j + i\pi/2) g(\theta_j + i\pi/2)}{\epsilon'(\theta_j + i\pi/2)} + \int \frac{d\theta}{2\pi} \frac{f(\theta) g(\theta)}{1 + e^{\epsilon(\theta)}}$$

Exponential operators $\mathcal{O}_\alpha = e^{\alpha(b+b^{-1})/2\phi(0,0)}$ [Negro, Smirnov 2013]

Fermionic structure $\{\beta_m, \beta_n^*\} = \{\bar{\gamma}_m, \bar{\gamma}_n^*\} = -\delta_{n,m} t_n(\alpha)$ $\quad 1/t_n(\alpha) = 2 \sin(\pi(\alpha - na))$

$$\frac{\langle \theta_1, \ldots, \theta_n | \beta_1^* \bar{\gamma}_1^* \Phi_\alpha | \theta_n, \ldots, \theta_n \rangle_L}{\langle \theta_1, \ldots, \theta_n | \Phi_\alpha | \theta_n, \ldots, \theta_n \rangle_L} = \omega_{1,1} - t_n(\alpha) \qquad \beta_1^* \bar{\gamma}_1^* \Phi_\alpha \propto \Phi_{\alpha - 2a}$$

$$\omega_{n,m} = e_n \bullet (1 + K_\alpha + K_\alpha \bullet K_\alpha + \ldots) \bullet e_m \qquad e_n(\theta) = e^{n\theta} \quad K_\alpha(\theta) = \frac{ie^{i\alpha}}{\sinh(\theta + i\pi a)} - \frac{ie^{-i\alpha}}{\sinh(\theta - i\pi a)}$$

Checked in the UV (Liouville 3pt functions) and IR (generalised LM) [Bajnok, Smirnov 2019]

Summary

Overview of last 5 years work

- Motivation: why finite volume form factors, AdS/CFT 3pt functions
- Finite volume corrections

$$x \equiv x + L$$

Small volume ← Large volume

Energy spectrum	Exact description, Thermodynamic Bethe Ansatz	Lüscher correction of multiparticle energies	Momentum quantisation, Bethe Yang equation	Masses, scatterings
Diagonal Form Factors	Linear integral equations based on the TBA	Leading exponential volume corrections	Sum for partitions, partial densities	Connected form factors
Non-diagonal form factors	Still missing	Leading exponential volume corrections	Change in the normalisation of states	Form factors