Analyticity and Unitarity for Cosmological Correlators

Victor Gorbenko
EPFL

Lorenzo Di Pietro, VG, Shota Komatsu

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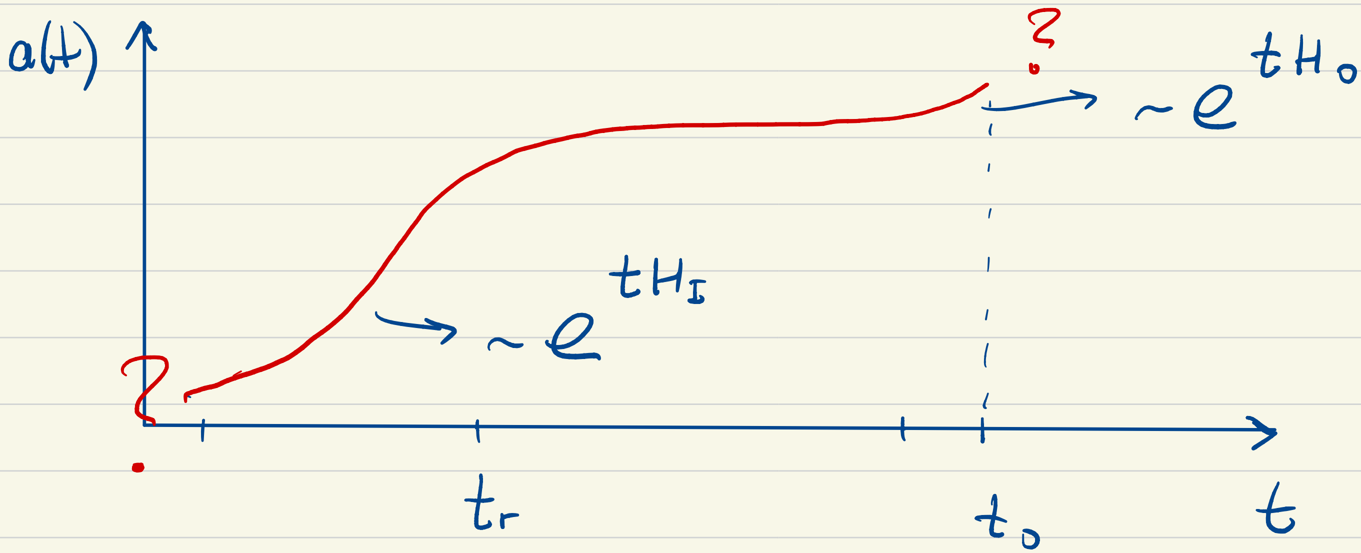
- Motivation and broader picture on fundamental Cosmology

- QFT on rigid dS:
  - Correlators via EAdS Lagrangian
  - Analytic properties
  - Positivity from unitarity

- Open problems

Motivation

- We live in an expanding cosmological spacetime!



Develop

Fundamental theory capable of describing expansion

Computational tools for phenomenological uses

- Our main fundamental theory is AdS/CFT suited for asymptotically static spacetimes

  see for attempts to embed cosmology into AdS/CFT:

  (not easy)

  Dionysios Anninos (Princeton, Inst. Advanced Study), Diego M. Hofman (Amsterdam U.) (Mar 14, 2017)
  Published in: *Class.Quant.Grav.* 35 (2018) 8, 085003 • e-Print: 1703.04622 [hep-th]

  Stefano Antonini (Maryland U.), Petar Simidzija (British Columbia U.), Brian Swingle (Maryland U. and Brandeis U.), Mark Van Raamsdonk (British Columbia U.) (Mar 21, 2022)
  e-Print: 2203.11220 [hep-th]

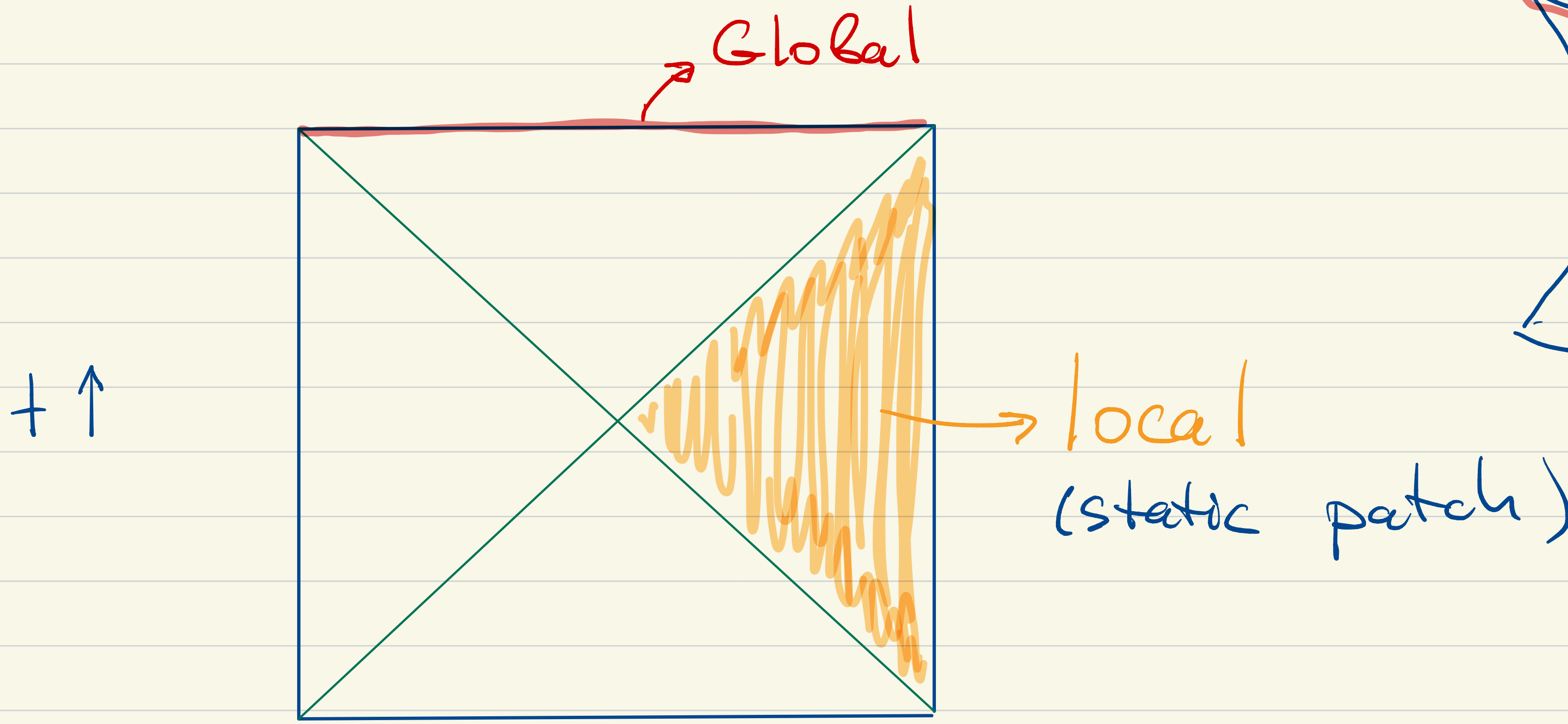
- We study expanding spacetimes directly

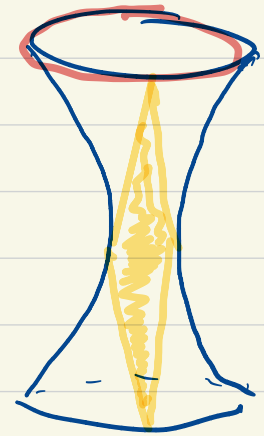
$$-dt^2 + a^2(t)\, d^2x\,, \quad a(t) \sim e^{Ht} \sim \text{de Sitter}$$

- Most symmetric ~ the simplest?

  (also relevant for observations)

Global vs local cosmology

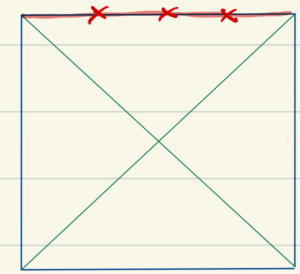




Penrose diagram of dS (e.g. AdS-Schw.)

- We consider Global observables

Global observables



Wave function

$$\Psi[\bar\varphi] = \int^{\bar\varphi} D\varphi\, e^{iS[\varphi]}$$

$$\frac{\delta}{\delta\bar\varphi}\cdots\frac{\delta}{\delta\bar\varphi}\Psi[\bar\varphi] = $$

$$= \langle O \dots O \rangle$$

$$\Psi[\bar\varphi] \approx Z_{AdS}[J]$$

Correlators

$$\langle \varphi(x_1)\varphi(x_2)\dots\varphi(x_n)\rangle\Big|_{t\to\infty} =$$

$$= \int D\bar\varphi\; \Psi[\bar\varphi]\,\Psi^*[\bar\varphi]\,\bar\varphi(x_1)\,\bar\varphi(x_2)\dots$$

No obvious analog in AdS

Path integral over sources looks strange from CFT point of view

- This is an important issue for dS/CFT
- If gravity is included, one of the sources is boundary metric → need to path integrate over it...
- In this talk we do not consider dynamical gravity. As a necessary step we want to understand the basic properties of cosmological correlators $\langle \varphi\varphi \ldots \varphi \rangle$
- Compared to AdS, not much is known

Why is it so much more complicated in dS?

- We consider the following theory

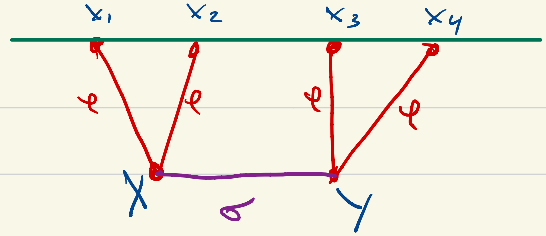
$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m_\varphi^2\varphi^2 - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{2}g\varphi^2\sigma$$

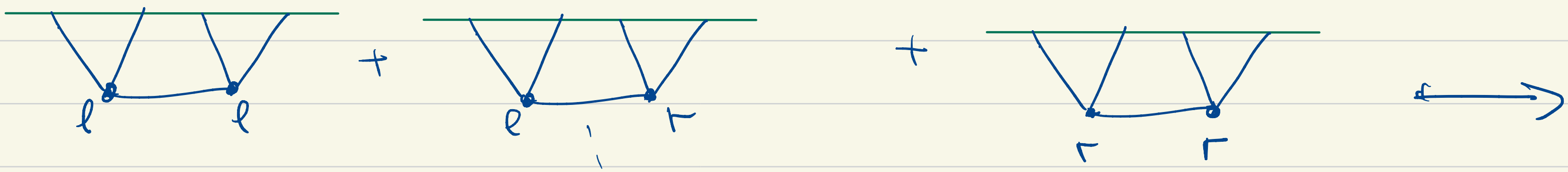
- Let us first review the "in-in" formalism:

$$\langle \bar{T} e^{\int^t iH_{int}\,dt} \varphi(x_1)\varphi(x_2)\dots T e^{-\int^t iH_{int}\,dt} \rangle$$

Hartle-Hawking or Bunch-Davies state

- Wick contractions involve three types of propagators:
G^{lr}, G^{rr}, G^{ll}
- Focus on the 4-pt. function: $\langle \varphi\varphi\varphi\varphi \rangle$

$\langle \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\rangle =$  $+ O(g^4)$



$\alpha = (ll, lr, rr)$

$$G^{\alpha}_{\nu}(X,Y) \simeq {}_2F_1\left(\frac{d}{2}+i\nu, \frac{d}{2}-i\nu, \frac{d+1}{2}, \frac{1+S+i\varepsilon^{\alpha}}{2}\right)$$

$$\longrightarrow \int_{dS} dX\, dY\, K_{\nu_\varphi} K_{\nu_\varphi} K_{\nu_\varphi} K_{\nu_\varphi} G^{\alpha}_{\nu_\sigma}(X,Y) \qquad \nu = \sqrt{m^2 - \frac{d^2}{4}}$$

- There isn't a simple Fourier transform that would factorize the integrals.

- The idea is to analytically continue the integrals to EAdS and use technology developed for AdS

see also Sleight, Tarrona

C. Sleight and M. Taronna, *Bootstrapping Inflationary Correlators in Mellin Space*, *JHEP* **02** (2020) 098 [1907.01143].

C. Sleight and M. Taronna, *From AdS to dS Exchanges: Spectral Representation, Mellin Amplitudes and Crossing*, 2007.09993.

C. Sleight, *A Mellin Space Approach to Cosmological Correlators*, *JHEP* **01** (2020) 090 [1906.12302].

C. Sleight and M. Taronna, *From dS to AdS and back*, *JHEP* **12** (2021) 074 [2109.02725].

$$ds^2_{EAdS} = \frac{dz^2 + dx^2}{z^2 k^2} \qquad ds^2_{dS} = \frac{-d\eta^2 + dx^2}{\eta^2 H^2}$$

$$\int d\eta \longrightarrow \int dz \qquad \eta_\ell \to e^{i\frac{\pi}{2}} z$$

$$\eta_r \to e^{-i\frac{\pi}{2}} z$$

$$\langle \bar{T} e^{\int^{t} i H_{int} dt} \varphi(x_1)\varphi(x_2)\ldots T e^{-\int^{t} i H_{int} dt} \rangle$$

- Every QFT on dS is equivalent to a FT on EAdS space with the doubled field content:

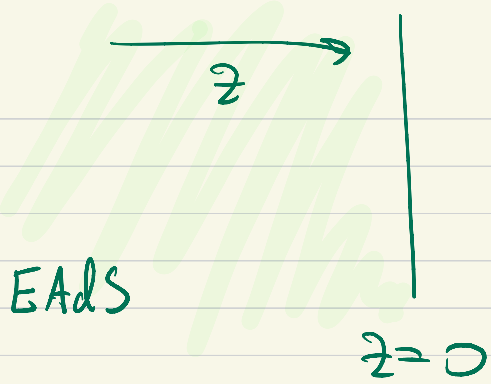
$$\boxed{\mathcal{L}^{dS}(\phi) = -\frac{1}{2}\partial\phi\partial\phi - \frac{1}{2}m^2\phi^2 - V(\phi)}$$

$$\downarrow$$

$$\boxed{\begin{aligned}\mathcal{L}^{AdS}(\phi_+,\phi_-) &= i\sinh\pi\nu\left(\partial\phi_+\partial\phi_+ - m^2\phi_+^2\right) - i\sinh\pi\nu\left(\partial\phi_-\partial\phi_- - m^2\phi_-^2\right) - \\ &- e^{-i\pi\frac{d-1}{2}}V\left(e^{i\frac{\pi}{2}\left(\frac{d}{2}+i\nu\right)}\phi_+ + e^{i\frac{\pi}{2}\left(\frac{d}{2}-i\nu\right)}\phi_-\right) - e^{i\pi\frac{d-1}{2}}V\left(e^{-i\frac{\pi}{2}\left(\frac{d}{2}+i\nu\right)}\phi_+ + e^{-i\frac{\pi}{2}\left(\frac{d}{2}-i\nu\right)}\phi_-\right)\end{aligned}}$$

$$\nu = \sqrt{\frac{m^2}{H^2} - \frac{d^2}{4}},$$

φ_+ and φ_- are two independent fields with different boundary conditions:



$$\varphi_+ \approx z^{-\frac{d}{2} - i\nu}$$

$$\varphi_- \approx z^{-\frac{d}{2} + i\nu}$$

$$ds^2 = \frac{dz^2 + dx^2}{z^2}$$

- There is a range of masses for which both boundary conditions are allowed (and unitary) in AdS space

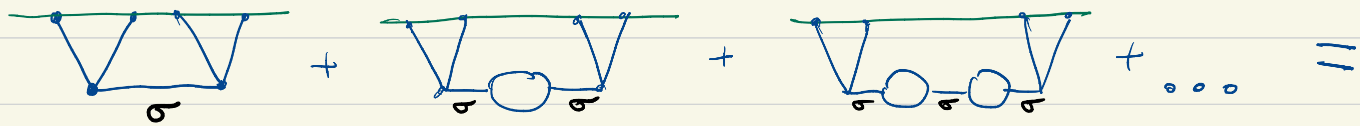
  e.g. Klebanov, Witten 9905104

- Here we are generically outside of this range, moreover, one of the fields has a wrong sign kinetic term. But this is OK (at least in perturbation theory) because we only deal with the Euclidean theory.

- A similar relation for the wave function is known but doesn't require the field doubling (Maldacena 0210603).

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m_\varphi^2\varphi^2 - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{2}g\varphi^2\sigma$$

- As in flat space, we need to resum bubble diagrams



$$\boxed{\begin{aligned} & g^2\left(\frac{\eta_c}{x_{12}}\right)^{d-2i\nu_\phi}\left(\frac{\eta_c}{x_{34}}\right)^{d-2i\nu_\phi}\frac{\Gamma^4(i\nu_\phi)}{\pi^{2d+6}2^{11}}\times \\ & \times\int_{-\infty}^{\infty} d\nu\, f(\nu)\sin^2\frac{\pi}{4}(d-4i\nu_\phi-2i\nu)\,\Gamma^2\left(\frac{d-4i\nu_\phi\pm 2i\nu}{4}\right)\Gamma^2\left(\frac{d\pm 2i\nu}{4}\right)\mathcal{F}_\nu^{-\nu_\phi-\nu_\phi-\nu_\phi-\nu_\phi}(z,\bar{z})\end{aligned}}$$

$$\boxed{f(\nu) = \frac{i\nu g^2}{\nu^2-\nu_\sigma^2+\frac{4\pi^2}{i\nu}g^2 B_{\nu_\phi}(\nu)}}$$

scalar Conformal partial wave

$$\boxed{B_{\nu_\phi}(\nu) = \frac{1}{16\pi^3}\left[\pi+\cot\pi i\nu_\phi\left(\psi\left(\frac{1}{2}+\frac{i\nu}{2}-i\nu_\phi\right)-\psi\left(\frac{1}{2}+\frac{i\nu}{2}+i\nu_\phi\right)\right)\right]}$$

(e.g in $d=2$)

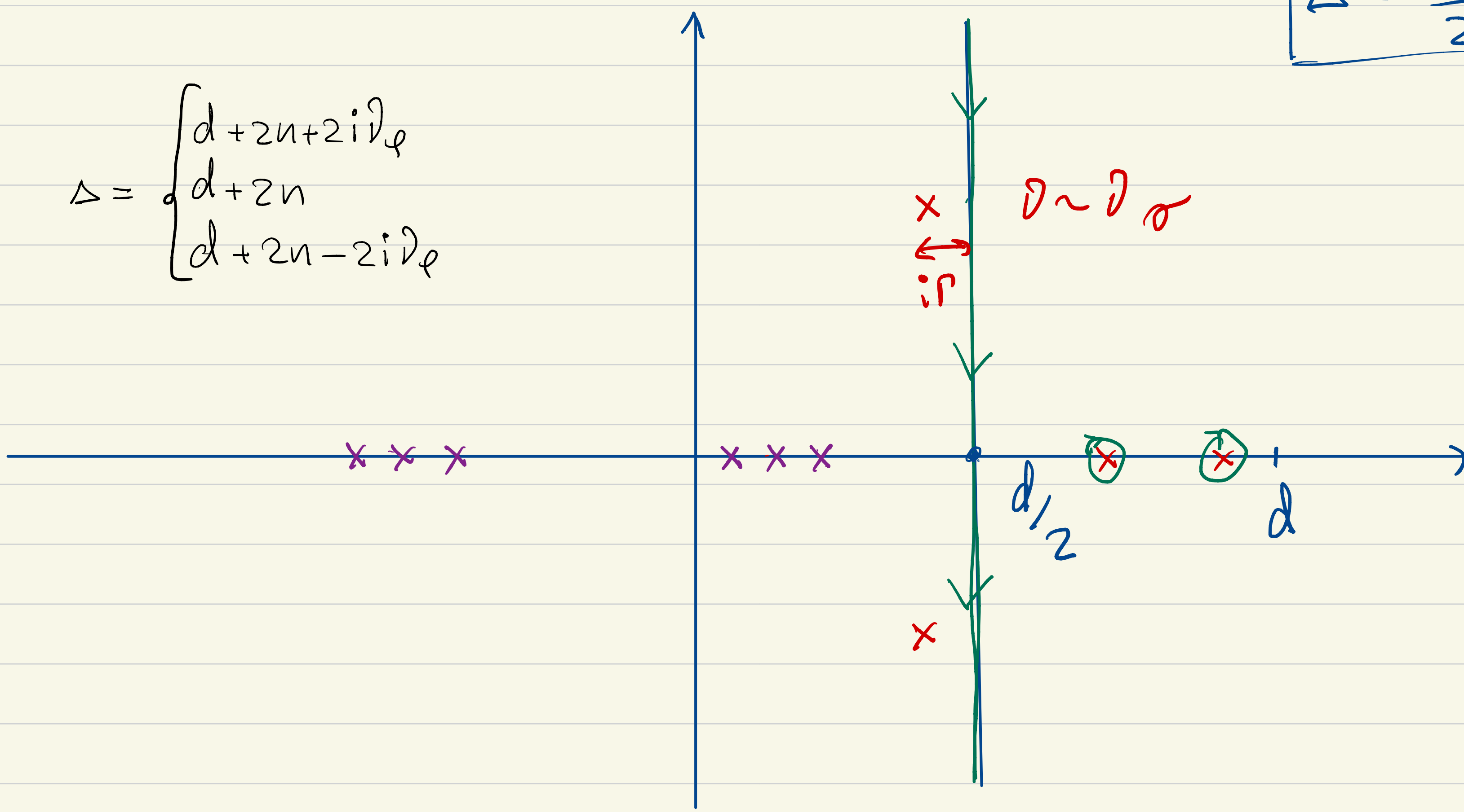
Analytic Properties

"spectral amplitude"

$$\langle \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\rangle\Big|_{t\to\infty} \simeq \int_{-\infty}^{\infty} d\nu\, \rho(\nu)\, F_\nu^{-\nu_\varphi,-\nu_\varphi,-\nu_\varphi,-\nu_\varphi}(x_1, x_2 \ldots)$$

$$\boxed{\Delta = \frac{d}{2} + i\nu}$$

$$\Delta = \begin{cases} d + 2n + 2i\nu_\varphi \\ d + 2n \\ d + 2n - 2i\nu_\varphi \end{cases}$$

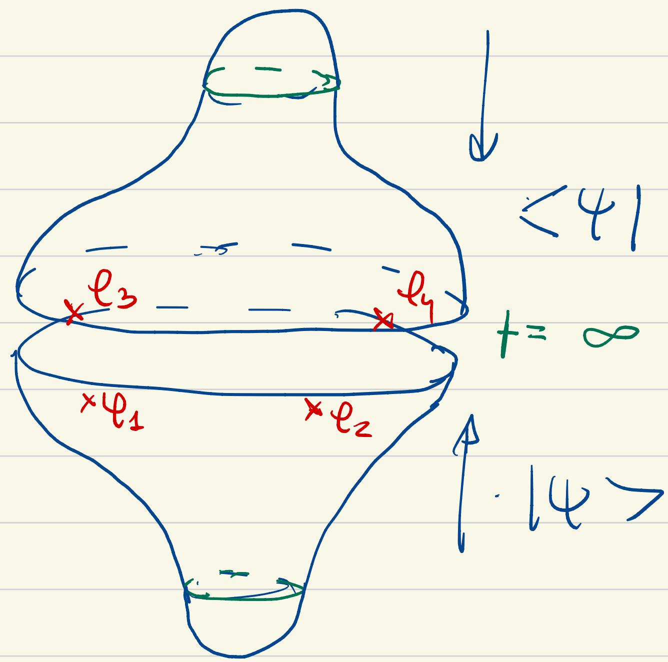


Unitarity and positivity

also → M. Hogervorst, J. Penedones and K.S. Vaziri 2107.13871.

- To derive constraints from unitarity, we reinterpret the correlator as an overlap:

$$\langle \varphi(x_1)\,\varphi(x_2) | \varphi(x_3)\,\varphi(x_4) \rangle =$$



- Unitarity is obscure in the EAdS formulation we derive it directly in dS, without relying on perturbation theory.

- We insert resolution of Id :

$$\langle \varphi\varphi\varphi\varphi \rangle = \int_0^\infty d\nu \sum_J \underbrace{\langle \varphi(x_1)\varphi(x_2)\, P_{\Delta,J}\, \varphi(x_3)\varphi(x_4) \rangle}_{C_{\Delta,J} \cdot F_{\Delta,J}^{\Delta_\varphi\Delta_\varphi\Delta_\varphi\Delta_\varphi}(z,\bar{z})}$$

- By studying limit $x_3 \to x_1$ $x_4 \to x_2$ we see

$$C_{\Delta,J} \approx \left| \langle \varphi(x_1)\varphi(x_2) | P_{\Delta,J} \right|^2 \geq 0$$

(careful procedure a bit more subtle)

- $C_{\Delta,J} = \rho_J(\nu) + \rho_J(-\nu) = 2 \cdot \mathrm{Im}\, \rho_J(\nu) \geq 0$

$$\langle \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) \rangle \Big|_{t\to\infty} \approx \int_{-\infty}^{\infty} d\nu\, \rho(\nu)\, F_\nu^{-\Delta_\varphi,-\Delta_\varphi,-\Delta_\varphi,-\Delta_\varphi}(x_1, x_2 ..)$$

- We checked positivity of $\text{Im}\,\rho$ in the $g\varphi^2\sigma$ model and in large N $O(N)$ model at finite coupling.

Summary: Properties of $\rho(\nu)$

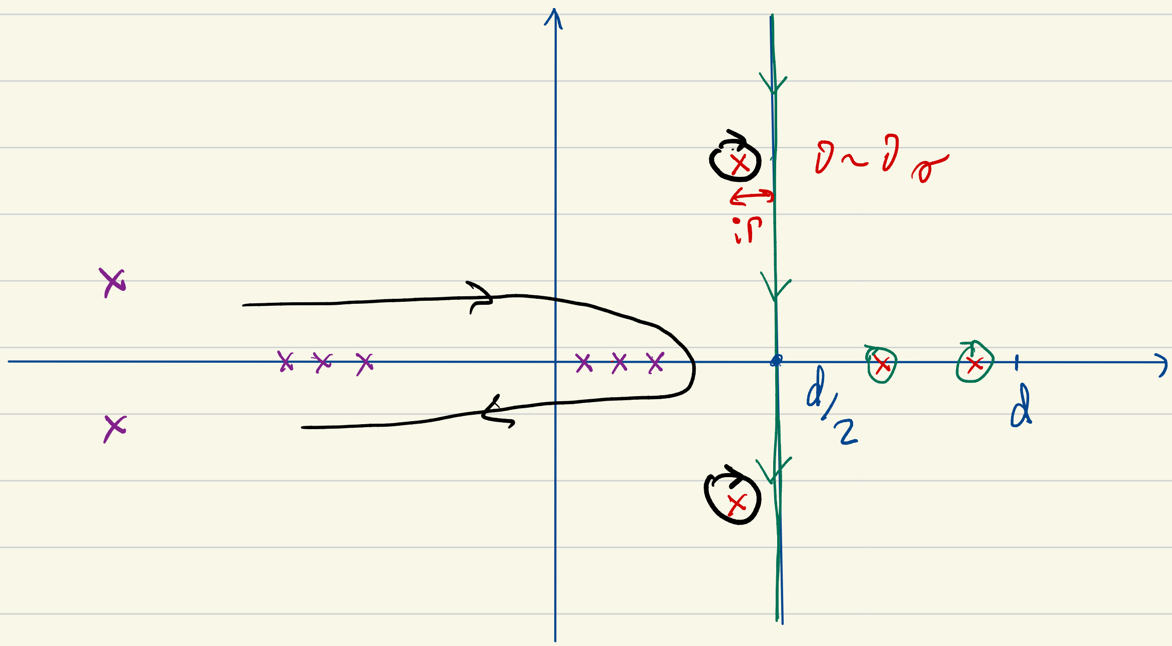
$$\langle \varphi\varphi\varphi\varphi \rangle = \sum_J \int_{-\infty}^{\infty} d\nu\, \rho_J(\nu)\, \mathcal{F}_\nu \quad \leftrightarrow \quad \rho_J(\nu) = \int dx_i \langle \varphi\varphi\varphi\varphi \rangle \mathcal{F}_{J,\nu}$$

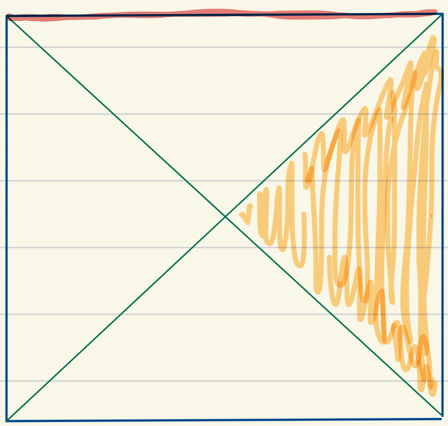
- Positive $[\rho(\nu) + \rho(-\nu)] \longrightarrow$ Non-perturbative
- Meromorphic, Analytic "on the physical sheet"
- Structure of a real non-unitary CFT with complex anomalous dimensions.

$\Big\} \rightarrow$ in P.T.

- Operators $\sim$ poles of ρ

- OPE can be generated by pulling the contour:





- There is no (boundary) operator – – (bulk) state correspondence

- Operators related to bulk QNMs?

Generalizations:

No gravity

- It is likely that analytic properties are true non-perturbatively (in QFT)
- We used $SO(1, d+1)$ inv. but conceptually the technology will work with smaller symmetry. $ISO(d)$ or $ISO(d) \times SO(1,1)$ (important for inflation)
- Bootstrap applications – ?

M. Hogervorst, J. Penedones and K.S. Vaziri 2107.13871.

positivity condition is the same as in the bootstrap of automorphic spectra

Kravchuk, Mazac, Pal '21 ; Bonifacio '21

Generalizations:

With gravity

- A big question is which of the structure we defined remains once gravity is made dynamical... (if not completely, in which approximation)
  → in inflation
  → in pure dS

- Generalization of our EAdS rotation would lead to a theory with two metrics, one of which has alternate boundary conditions...