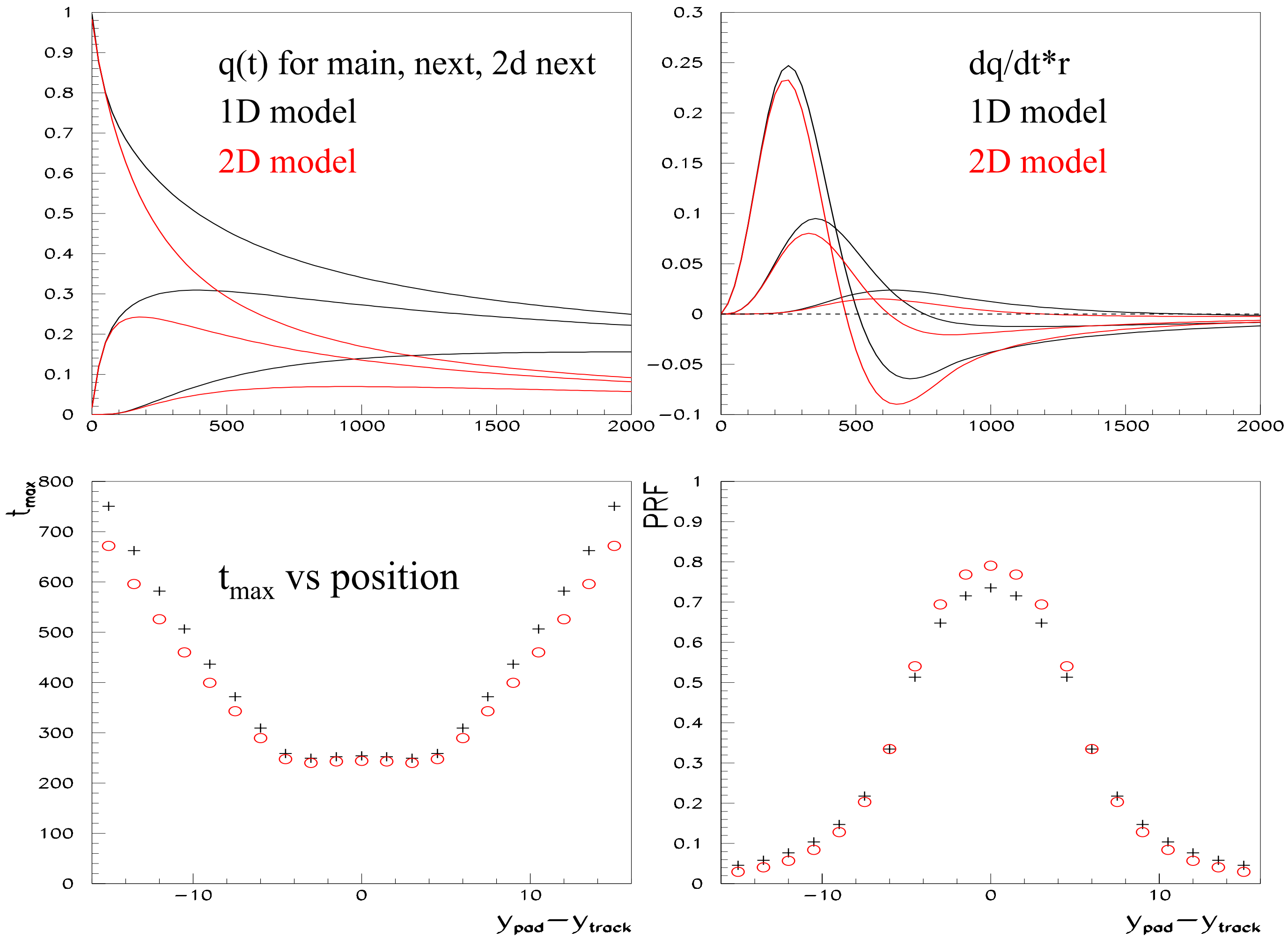
electronic response

assuming a convolution of i(t) = **dq/dt** with the response to a pulse



better agreement with real data than using q*r

and less differences between 1D and 2D models

a simple way to “solve” the diffusion equation on a rectangle with boundary conditions

$$\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} = RC\frac{\partial \rho}{\partial t}$$

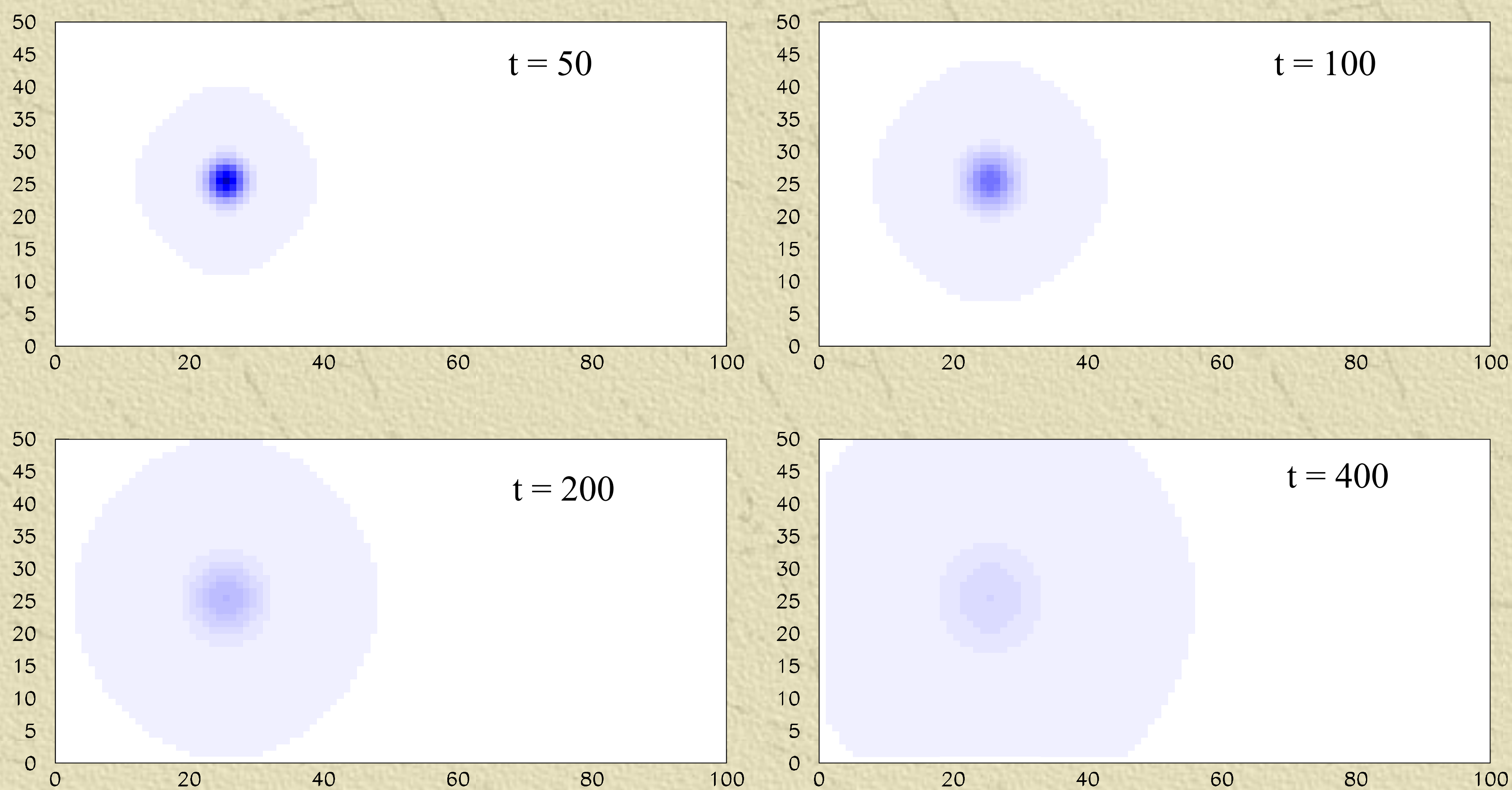
- describe the densities by values ρ_{ij} on a rectangular grid (x_i, y_j)
- maintain $\rho = 0$ on the edges of the plate
- elsewhere: approximate the laplacian $\Delta\rho$ by
  $(\rho_{i+1,j} + \rho_{i-1,j} - 2\,\rho_{i,j})/L_x^{2} + (\rho_{i,j+1} + \rho_{i,j-1} - 2\,\rho_{i,j})/L_y^{2}$
- make a step as $\rho_{i,j}$ += $\Delta\rho_{i,j}\,\delta t/RC$

comments:

- the density is « self-smoothing », so after a few steps, the laplacian is very well approximated, and inaccuracies in the first steps are
- the size of the time step may be increased by using the Rung-Kutta method
- in practice: after a short time, the results are in excellent agreement with the Fourier expansion
- *the method may be extended to other geometries, including non-uniform conditions (e.g. non-homogeneous value of RC)*

evolution from a pointlike charge (far from the edges)

rectangle of 100×50 cells of 1×1 mm RC = 25 ns/mm^2 time step 5 ns (RK method)



evolution from a pointlike charge
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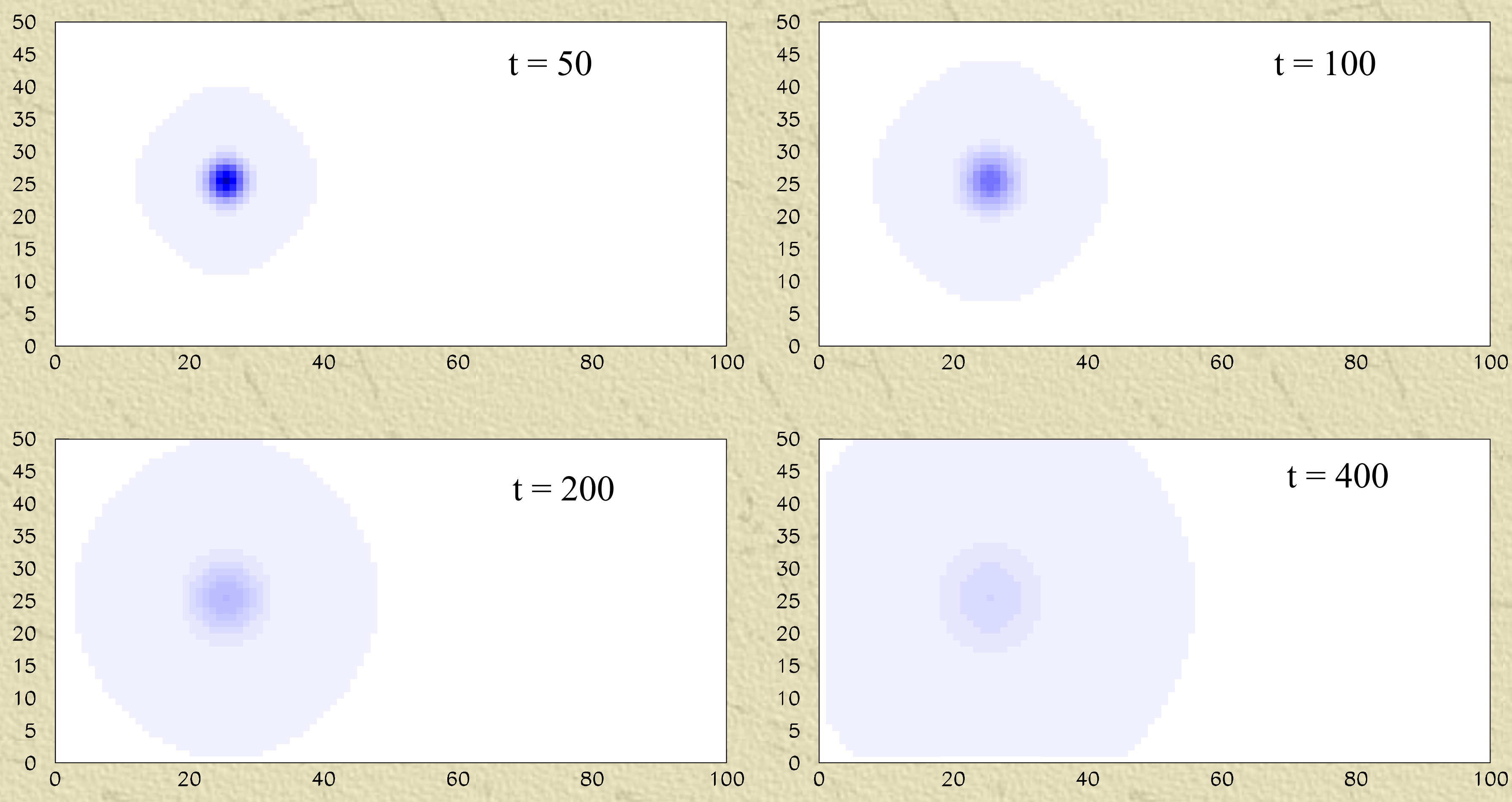
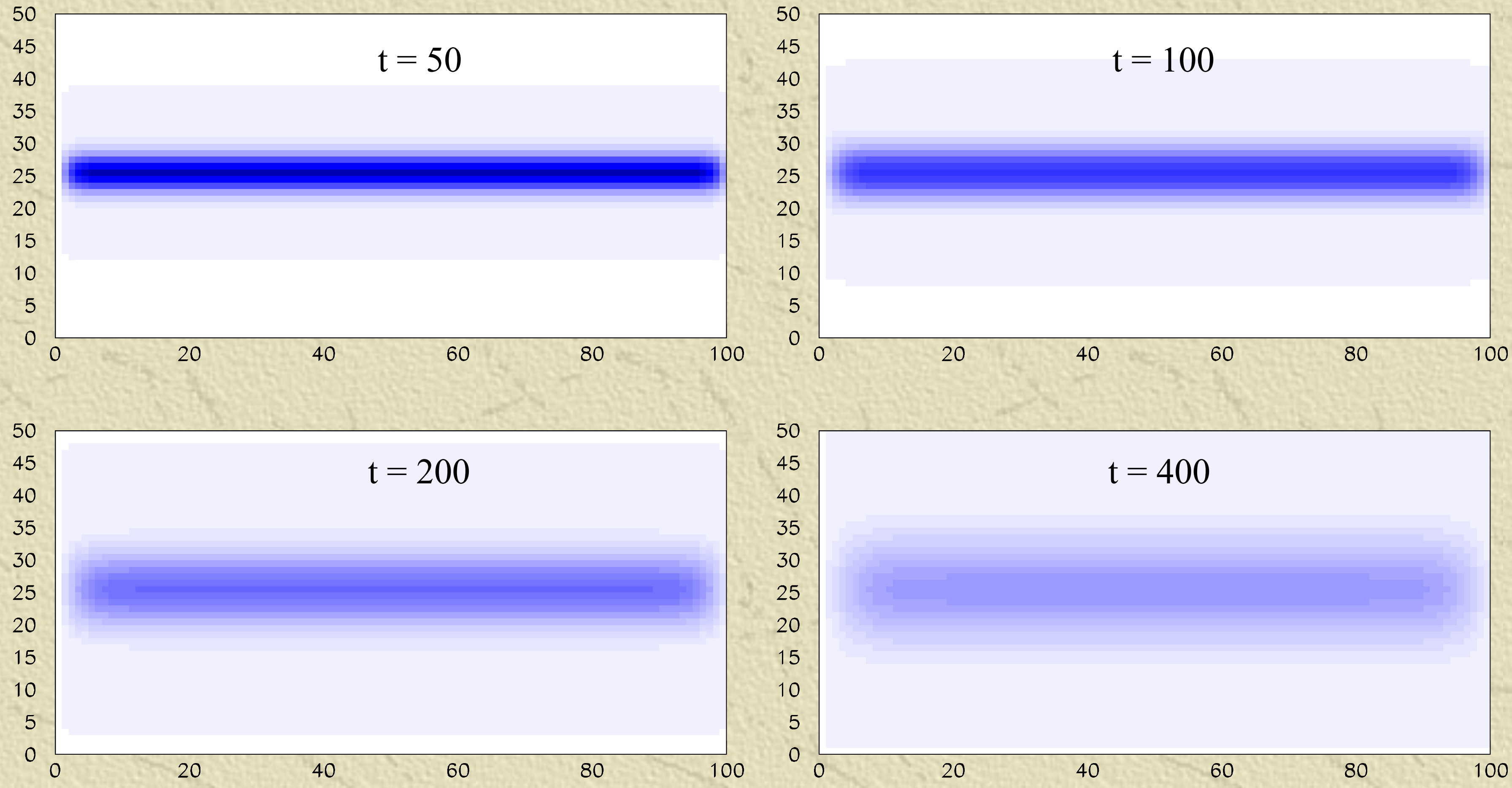
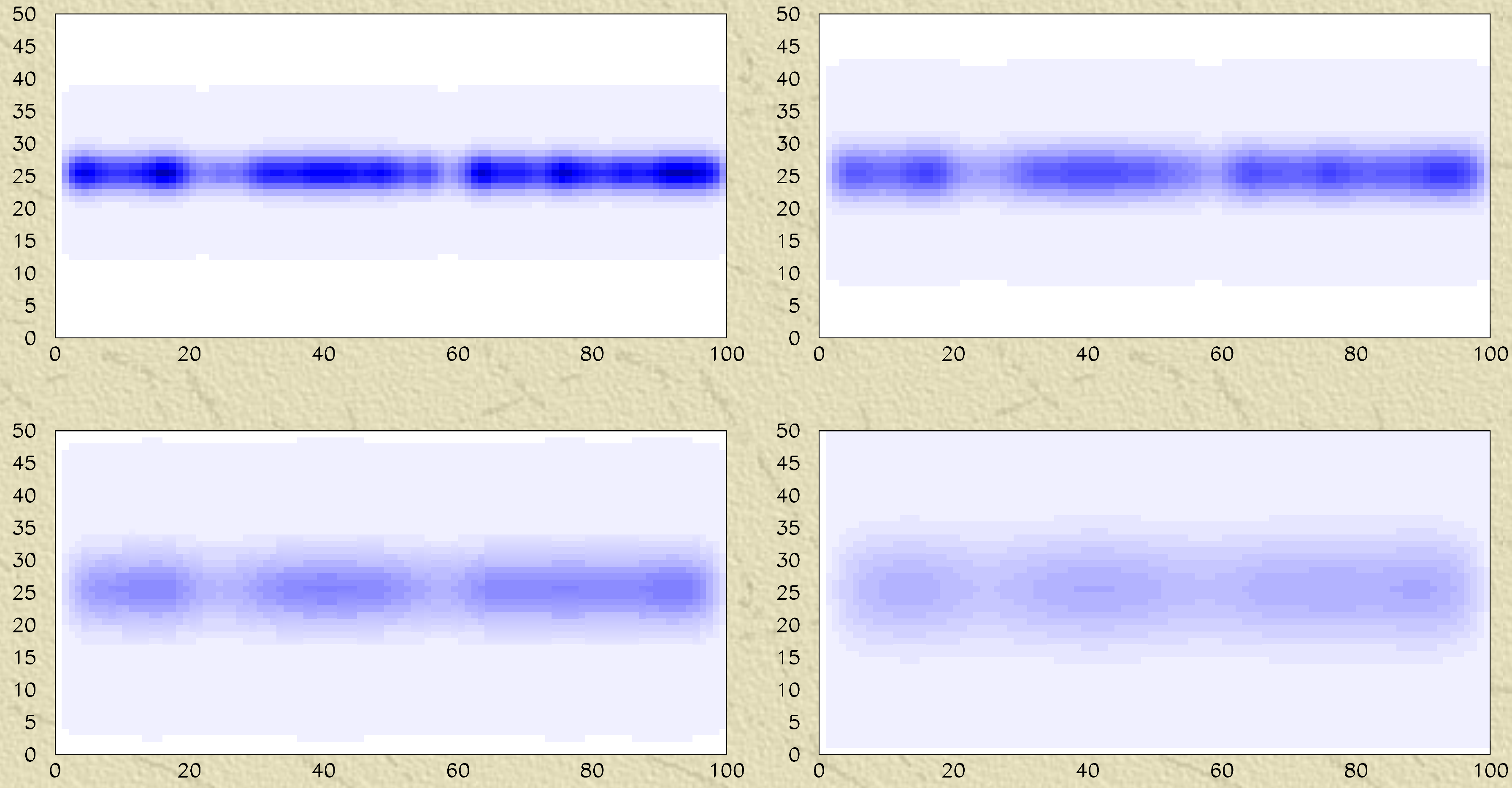


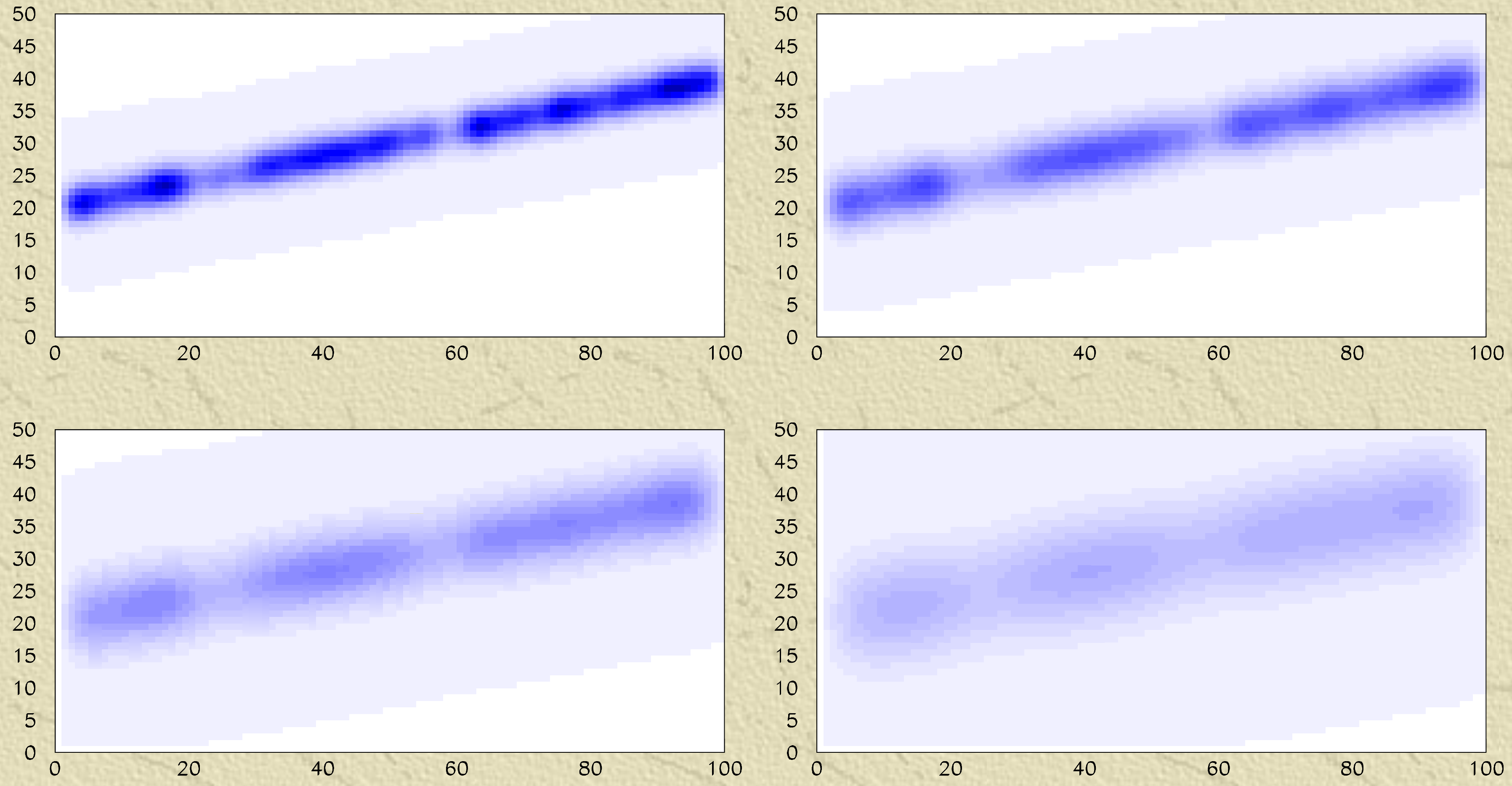
evolution from a uniform linear distribution



evolution from a random set of pointlike sources



the same along an oblique track



“analytic” solution with a Fourier expansion

$$\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} = RC\frac{\partial \rho}{\partial t}$$

aim: solve the equation in a rectangle *(0,a)*×*(0,b)* with $\rho = 0$ on the edges

principle of the computation:

- expand $\rho(x,y,t)$ in a Fourier series in x,y obeying the boundary condition

$$f_{\alpha,\beta}(x,y) = \frac{2}{\sqrt{ab}}\sin(\frac{\alpha\pi x}{a})\sin(\frac{\beta\pi y}{b})$$

- find the dependence in t of each term

$$\exp\left(-\pi^2\left(\frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}\right)\frac{t}{RC}\right)$$

- determine the coefficients from the density at $t = 0$

for example : unit charge at (x_0,y_0) : $\rho(x,y,0) = \delta(x-x_0)\,\delta(y-y_0)$

$$C_{\alpha,\beta}(x,y) = \frac{2}{\sqrt{ab}}\sin(\frac{\alpha\pi x_0}{a})\sin(\frac{\beta\pi y_0}{b})$$

- obtain the solution :

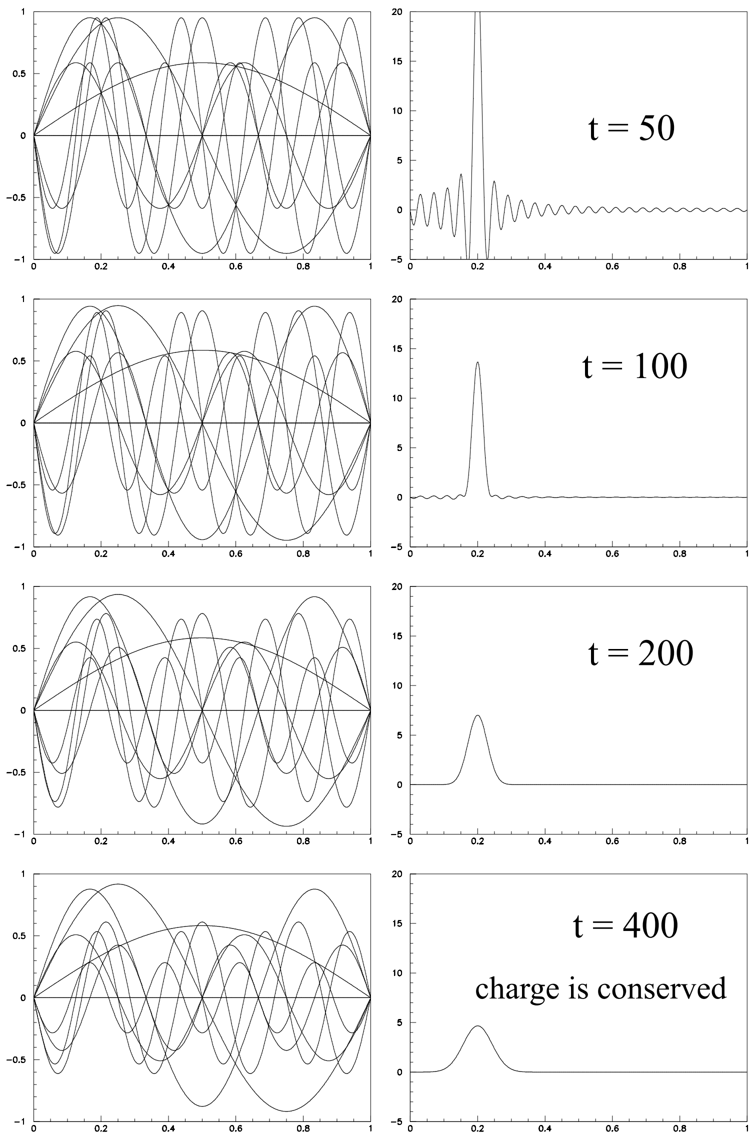
$$\frac{4}{ab}\sum_{\alpha,\beta}\sin\left(\frac{\alpha\pi x_0}{a}\right)\sin\left(\frac{\alpha\pi x}{a}\right)\sin\left(\frac{\beta\pi y_0}{b}\right)\sin\left(\frac{\beta\pi y}{b}\right)\exp\left(-\pi^2\left(\frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}\right)\frac{t}{RC}\right)$$

similar to Riegler’s result, except time dependence of the modes

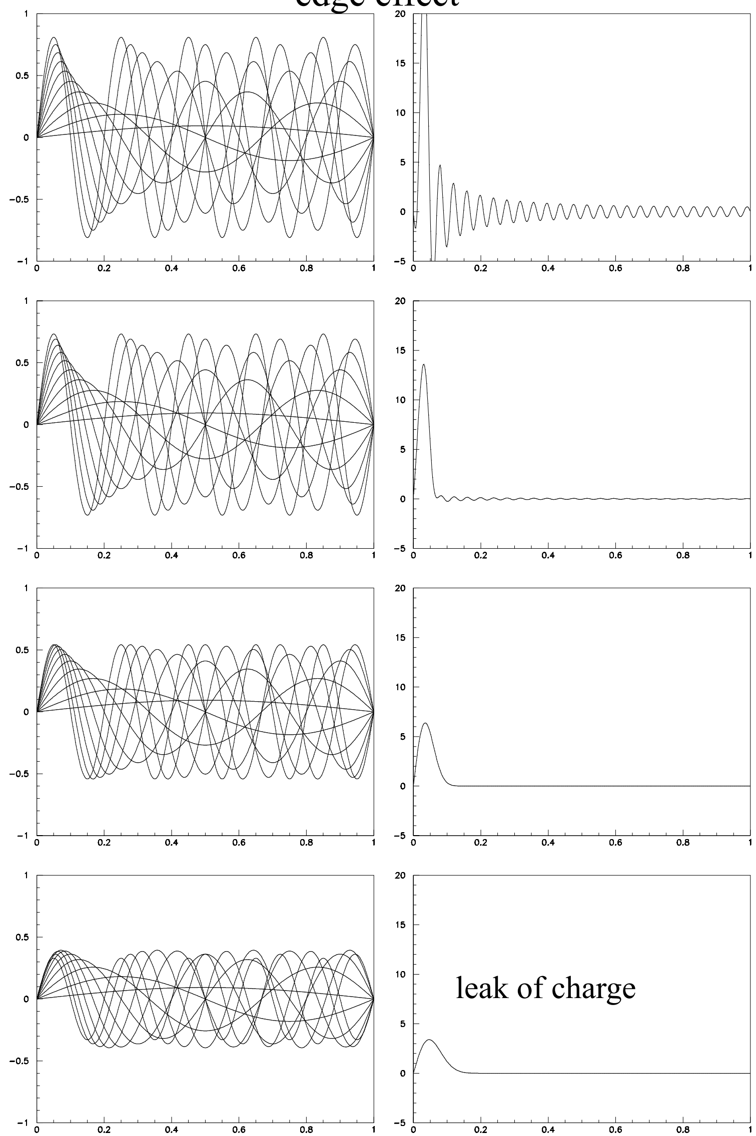
- integrate over a pad to obtain $q(t)$

illustration of the summation in 1D

first 10 terms | sum of 50 terms



edge effect

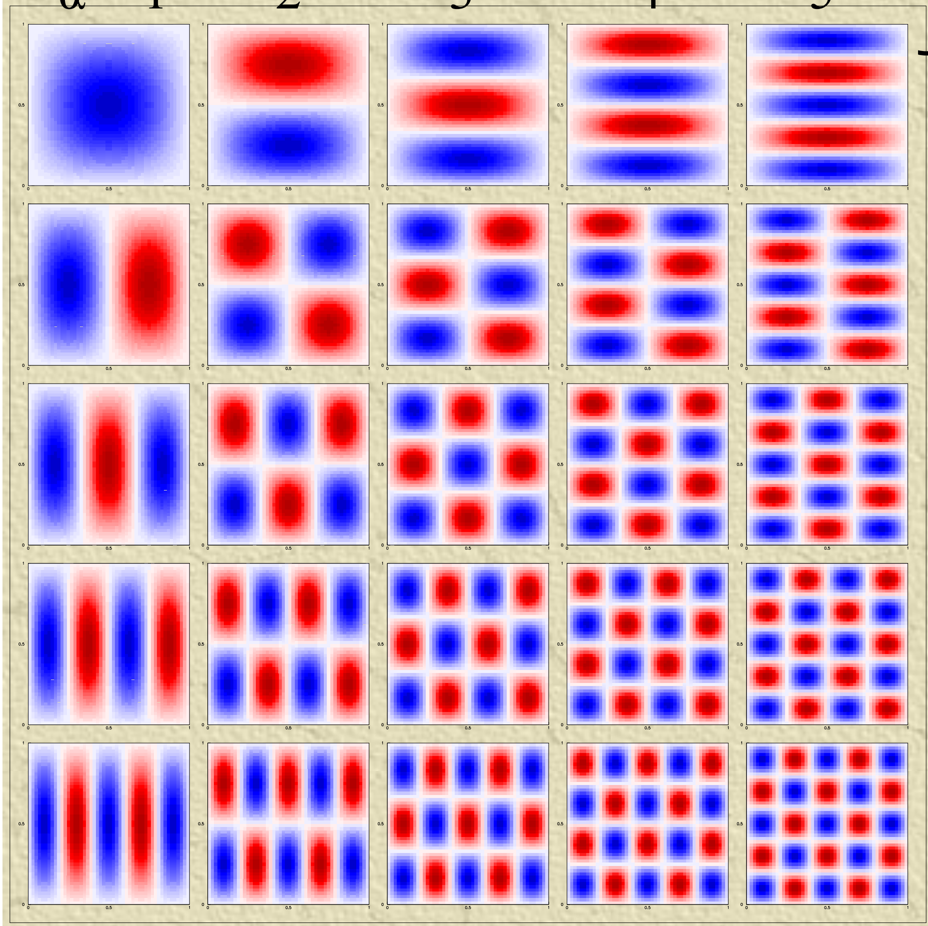


evolution of the modes for the summation in 2D

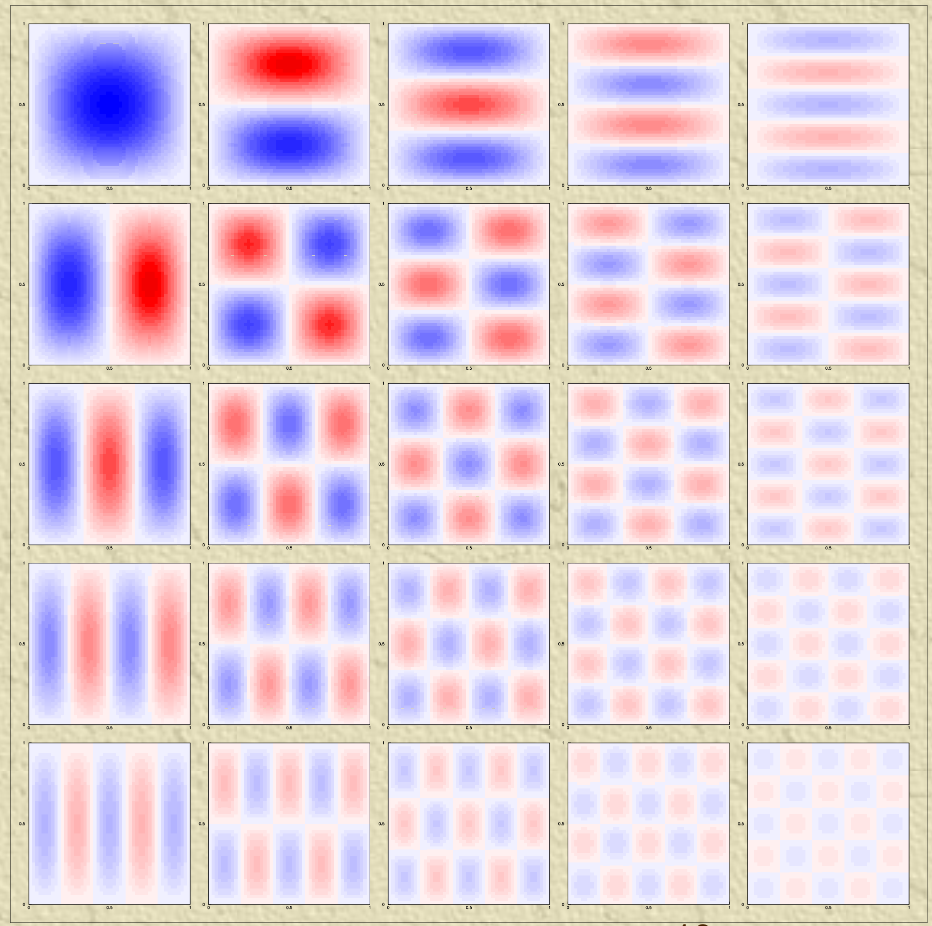
(levels of blue for positive values, red for negative ones)

densities at t = 0

$\alpha = 1$ 2 3 4 5

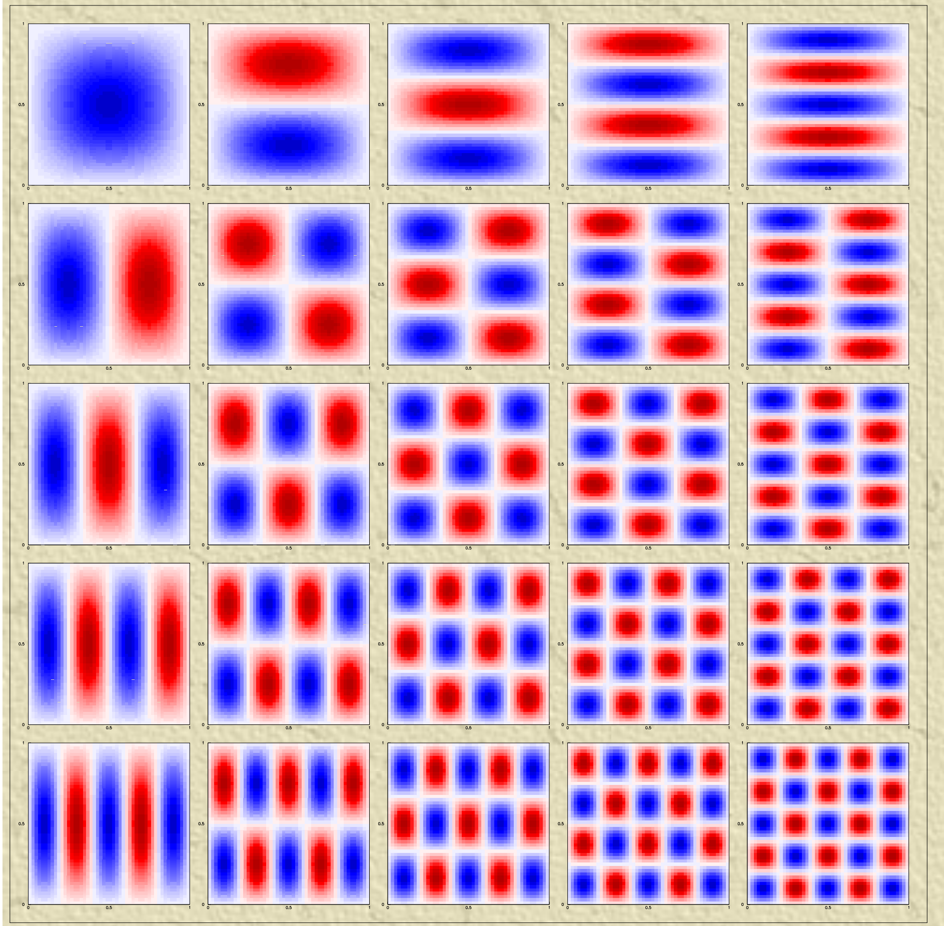


densities at some t > 0

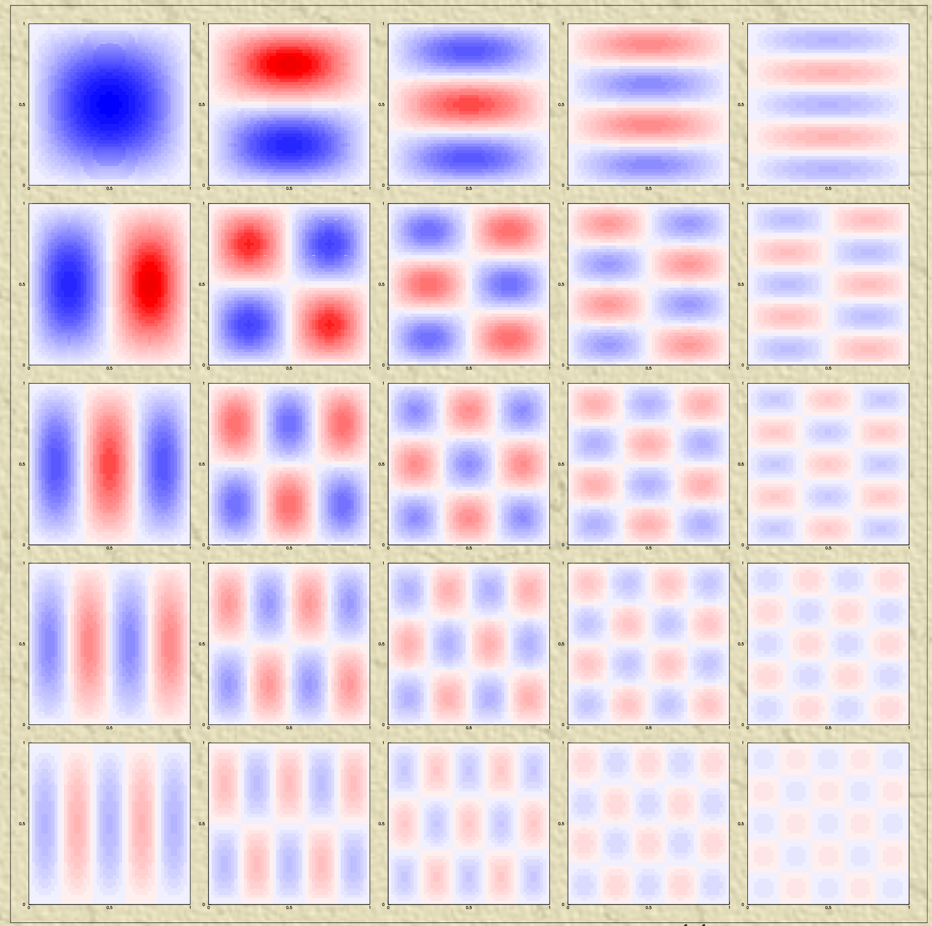


modes for the summation in 2D

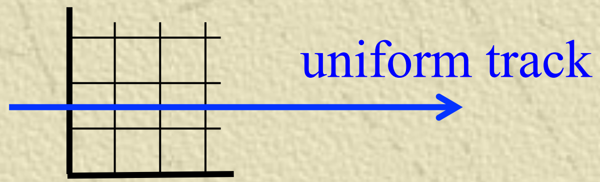
initial densities for $\alpha,\beta = 1$ to 5

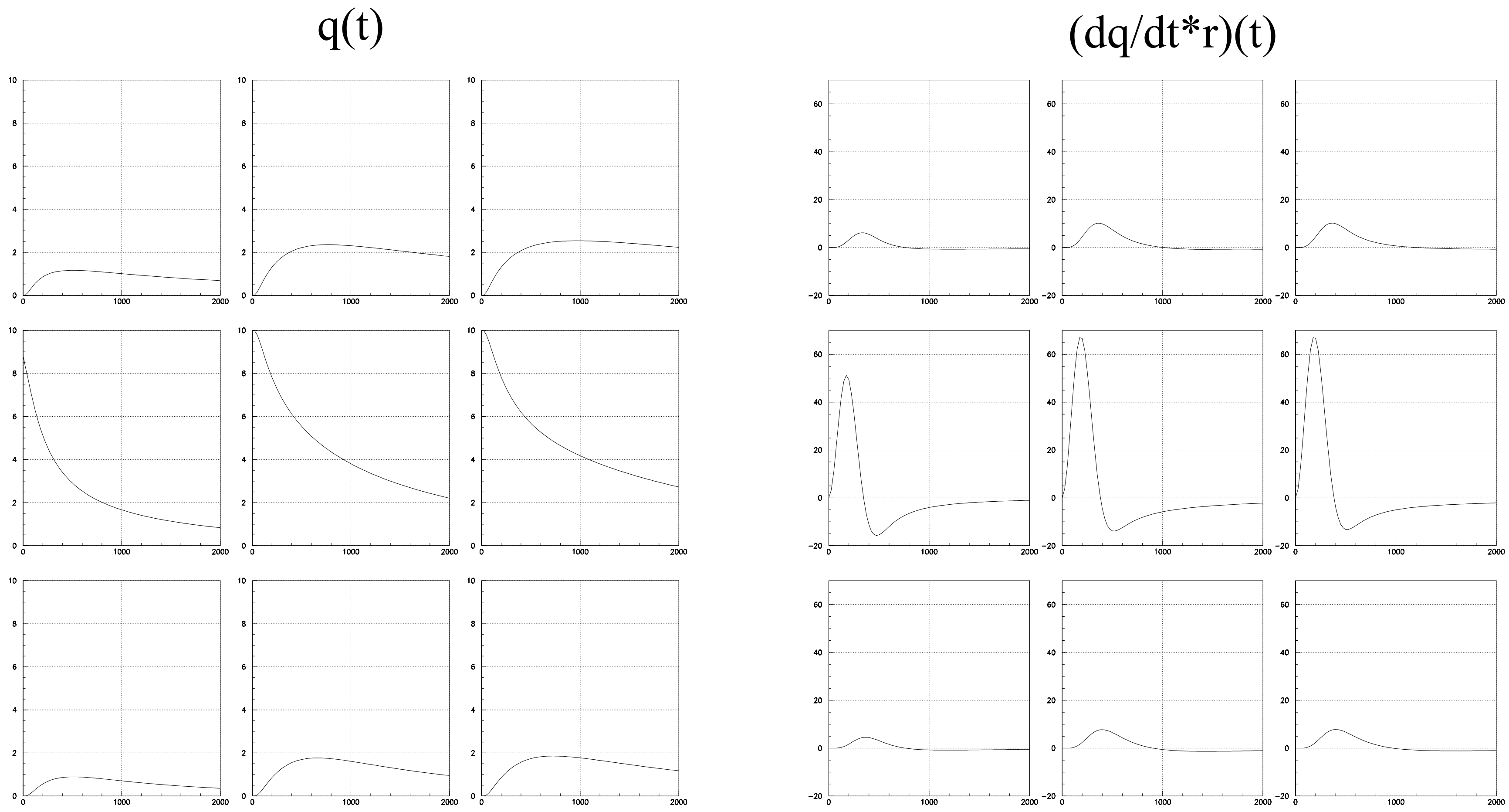


densities at some $t > 0$



waveforms for 9 pads in the lower left corner





evidence for the attenuation on the edges

approximation of Riegler's formula

With Riegler's notations: functions $f(x,y)$ in a rectangle $(0,a)\times(0,b)$
with boundary condition $f=0$ on the edges ($x=0$ or $x=a$, $y=0$ or $y=b$)
complete orthonormal basis: $f_{\alpha,\beta}=2\sin(\alpha\pi x/a)\sin(\beta\pi y/b)/\sqrt{ab}$ ($\alpha=1,2,\cdots\infty$, $\beta=1,2,\cdots\infty$)
initial state: a charge Q at (x_0,y_0): $\rho(x,y,0)=Q\,\delta(x-x_0)\,\delta(y-y_0)$
Projecting onto the basis:

$$\rho(x,y,0)=\frac{4Q}{ab}\sum_{\alpha=1}^{\infty}\sin(\alpha\pi x_0/a)\sin(\alpha\pi x/a)\sum_{\beta=1}^{\infty}\sin(\beta\pi y_0/b)\sin(\beta\pi y/b)$$

particular solution of the equation $\frac{\partial^2\rho}{\partial x^2}+\frac{\partial^2\rho}{\partial y^2}=RC\frac{\partial\rho}{\partial t}$

$$F_{\alpha,\beta}(x,y,t)=\sin(\frac{\alpha\pi x}{a})\sin(\frac{\beta\pi y}{b})\exp\left(-\frac{k_{\alpha\beta}^2\,t}{RC}\right)\qquad \text{with } k_{\alpha\beta}^2=\pi^2\left(\frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}\right)$$

Hence the evolution in time of ρ:

$$\rho(x,y,t)=\frac{4Q}{ab}\sum_{\alpha,\beta}\sin\left(\frac{\alpha\pi x_0}{a}\right)\sin\left(\frac{\alpha\pi x}{a}\right)\sin\left(\frac{\beta\pi y_0}{b}\right)\sin\left(\frac{\beta\pi y}{b}\right)h(k_{\alpha\beta},t)\qquad \text{with } h(k_{\alpha\beta},t)=\exp\left(-\frac{k_{\alpha\beta}^2 t}{RC}\right)$$

In Riegler's formula:

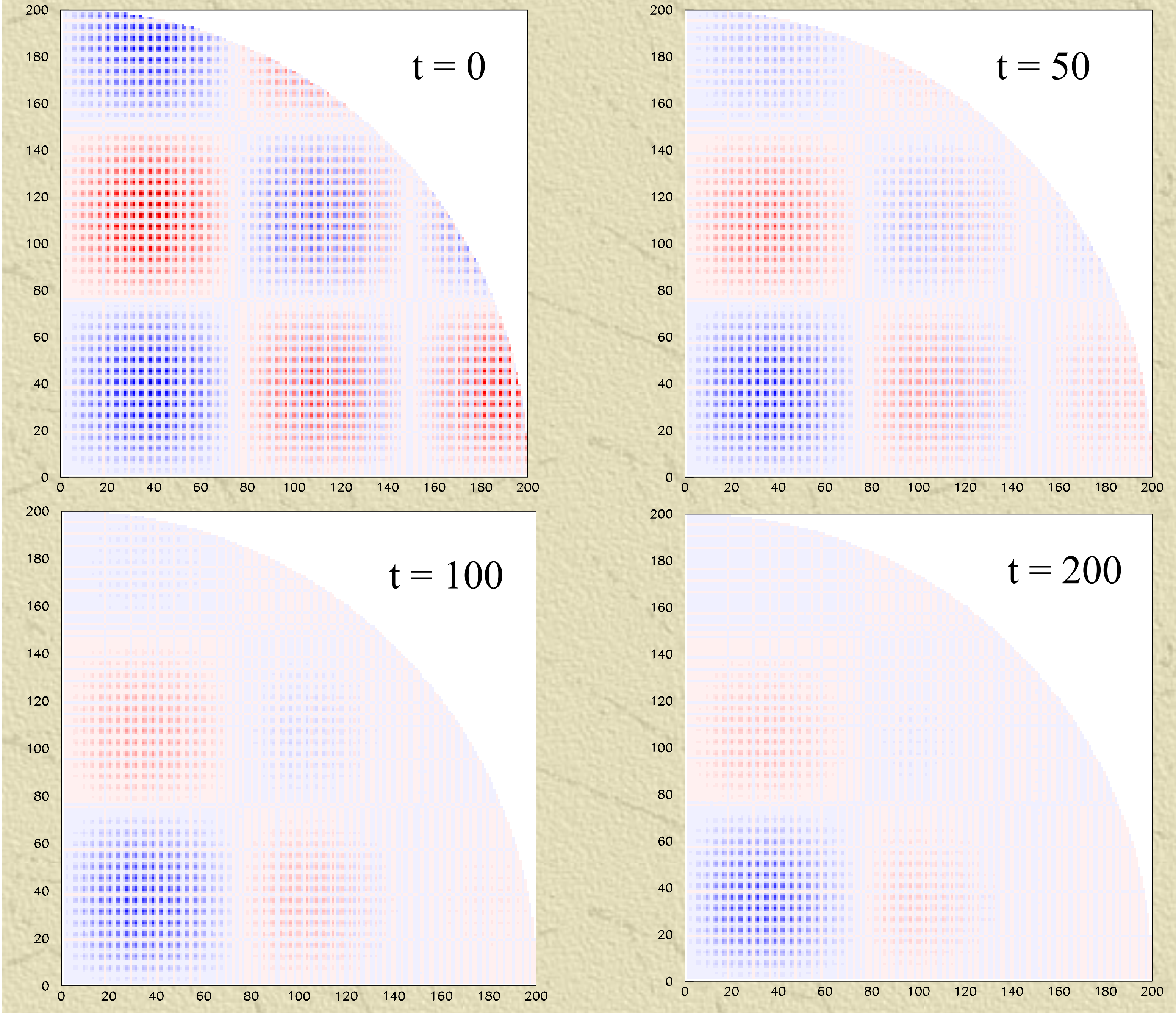
$$h(k,t)=\frac{\varepsilon_1 e^{-t/\tau(k)}}{\varepsilon_1\cosh(kd_1)+\varepsilon_3\coth(kd_3)\sinh(kd_1)}\qquad \text{with } \tau(k)=\frac{R}{k}(\varepsilon_1\coth(kd_1)+\varepsilon_3\coth(kd_3))$$

With the approximation $kd_1\ll 1$ and $kd_3\ll 1$ we obtain:

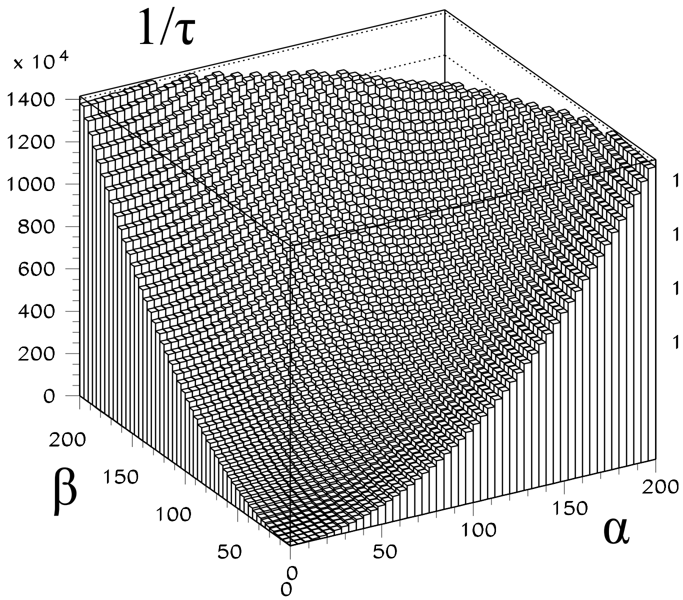
$$h(k,t)=\frac{\varepsilon_1 e^{-t/\tau(k)}}{\varepsilon_1+\varepsilon_3 d_1/d_3}\qquad \text{with } \tau(k)=\frac{R}{k^2}\left(\frac{\varepsilon_1}{d_1}+\frac{\varepsilon_3}{d_3}\right)=RC/k^2\quad \text{with } C=\frac{\varepsilon_1}{d_1}+\frac{\varepsilon_3}{d_3}\quad \text{(capacities in parallel)}$$

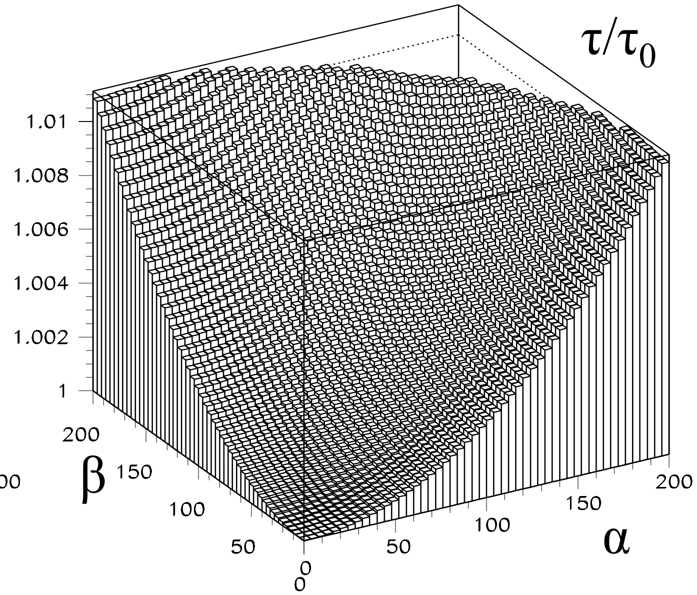
(valid approximation at large t if $d_1,d_3\ll a,b$ because the terms with large α and/or β decrease very rapidly with t)

evolution of Riegler coefficients with time

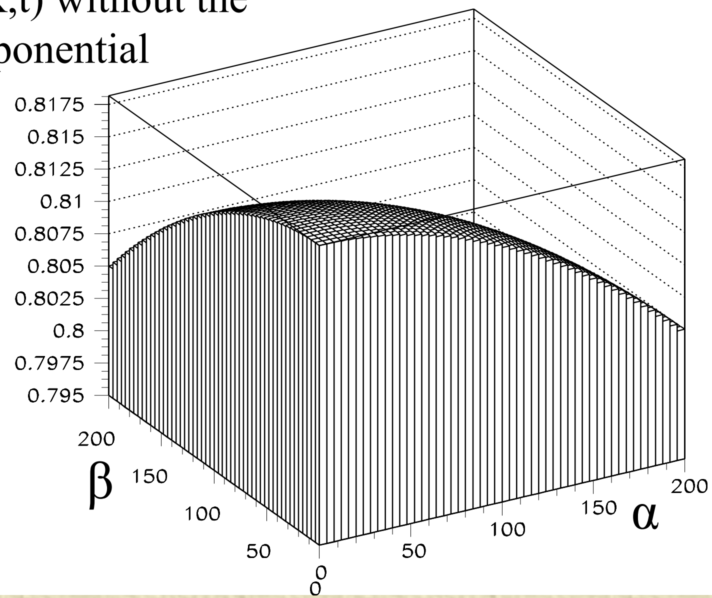


quasi equivalence with « telegraphist »





h(k,t) without the exponential



in a large domain of α,β :

$1/\tau$ is quadratic in α,β

τ is close to τ_0 (value in the approximation of small kd1,kd3)

h(k,t) is practically cst.exp($-t/\tau$)

but: cst is not 1

model "1.5D" for linear tracks (in infinite plane)

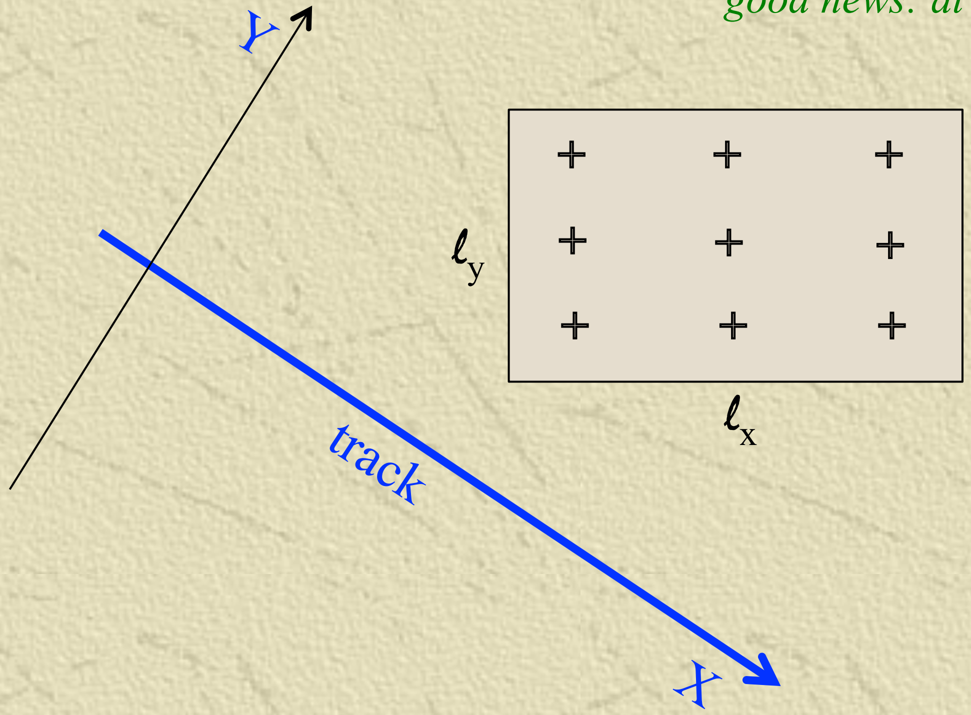
$$\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} = RC\frac{\partial \rho}{\partial t} \xrightarrow{\text{particular solution}} \rho_\alpha(t) = \frac{1}{\sqrt{t}} \exp\left(-\frac{y^2 RC}{4t}\right) \exp(i.2\pi\alpha x) \exp\left(-\frac{4\pi^2\alpha^2 t}{RC}\right)$$

at t = 0: distribution of charges along x axis over a length L ➔ Fourier series

density at time t > 0

$$\sigma(x) = \sum_{k=0}^{\infty} (c_k \cos(2\pi kx/L) + s_k \sin(2\pi kx/L)) \rightarrow \rho(x,y,t) = \sqrt{\frac{RC}{4\pi t}} \exp\left(-\frac{y^2 RC}{4t}\right) \sum_k (c_k \cos(2\pi kx/L) + s_k \sin(2\pi kx/L)) \exp\left(-\frac{4\pi^2 k^2 t}{RCL^2}\right)$$

good news: at large t, the series can be truncated



$$Q_{pad}(t) = \iint_{pad} \rho(X,Y,t)\, \mathrm{d}x\, \mathrm{d}y$$

may be computed with a Gauss-Legendre quadrature method extended to a double integral

$$\iint_{pad} f(x,y)\, \mathrm{d}x\, \mathrm{d}y = \frac{\ell_x \ell_y}{4} \sum_i \sum_j w_i w_j f(x_i, y_j)$$

x_i ,y_i : zeroes of Legendre polynomials

a possible ideal fitting strategy

- choose as free parameters the Fourier coefficients c_k , s_k + geometry of the track (y_0, dy/dx, curvature, t_0…)
- select a few « significant » points t_i in the waveforms of activated pads

  (may be different in different pads)
- fit the predictions (dq/dt*r)(t_i) to the measured values
- deduce the geometry of the track + estimation of dE/dx

technical remarks:

- how many Fourier terms ? Intuitively, no more than the number of rows/columns covered by the track; to be tuned… in any case, the series may be truncated at large t

- the prediction is linear w.r.t. the c_k , s_k and may be linearized w.r.t the geometrical parameters once a good approximation is found

➔ possible strategy: make a fit of the geometry with existing methods, then linearize and make iterations only if really needed (in principle, very fast convergence)

- big amount of computation to obtain the predictions: search for valid approximations !

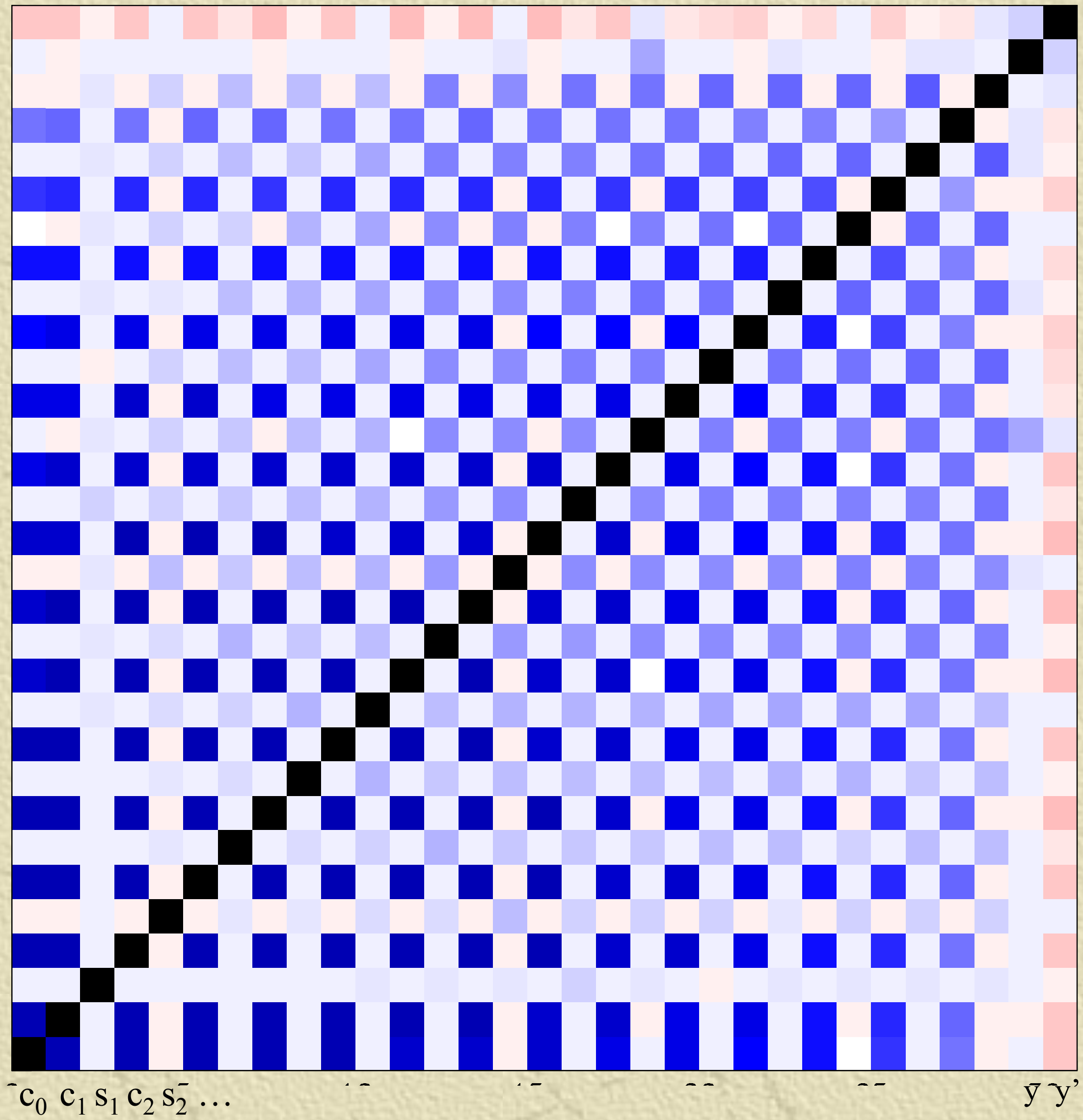
(e.g. Gauss-Legendre with few points at large t)

- no edge effect in this model: discard the first and last row/column ? find an empirical approximation for these pads ?

- in principle, the waveforms can be fully exploited (not only t_{max}, q_{max})

but : sensitivity to possible defects of the model; more difficult to tune it from the data

let us be optimistic...



correlations between fitted parameters

Fourier coefficients:

c_0 , c_1,s_1 , c_2,s_2 … c_{14},s_{14}

position of the track

y, y' = dy/dx

weak correlation between geometry and Fourier coefficients:

good news for convergence