Search for stochastic gravitational wave background with LIGO-Virgo interferometers

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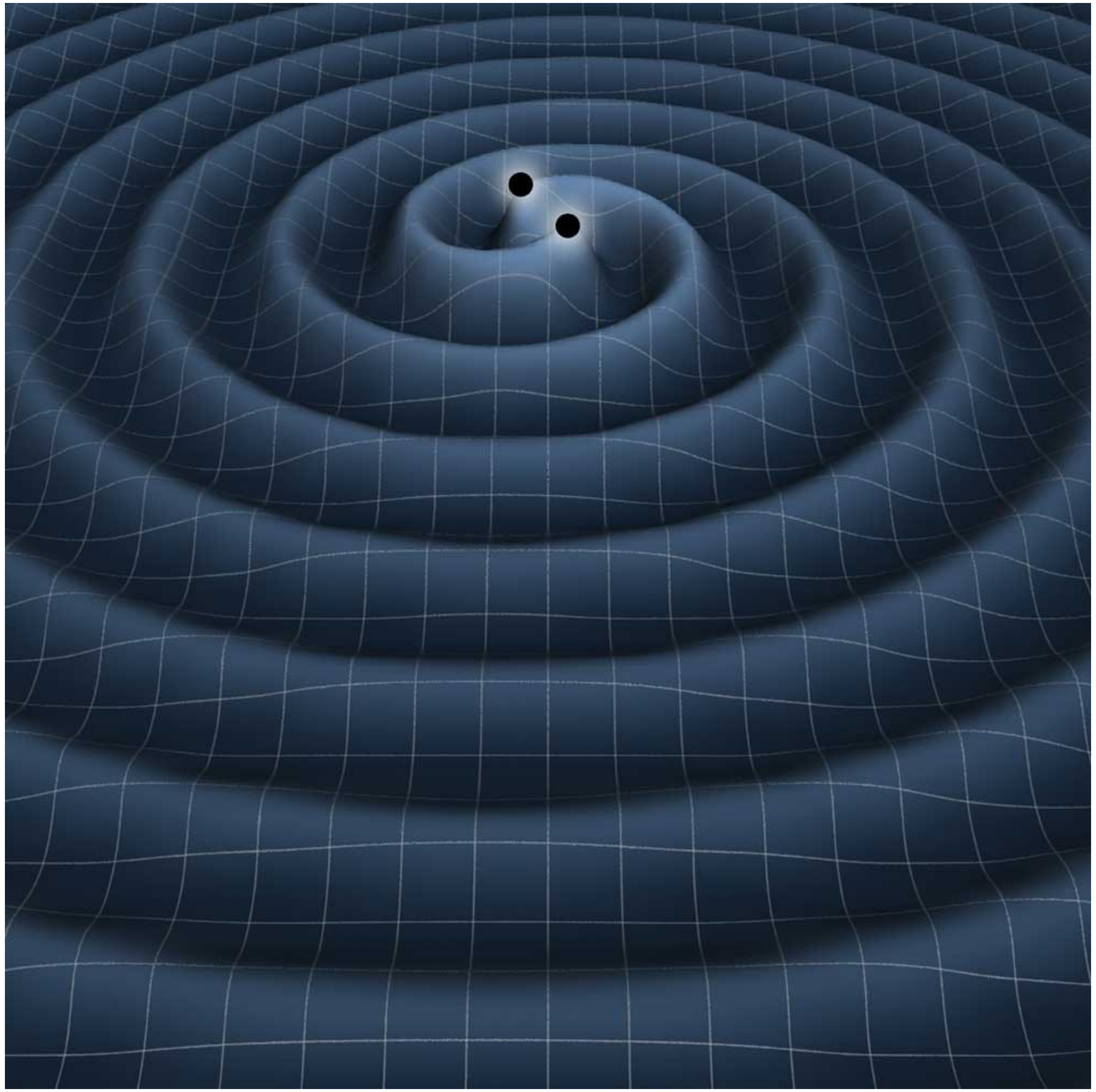
Overview

1. Introduction to the stochastic gravitational wave background.
2. Search strategies
3. Toy-model simulation
4. Construction of a realistic MC simulation



The stochastic gravitational wave background

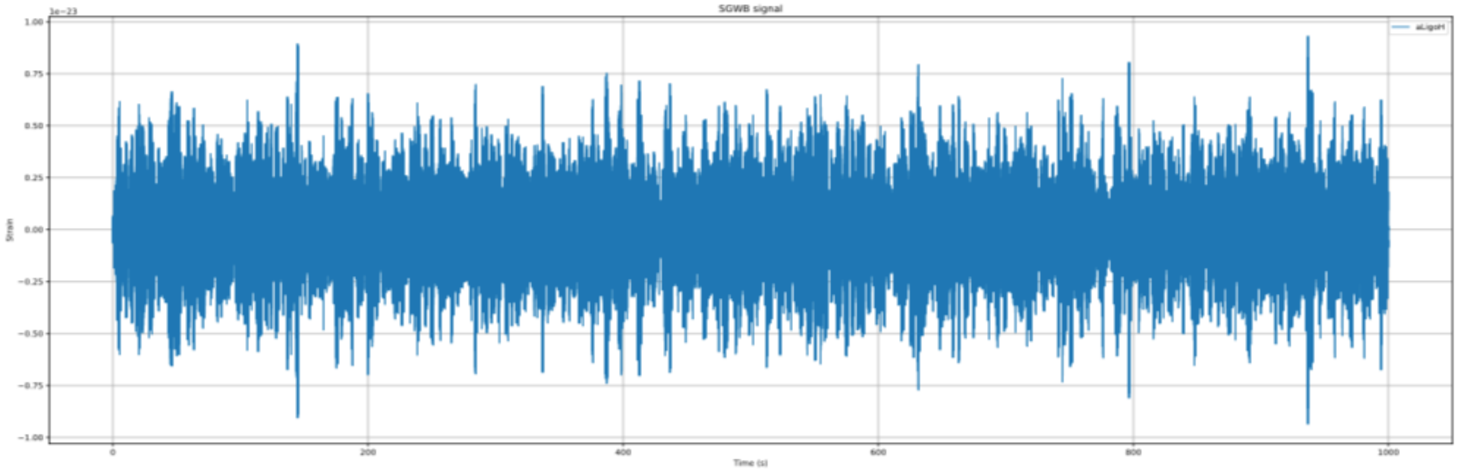
The SGWB is the superposition of a large number of independent GW signals too weak to be detected individually. We denote two types of such signal :

Cosmological origin

- ➢ Phase transition in the early universe
- ➢ Analog to the CMB

Astrophysical origin

- ➢ Compact binaries merge BBH + BNS + BHNS
- ➢ Unresolved sources
- ➢ Dominant in the SGWB



Study hypothesis :

- Isotropic
- Unpolarized
- Gaussian

Motivations and problematics

SGWB shall provide information about :

- The early moment of this universe (inflation, phase transitions, gravity theory, etc…)
- Astrophysical processes, populations of compact objects.

Goal of the internship :

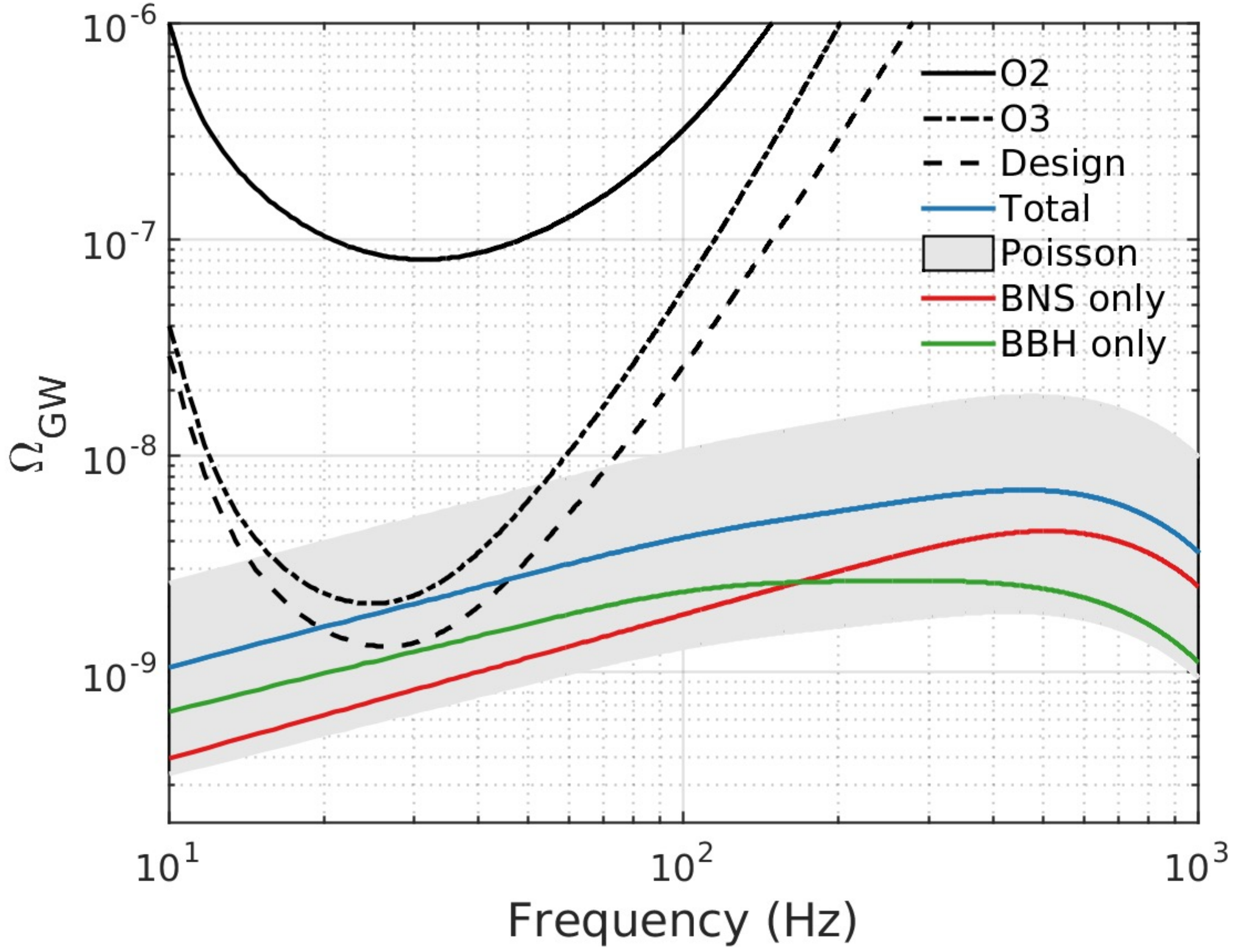
- Create an accurate and realistic (Monte-Carlo) simulation of astrophysical SGWB based on models and observations.
- Develop methods to extract this signal from LIGO-Virgo data stream.

This search work aims to propose a flexible tool for stochastic background analysis via a wide range of input parameters and distributions.

Prospective with LIGO-Virgo

- From the SGWB, we can derive the energy density of the stochastic astrophysical background.

- The astrophysical stochastic background should be visible during the next observing runs.



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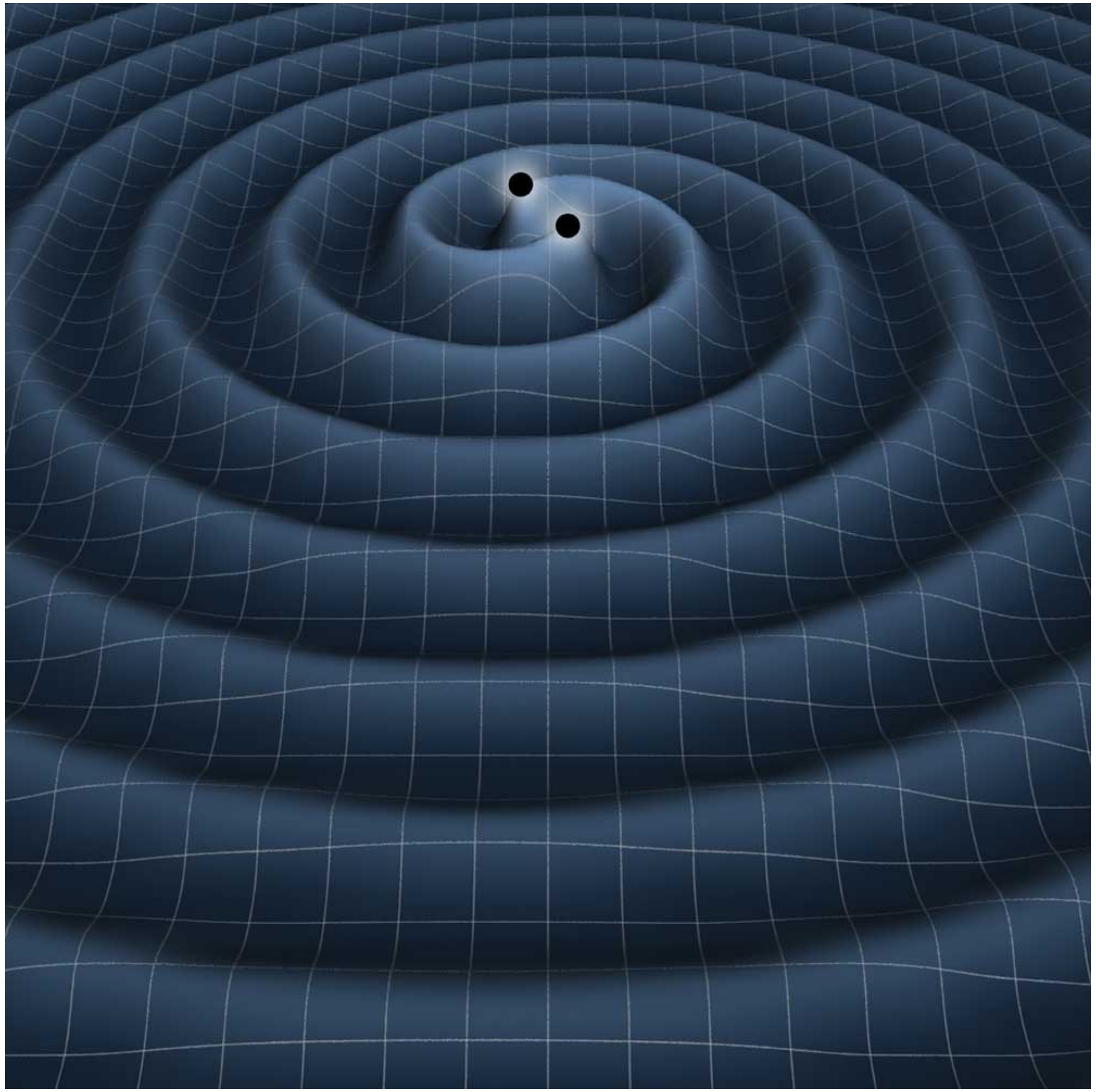


How to detect the SGWB ?

SGWB would just appear as noise in a single detector

⇒ Impossible to use “match filtering” !

⇒ Look for correlation of the SGWB signal in two detectors.

In each detector, a signal with two component will be detected :

$$s_i(t) = h_i(t) + n_i(t)$$
$$n_i(t) >> h_i(t)$$

The idea is to superimpose both signals in the Fourier domain via two methods :

Cross correlation method
Optimal filtering method

Cross correlation analysis

Find a way to look for correlation between two detectors :

hypothesis : only the SGWB is correlated in the two detectors + co-aligned and co-located

$$< \hat{C_{12}} > = < s_1(t) s_2(t) > \simeq < h_1(t) h_2(t) >$$

Correlation of both signals

$$< \hat{C_{12}} > = \int_{-\infty}^{+\infty} df \int_{-\infty}^{+\infty} df' \, \tilde{s_1}(f) . \tilde{s_2}^*(f') = S_h(f)$$

Strain power spectral density of SGWB

- SNR of the signal :

$$SNR = \frac{\mu}{\sigma} = \sqrt{T} \frac{S_h(f)}{\sqrt{S_1(f) S_2(f)}}$$

and $S_i(f) \simeq < \hat{C_{ii}} > \simeq < n_i^2 >$

Observation time

Power of detectors

Optimal filtering method

- The correlation between $s_1(f)$ and $s_2(f)$ is given by :

$$< \hat{C_{12}} >= \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df^{'} \tilde{s_1}(f).\tilde{s_2}(f^{'}).\tilde{Q}(f, f^{'})$$

- Introduce an “a priori” filter function :

Spectral shape of $S_h(f)$

Overlap reduction function

$$Q(f) = \frac{H(f).\Gamma_{12}(f)}{P_1(f).P_2(f)}$$

Total power in a detector

⇒ $Q(f)$ is the optimal way to correlate data from two detectors while maximizing the SNR.

Energy density spectrum

$\rho_{gw} = \frac{c^2}{32\pi G} < \dot{h}_{ab}\dot{h}^{ab} >$ is the quadratic expectation value of the metric perturbation.

$S_h(f) \equiv$ Strain power spectral density of the SGWB and it is related to $\Omega_{gw}(f)$ s.t. :

$$\Omega_{gw}(f) = \frac{1}{\rho_c} \cdot \frac{d\rho_{gw}}{dln(f)} \quad \Leftrightarrow \quad S_h(f) = \frac{3H_0^2}{2\pi^2} \cdot \frac{\Omega_{gw}(f)}{f^3}$$

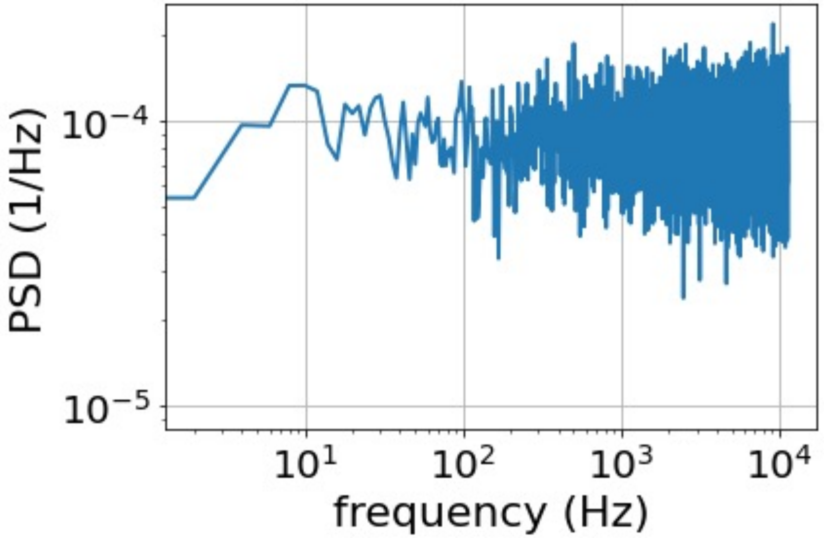
For a binary inspiral :

$$\Omega_{gw}(f) \propto f^{\frac{2}{3}}$$

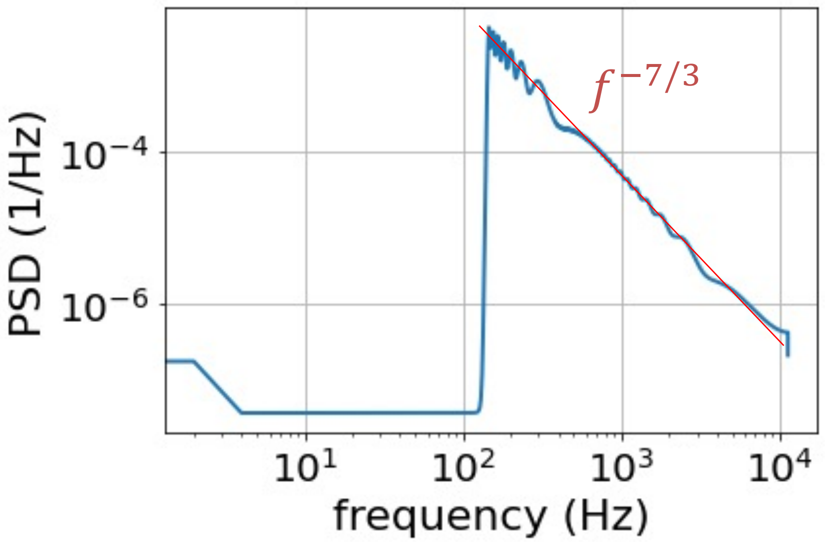
$$S_h(f) \propto \frac{\Omega_{gw}}{f^3} \quad \Rightarrow S_h(f) \propto f^{-\frac{7}{3}}$$

Spectral shape $H(f)$

- The spectral shape is based on the waveform of a GW in frequency domain $\Leftrightarrow$ Tuned to a particular SGW
- I generate a white noise and a single chirp :



White noise spectrum



Chirp spectrum

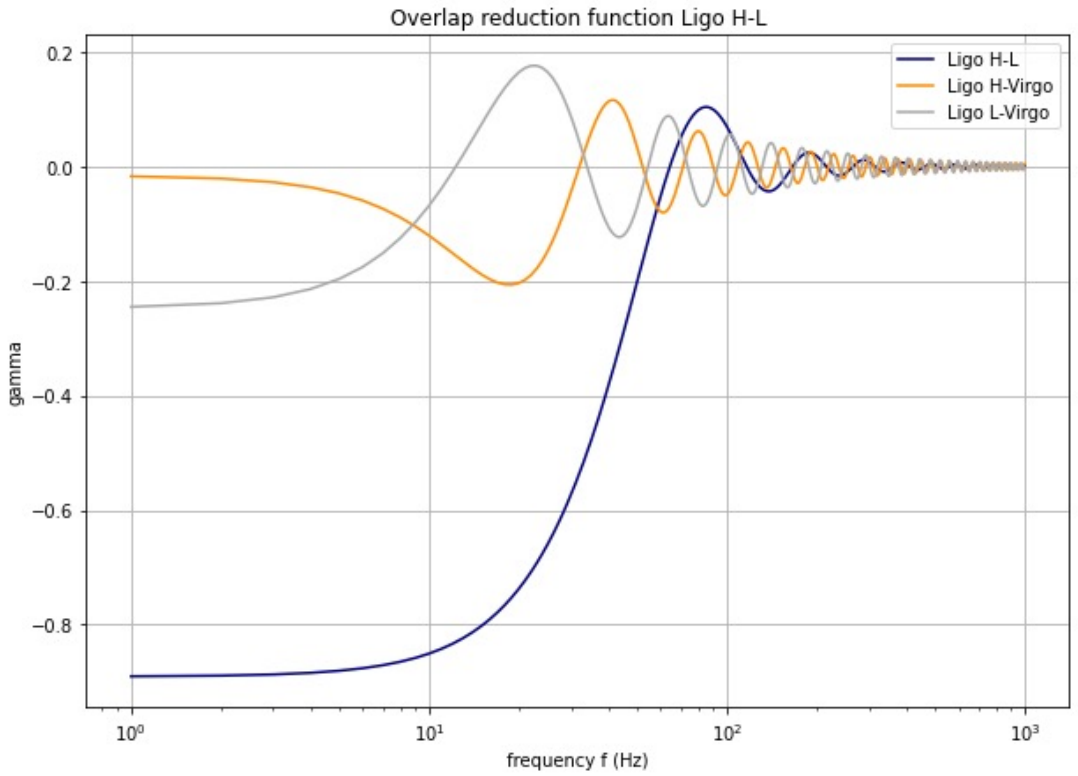
- Hence, the spectral shape of the filter is chosen to be : $H(f) = (\frac{f}{f_{ref}})^{-\frac{7}{3}}$

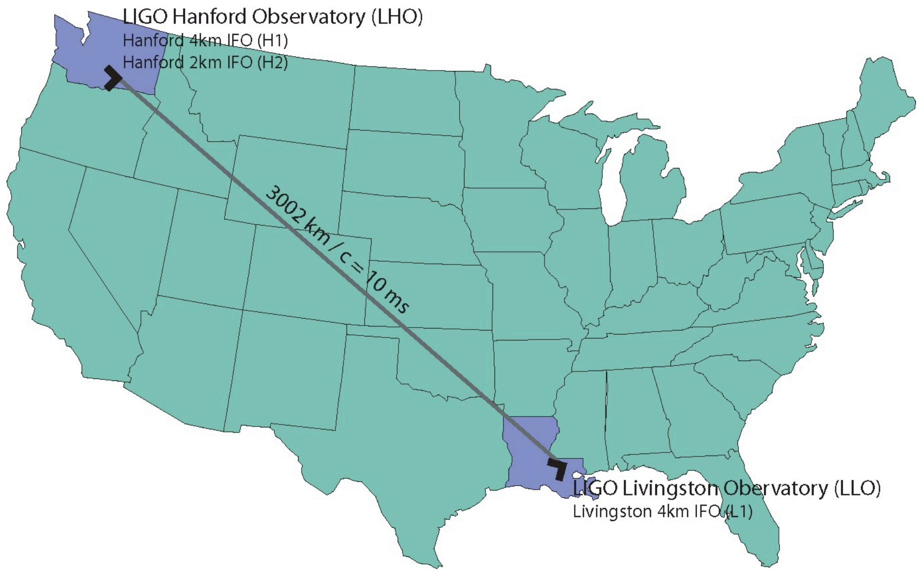
Overlap reduction function $\Gamma_{12}(f)$

- Detectors are *misaligned* / not *collocated*

  $\Leftrightarrow$ different detector response !

- Introduce the overlap function $\Gamma_{12}(f)$:





Tell us how the common sensibility of two detector varies as a function of the frequency

$$< s_1(f) s_2(f) >= \Gamma_{12}(f) . S_h(f)$$

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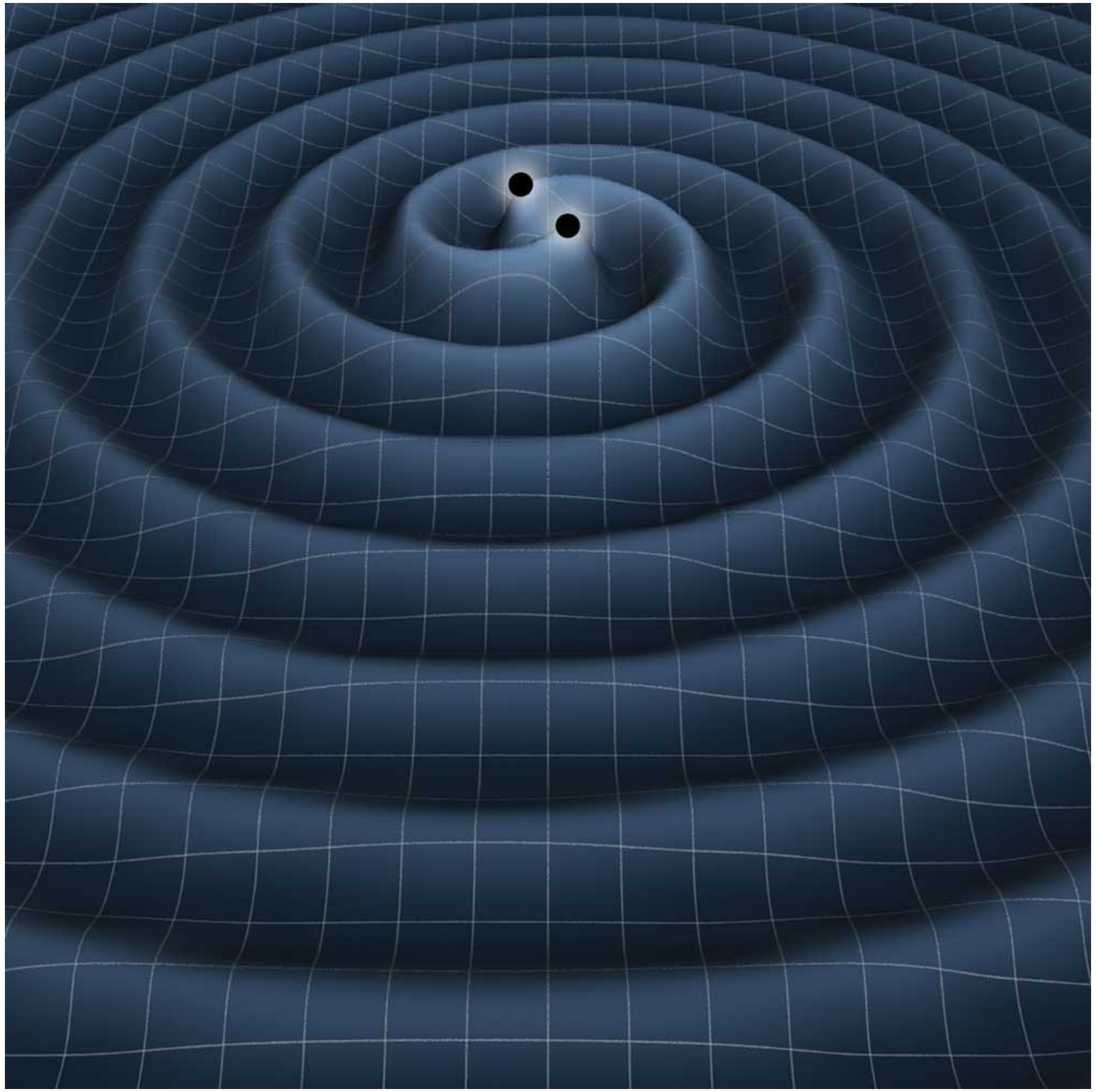
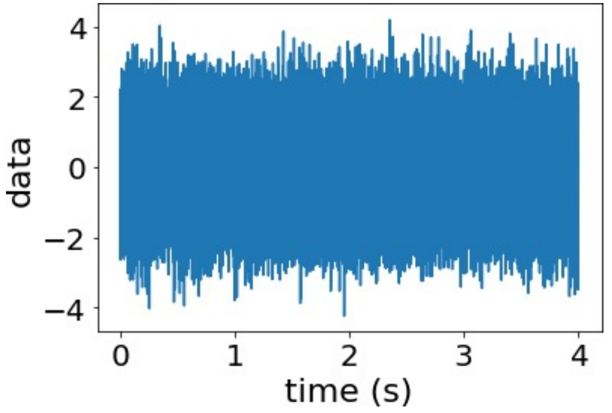
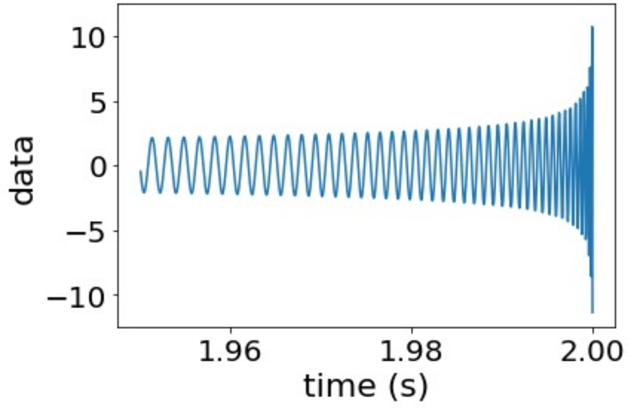


Toy-model and first analysis

SGWB toy-model

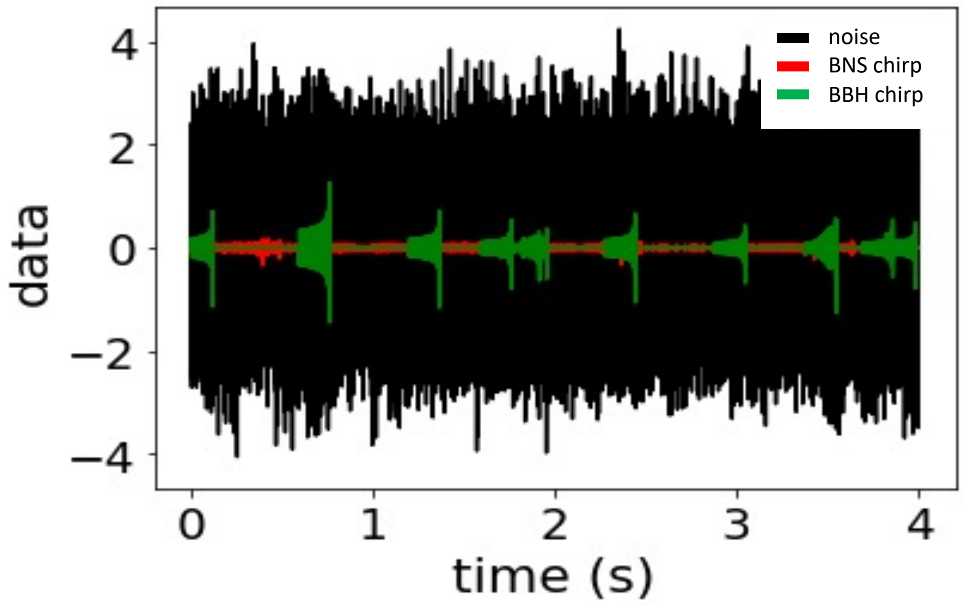
- White instrumental noise
  - Y = numpy.random.normal(0,var,N)
- BBH/BNS chirp signals
  - Y= A.sin(ϴ)
- Chosen by hand rate/amplitudes
- Random arrival times
- Co-aligned/co-located detectors
- All mergers are at the same distance.



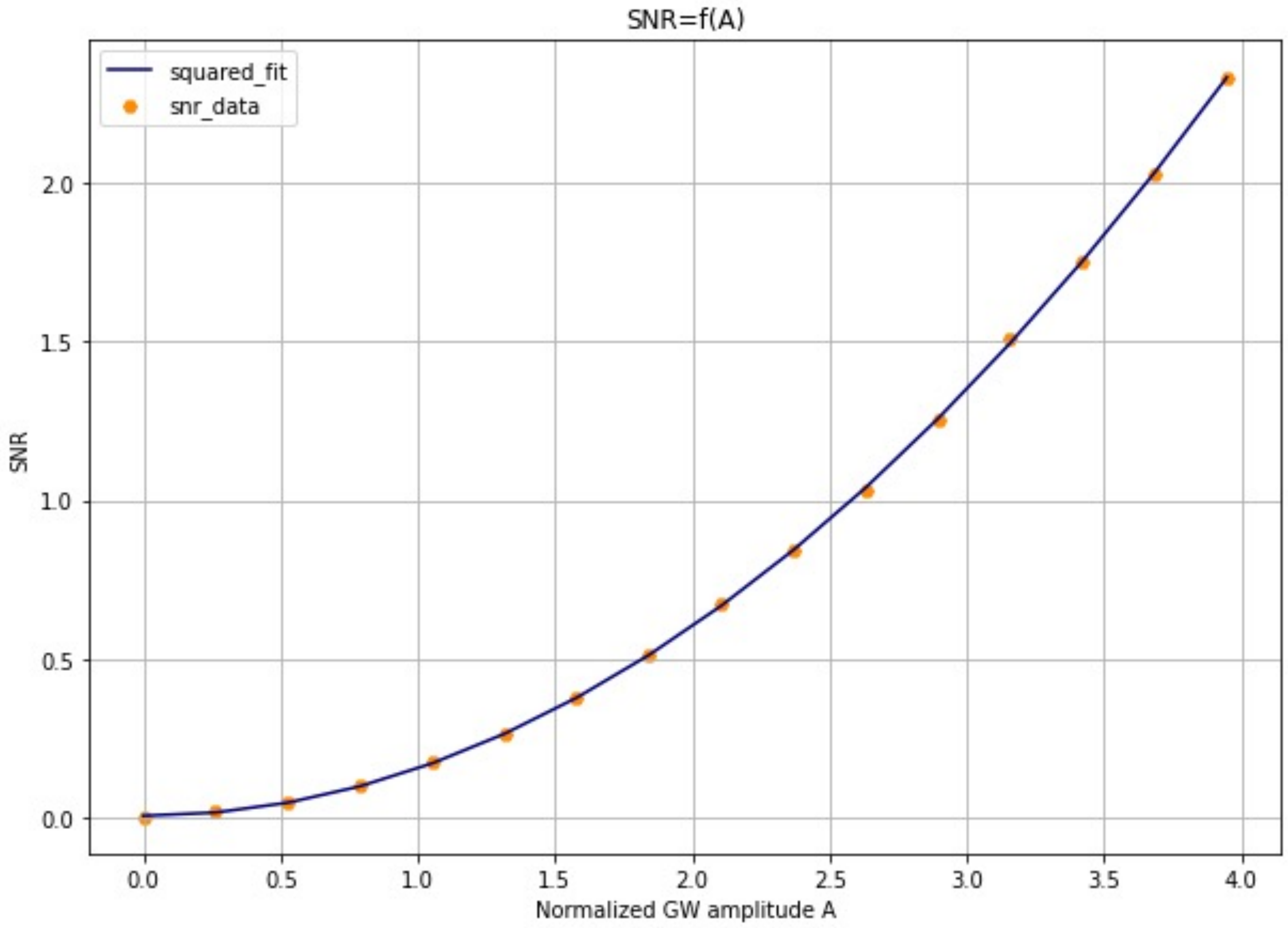


We inject **10 BBH** chirp and **10 BNS** chirp into a **4s** instrumental noise, and apply the simple cross-correlation algorithm :

$$SNR = 5.1$$



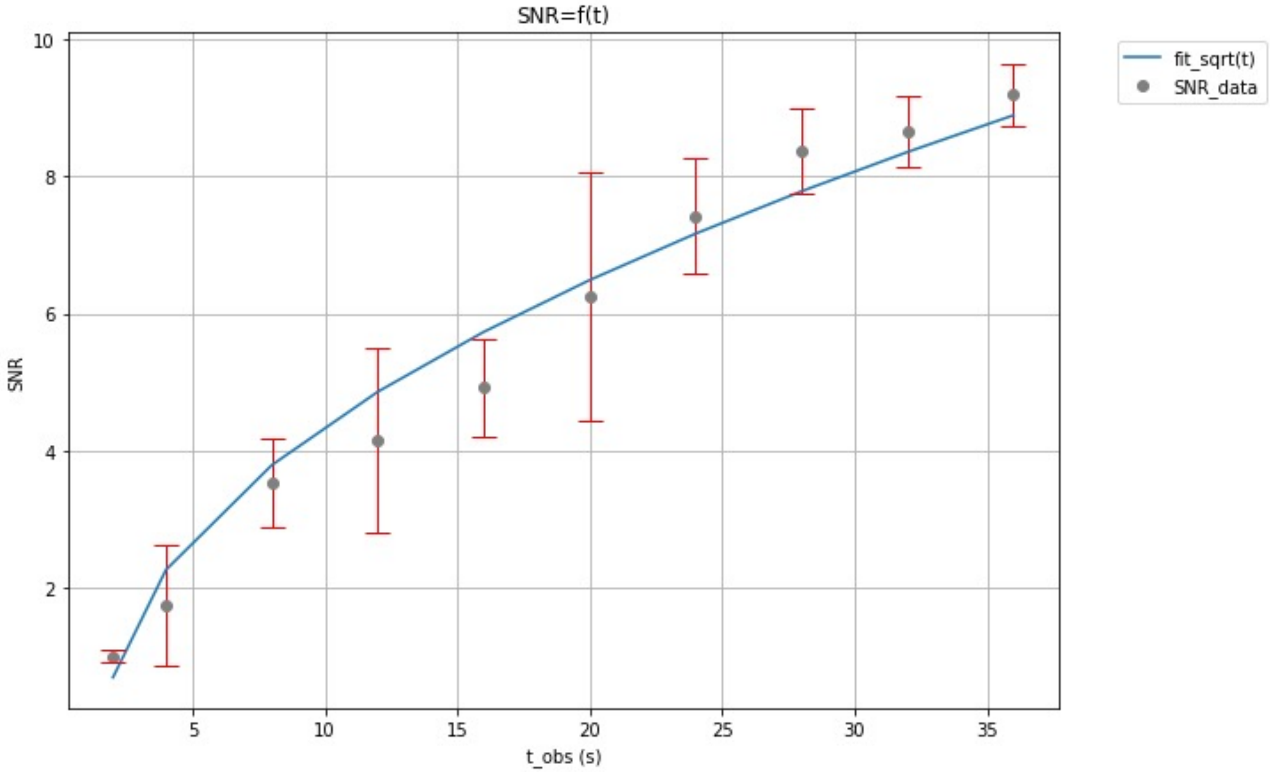
SNR variation with simulation parameters

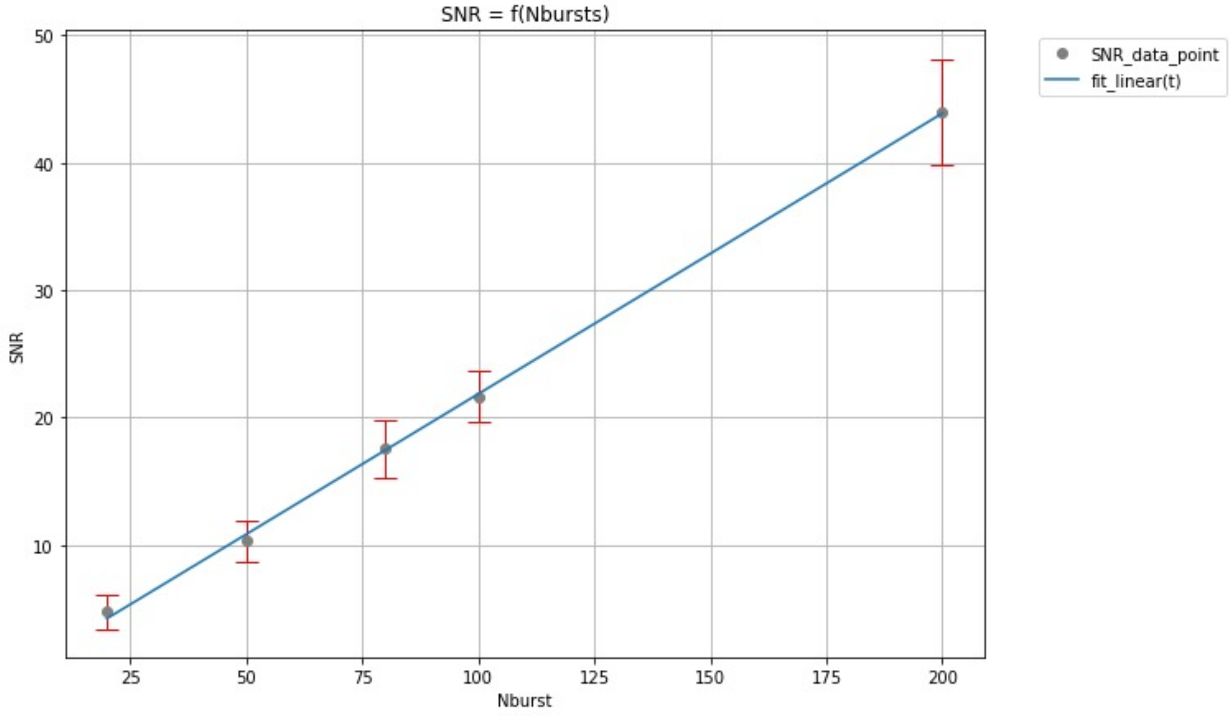


$$SNR = \frac{\mu}{\sigma} = \sqrt{T}\frac{S_h(f)}{\sqrt{S_1(f)S_2(f)}}$$

$$SNR \propto S_h(f)$$

$$S_h(f) = < h^2 > \propto A^2$$

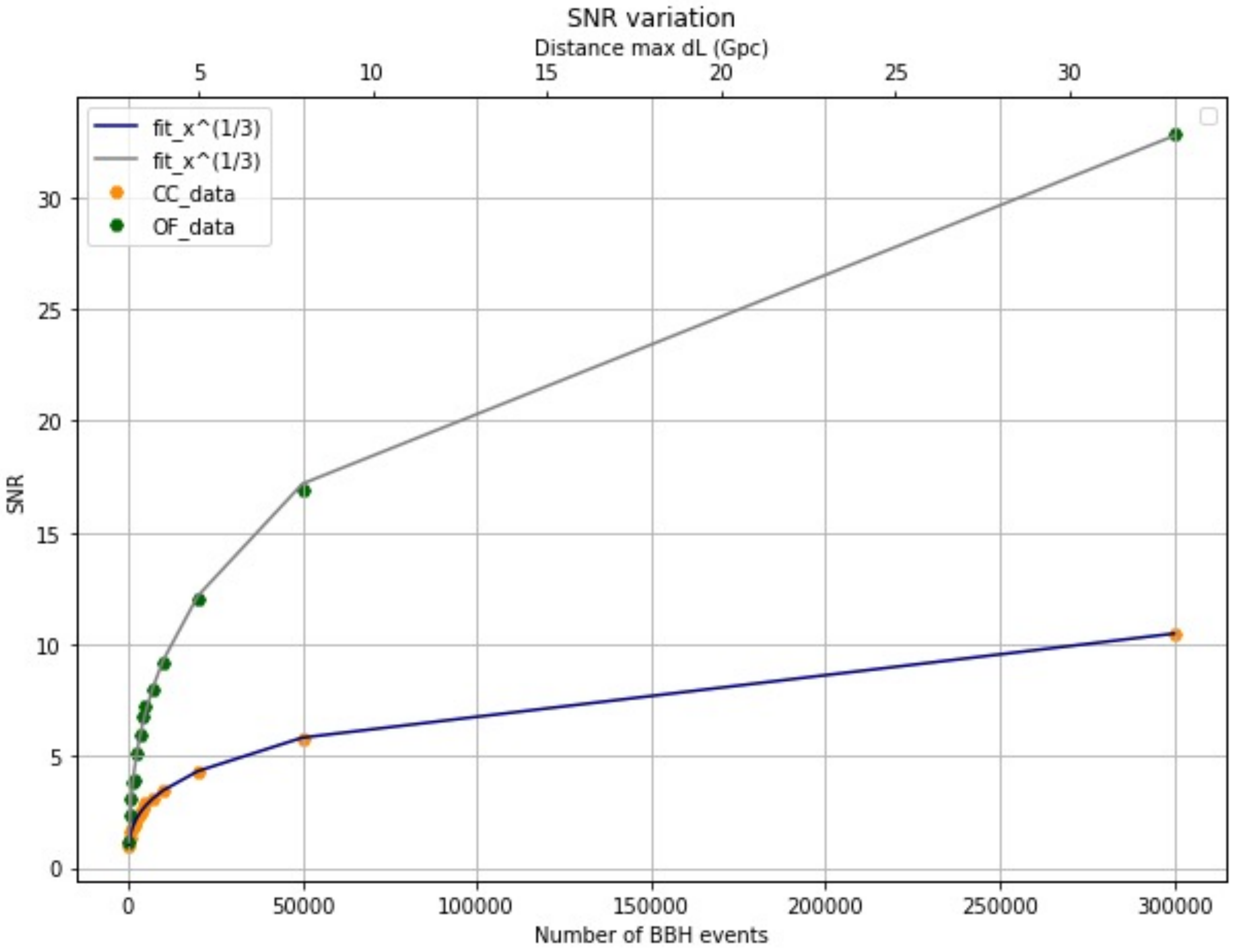




$$SNR \propto \sqrt{T}$$

$$SNR \propto A.Nbursts + B$$

Comparison : Cross-correlation / Optimal-Filtering



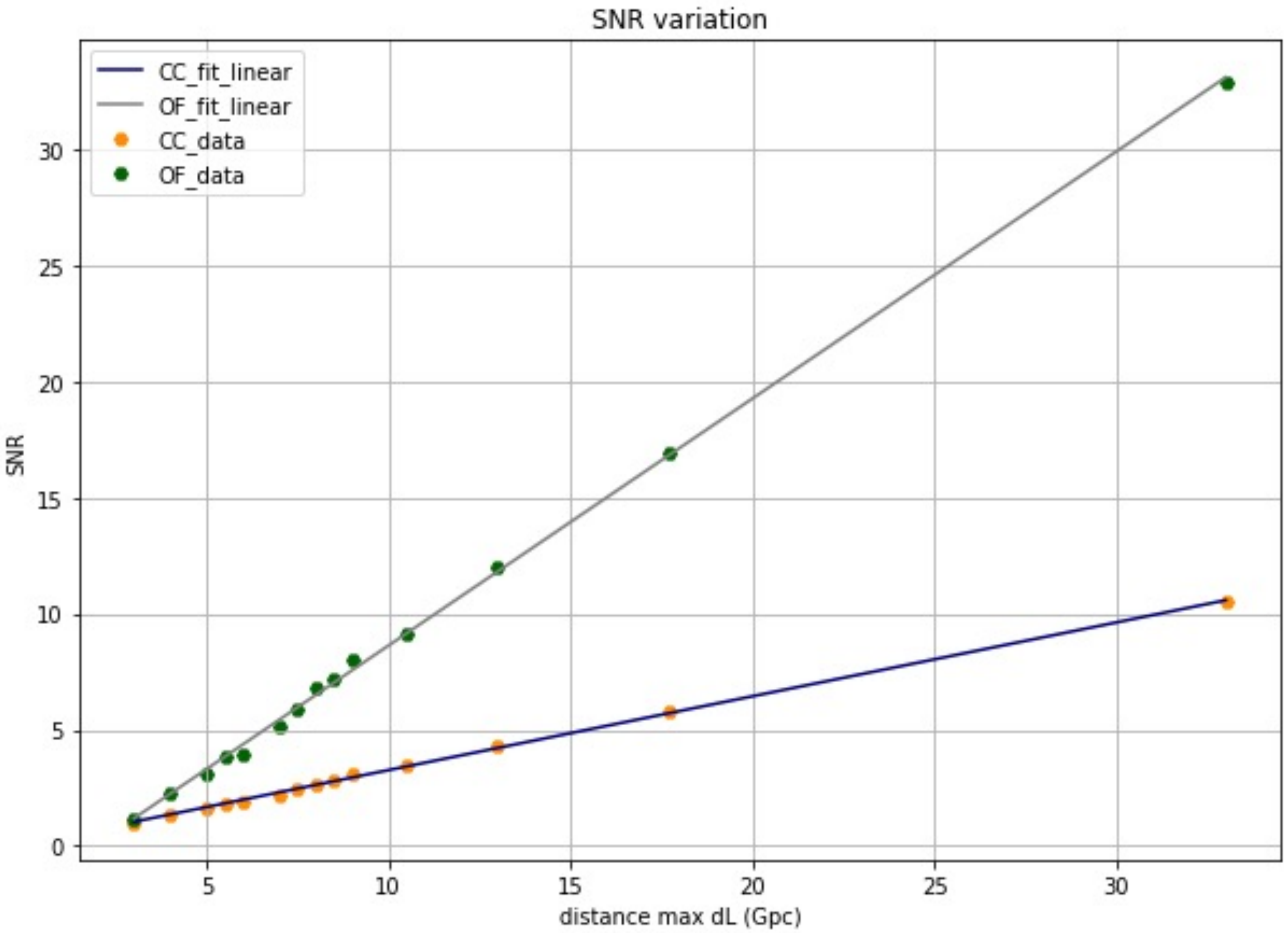
Introducing a distance distribution of mergers :

- Distance distribution d_L is linear.
- Fixed primary masses at $10M_{\odot}$
- 2 sets of SNR data :
  - Cross correlation
  - Optimal filtering

$SNR = f(Nburst)$ is no longer linear.

The Optimal filtering manages to detect the signal at larger distances, hence smaller amplitudes.

SNR evolution with the luminosity-distance



Expecting a threshold effect of the SNR with the distance :

- The evolution of $SNR = f(dL)$ is linear.
- Up to 30Gpc, the SNR keeps increasing.
- The Optimal filtering is still more efficient.

→ The amplitude of distant sources is still contributing to the SNR.

Maybe we did not simulate mergers far enough to see any effect. Or the distribution of merger is not physically accurate.

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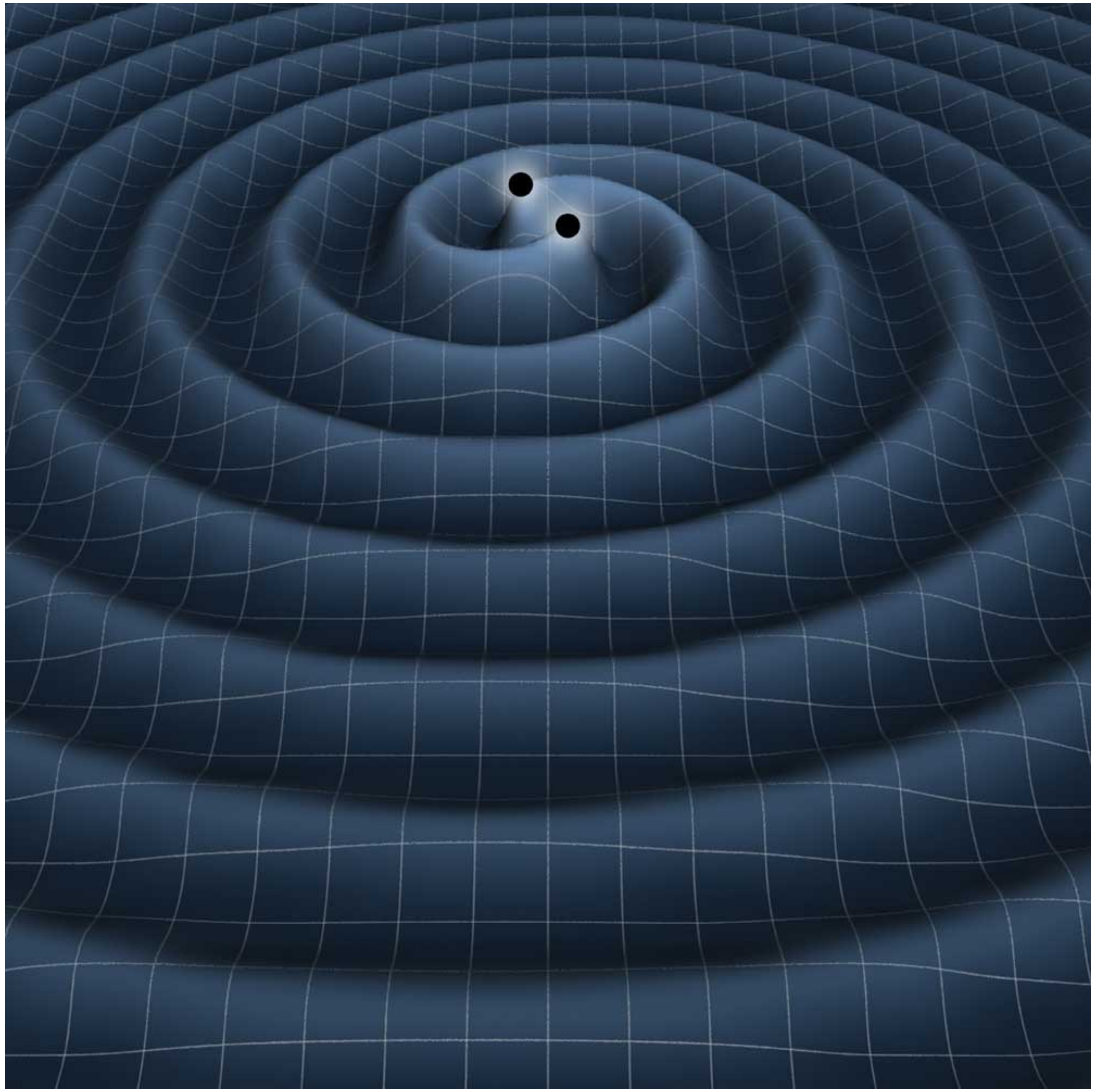


Physical characteristics of the SGWB

Features :

- Waveform simulated with PyCBC and Numerical General Relativity : SEOBNRv4_opt
- Coloured instrumental noise
- Only BBH systems
- Angular distribution of merger
- Cosmological Redshift and rate density distribution
- Overlap function for miss-aligned detectors

Goals :

All these features aim to switch from our Toy-model into a complex simulation, able to generate a realistic SGWB.

The SGWB will be used as an input in our work, to perform analysis.

Coloured noise from PSD

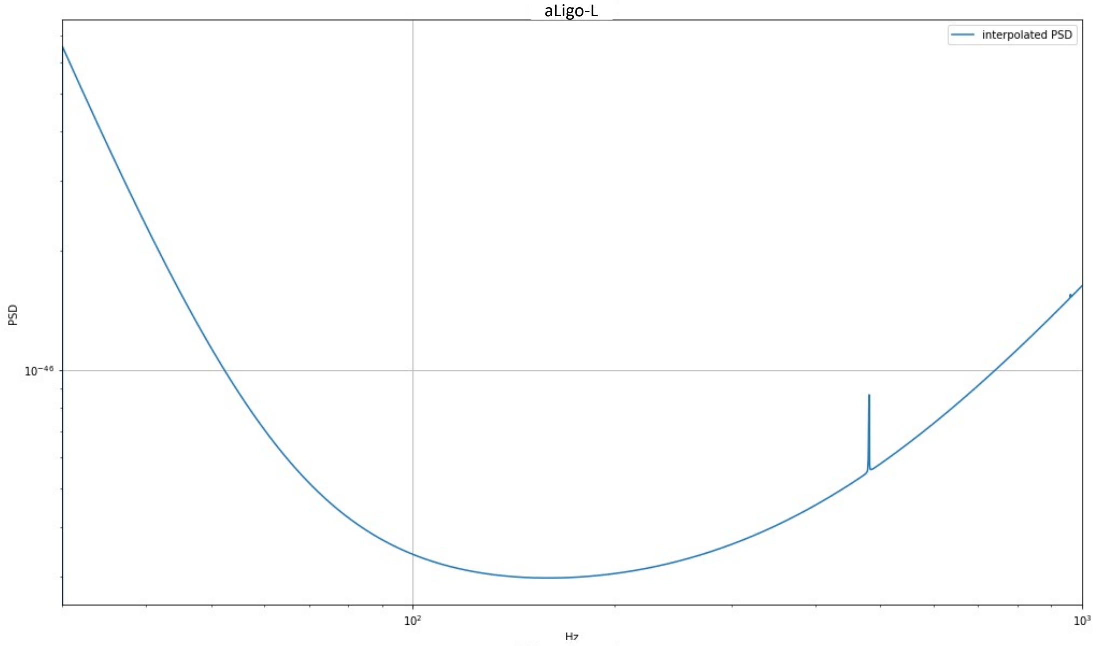
- We construct the Power spectral density (PSD) of a specific detector.

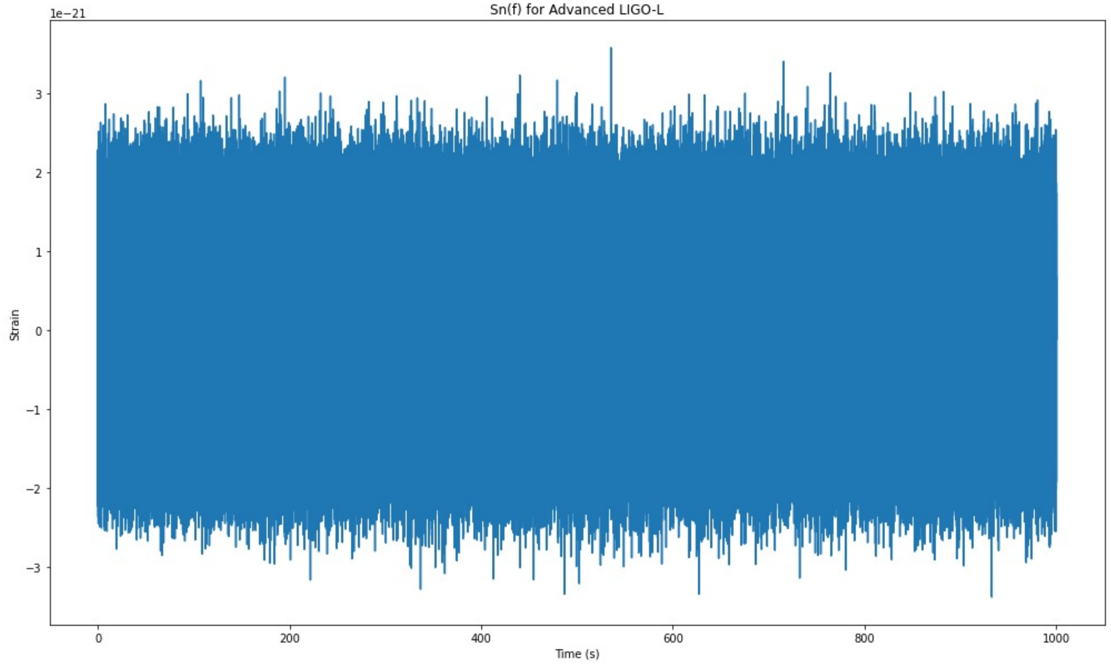
  (advanced Ligo Hanford on the graphic)

- Generate the associated coloured noise, interpolated to $4096\ H_Z$.

Parameters :

$$F_s = 4096\ H_z$$
$$f_{cut} = 25\ H_z$$
$$T = 1000s$$



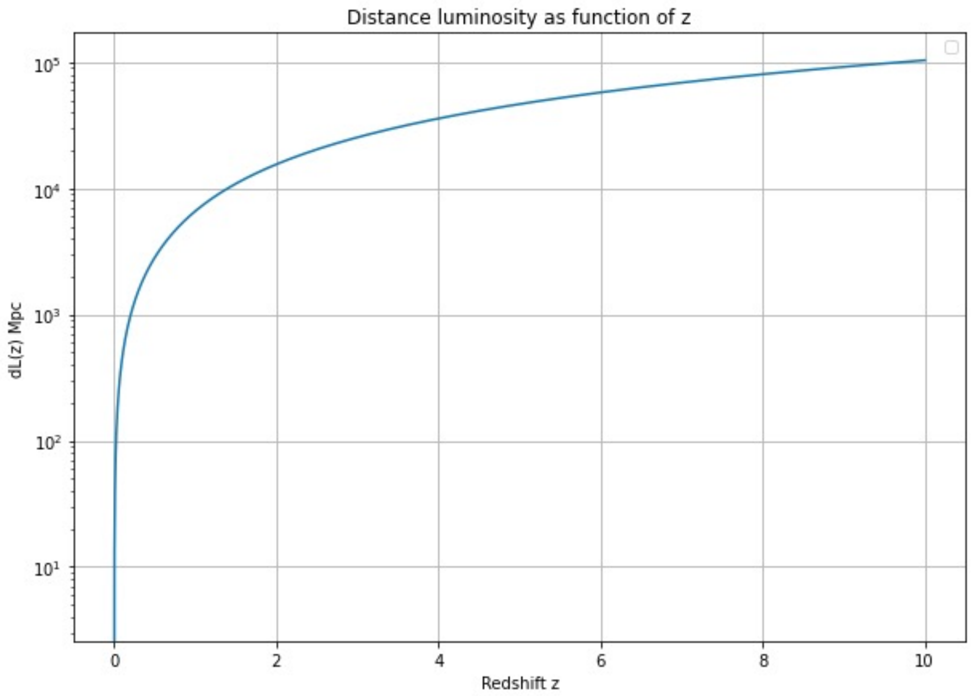


Cosmological Redshift of GW's

The cosmological redshift z is due to the expansion of the universe and is defined as :

$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}}$$

$$d_L = d(1+z)$$



Impact of z on the GWs :

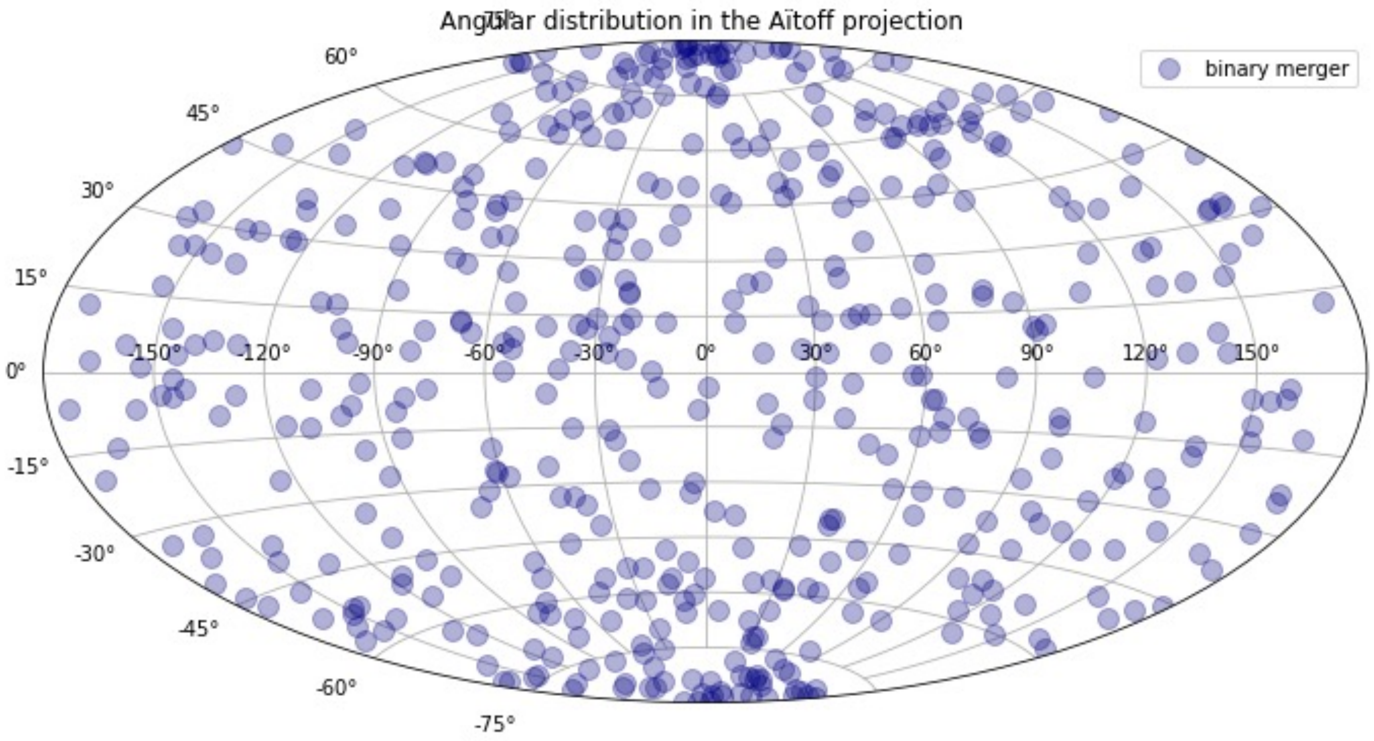
- Shifts the GW wavelength
- Induces a mass-redshift degeneracy

Low-masses sources at high z $\Leftrightarrow$ High-masses at low z

$$(m_1, m_2) \Leftrightarrow (m_1, m_2).(1+z)$$

Angular distribution of binary systems

- As we are looking at a extragalactic astrophysical stochastic background, one can assume that the flux is isotropic and homogeneous over the sky.



Parameters of the distribution :

$$T = 1000s$$

$$\text{Redshift} \;\; 0 < z < 3$$

$$(\alpha, \delta) \propto \text{uniform}$$

- ➢ Impacts the amplitude of each GW due to the detector response.
- ➢ Impact the time delay between detectors.
- ➢ Impact the polarization of each GW.

N.B: Isotropy and homogeneity are verified if we look far enough (z) and a for long enough observation time (T).

Merger-rate and redshift variation $R(z)$

The merger-rate density of BBH can be approached by the following equation :

$$R(z) = A.\frac{dV}{dz}.\frac{1}{1+z}.\psi_{MD}(z)$$

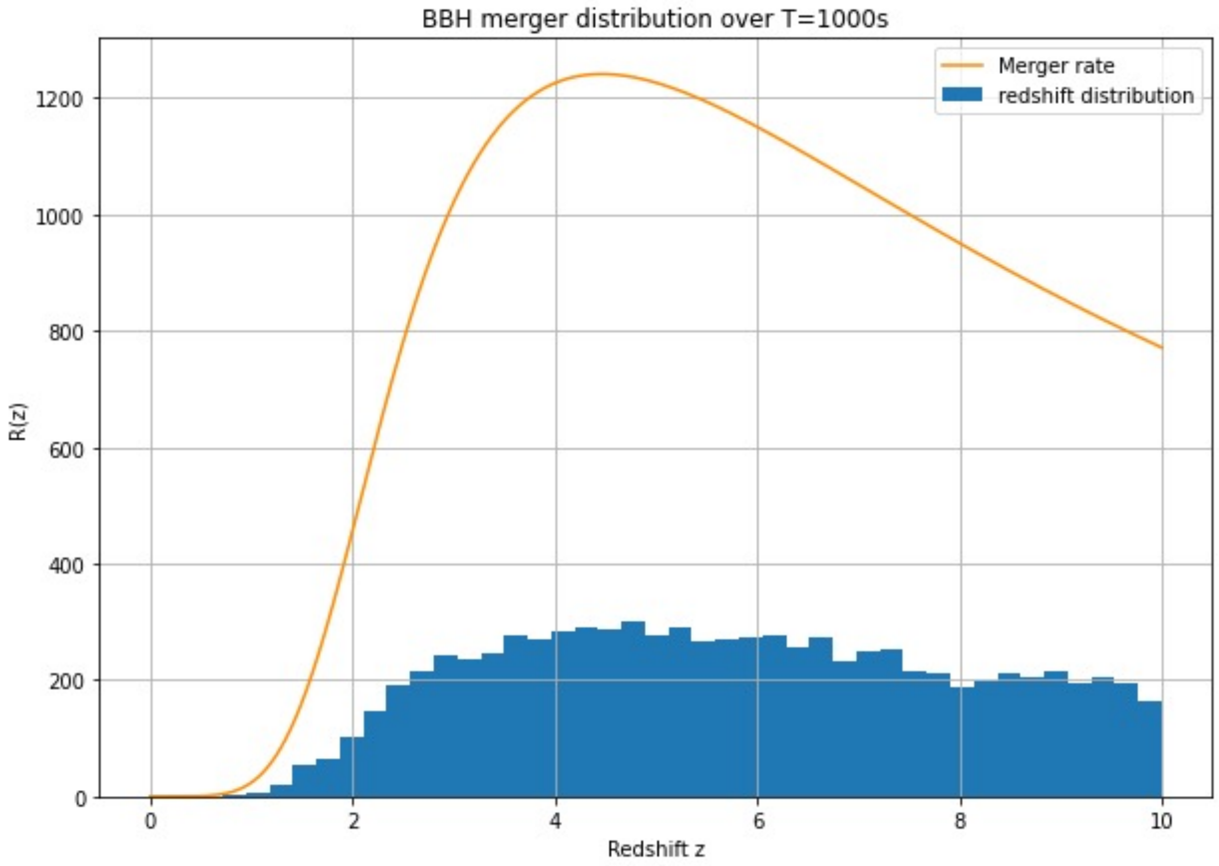
Comoving volume element of universe $\quad \frac{dV}{dz} = 4\pi.\frac{c}{H_0}.\frac{r(z)^2}{E(\Omega,z)}\,, \quad E(\Omega,z) = \sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}$

Transform from the detector frame to the sources frame

Based on the star formation rate of Madau et Dickinson 2014 $\quad \psi_{MD}(z) = \psi_0.\frac{(1+z)^\alpha}{1+(\frac{1+z}{C})^\beta}$

BBH system distribution up to z_{max}

An optimized rejection method is used to construct the BBH distribution up to a certain redshift :



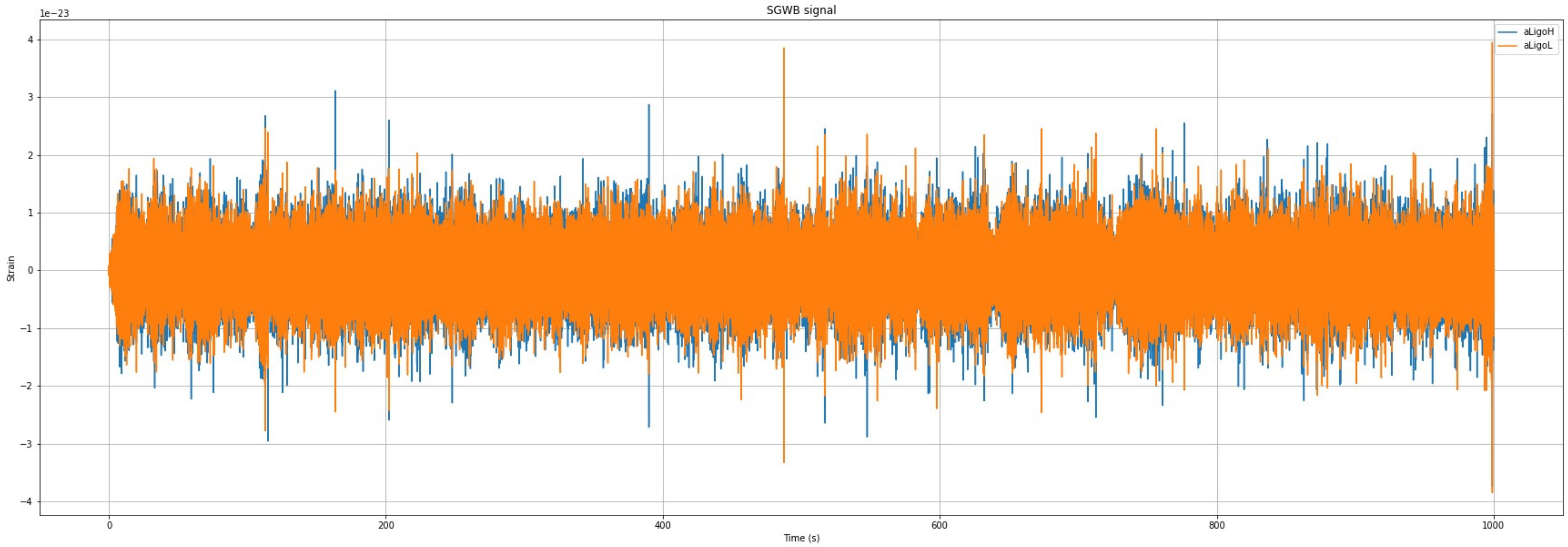
- Here, we have simulated up to $z = 10$ with an observation time $T = 1000s$.

- Cosmological parameters :

$$\Omega_\Lambda = 0.7$$
$$\Omega_m = 0.3$$

- The total number of BBH merger contributing to the SGWB is equal to 8355

First result of stochastic GW background



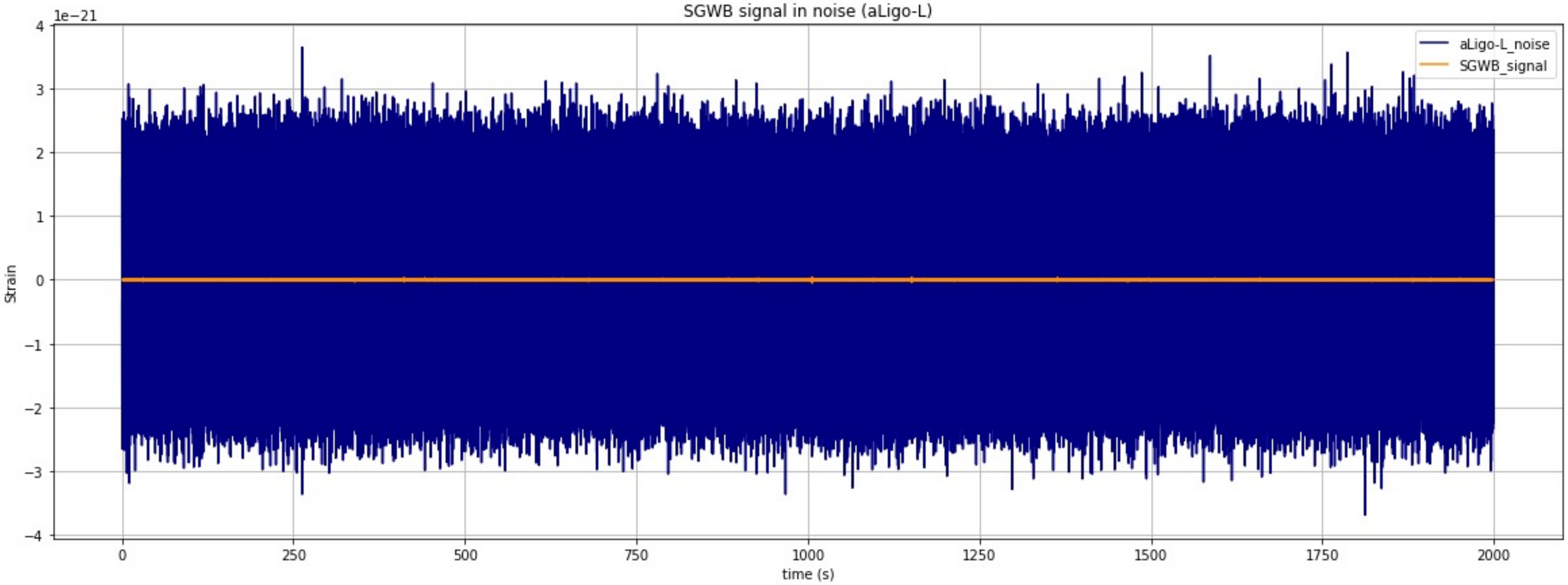
- This type of SGWB is called a “confusion limited” background due to the overlap between individual signals.
- The amplitude modulation of the strain is due to all parameters distributions we have implemented.

$$T = 1000s \quad f_{cut} = 20H_z$$

$$BBH = 8355 \quad z = 10 \quad m_1, m_2 = 10M_\odot$$

WARNING : Primary masses are fixed to $10M_\odot$ $\Longrightarrow$ Need a mass distribution.

Orders of magnitude and SNR



$$T = 2000s$$

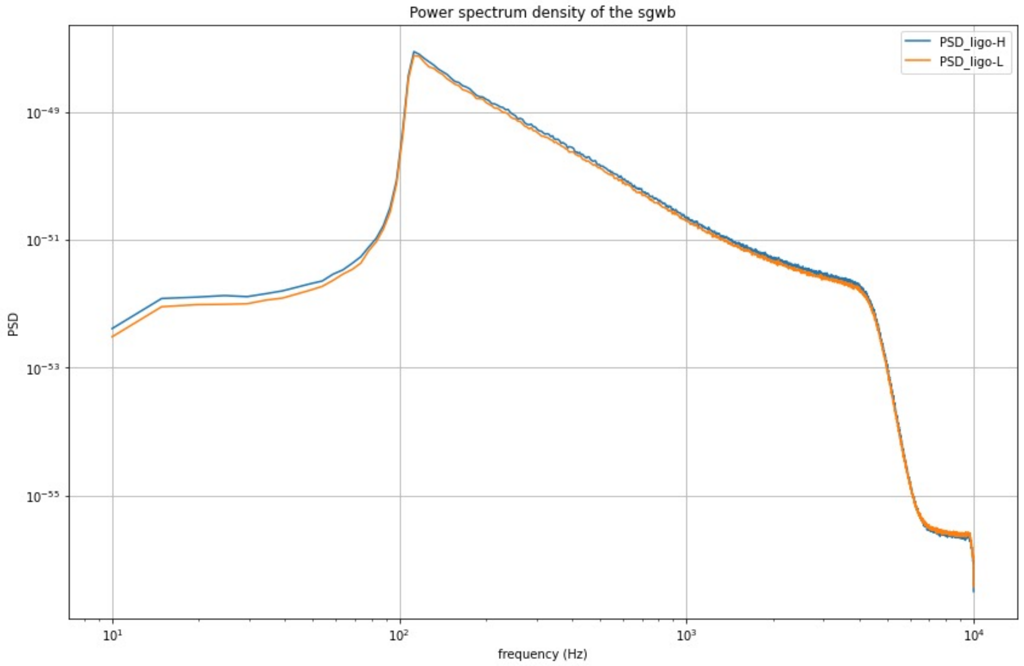
$$0 < z < 10$$

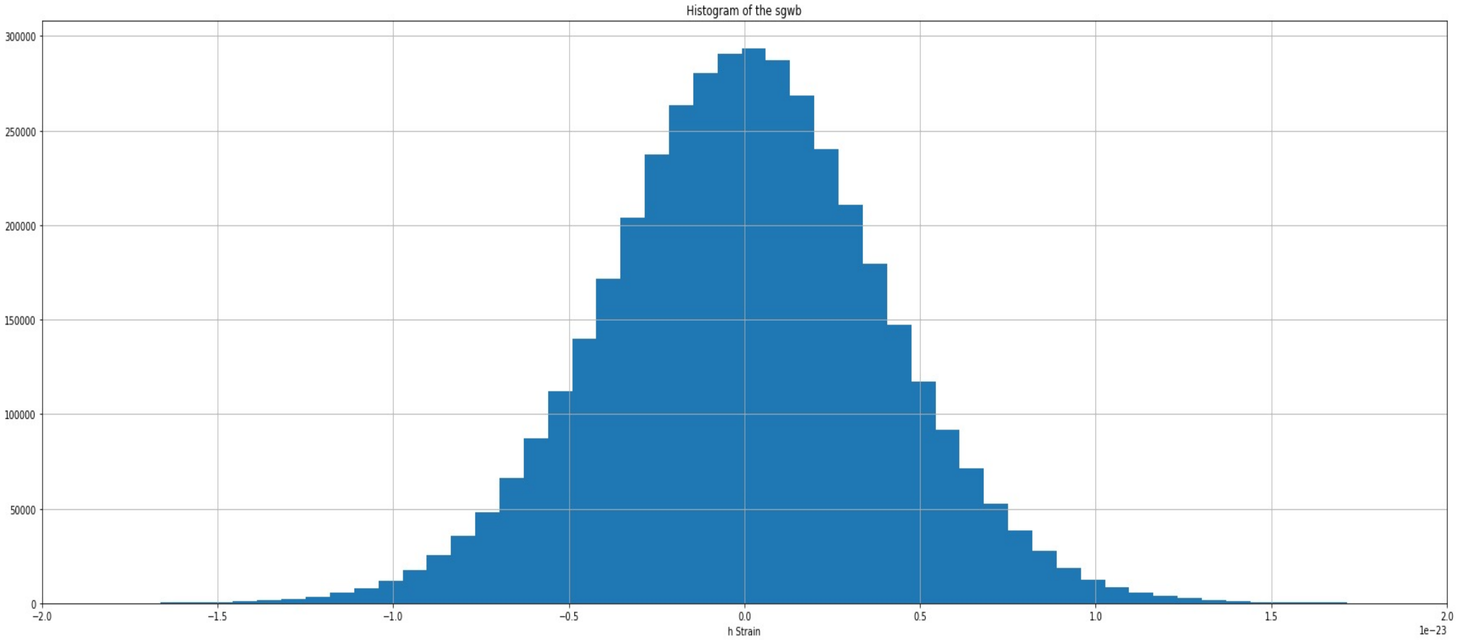
$$m_1 = m_2 = 10M_\odot$$

$$\text{Optimal filtering :} \quad SNR = 0.036 \pm 0.007$$

(work in progress)

Physical properties of the SGWB





The PSD of the signal has the same spectral shape as a single waveform.

$$H(f) = \left(\frac{f}{f_{ref}}\right)^{-\frac{7}{3}}$$

The overall signal verified the gaussianity hypothesis, due to the non-zero overlap and the global unpolarized GW.

Outlook for my internship

- Implement a realistic mass distribution of BBH mergers.

- Study the SNR evolution with various parameters such as the maximal redshift.

- Compare with other geometry detectors (CE, ET, etc...)

Thank you

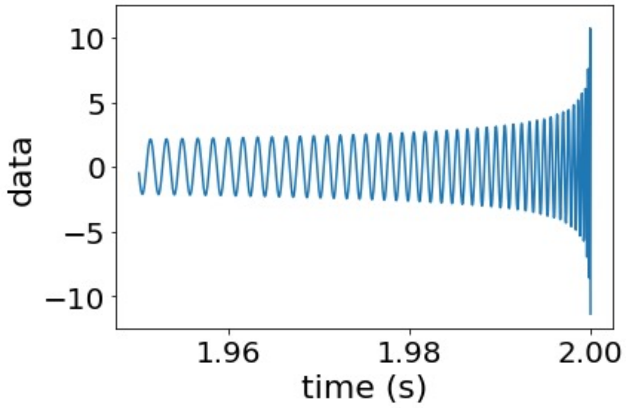
Gravitational wave signals

- A GW is a time-dependent perturbation of the geometry of space-time :

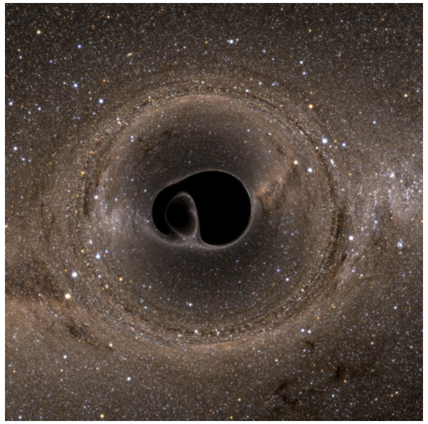
$$h_{ab}(t,\vec{x}) = \int_{-\infty}^{\infty} df \int d^2\Omega_{\vec{k}} \sum_{A=+,\times} h_A(f,\vec{k}) e^A_{ab}(\vec{k}) e^{i2\pi f (t - \frac{\vec{k}.\vec{x}}{c})}$$

- The entire signal is called a ”waveform” and can be described with 3 phases :

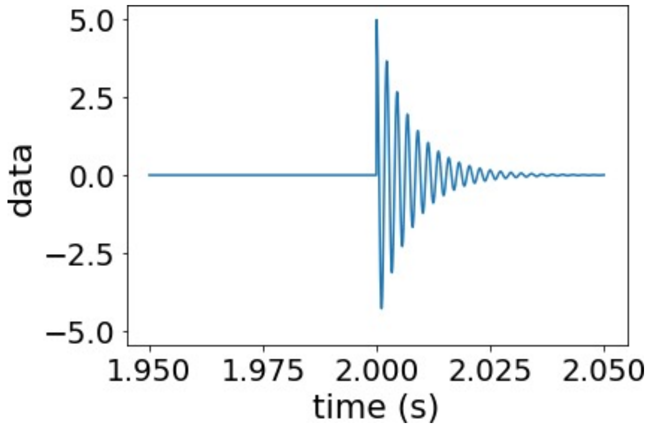
Chirp (in-spiral)



Merger phase



Ringdown



Signals and gravitational background

- LIGO and Virgo have recently detected signals from BBH and BNS mergers.
- The sum of all unresolved signals from BBH and BNS = GWB
- The combined detection and analysis of this GWB can lead to precious measurments :

$$! \quad \begin{matrix} statistical\ distribution\ of\ sources \\ phenomenology\ of\ the\ GW \\ cosmological\ GWB \end{matrix}$$

⇨ Very important to produce a consistent toy-model with accurate simulations of these signals.

Gravitational wave : waveform and amplitude

- GW = time dependent perturbations of the geometry of space time.

$$h_{ab}(t,\vec{x}) = \int_{-\infty}^{\infty} df \int d^2\Omega_{\vec{k}} \sum_{A=+,\times} h_A(f,\vec{k}) e_{ab}^A(\vec{k}) e^{i2\pi f(t-\frac{\vec{k}.\vec{x}}{c})}$$

- It is related to the strain power spectral density function $S_h(f)$:

$$< h_A(f,\vec{k}) h_{A'}^*(f^{'},\vec{k}^{'}) >= \frac{1}{16\pi} S_h(f) \delta(f-f^{'}) \delta_{AA'} \delta^2(\vec{k},\vec{k}^{'})$$

Ground-based detector

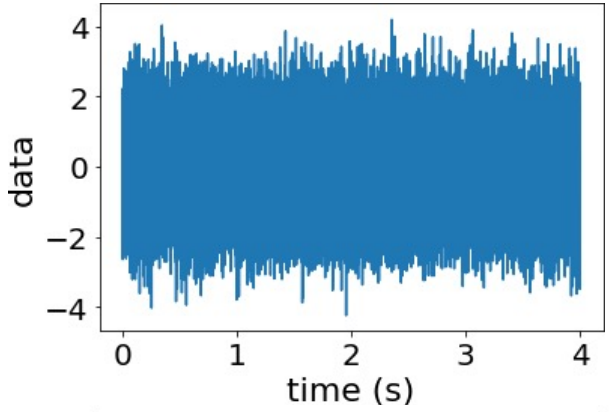
- Background noise detector ⇔ « White noise».
- Simulation :

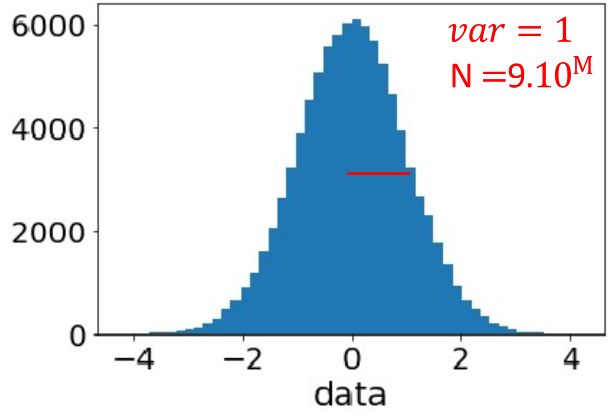
Sampling rate $F_{:}$ = 22500 $H_{<}$

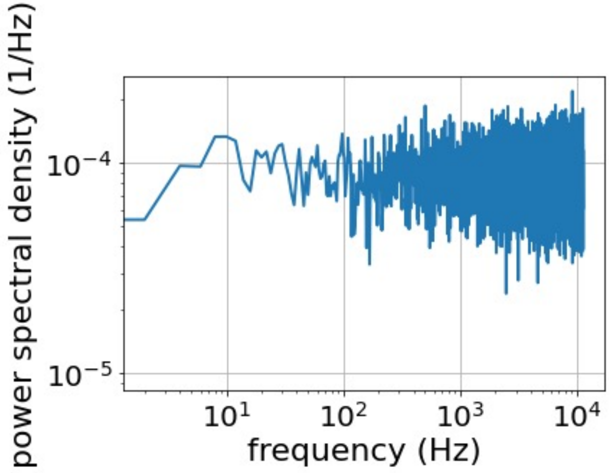
Time limits of the sample = $t_{>?@} = 0s$, $t_{>CD} = 4s$

Waveform :

y = $random.normal(0, var, N)$







Chirp signal : inspiral phase

- First phase of the coalescence ⬄ « Chirp ».
- Simulation :

  Pulse duration ~ 100s

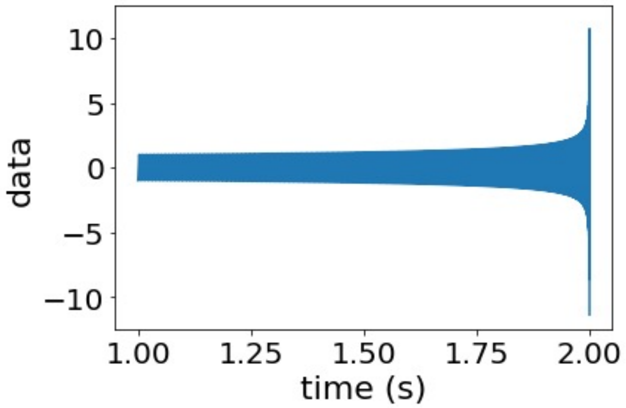
  tau = $t_0 - t$= 2-t

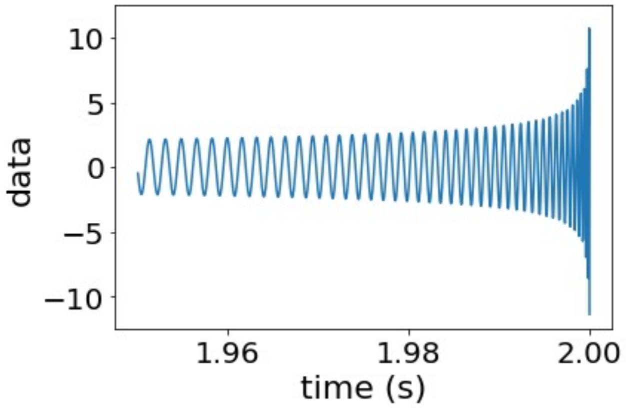
  $\phi = -(tau/M_T)^{U/V}$, s.t. $M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} = \frac{m}{2^{1/5}}$

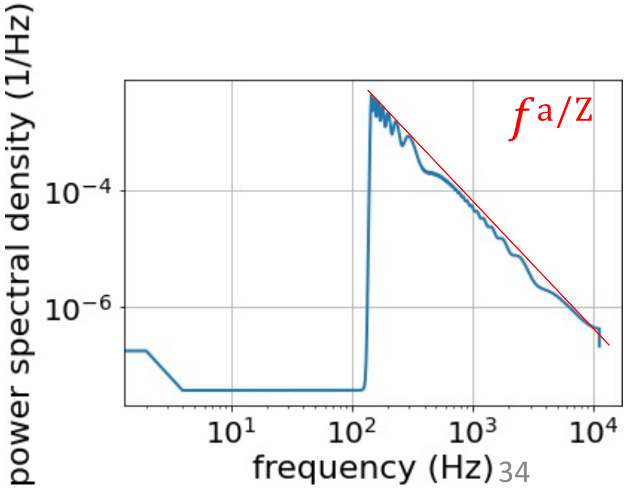
  $m = 1{,}4 M_\odot$

  $amp \propto A(M_T^{U/Z}) tau^{[\backslash/M}$, s.t. $A = 1$
- Waveform :

$$y = amp * \sin(\phi)$$







Ringdown signals : relaxation

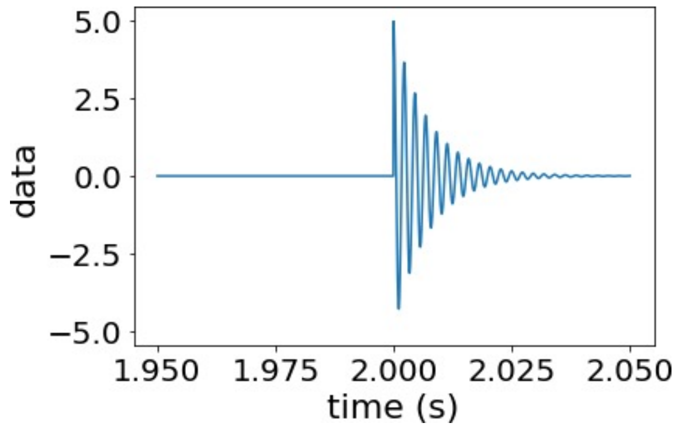
- 2nd phase after the merger ⇔ « Ringdown ».
- Simulation :

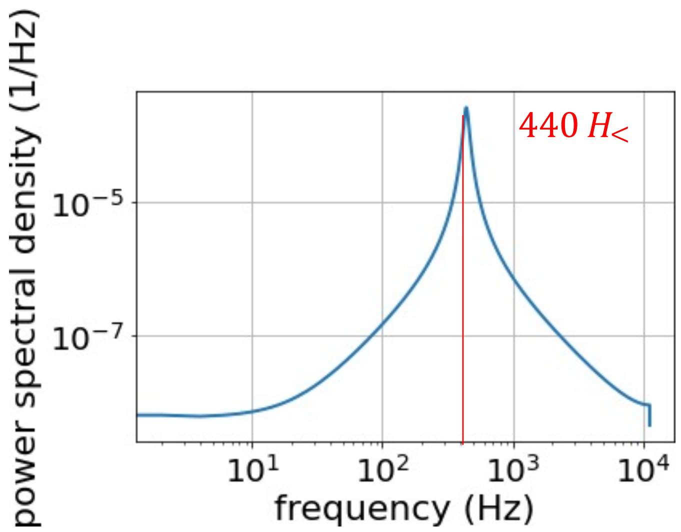
  Pulse duration ~ 1s

  Amplitude A=1

  Frequence $f_0 = 440\ H_<$
- Waveform :

$$y = Ae^{[bcd_e f/g}\cos(2\pi f_0 t)$$





Derivation method of the density spectrum

- The gravitational stochastic background also differ if we look at the « power density spectrum ».
- The numpy function *scipy.signal.welch*

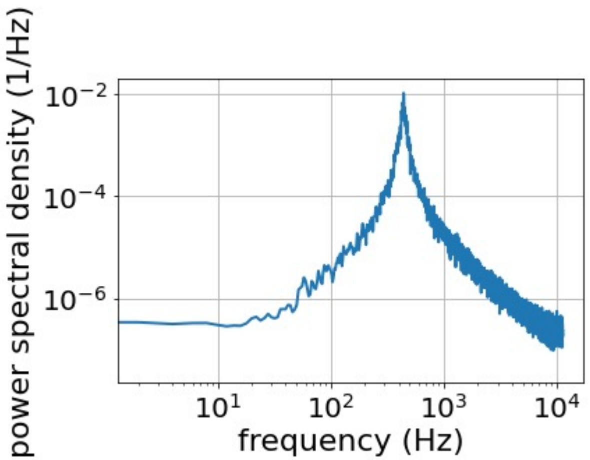
- Cut the whole signal into overlapping intervals.
- Windowing of the signal $s \otimes f$, such that $f = \begin{cases} 1 \; if \; t \in [T_\backslash, T_b] \\ 0 \; else \end{cases}$
- DFT over each segment of the periodogram we constructed.
- Overall average of the DFT coefficients.

⇒ Unbiased estimator, mostly due to the temporal average.

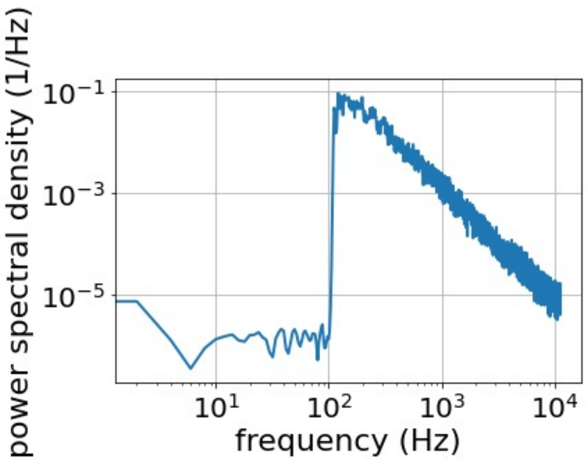
Multi-burst and power density spectrum

- When we look at the stochastic GWB, it is crucial to understand that it is formed by a combination of chirp and ringdown signals.

Confusion-limited



Pop-corn



The power density spectrum keeps the same shape.

How to combine signals ?

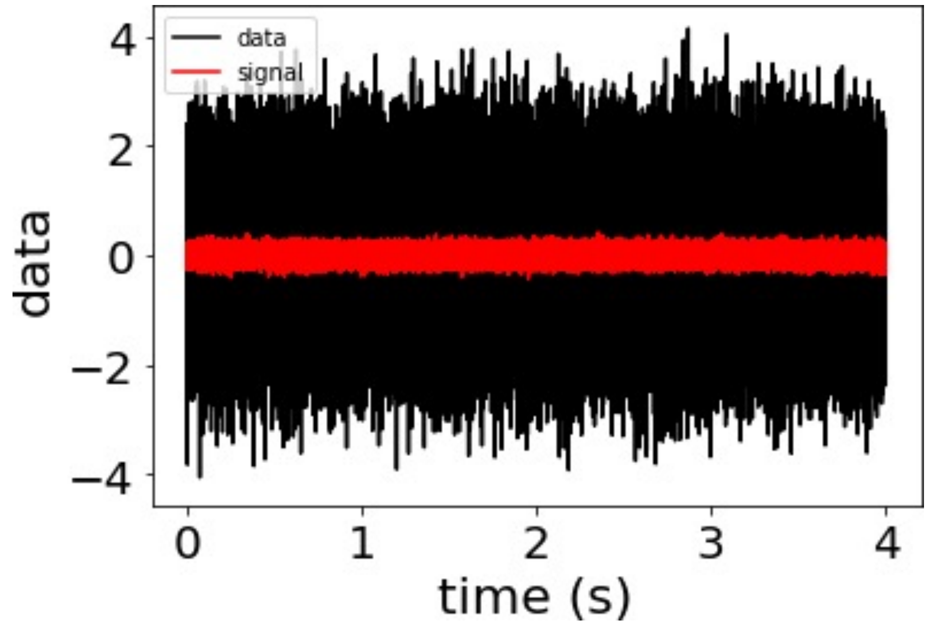
- One can say that a complete signal should be the sum of :
  - White noise detector
  - Multiple chirp + Ringdown

- Let's simulate a simple example :

white-noise detector

+

White GWB uncorrelated



Multi-burst signal and coaligned detectors

⇨ Let's consider two type of signals from BNS and BBH :

BNS chirp :

$$A = 5$$
$$m = 1.4M_{\odot}$$
$$Nburst = 10$$
$$chirpduration = 4.84\ s$$
$$N = 9.10^{M}$$

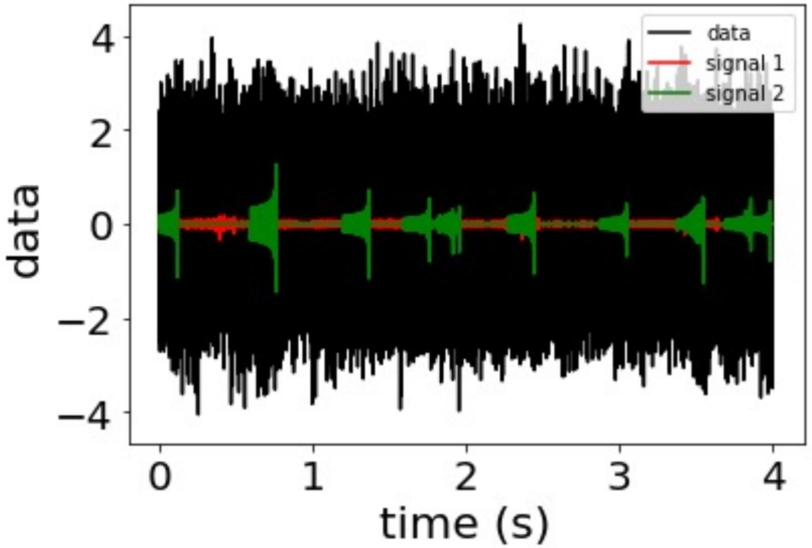
BBH chirp :

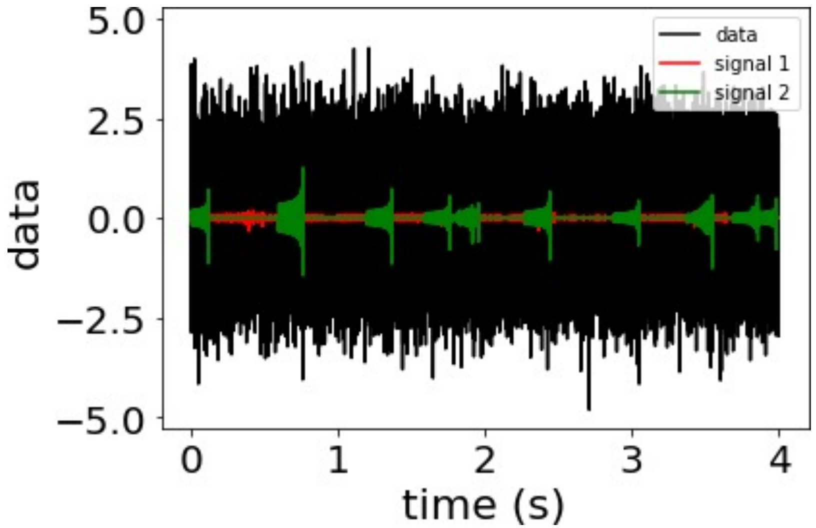
$$A = 1$$
$$m = 10.4M_{\odot}$$
$$Nburst = 10$$
$$chirpduration = 0.17\ s$$
$$N = 9.10^{M}$$

Multi-burst signal and coaligned detectors

- We can now create time-domain data with several detectors :

Confusion-limited BNS ($m_? = 1.4M_\odot$) + Pop-corn BBH ($m_? = 10M_\odot$)





Remark : The amplitude, the masses, the correlation are all parameters that can be changed according to what we are interested in.

Analysis of the previous example

- Signal component in the two detectors are identical.
- The noise (data) in the two detectors are different !

⇒ The stochastic GWB should be visible !

The stochastic GWB is like a random signal (noise)… So we can not use a waveform filter to extract it, nor subtract the instrumental noise.

⇒ Cross correlation method.

Cross correlation method

- 2 coaligned and collocated detectors = $d_\backslash = h + n_\backslash$; $d_b = h + n_b$

  h≡ common GW signal

  $n_?$≡ instrumental noise component

- Expected value of $\hat{C}_{12}$ $< \hat{C}_{12} >=< d_1 d_2 >=< h^2 >$

$$= S_h(f)$$

Unbiased estimator of the power of the GW.

Hypothesis : separated enough + not correlated + no Schuman resonances

Cross correlation applied to ChirpBNS GWB

- Parameters : $A = 0.05$

  $Nburst = 20$

  $var = 1$

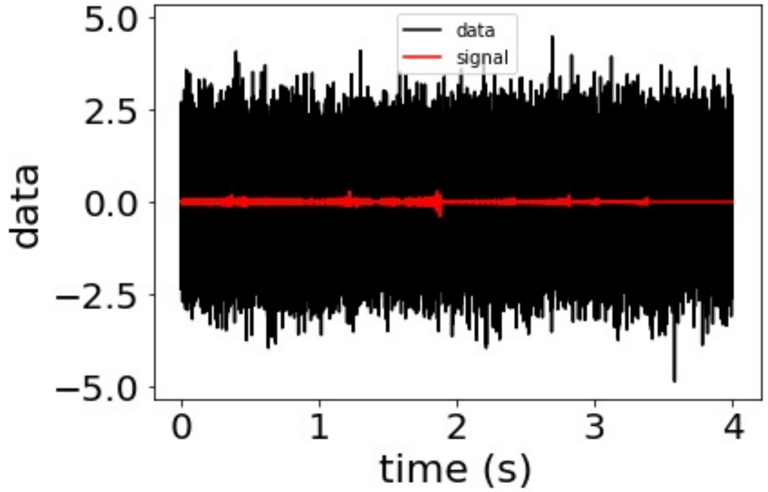
Results :

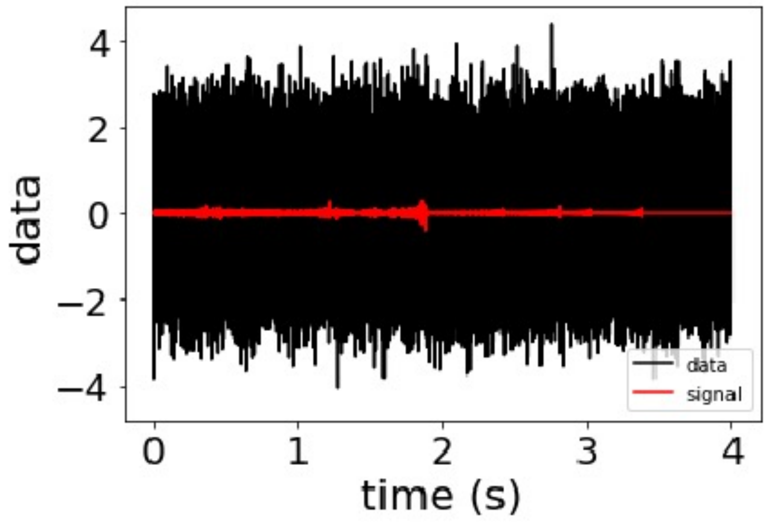
$S_{x} = 0.00353944$

$S_{@\backslash} = 0.99985473$

$S_{@b} = 0.99396017$

$SNR = 1.68651334$

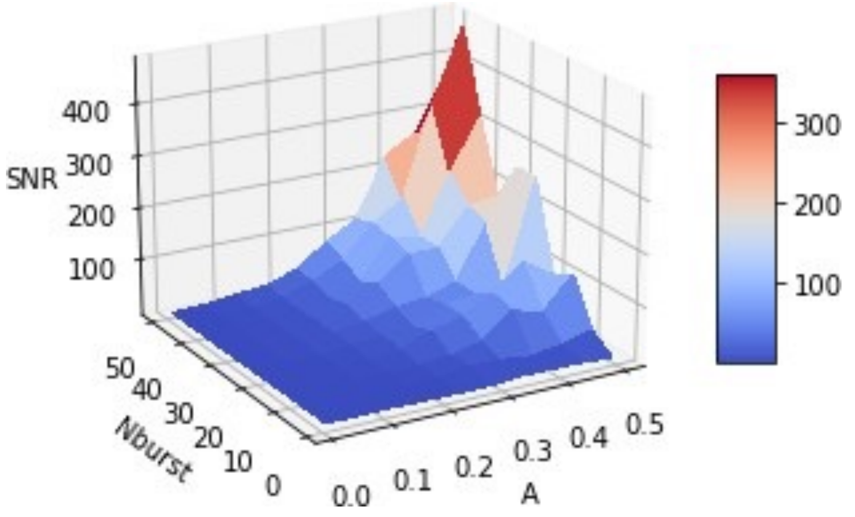




SNR's evolution of the cross correlation

- To get a better idea of how the code works, one can look at the SNR for several couples :

$$(Nburst, A)$$



From this plot, we can conclude that A has a greater impact on the SNR than Nburst.

- The next thing to do would be to look how this method works over a longer period of time ($t_{>\mathrm{CD}} > 4s$), and also with other signals.

Optimal filtering theory

- What if both detectors are misaligned ?

⇨ Introduce an overlap function $\Gamma_{12}(f)$ such that :

$$\hat{C_{12}}(f) \equiv \Gamma_{12}(f) S_h(f)$$

- Optimal way to correlate data :

In in Fourier space, one can write the general form of the cross corelation

$$\hat{C}_{12}(f) = \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df' \, \delta_T(f - f') \tilde{d}_1(f) \tilde{d}_2^*(f') \tilde{Q}^*(f')$$

$Q(f)$ is an apriori filter that only depends on $\Delta t = t - tí$

Optimal filtering theory

- Optimizing the filter $\Leftrightarrow$ Maximizing the SNR of $\hat{C}_{12}$ such that :

$$SNR(\hat{C_{12}}) = \frac{\mu}{\sigma} = \frac{< \hat{C_{12}} >}{< \hat{C_{12}}^2 > - < \hat{C_{12}} >^2}$$

- I will use the analytical form of the overlap function between LIGO/Virgo.
- During the week, I shall use this method of optimal filtering versus the classical one, and compare the SNR values.

Appendix

- Arrival time for chirp signals :

$t_0 = random.uniform(0, t_{>\mathrm{CD}}, Nburst)$ ⇔ Flat distribution

$t_0 = np.sort(t_0)$

- Amplitude of the chirps :

$AI = random.uniform(A + 0.5, A - 0.5, Nburst)$