



QCD CORRECTIONS FOR SM PROCESSES

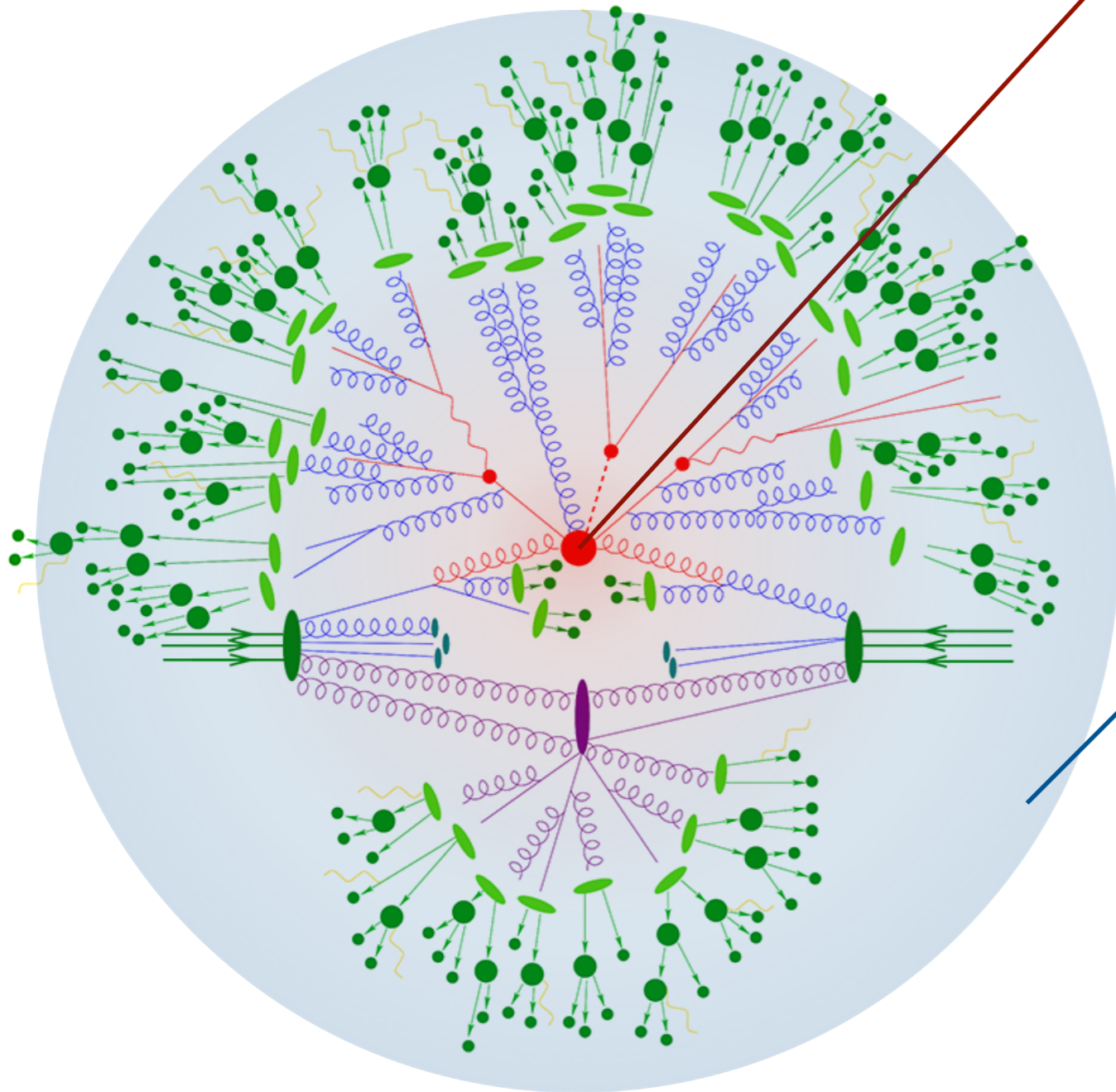
SM – techniques, calculations & phenomenology **WG**

Alexander Huss

DISCLAIMER

- *impossible to cover everything*
- *personal selection of recent results*
- *highlight some potentially interesting topics*
(more at: <https://phystev.cnrs.fr/wiki/2021:topics>)
- *QCD, but dedicated talks:*
Higgs, PDF & jets, substructure, MC, ...

SCATTERING REACTIONS @ LHC!



Short distance “hard”

- high scales: 10^2 — 10^3 GeV



*evolution towards a
physical observable state*

Long distance “soft”

- low scales: $\mathcal{O}(\text{few GeV})$

{Tools & MC}

SCATTERING REACTIONS @ LHC!

● Short distance “hard”

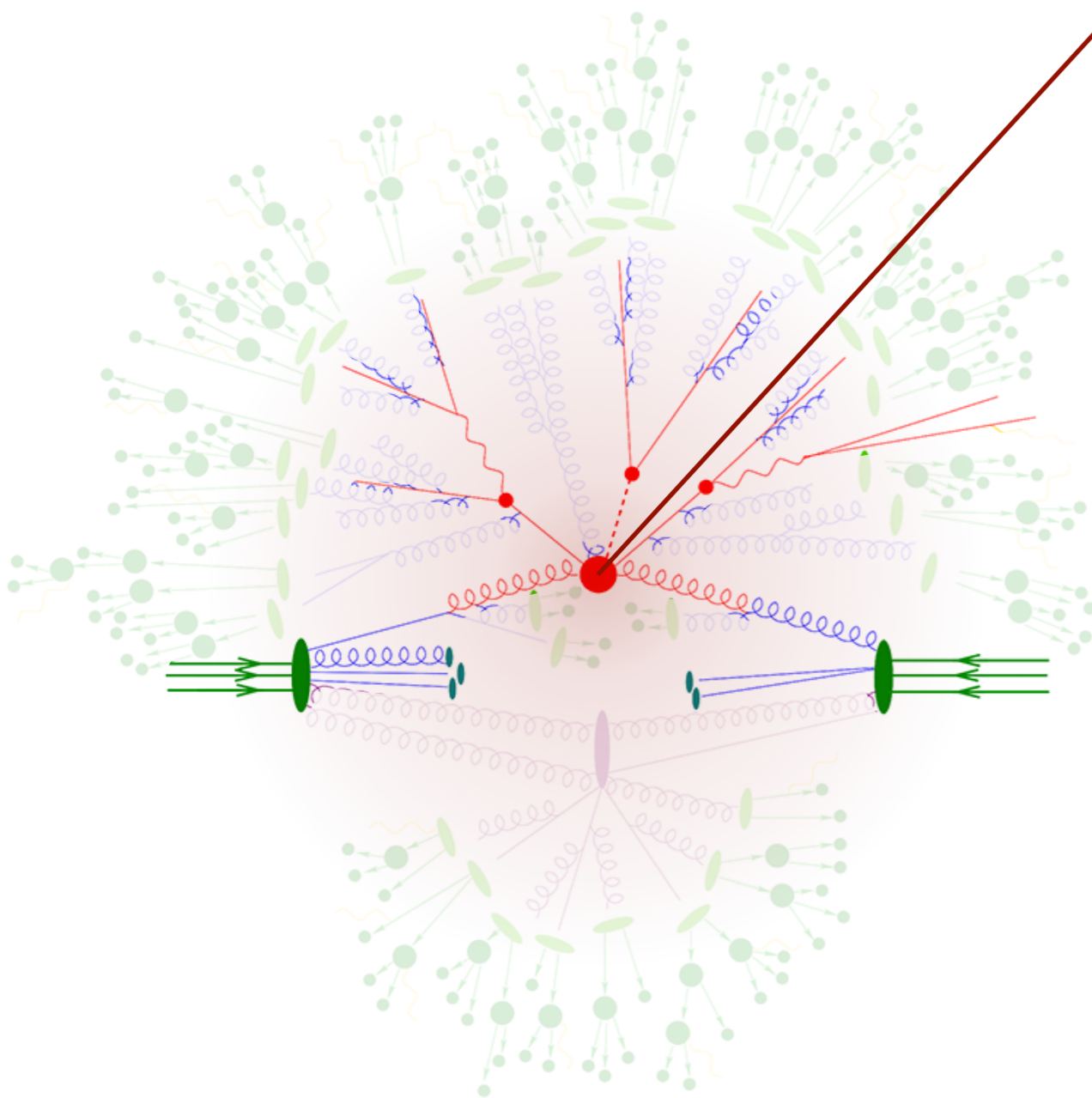
- ▶ high scales: 10^2 — 10^3 GeV

$$\sigma = \sigma_0 \times (1 + \alpha_s + \alpha_s^2 + \alpha_s^3 + \dots)$$

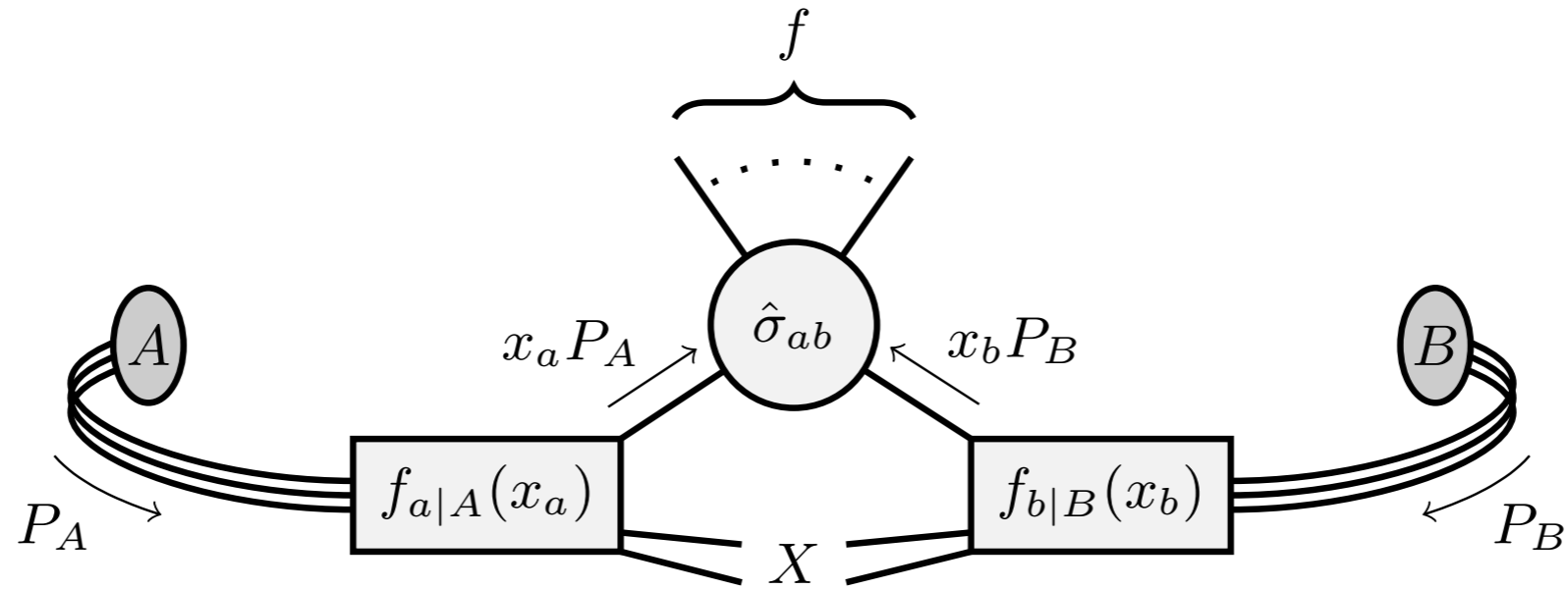
fixed order: LO NLO NNLO N³LO ...

● Long distance “soft”

- ▶ low scales: $\mathcal{O}(\text{few GeV})$



THE MASTER FORMULA.



$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b f_{a|A}(x_a) f_{b|B}(x_b) \hat{\sigma}_{ab}(x_a, x_b) (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

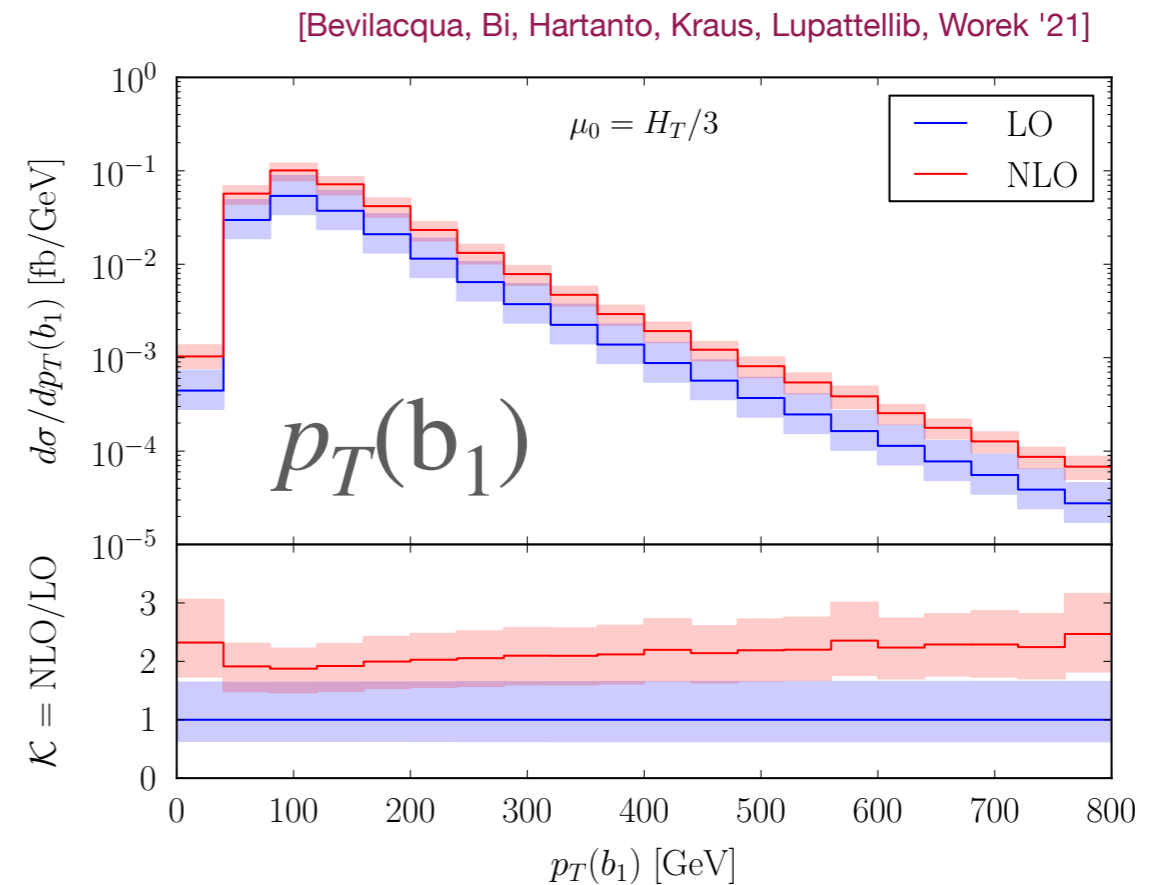
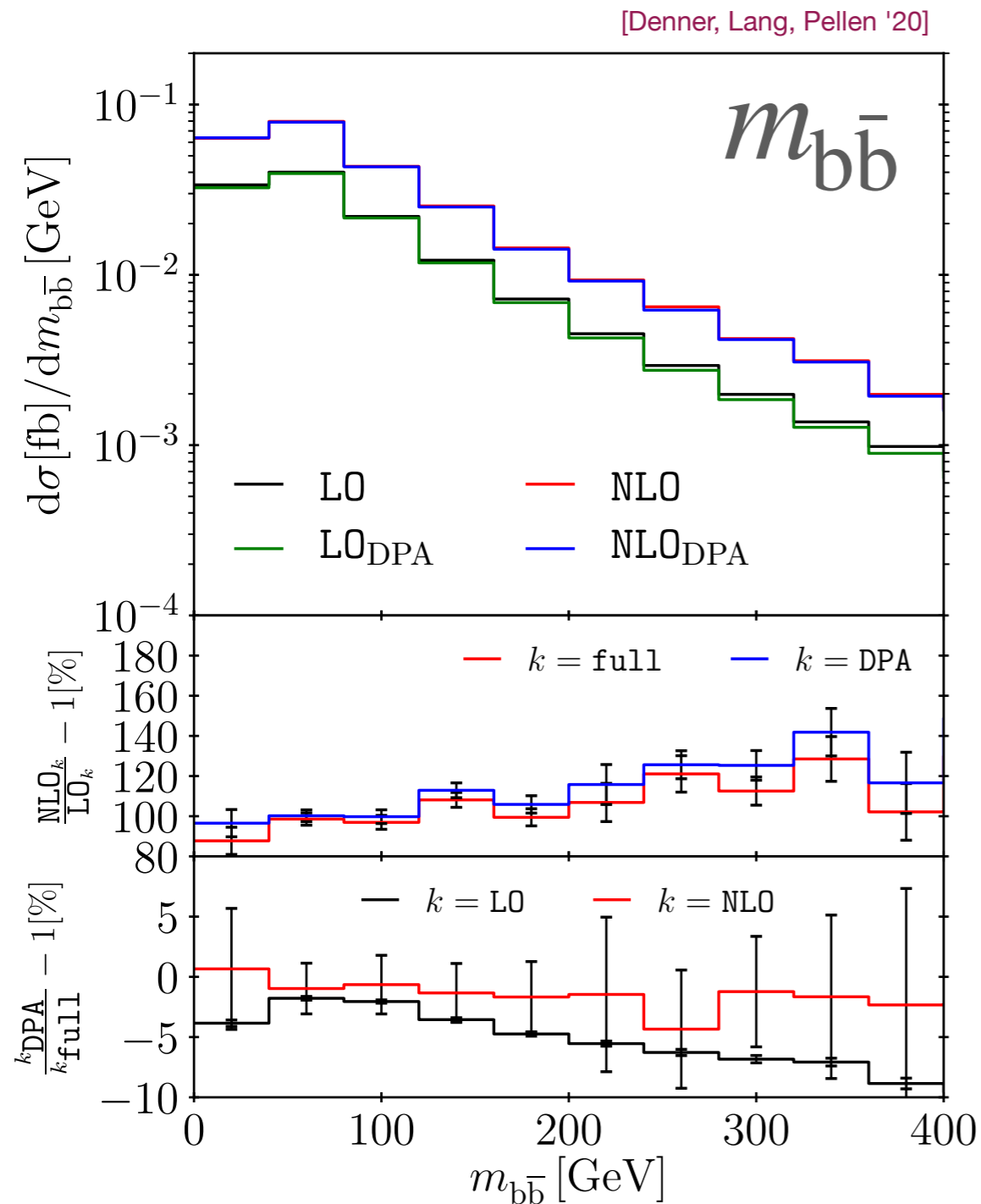
parton distribution functions (PDFs)
(non-perturbative, universal)

hard scattering
(perturbation theory)

non-perturbative effects
(power suppressed)
ultimately, limiting factor?

NLO — PUSHING THE LIMITS*

➤ $2 \rightarrow 8$ (6 coloured particles): $pp \rightarrow \mu^- \bar{\nu}_\mu e^+ \nu_e b \bar{b} b \bar{b}$ (off-shell $t\bar{t}b\bar{b}$)

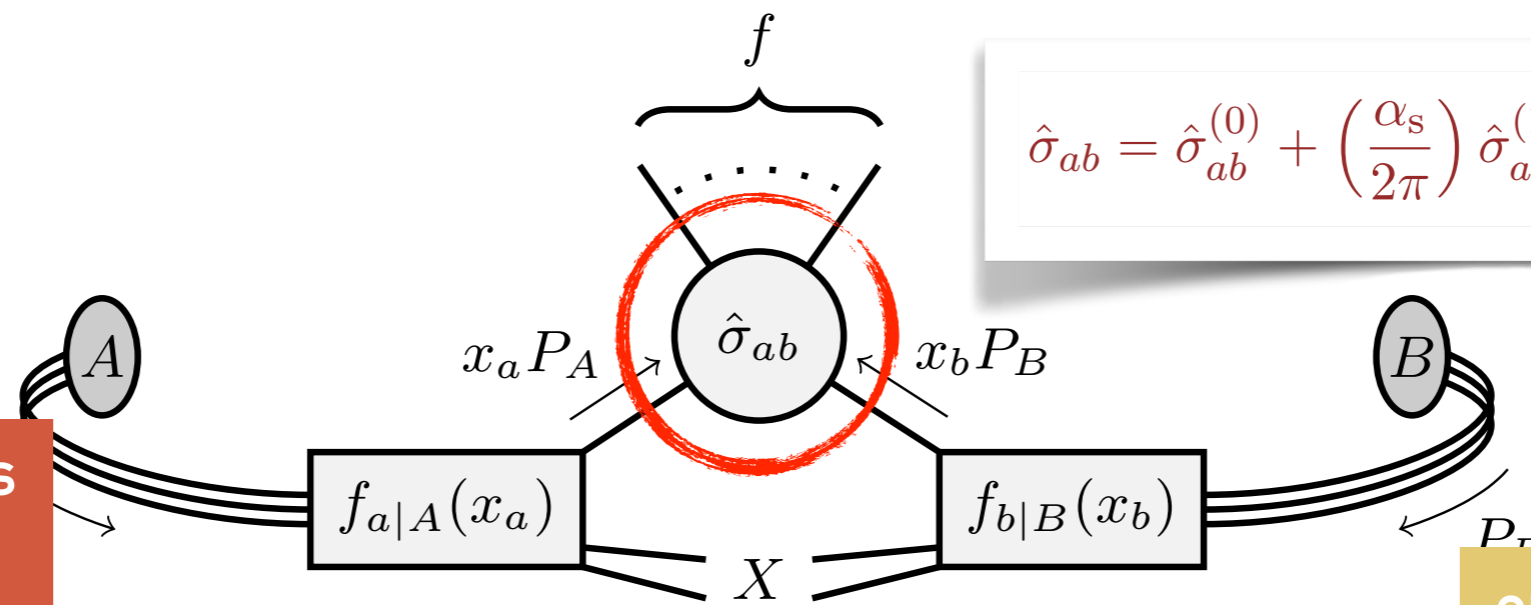


- large **NLO** corr. & impact on shape
- full v.s. on-shell: 5 - 10 %

* another frontier: NLO loop-induced {S. Jones}

NNLO — BOTTLE NECKS

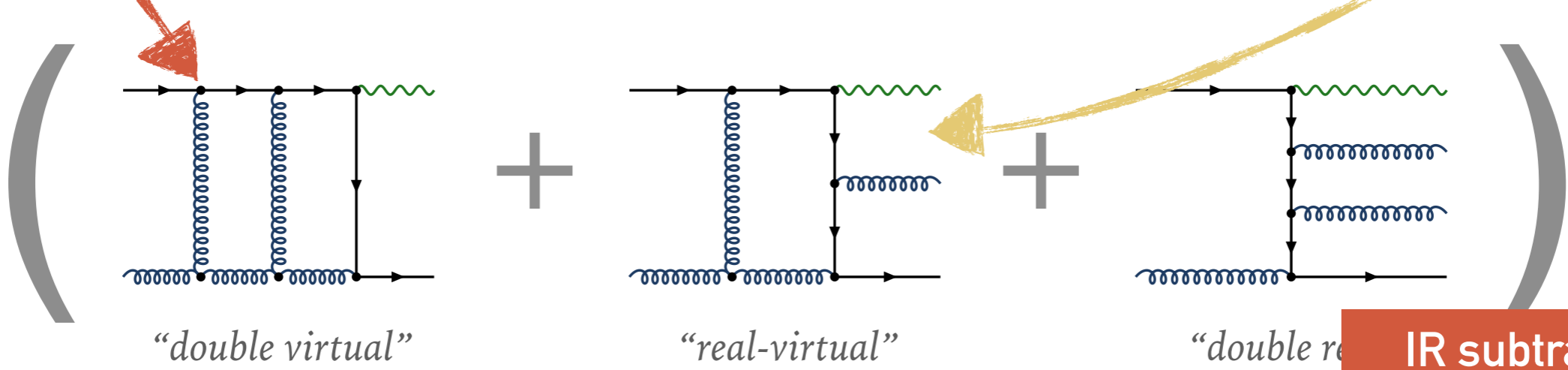
$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \dots$$



two-loop amplitudes
(new class of functions,
combinatoric &
algebraic complexity)

one-loop amplitudes
(evaluation in singular
& unstable regions)

next-to-next-to-leading order (NNLO)

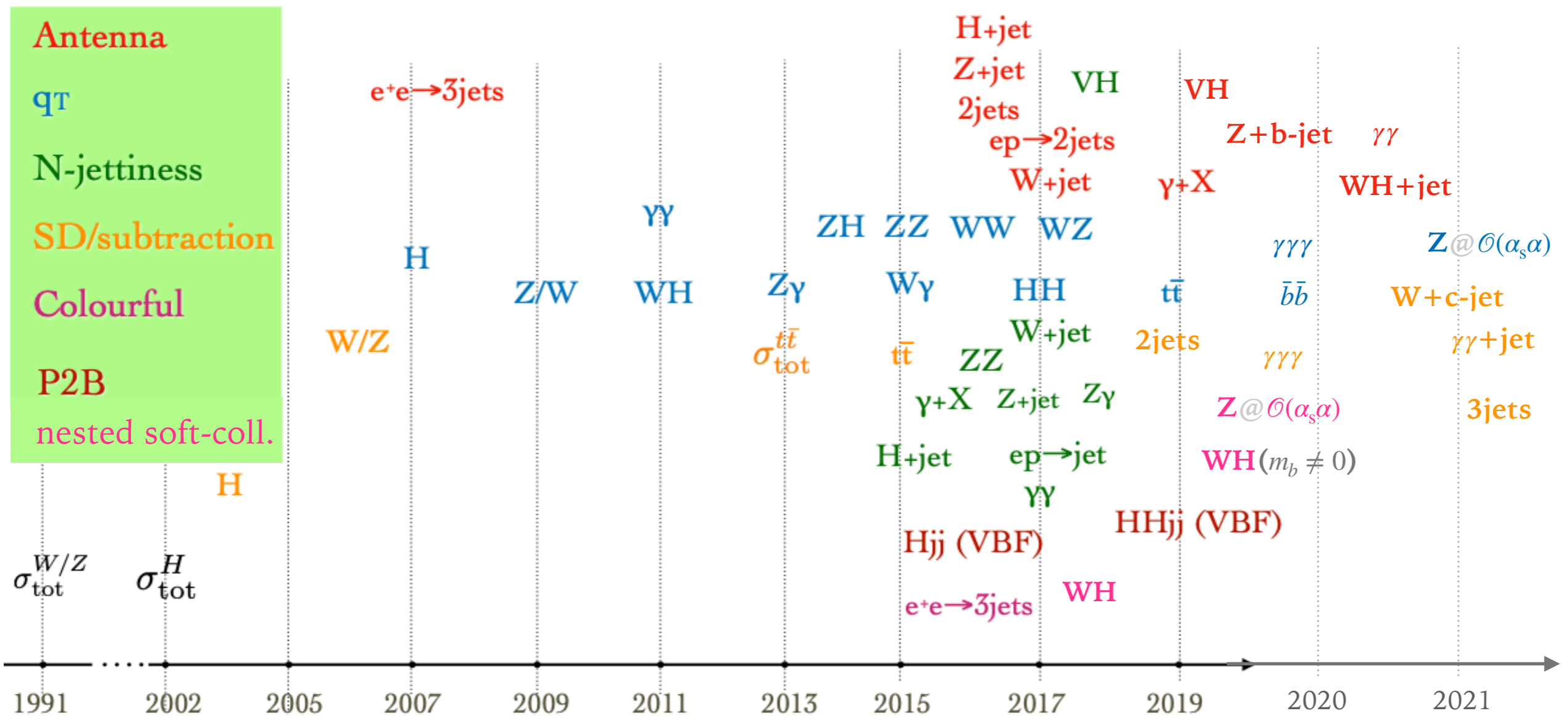


IR subtraction
(involved IR structure,
numerical stability,
construction)

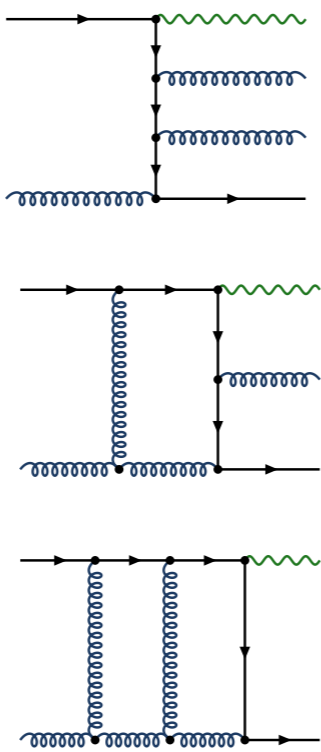
infrared singularities

TIMELINE FOR NNLO

► Remarkable progress in the development of methods to perform NNLO computations!



[based on slide by M. Grazzini; QCD@LHC 2019]

$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} d\sigma_{\text{NNLO}}^{\text{RR}} + \int_{\Phi_{Z+2}} d\sigma_{\text{NNLO}}^{\text{RV}} + \int_{\Phi_{Z+1}} d\sigma_{\text{NNLO}}^{\text{VV}} + \dots$$


Σ finite

- in general: **measurement function**
 - fiducial cross sections
 - differential distributions
 - reconstruction (jets, γ , ...)

}

$\mathcal{F}_{\text{obs}}^{(n)}$

→ massage expression to render intermediate objects finite (suitable for MC integration)

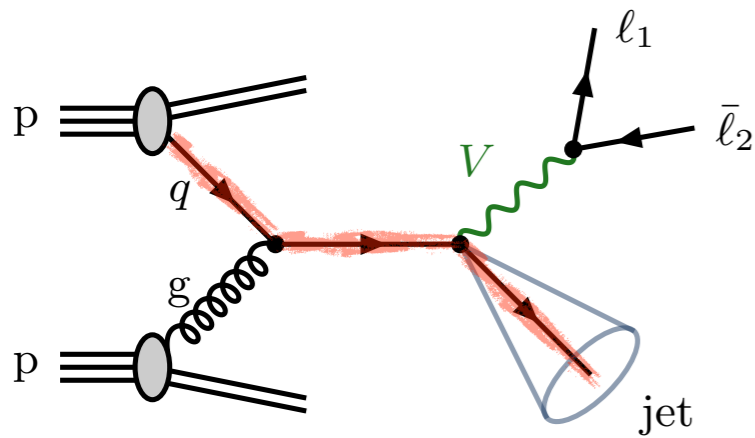
DIFFERENT METHODS*

- ▶ Antenna [Gehrmann–De Ridder, Gehrmann, Glover '05]
- ▶ CoLorFul [Del Duca, Somogyi, Trocsanyi '05]
- ▶ qT-subtraction [Catani, Grazzini '07; MATRIX]
- ▶ STRIPPER (sector-improved residues) [Czakon '10]
- ▶ nested soft-collinear [Caola, Melnikov, Röntschi '17]
- ▶ N-jettiness [Gaunt, Stahlhofen, Tackmann, Walsh '15; Boughezal, Focke, Liu, Petriello '15; MCFM]
- ▶ Projection-to-Born [Cacciari, et al. '15]
- ▶ Geometric, Local analytical Sectors [Herzog '18] [Magnea et al. '18]

* Subtraction & Slicing

V + jet PRODUCTION @ NNLO QCD WITH FLAVOUR

.....
NNLO QCD now well-established with 2 independent calculations:



- **Z+jet:** {
 - Antenna: [Gehrmann-De Ridder, Gehrmann, Glover, AH, Morgan '15]
 - N-jettiness: [Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello '15]
 - **W+jet:** {
 - N-jettiness: [Boughezal, Liu, Petriello '15]
 - Antenna: [Gehrmann-De Ridder, Gehrmann, Glover, AH, Walker '17]
 - **γ+jet:** {
 - N-jettiness: [Campbell, Ellis, Williams '16]
 - Antenna: [Chen, Gehrmann, Glover, Höfer, AH '19]

... now comes in different flavours:

- **Z+b-jet**

[Gauld, Gehrmann-De Ridder, Glover, AH, Majer '20]

- **W+c-jet**

[Czakon, Mitov, Pellen, Poncelet '20]

- identify flavour of a jet (“tag”)

⇒ test of perturbative QCD

⇒ flavour structure of protons

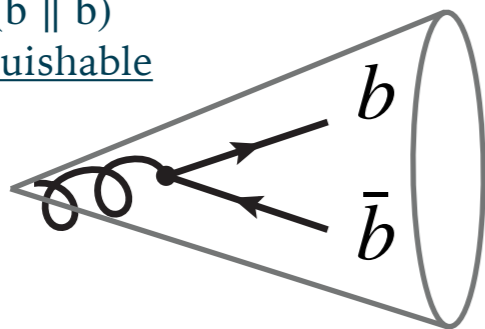
- $m_q \equiv 0 \Leftrightarrow$ IR unsafe with anti- k_T

FLAVOUR TAGGING & IR SAFETY

anti- k_T used in experiment $\begin{cases} m_b \neq 0 \text{ (4FS): finite, but sensitive to } \ln(Q^2/m_b^2) \\ m_b \equiv 0 \text{ (5FS): divergent} \end{cases}$

1. Collinear (NLO)

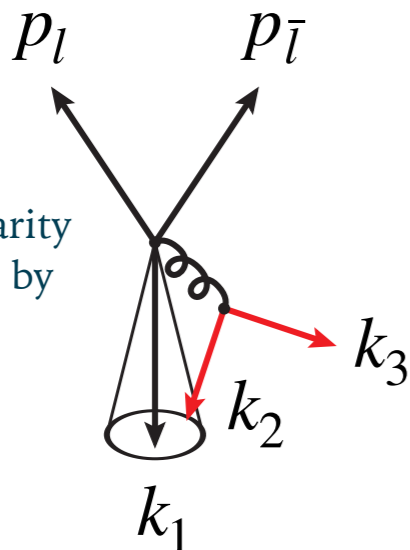
$g \leftrightarrow (b \parallel \bar{b})$
indistinguishable



\rightsquigarrow assign tag using: $b \leftrightarrow +1$ & $\bar{b} \leftrightarrow -1$
(alternatively reject even tags)

2. Soft (NNLO)

$E_g \rightarrow 0$ singularity
not protected by
 $E_g \geq 2m_q$



\rightsquigarrow modify the clustering: **flavour- k_T** [Banfi, Salam, Zanderighi '06]

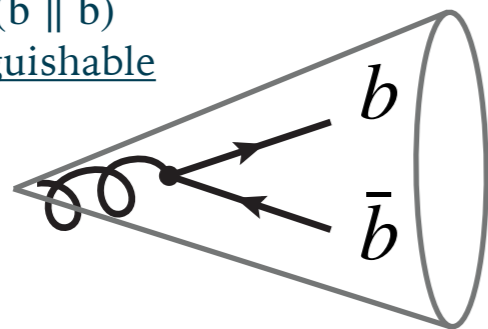
$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta \phi_{ij}^2}{R^2} \begin{cases} \max(k_{ti}, k_{tj})^\alpha \min(k_{ti}, k_{tj})^{2-\alpha} & \text{softer of } i, j \text{ is flavoured,} \\ \min(k_{ti}, k_{tj})^\alpha & \text{softer of } i, j \text{ is unflavoured,} \end{cases}$$

FLAVOUR TAGGING & IR SAFETY

anti- k_T used in experiment \rightarrow $m_b \neq 0$ (4FS): **finite**, but sensitive to $\ln(Q^2/m_b^2)$
 \rightarrow $m_b \equiv 0$ (5FS): **divergent**

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$g \leftrightarrow (b \parallel \bar{b})$
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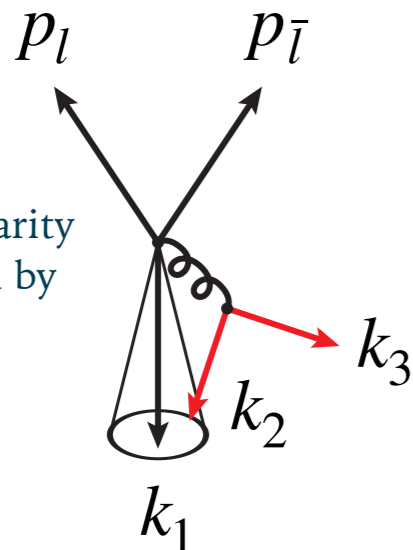


issue for any fixed-order & jet algo!

\rightsquigarrow **assign tag** using: $b \leftrightarrow +1$ & $\bar{b} \leftrightarrow -1$
 (alternatively reject even tags)

2. Soft (NNLO)

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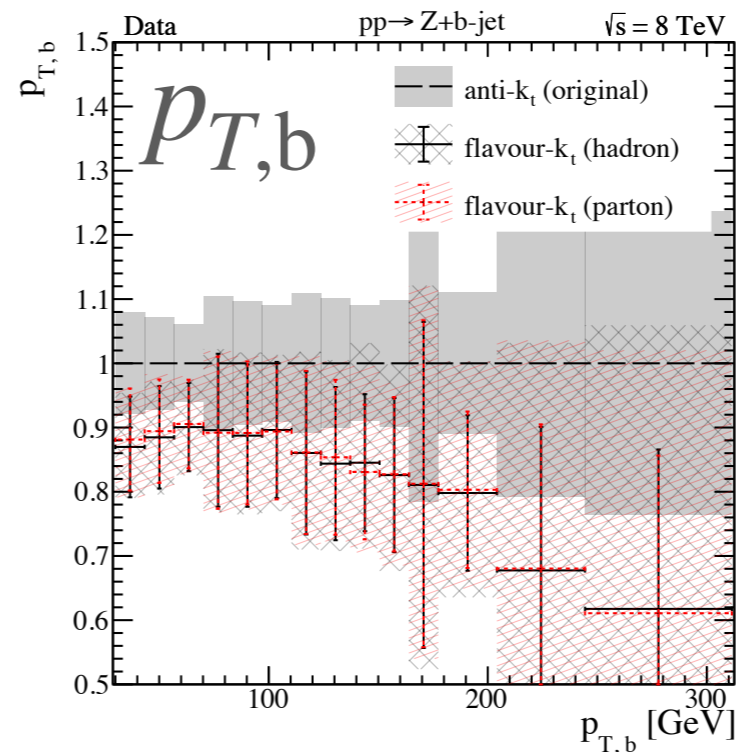
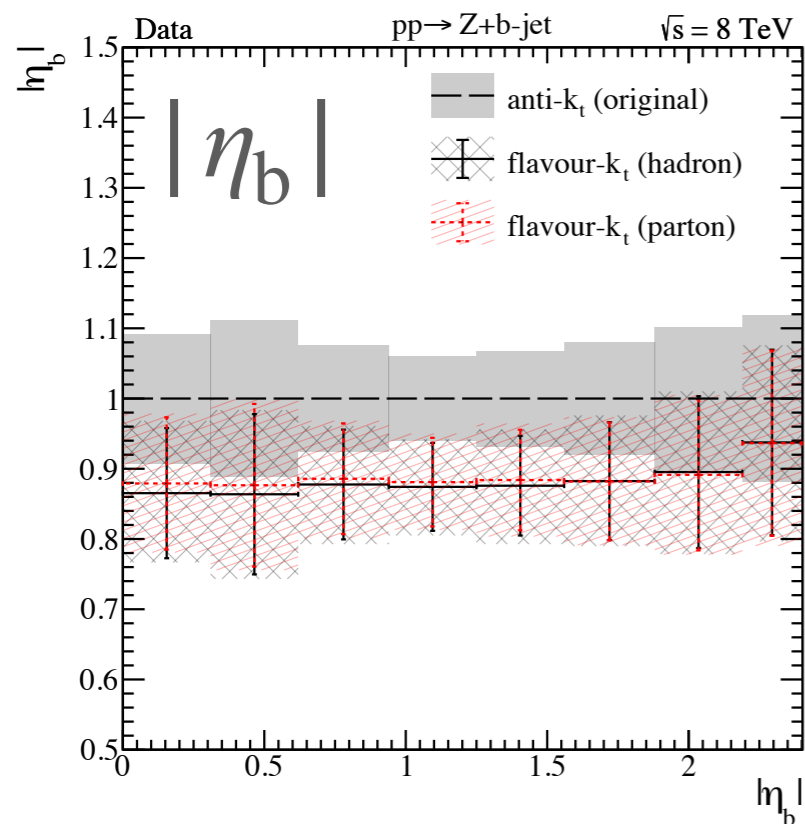
mismatch to experiment!

\rightsquigarrow modify the clustering: **flavour- k_T** [Banfi, Salam, Zanderighi '06]

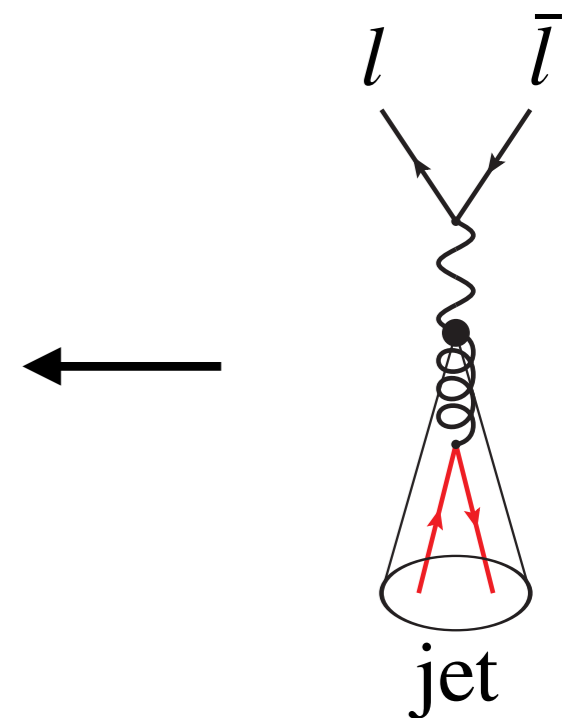
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FLAVOUR TAGGING — TH V.S. EXP

- Unfolding $\sim 10\%$ sizeable for Z+b-jet (larger @ high- p_T)
 - ↪ mainly “background subtraction”



“Collinear”



- flavour- k_T in experiment? At least resolve the collinear? How reliable is NLO+PS?
- alternatives to flavour- k_T ? Can substructure techniques help (soft drop, ...)?

TOP QUARK PAIRS @ NNLO WITH THE $\overline{\text{MS}}$ MASS

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli '20]

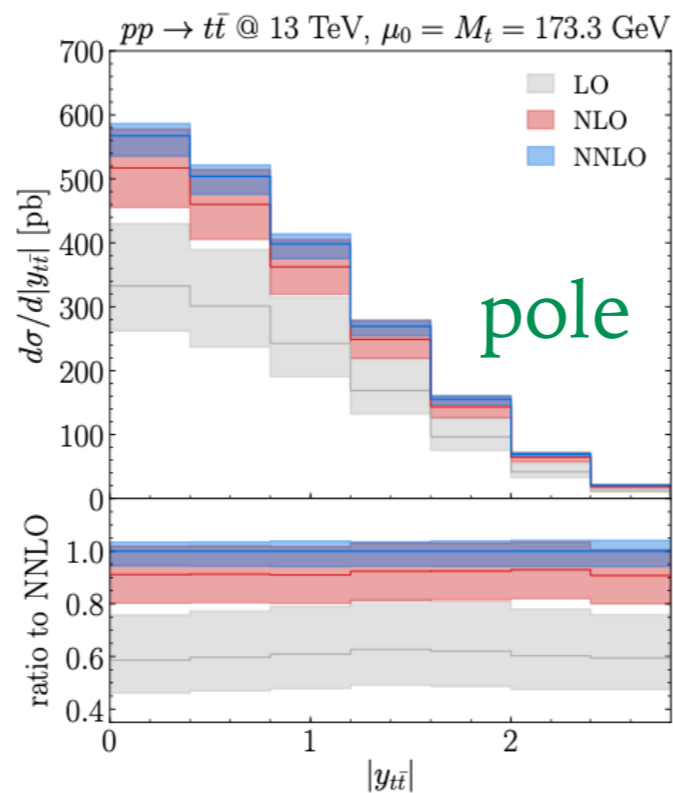
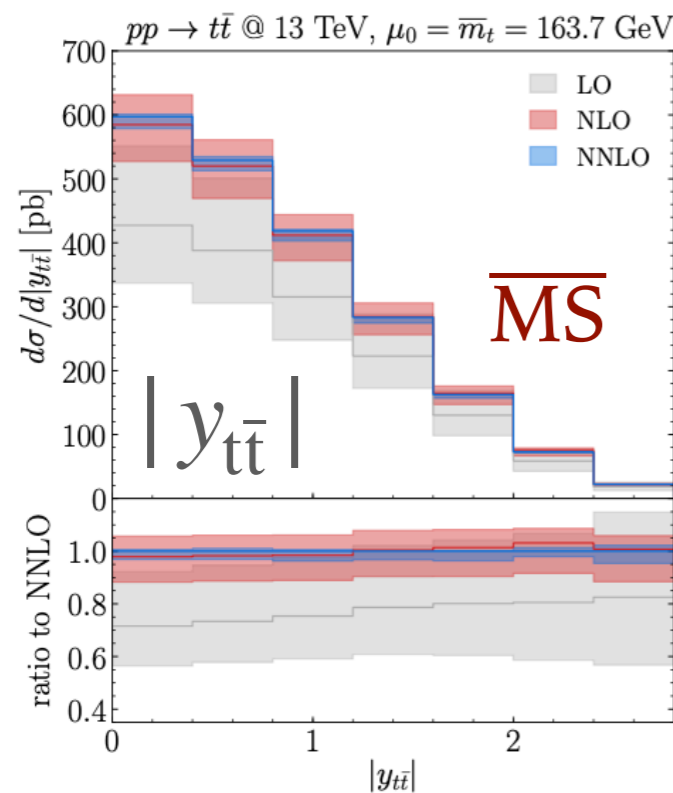
- relation between the **pole mass** and the $\overline{\text{MS}}$ mass:

$$M_t = m_t(\mu_m) d_m(m_t(\mu_m), \mu_m) = m_t(\mu_m) (1 + \alpha_s d^{(1)}(\mu_m) + \dots)$$

- formal replacement ($M_t \rightarrow m_t(\mu_m)$) & expand \Rightarrow $\overline{\text{MS}}$ cross section:

$$\bar{\sigma}(\alpha_S(\mu_R), \mu_R, \mu_F; \mu_m, m_t(\mu_m); X) = \sigma(\alpha_S(\mu_R), \mu_R, \mu_F; M_t = m_t(\mu_m) d(m_t(\mu_m), \mu_m); X)$$

new scale $\mu_m \rightsquigarrow$ 15-point scale variation



- good perturbative behaviour
- difference between schemes reduce at higher orders

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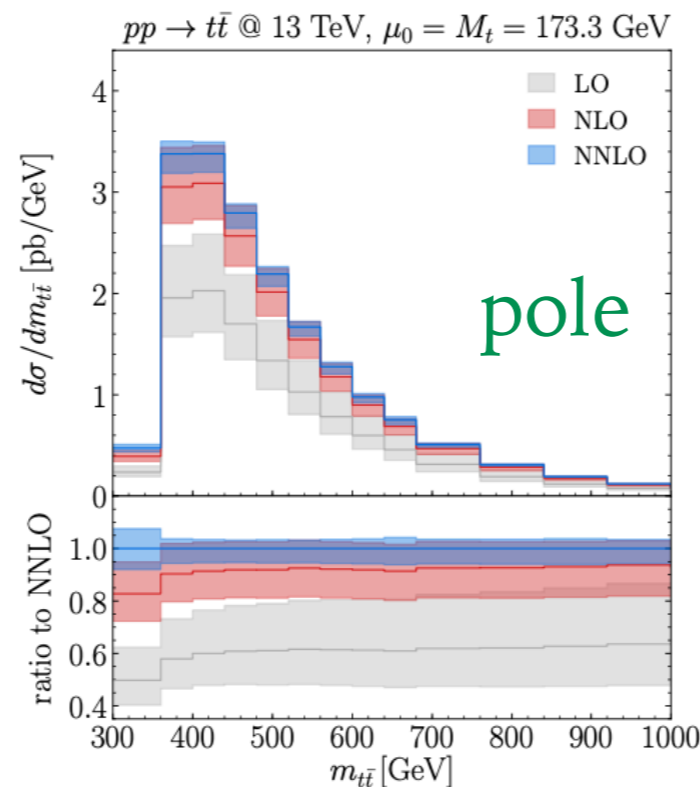
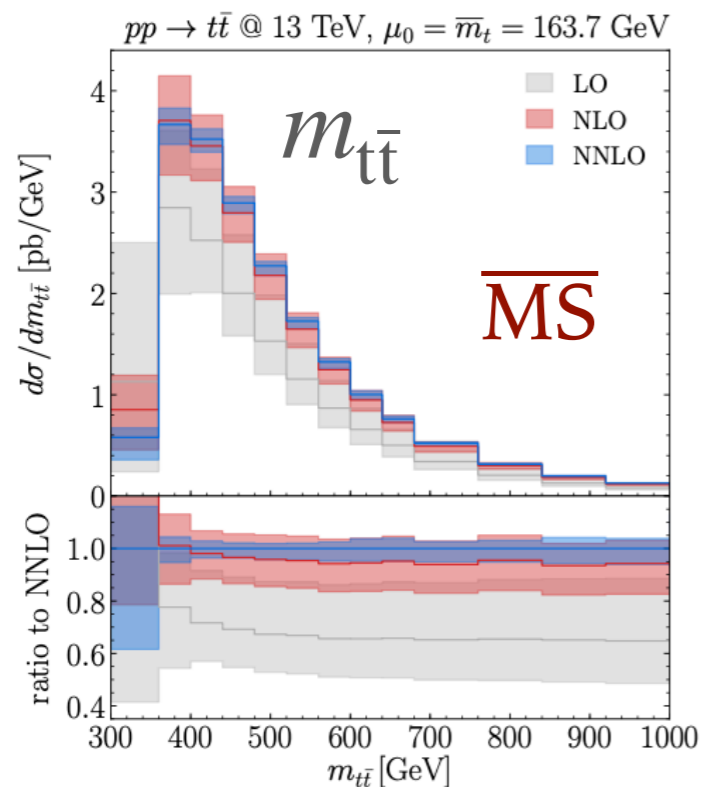
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new scale μ_m \rightsquigarrow 15-point scale variation



- caution:** threshold region \rightsquigarrow large K -factor & uncertainties

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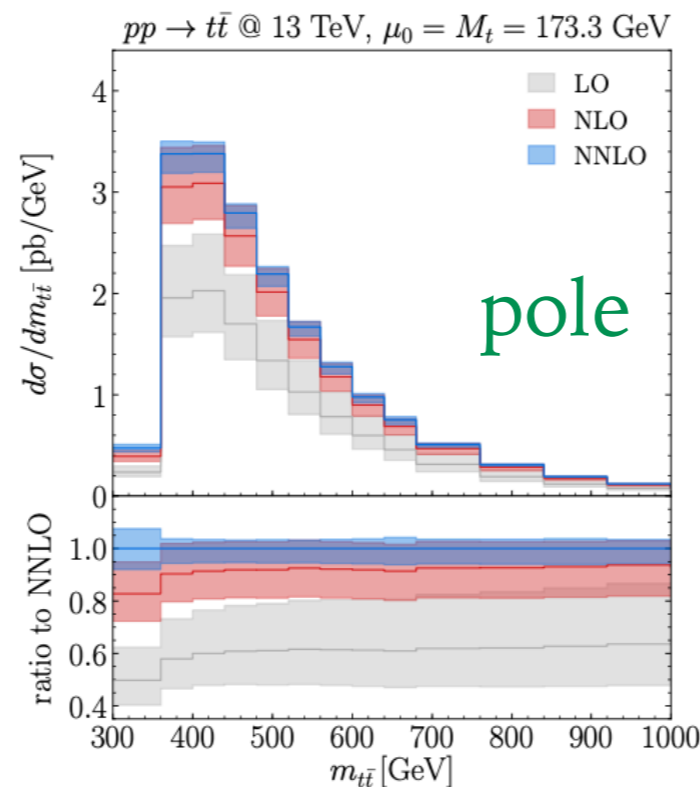
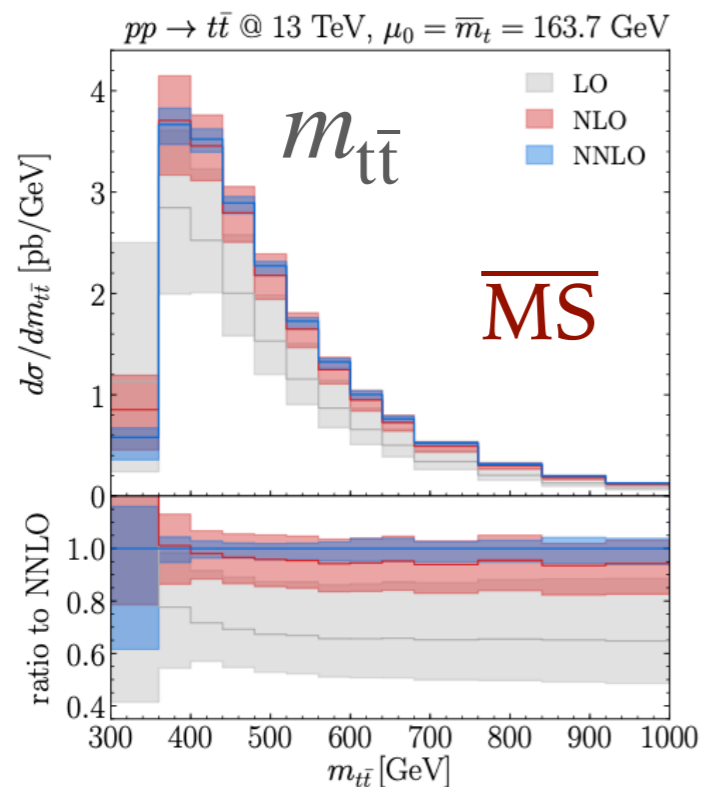
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new scale $\mu_m \rightsquigarrow$ 15-point scale variation



- caution: threshold region \rightsquigarrow large K -factor & uncertainties

- difference between $\overline{\text{MS}}$ & pole mass extraction?
- running-mass effects found to be small

TOP QUARK SPIN CORRELATION AT NNLO

[Behring, Czakon, Mitov, Papanastasiou, Poncelet '19]

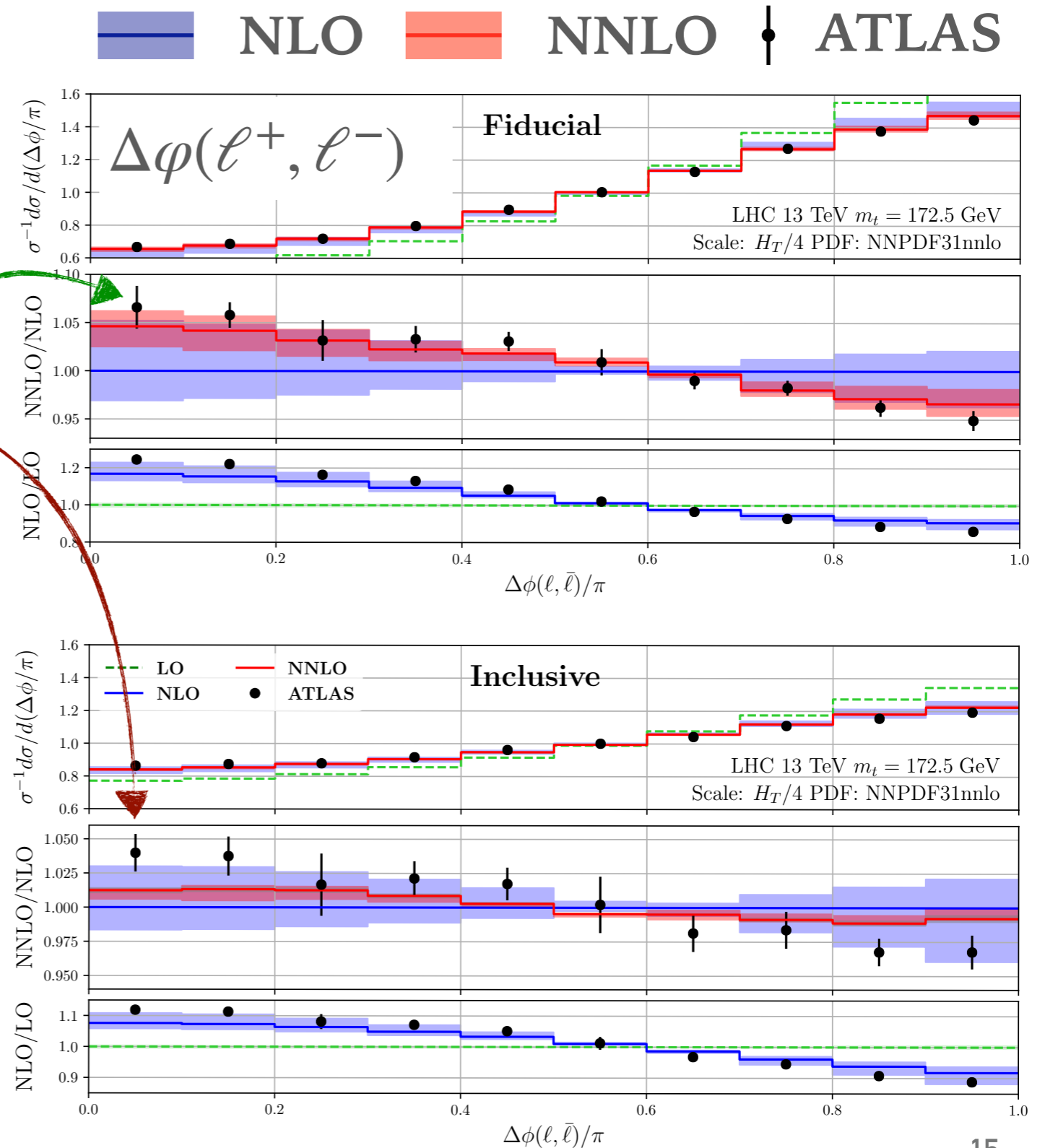
- leptons carry spin information of the tops
- fiducial: good agreement
- inclusive: some tension
- possible sources:

✓ scale choice, m_t , PDF, finite width, EW corrections

? extrapolation \rightsquigarrow need better understanding of modelling?

- full spin density matrix

[Czakon, Mitov, Poncelet '20]



TOP QUARK SPIN CORRELATION AT NNLO

[Czakon, Mitov, Poncelet '20]

- choice in expanding the ratio:

$$\frac{1}{\sigma} \frac{d\sigma}{dX} \quad \text{expanded} \quad \text{ATLAS}$$

$$R \equiv \frac{1}{\sigma} \frac{d\sigma}{dX} = R^{(0)} + \alpha_s R^{(1)} + \alpha_s^2 R^{(2)} \dots$$

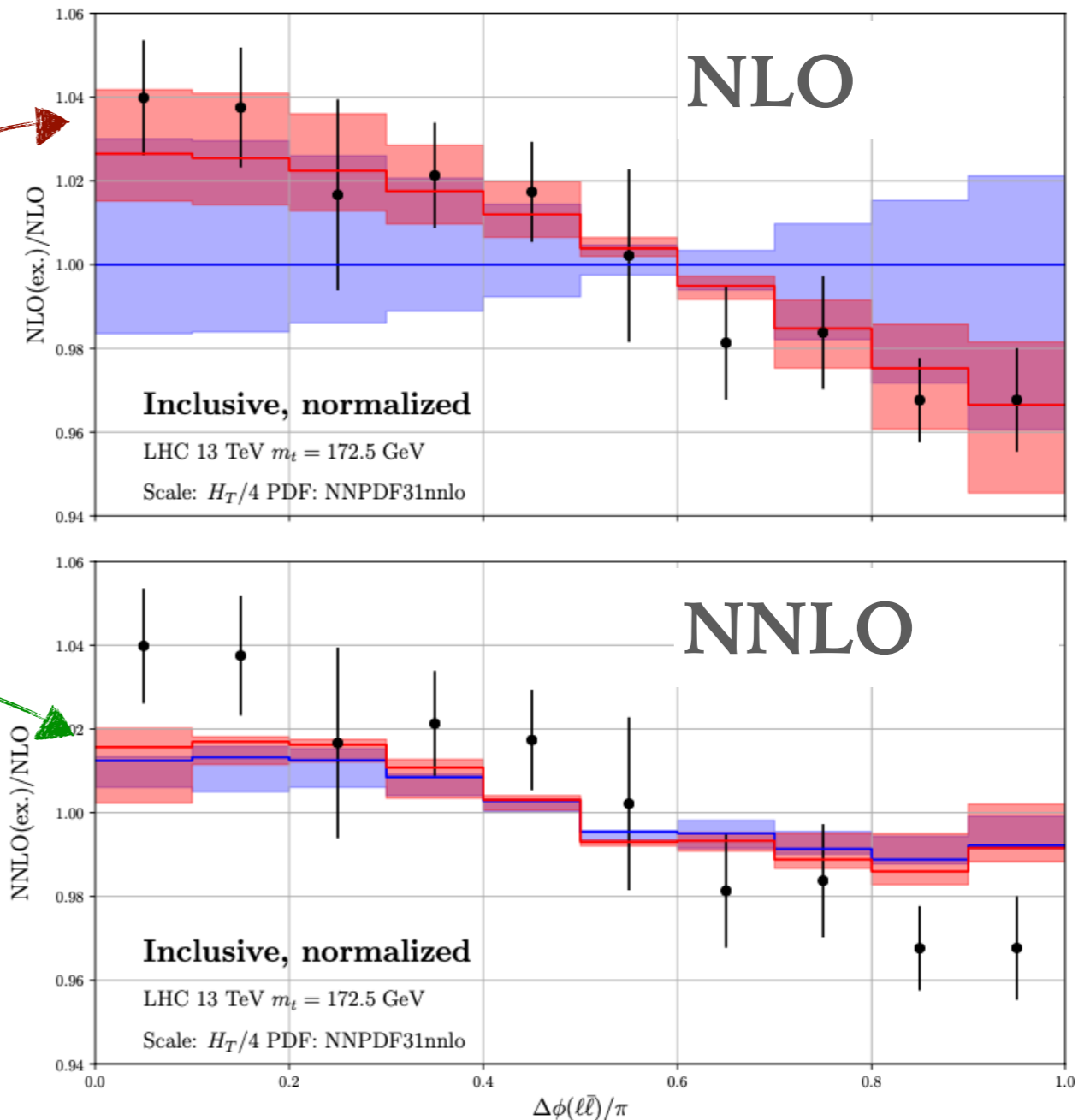
(formally equivalent)

- fake “agreement” at NLO

- ▶ difference \sim NLO uncertainty!

- both agree @ NNLO

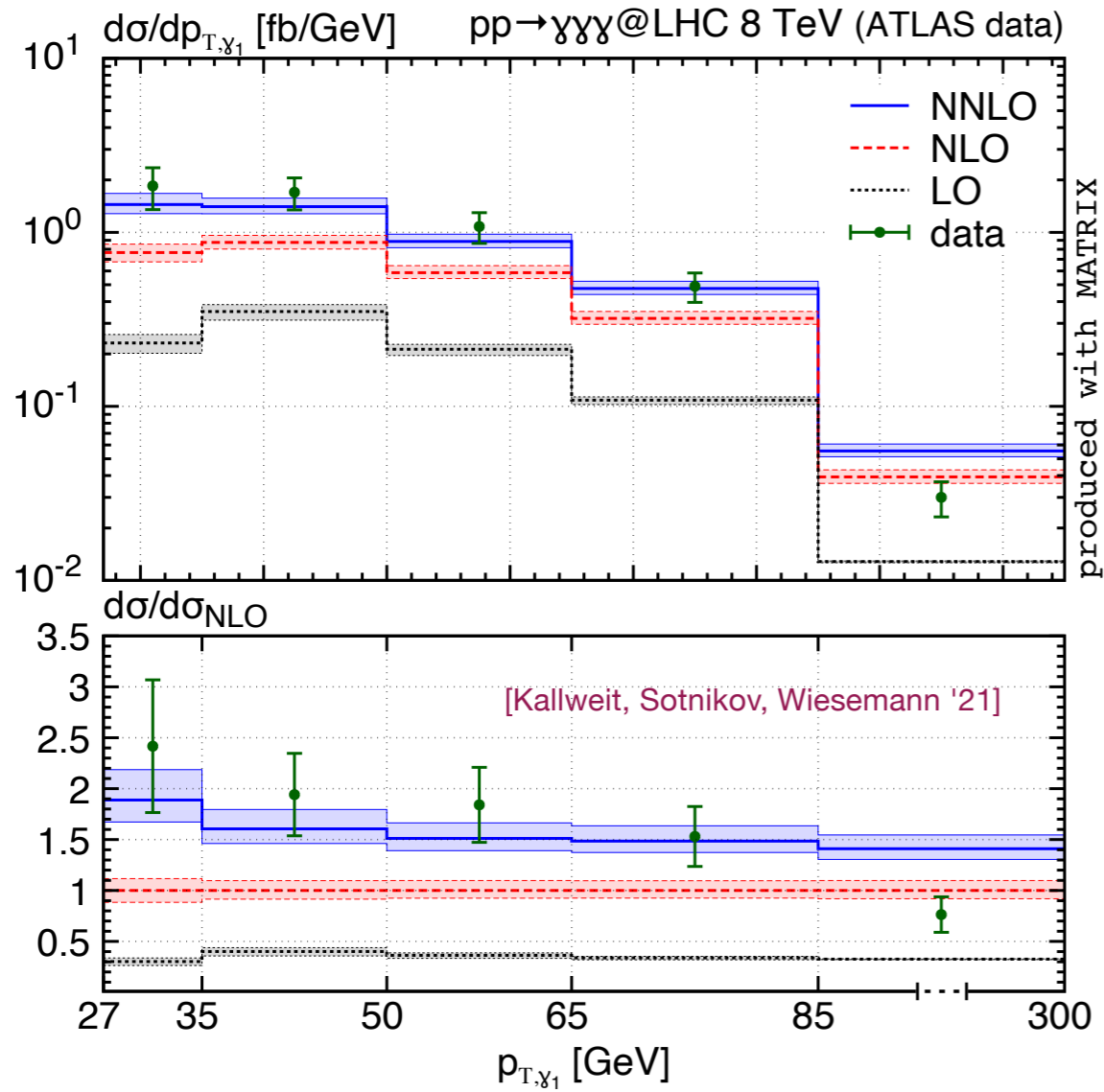
- ▶ predictions robust
- ▶ tension persists in “inclusive”



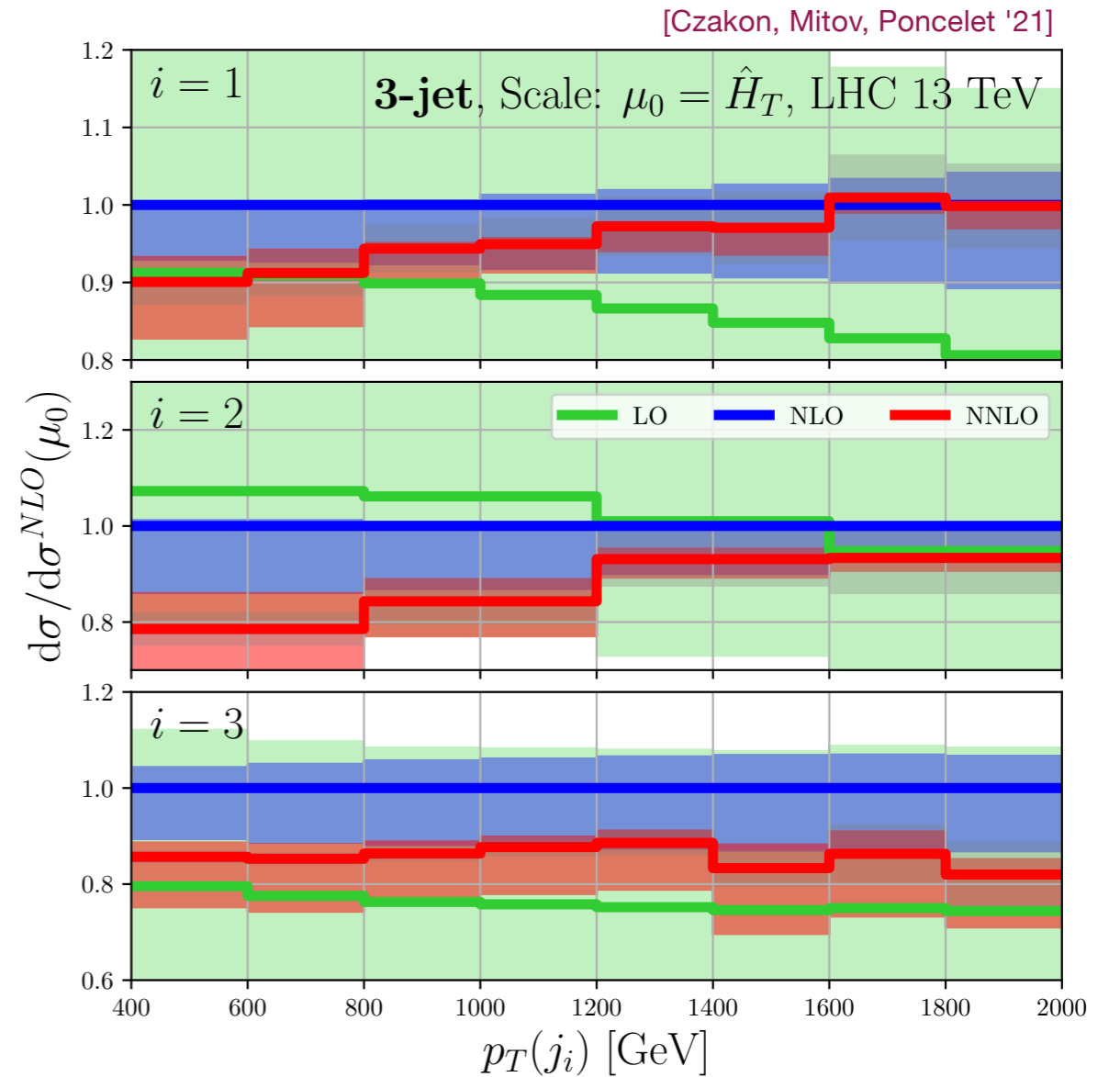
THE 2 \rightarrow 3 FRONTIER

► First results with massless external states:

LH '17 wishlist		
$pp \rightarrow 3 \text{ jets}$	NLO _{QCD}	N ² LO _{QCD}



$pp \rightarrow \gamma\gamma\gamma$



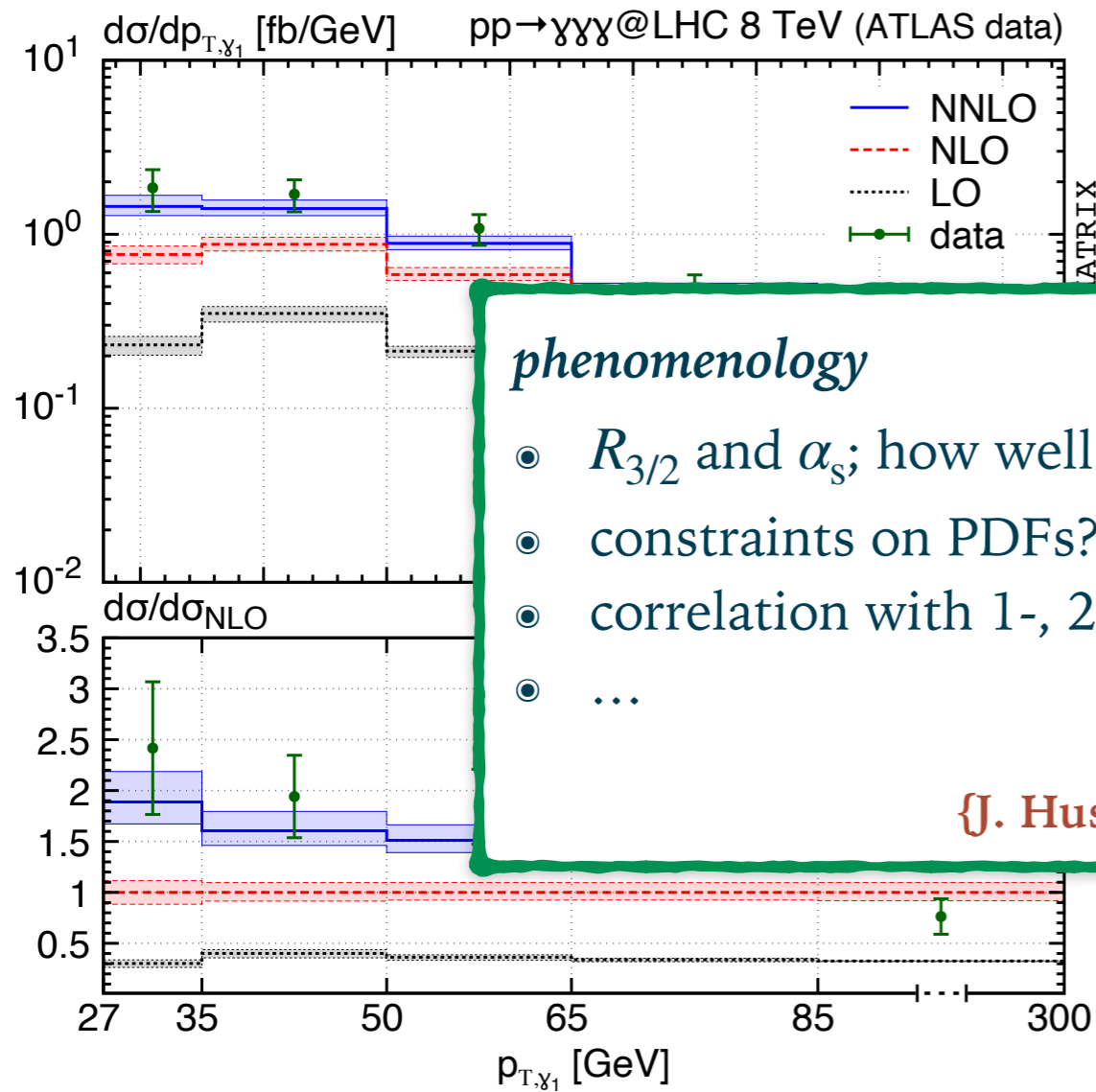
$pp \rightarrow 3 \text{ jets}$

* see also: [Chawdhry, Czakon, Mitov, Poncelet '19]

THE 2 → 3 FRONTIER

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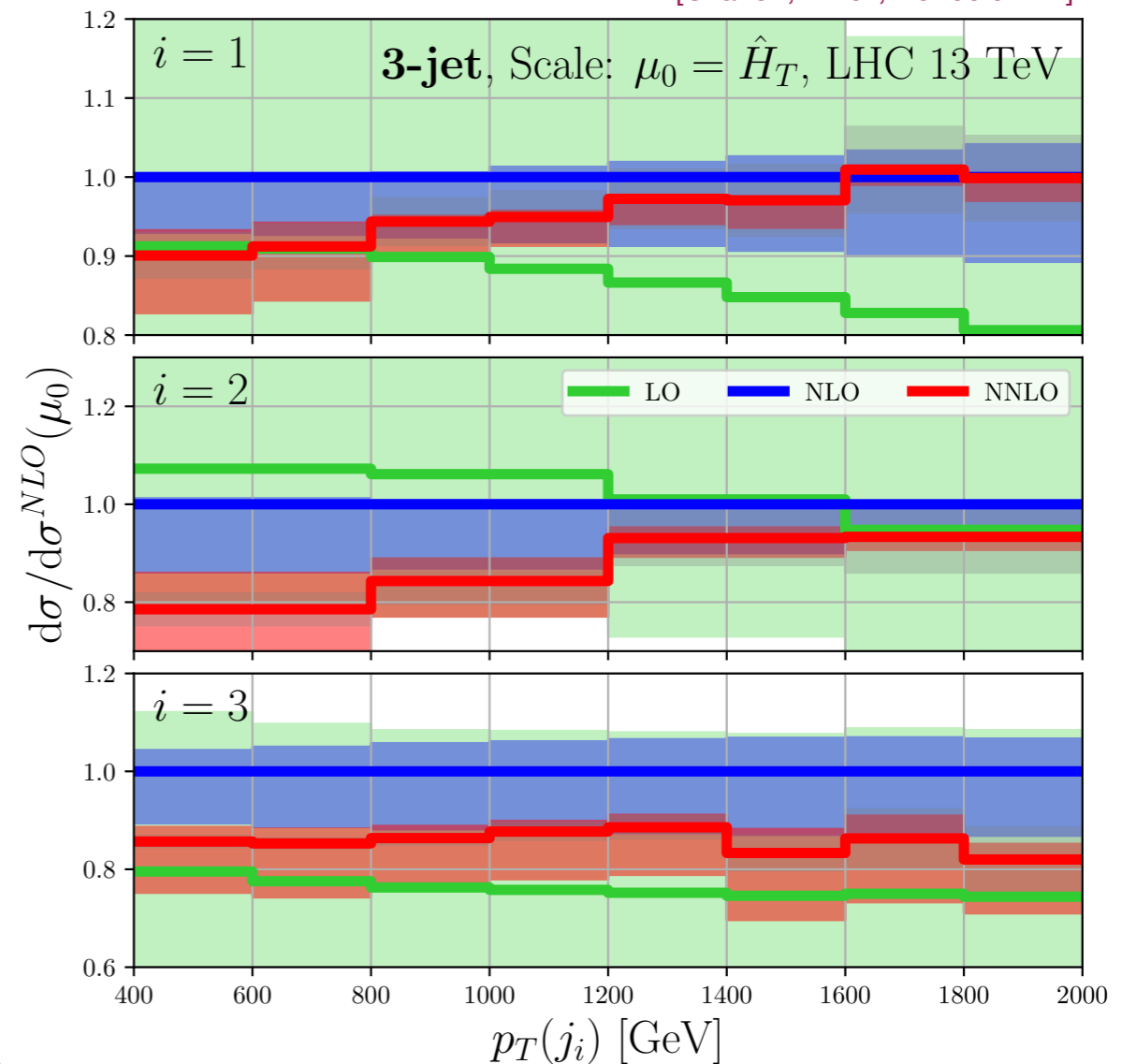


phenomenology

- $R_{3/2}$ and α_s ; how well?
- constraints on PDFs?
- correlation with 1-, 2-jet?
- ...

{J. Huston}

[Czakon, Mitov, Poncelet '21]



$pp \rightarrow \gamma\gamma\gamma$

$pp \rightarrow 3 \text{ jets}$

* see also: [Chawdhry, Czakon, Mitov, Poncelet '19]

TWO-LOOP AMPLITUDES

➤ What we can do: $2 \rightarrow 1$, 2 (also masses), 3 (massless)

➤ $2 \rightarrow 3$ amplitudes ready for phenomenology

• $pp \rightarrow \gamma \gamma \gamma$: (LC) [Abreu, Page, Pascual, Sotnikov '20]
[Chawdhry, Czakon, Mitov, Poncelet '20]

• $pp \rightarrow \gamma \gamma j$: (LC) [Chawdhry, Czakon, Mitov, Poncelet '21] (full) [Agarwal, Buccioni, von Manteuffel, Tancredi '21]
[Agarwal, Buccioni, von Manteuffel, Tancredi '21]

• $pp \rightarrow jjj$: (LC) [Abreu, Febres Cordero, Ita, Page, Sotnikov '21]

➤ 5-point with one external mass

soon within reach?

- $pp \rightarrow Z/W^\pm/H + jj$
- ...

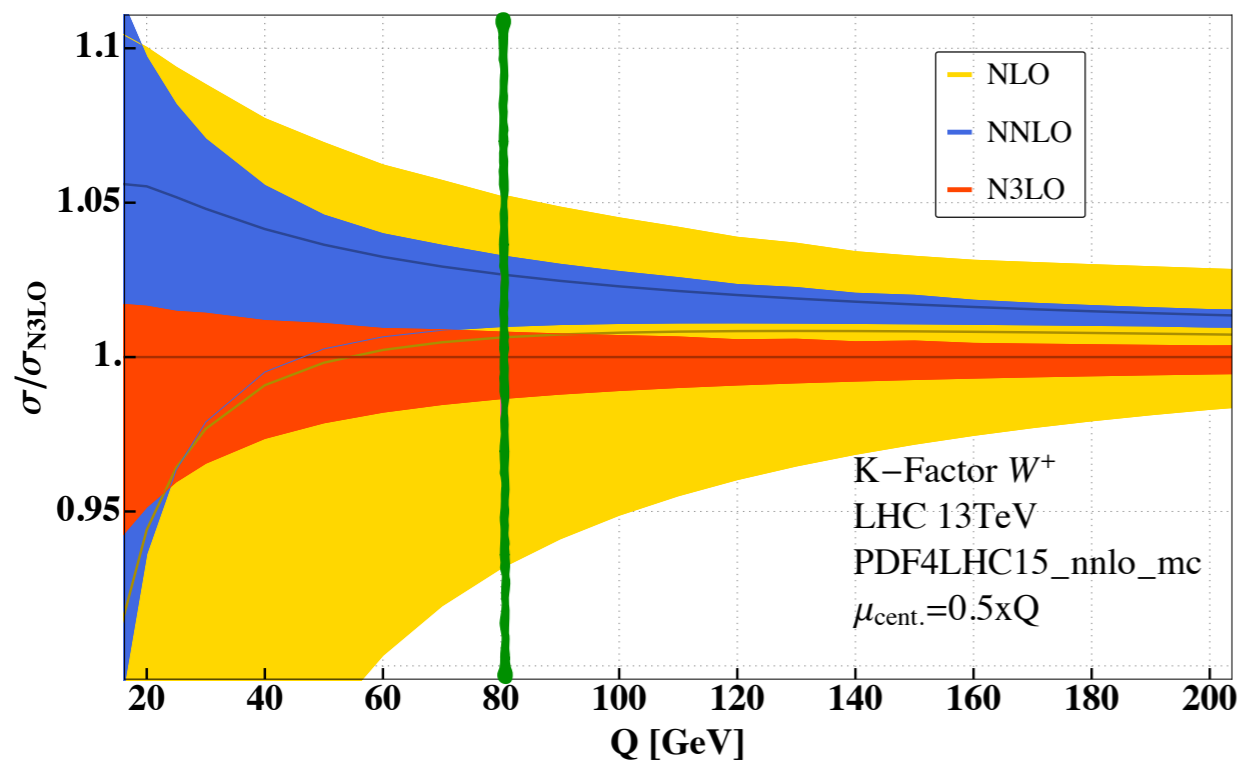
• (planar) [Abreu, Ita, Moriello, Page, Tschernow, Zeng '20] (Wb \bar{b}) [Badger, Bayu, Zoia '21]
[Papadopoulos, Tommasini, Wever '15] [Canko, Papadopoulos, Syrrakos '20]

NNLO SHORT SUMMARY

- ▶ fully differential NNLO: the new standard for $2 \rightarrow 2$ (towards $2 \rightarrow 3$)
- ▶ These calculations are ...
 - ... very **complex**
 - ↔ independent methods and/or implementation
 - ↔ validation & benchmark: Drell-Yan (sizeable differences) [Alekhin, Kardos, Moch, Trócsányi '21]
 - ↔ challenges for code releases: usability, quality assurance, ...
 - ... **CPU-cost intensive**
 - ↔ prohibitive for e.g. PDF & α_s fits
 - ↔ fast interpolation grids: APPLfast [APPLgrid, fastNLO, NNLOJET '19]
 - ↔ n-Tuples [LH '15, '17, '19]

N³LO — INCLUSIVE

H [Anastasiou et al. '15] [Mistlberger '18], H(VBF) [Dreyer, Karlberg '16], HH(VBF) [Dreyer, Karlberg '18], DY [Duhr, Dulat, Mistlberger '20]



[Duhr, Dulat, Mistlberger '20]

Q/GeV	$K_{\text{QCD}}^{\text{N}^3\text{LO}}$	$\delta(\text{scale})$	$\delta(\text{PDF}+\alpha_S)$	$\delta(\text{PDF-TH})$	$\frac{\sigma_{Z+\gamma^*}^{(0)}}{\sigma_{\gamma^*}^{(0)}}$
30	0.952	+1.5% -2.5%	$\pm 4.1\%$	$\pm 2.7\%$	1.01
50	0.966	+1.1% -1.6%	$\pm 3.2\%$	$\pm 2.5\%$	1.09
70	0.973	+0.89% -1.1%	$\pm 2.7\%$	$\pm 2.4\%$	2.16
90	0.978	+0.75% -0.89%	$\pm 2.5\%$	$\pm 2.4\%$	415
110	0.981	+0.65% -0.73%	$\pm 2.3\%$	$\pm 2.3\%$	7.4
130	0.983	+0.57% -0.63%	$\pm 2.2\%$	$\pm 2.2\%$	3.5
150	0.985	+0.50% -0.54%	$\pm 2.2\%$	$\pm 2.2\%$	2.6

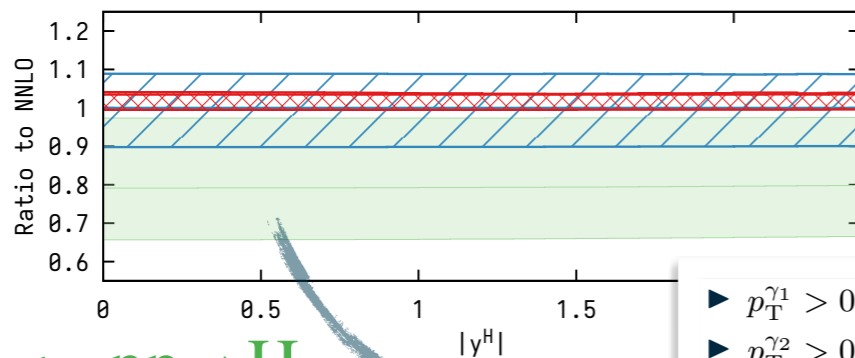
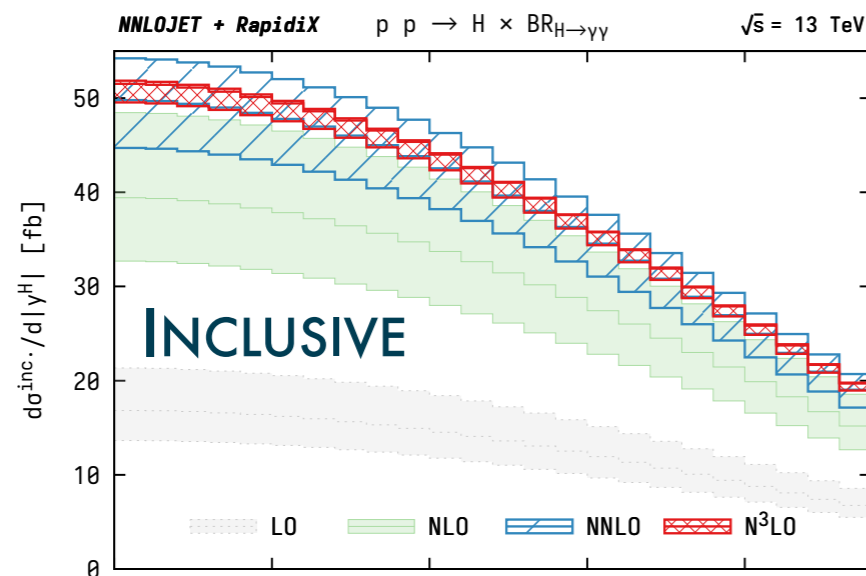
- ⊙ NNLO → N³LO:
↔ outside & similar size band?!

- ⊙ Missing N³LO PDFs!
- ⊙ Do we need to worry about fitting h.o. corrections?

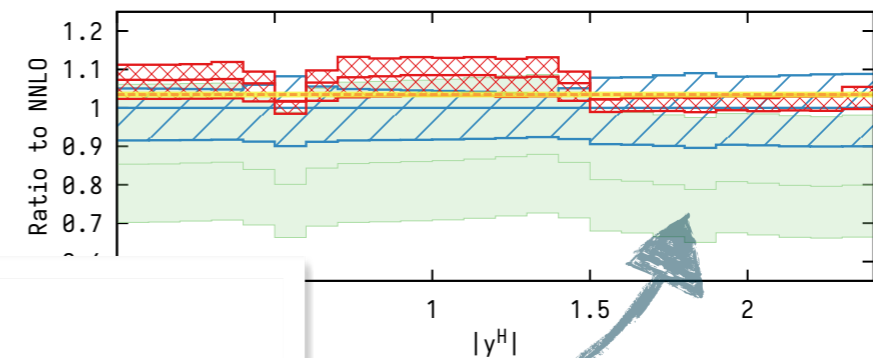
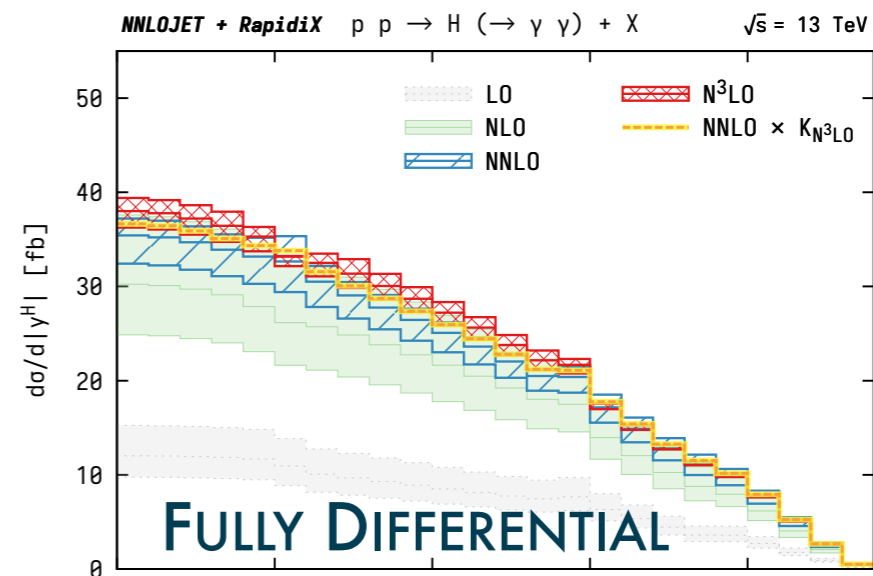
N³LO — GOING DIFFERENTIAL

DIS [Currie, Gehrmann, Glover, AH, Niehues, Vogt, Walker], H[y^H] [Cieri, et al. '18, Dulat, Mistlberger, Pelloni '18], H \rightarrow bb [Mondini, Schiavi, Williams '19],

H [Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21], H[$\sigma^{\text{fid.}}$] [Billis, et al. '21], DY[$\sigma^{\text{fid.}}$] [Cieri, et al. '21]



$d\sigma^{pp \rightarrow H}$
dY



- ▶ $p_T^{\gamma 1} > 0.35 \cdot m_{\gamma\gamma}$
 - ▶ $p_T^{\gamma 2} > 0.25 \cdot m_{\gamma\gamma}$
 - ▶ $|y^\gamma| < 2.37$
 - ▶ reject $1.37 < |y^\gamma| < 1.52$ (barrel-endcap)
 - ▶ photon isolation in $\Delta R < 0.2$
- $\hookrightarrow \sum_{\Delta R_{i\gamma} < 0.2} p_{T,i} < 0.05 \cdot E_T^\gamma$

$d\sigma^{pp \rightarrow H}$

[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

N³LO — GOING DIFFERENTIAL

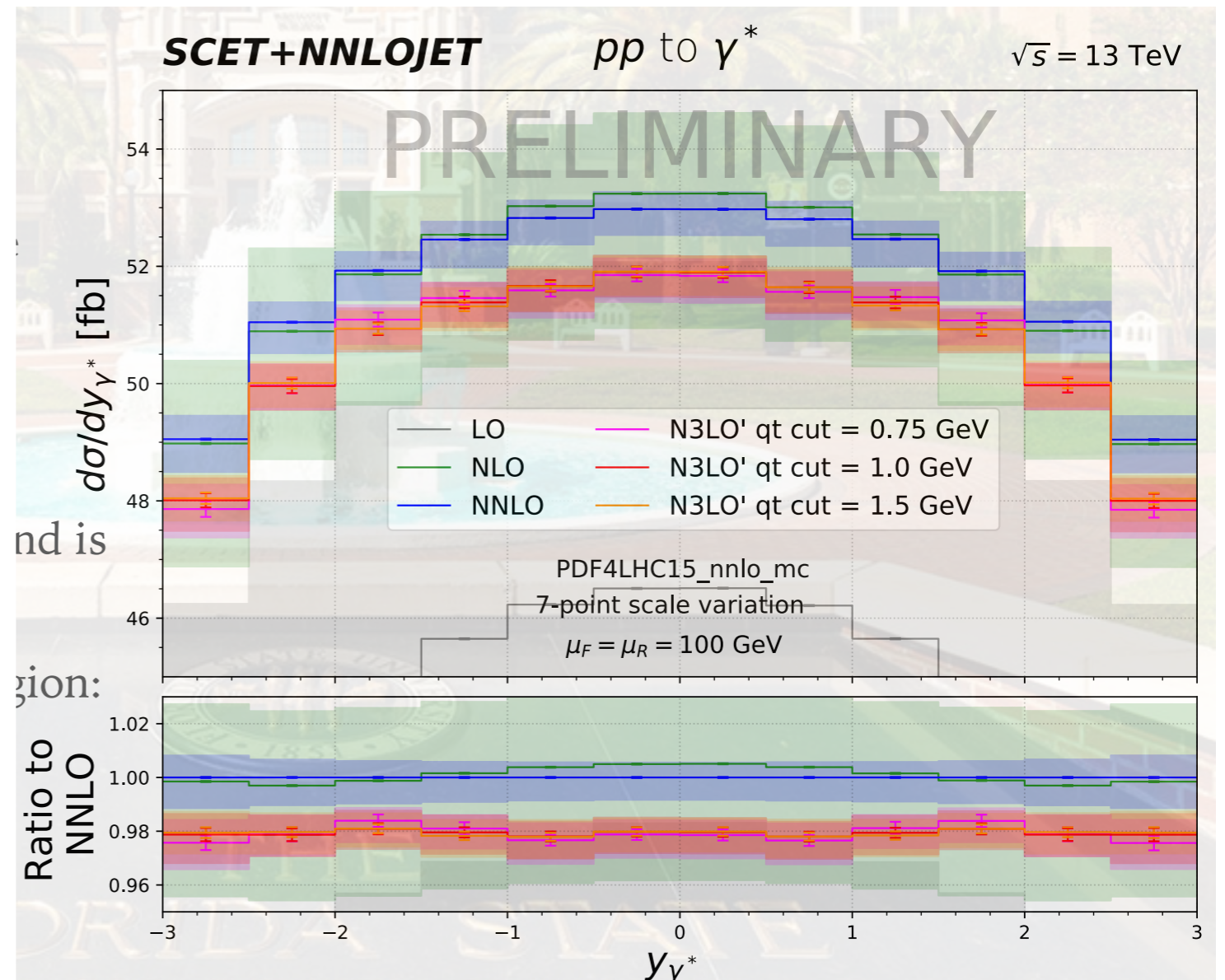
What is next?

- Drell-Yan !
- $pp \rightarrow \gamma\gamma$.
ingredients:
 q_T subtraction
 $pp \rightarrow \gamma\gamma j$ @ NNLO
 $pp \rightarrow \gamma\gamma$ @ 3-loops

[Caola, von Manteuffel, Tancredi '20]

• ... ?

[talk by X. Chen — Radcor LoopFest 2021]



dY

► photon isolation in $\Delta R < 0.2$

$$\hookrightarrow \sum_{\Delta R_{i\gamma} < 0.2} p_{T,i} < 0.05 \cdot E_T^\gamma$$

SCATTERING REACTIONS @ LHC!

- **Short** distance “hard”

- high scales: 10^2 — 10^3 GeV

$$\sigma = \sigma_0 \times (1 + \alpha_s + \alpha_s^2 + \alpha_s^3 + \dots)$$

fixed order: LO NLO NNLO N³LO ...



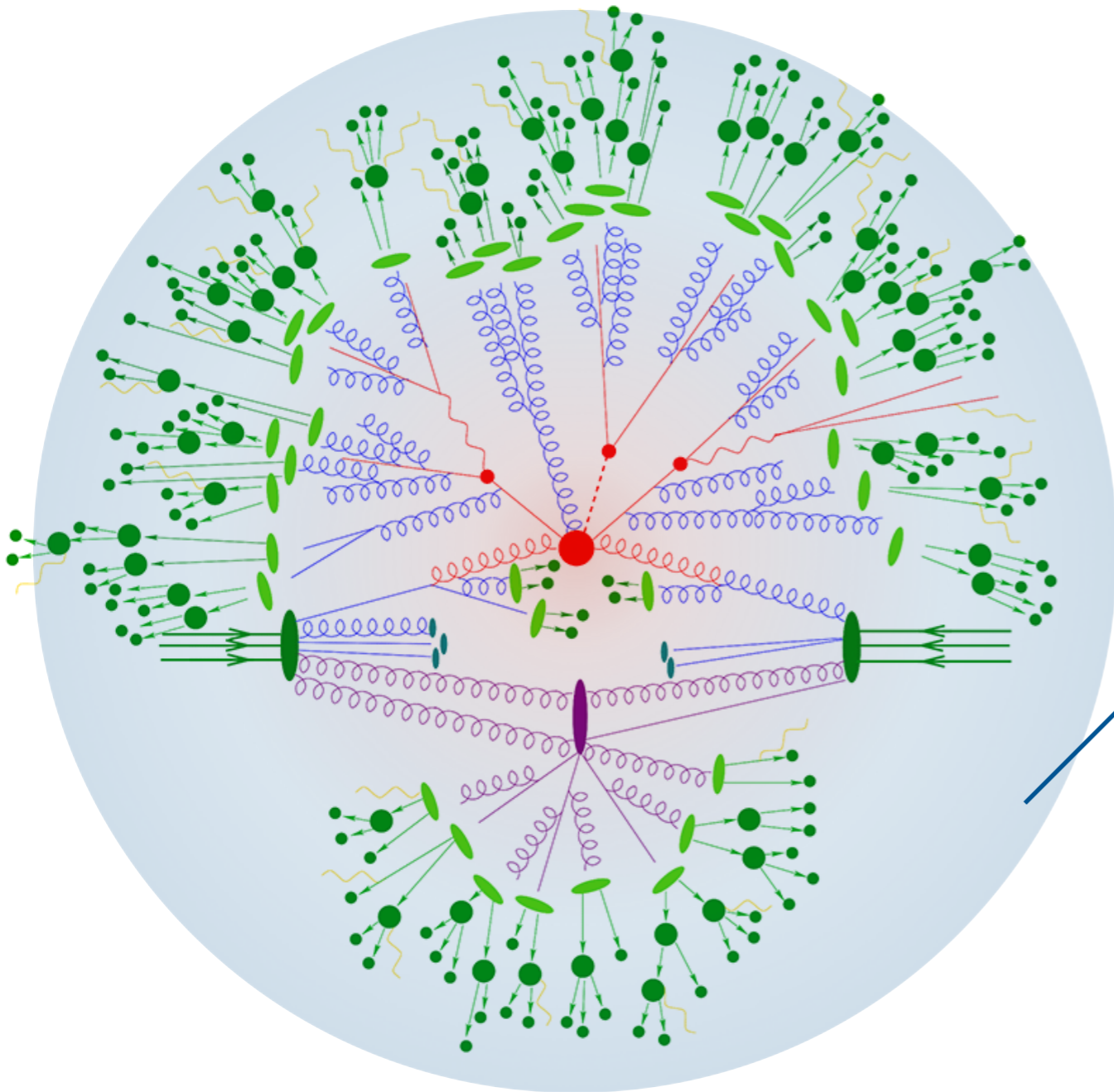
large logs $L!$

- **Long** distance “soft”

- low scales: $\mathcal{O}(\text{few GeV})$

$$\sigma = \sigma_0 \cdot \exp(\alpha_s^n L^{n+1} + \alpha_s^n L^n + \alpha_s^n L^{n-1} + \dots)$$

resummation: LL NLL NNLL ...



q_T DISTRIBUTION @ N3LL'

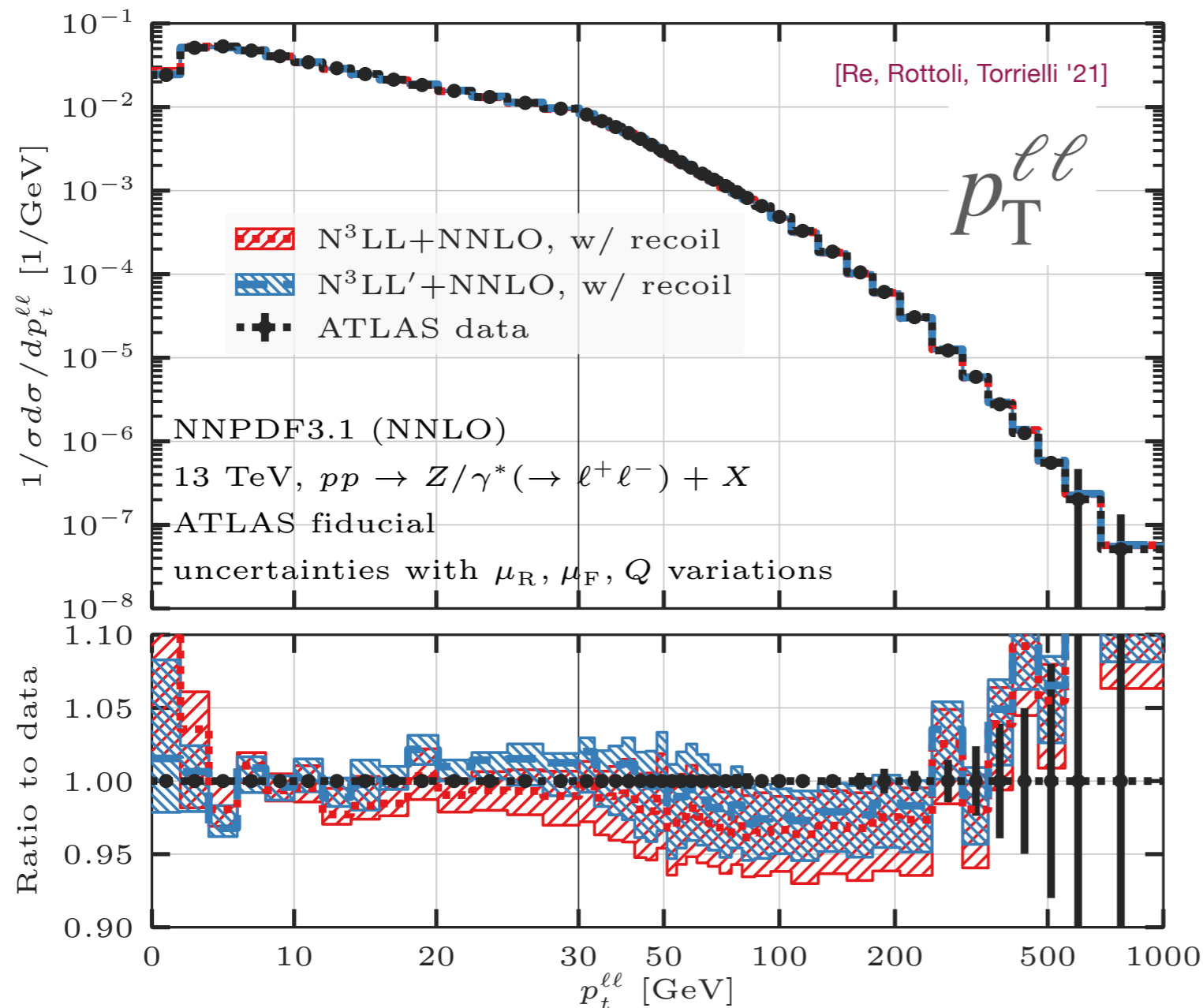
[Becher, Neumann '20; Billis et al. '21; Camada et al. '21; Re et al. '21]

➤ N3LL' = supplement N3LL with all constant terms of $\mathcal{O}(\alpha_s^3)$

↪ predict N3LO fiducial cross section

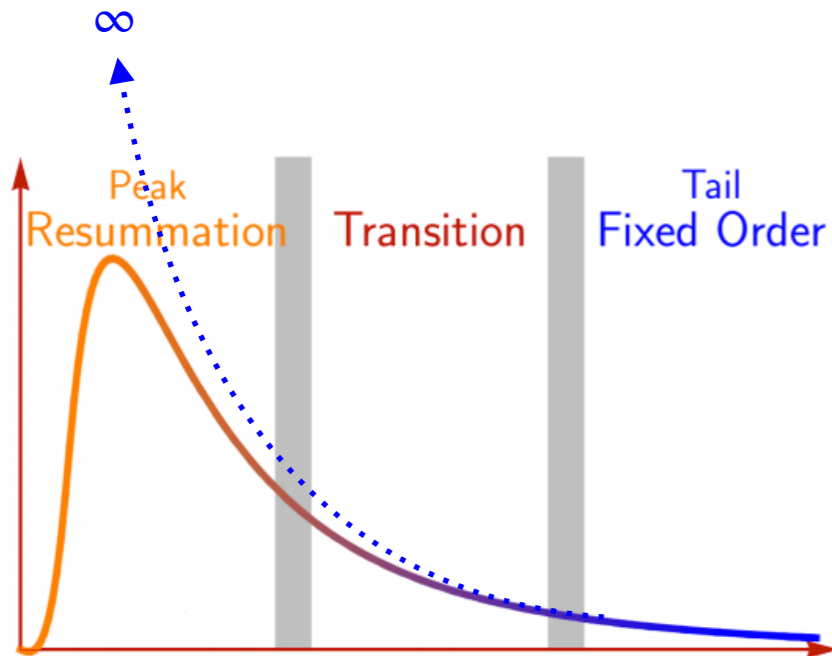
[Billis, Dehnadi, Ebert, Michel, Tackmann '21]

[Camarda, Cieri, Ferrera '21]



- **N3LL'+NNLO** improved description of **data** w.r.t **N3LL+NNLO**
- linear power corrections \leftrightarrow recoil [Ebert, Tackmann '19] ($\pm 1-2\%$ after matching to NNLO)
- reduced uncertainties
few-% level across $p_T^{\ell\ell}$

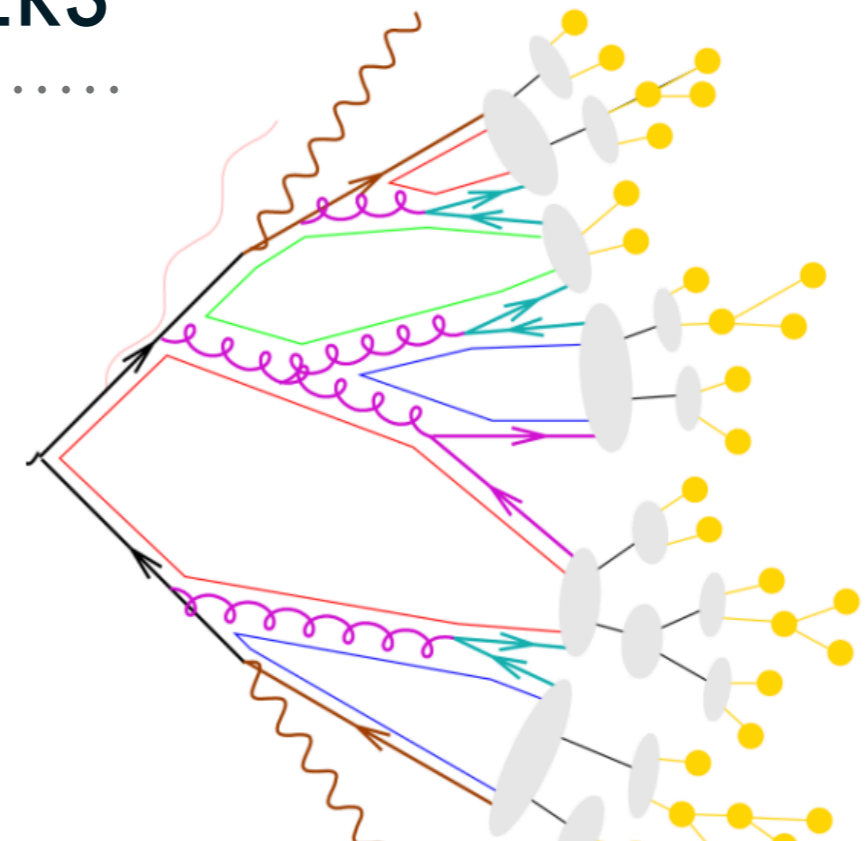
RESUMMATION & PARTON SHOWERS



Resummation

inclusive (analytic), tailored to specific observable with high logarithmic accuracy

- q_T N3LL^(*)
[Becher, Neumann '20; Billis et al. '21; Carmada et al. '21; Re et al. '21]
- NLL non-global logs
[Banfi, Dreyer, Monni '21]
- ...



Parton Showers (PS)

exclusive (MC algorithm), general purpose + non-perturbative models

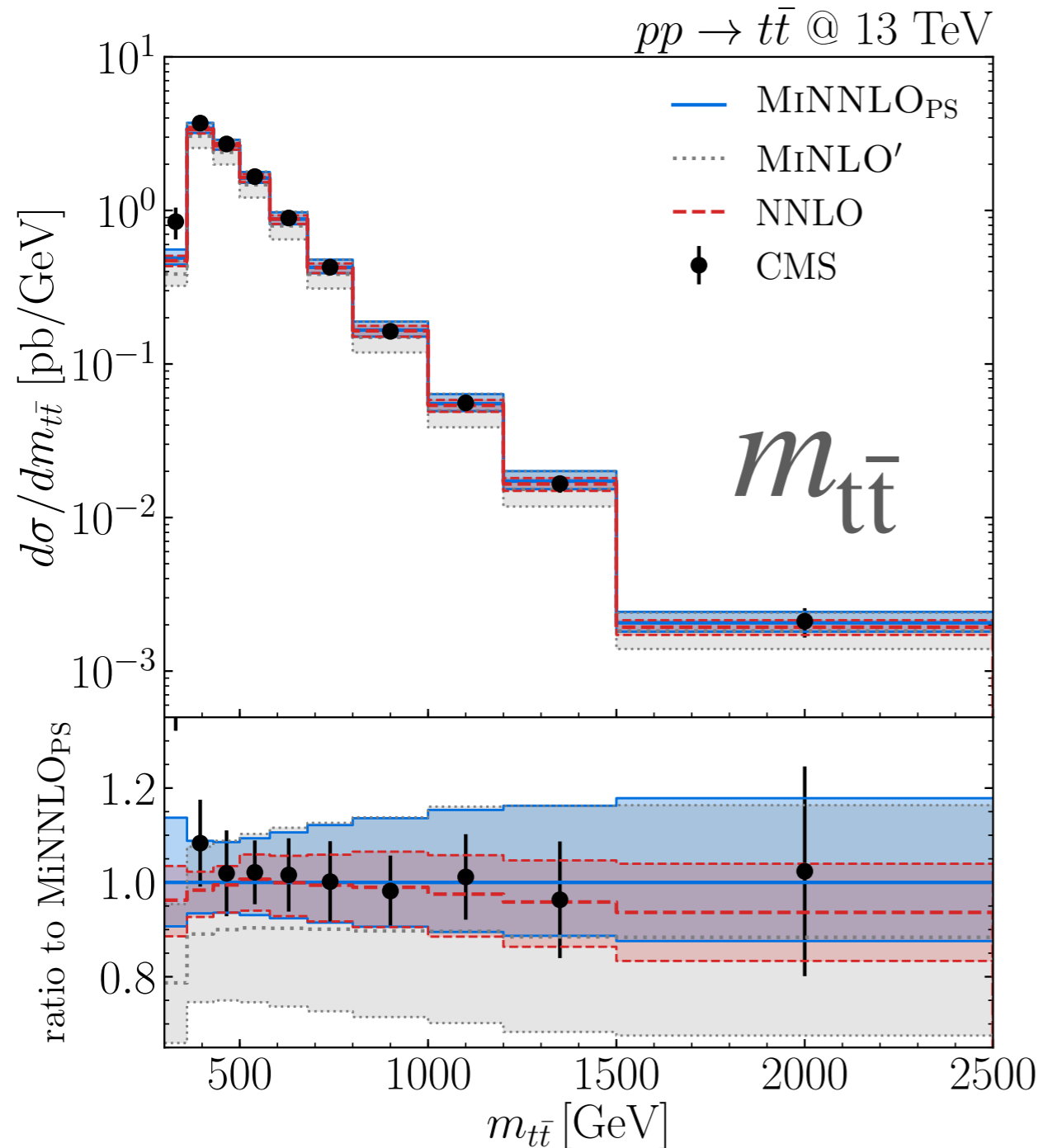
- NNLO + PS: NNLOPS [Hamilton, et al. '12,...] UNNLOPS [Höche, Li, Prestel '14,...] / [Plätzer '12] Geneva [Alioli, Bauer, et al. '13,...] MiNNLOPS [Monni, Nason, Re, Wiesemann, Zanderighi '19,...]

{substructure}

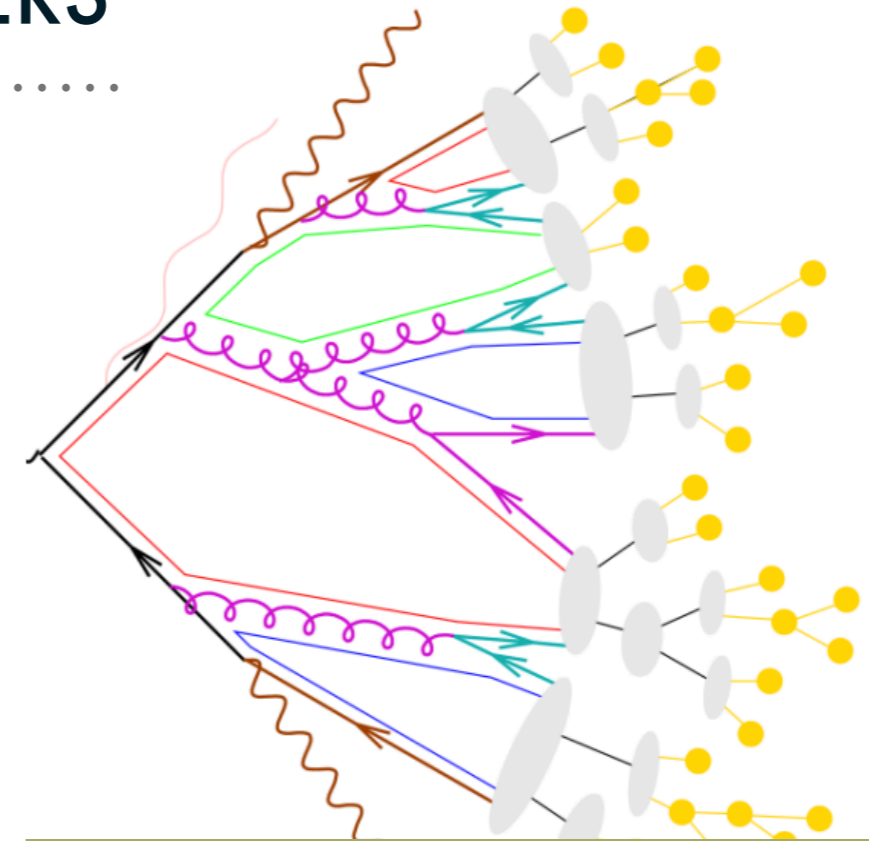
{Tools & MC}

RESUMMATION & PARTON SHOWERS

[Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi '21]



● first NNLO+PS with coloured final state



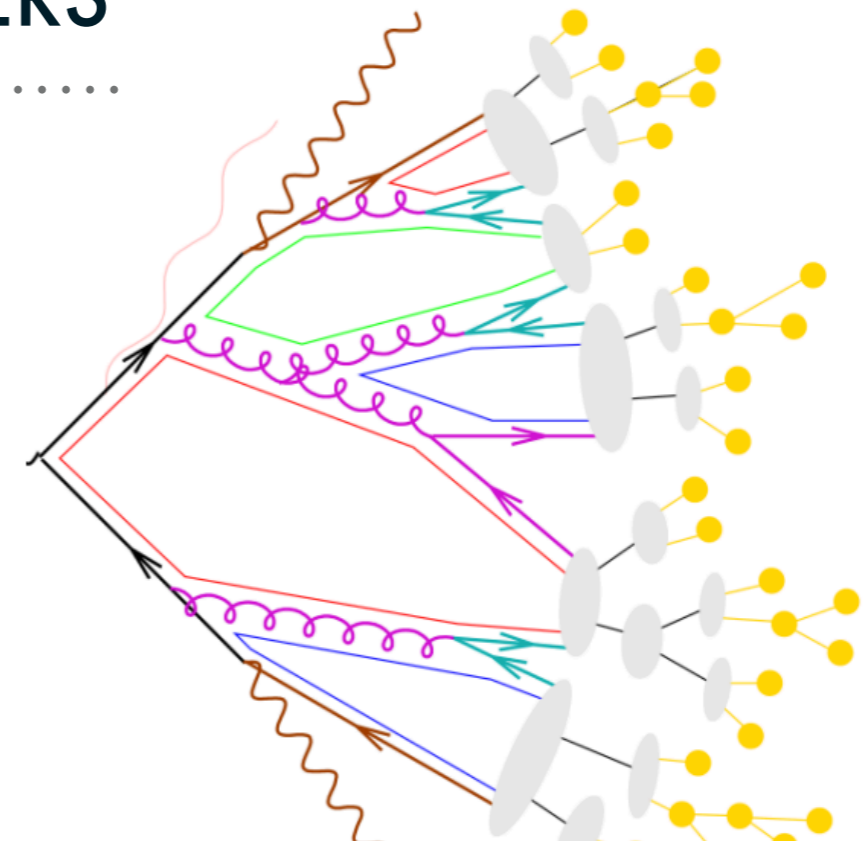
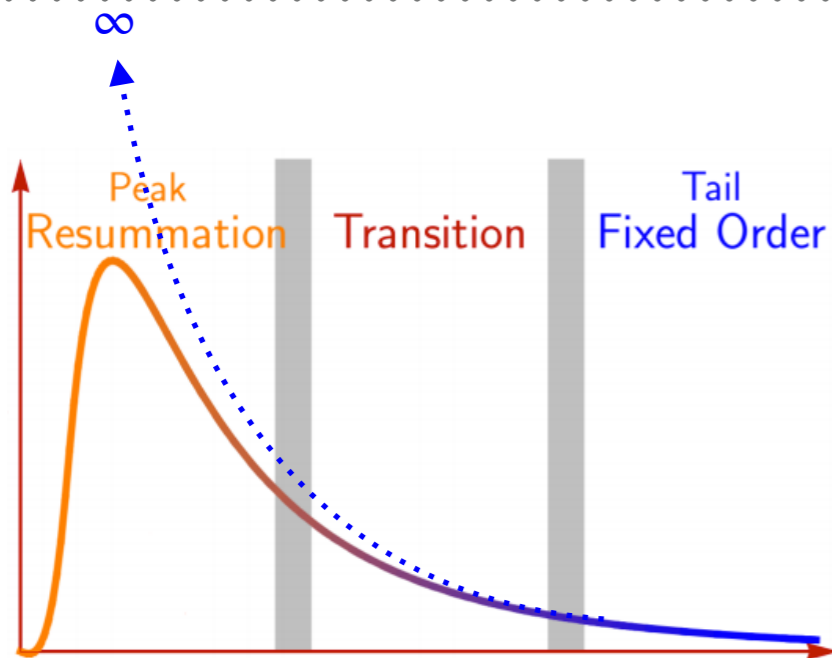
Parton Showers (PS)

exclusive (MC algorithm),
general purpose
+ non-perturbative models

{Tools & MC}

- NNLO + PS: NNLOPS [Hamilton, et al. '12,...] UNNLOPS [Höche, Li, Prestel '14,...] / [Plätzer '12] Geneva [Alioli, Bauer, et al. '13,...] MiNNLOPS [Monni, Nason, Re, Wiesemann, Zanderighi '19,...]

RESUMMATION & PARTON SHOWERS



?

Resummation

inclusive (analytic), tailored to specific observable with high logarithmic accuracy

Parton Showers (PS)

exclusive (MC algorithm), general purpose + non-perturbative models

• What is the (logarithmic) accuracy of parton showers?

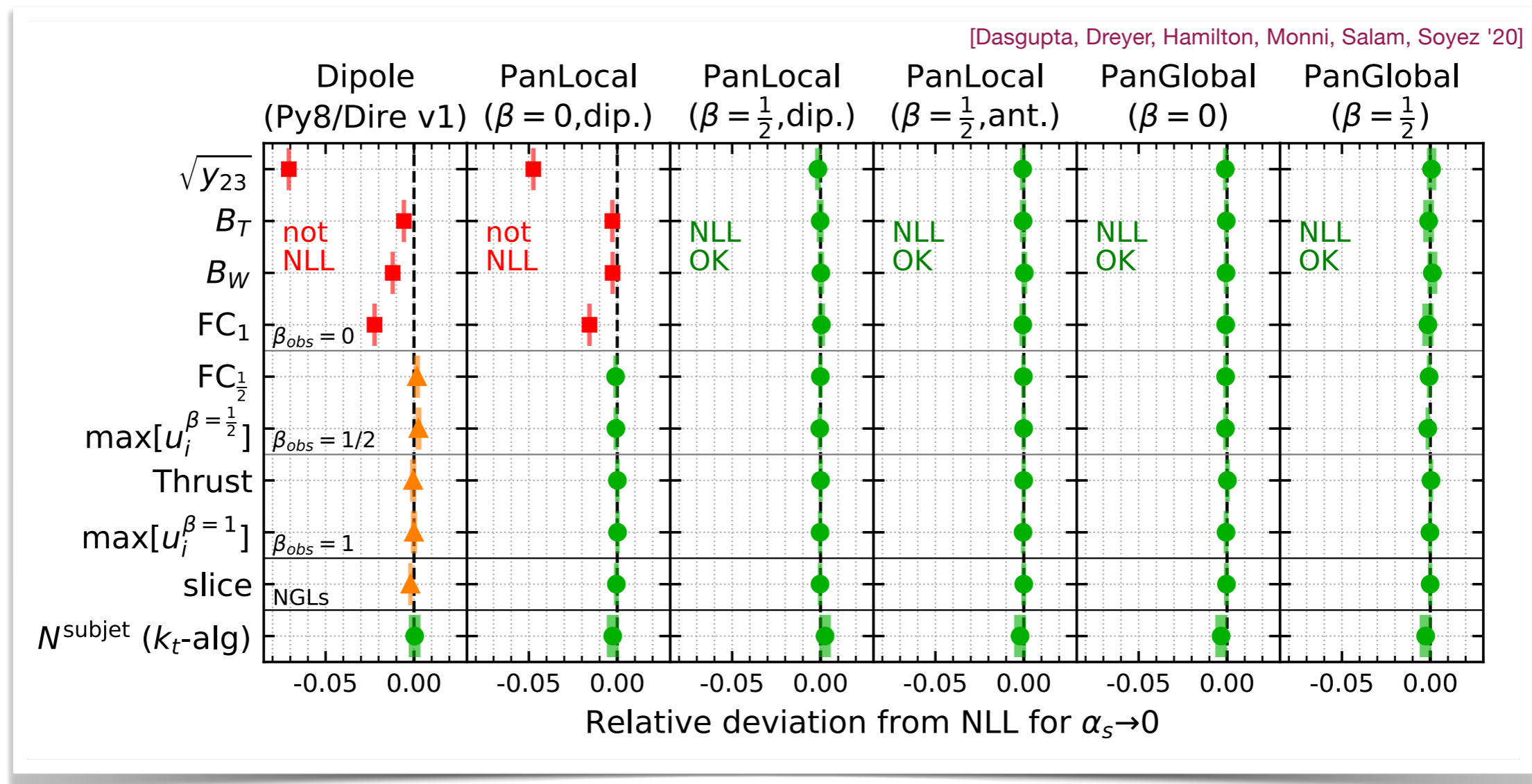
• Is it (N)LL? For what observable(s)?

• crucial to understand \rightsquigarrow design new PS

[Forshaw, Holguin, Plätzer '20] [Nagy, Soper '19] [...] [Dasgupta, et al. '20; Hamilton, et al. '20; Karlberg, et al. '21]

INVESTIGATING SHOWER ACCURACY*

➤ Compare PS to NLL observables: $\alpha_s \rightarrow 0$ for fixed $\alpha_s L$



- Is this observables set “complete”? How to extend it for pp?
- Can this test be adopted by other groups?

* see also: [Nagy, Soper, '20]

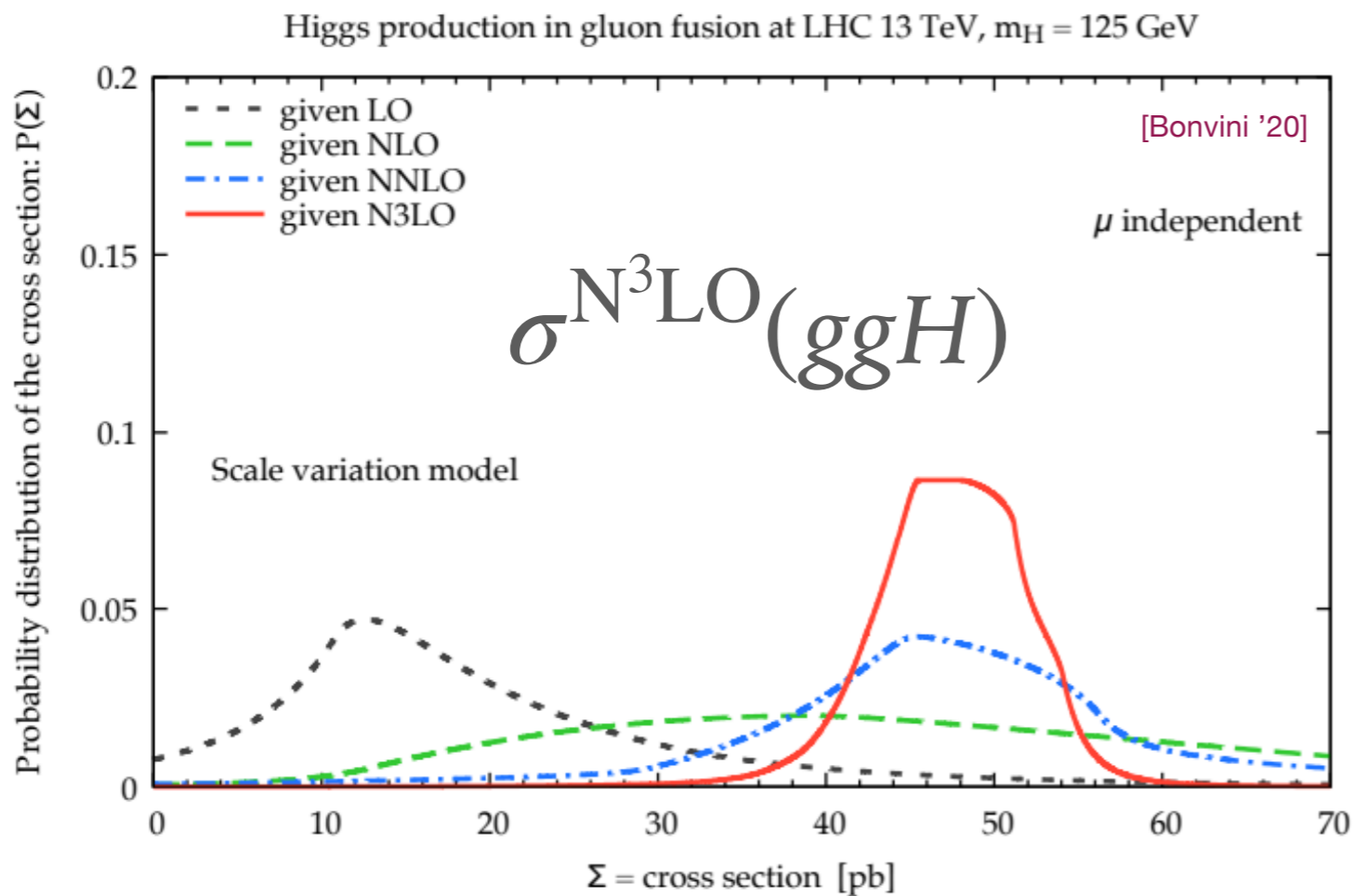
THEORY UNCERTAINTIES PT.1

- increasingly urgent to have more robust uncertainty estimates
 - ↪ e.g. for theory uncertainties in PDF fits
- ❖ scale ambiguities in jets ($p_{T,j}$ v.s. \hat{H}_T)
- ❖ scales in ratios:
 - top spin correlation (to expand or not @ NLO)
 - $p_T(Z) / p_T(W)$ (correlate: very small residual uncertainties)
 - ang. coefficients A_i (un-correlate in the *same process*)
- ❖ nuisance parameters in p_T res.

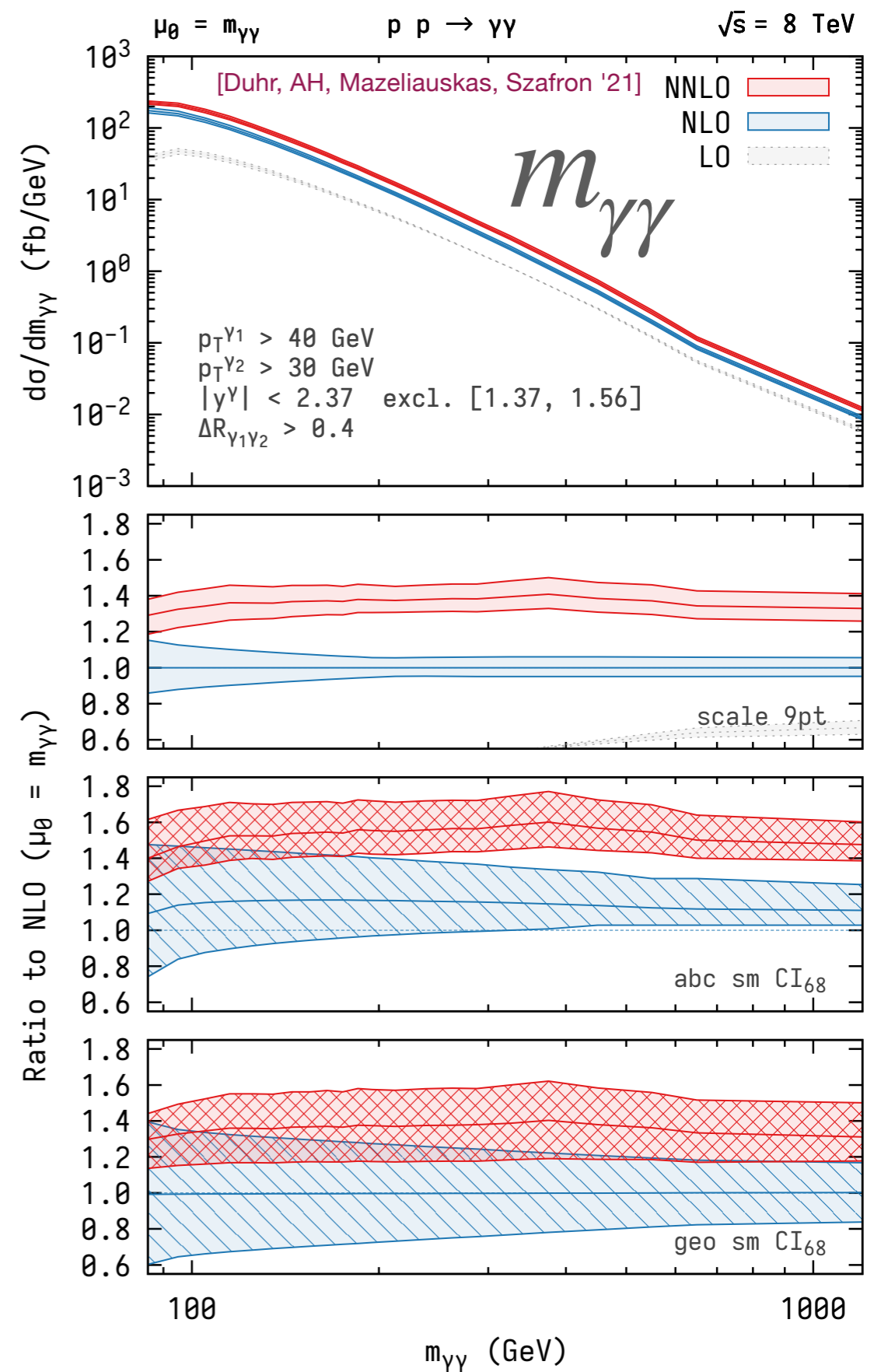
THEORY UNCERTAINTIES PT.2

- alternative approaches
- ↔ statistical interpretation?

Bayesian model [Cacciari, Houdeau '11]



- ⦿ PDF ↔ model and priors



- ⦿ treatment of correlations?

FINAL REMARKS.

- **Remarkable progress** in precision calculations:
 - $2 \rightarrow 8$ @ NLO, $2 \rightarrow 3$ @ NNLO, $2 \rightarrow 1$ @ N³LO
- still many **issues & challenges** \Rightarrow **discussions sessions!**
- **More topics to consider:** <https://phystev.cnrs.fr/wiki/2021:topics>
 - mixed QCD-EW corrections.
 - $p_T(Z)$ —in the world of per-cent precision
 - ➔ m_b effects, QED ISR, NP effects, ...
 - wishlist: what is needed for HL-LC, 100 TeV, EIC?
 - ... **your ideas!**

ENJOY (VIRTUAL) LES HOUCHES!