

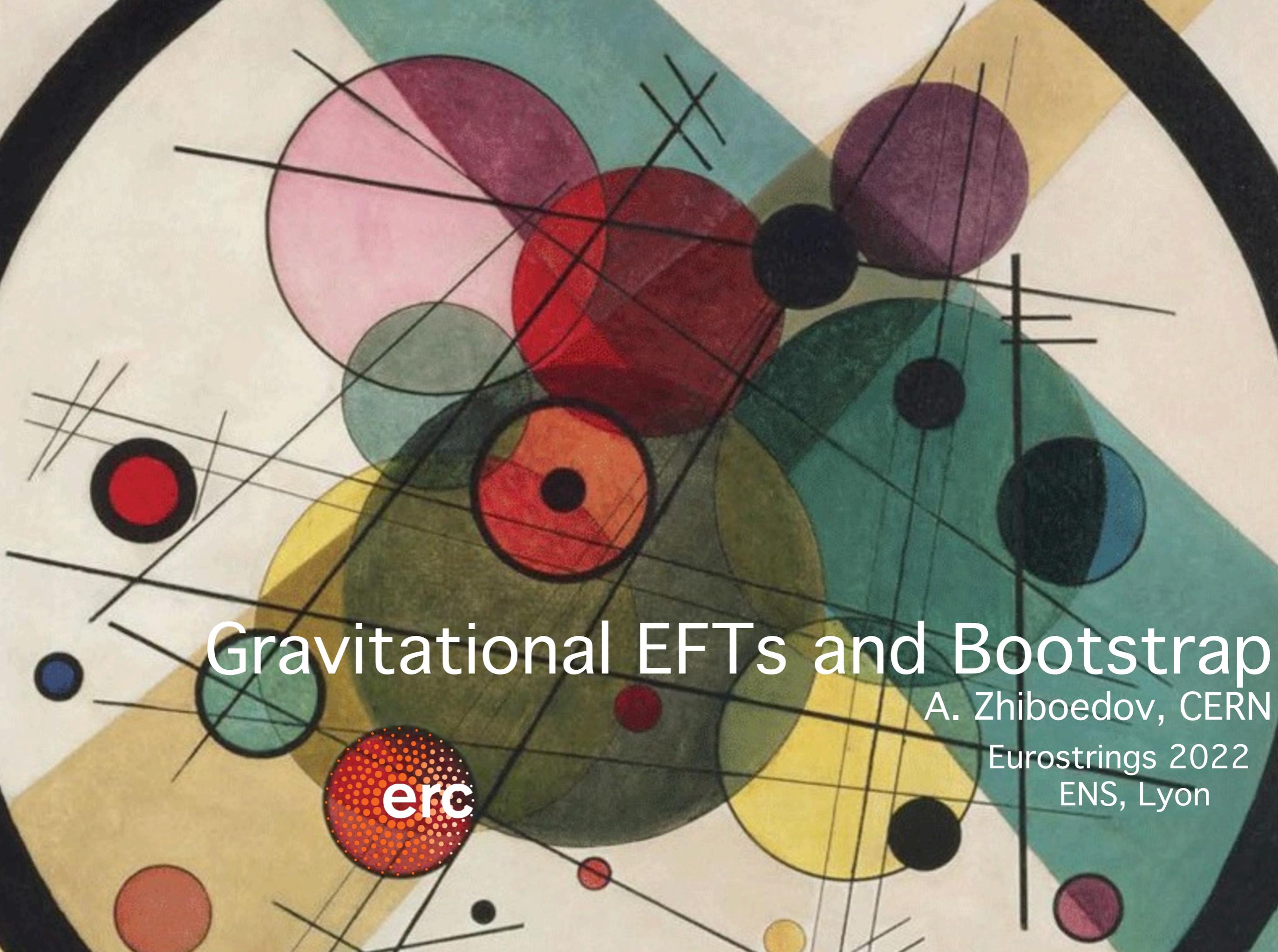
Review talk: EFTs and Bootstrap

A. Zhiboedov, CERN

Eurostrings 2022

ENS, Lyon

erc



Gravitational EFTs and Bootstrap

A. Zhiboedov, CERN

Eurostrings 2022

ENS, Lyon

erc

Modern understanding:

SM and GR are leading terms in the EFT expansion.

Traditional view:

$$\mathcal{L} = \mathcal{L}_{\text{IR}} + \sum_i g_i \frac{\mathcal{O}_i}{M^{\Delta_i - d}}, \quad g_i \sim O(1)$$

- symmetry

M - EFT cut-off

- unitarity (QM)

g_i - Wilson coefficients

The space of possibilities is huge!



- Which EFTs can be UV completed?
- What justifies the traditional view?

Bootstrap assumption:

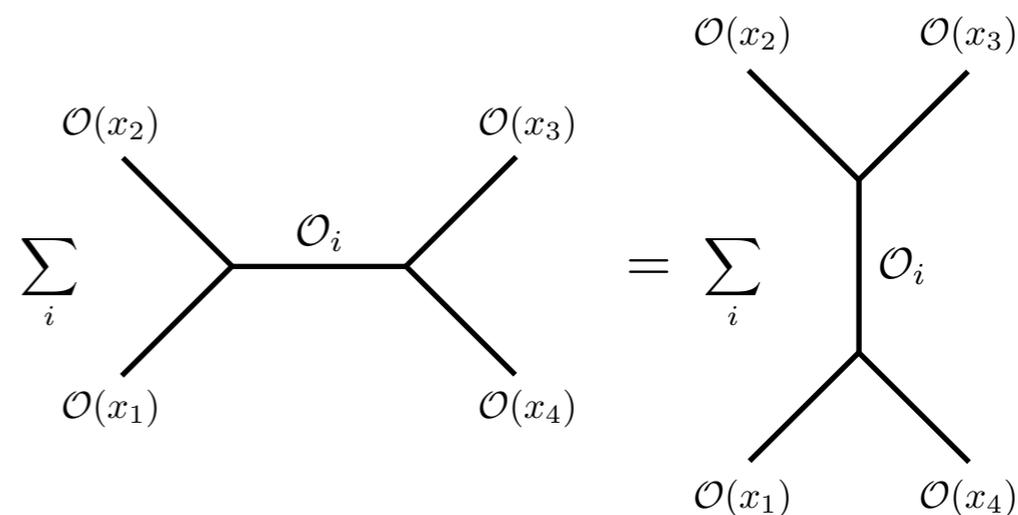
Known fundamental principles hold nonperturbatively at all scales.

1. Symmetry (conformal, Lorentz, ...)
2. Unitarity (quantum mechanics)
3. Causality

There could be new fundamental principles (e.g. swampland conjectures). We will not assume them in this talk.

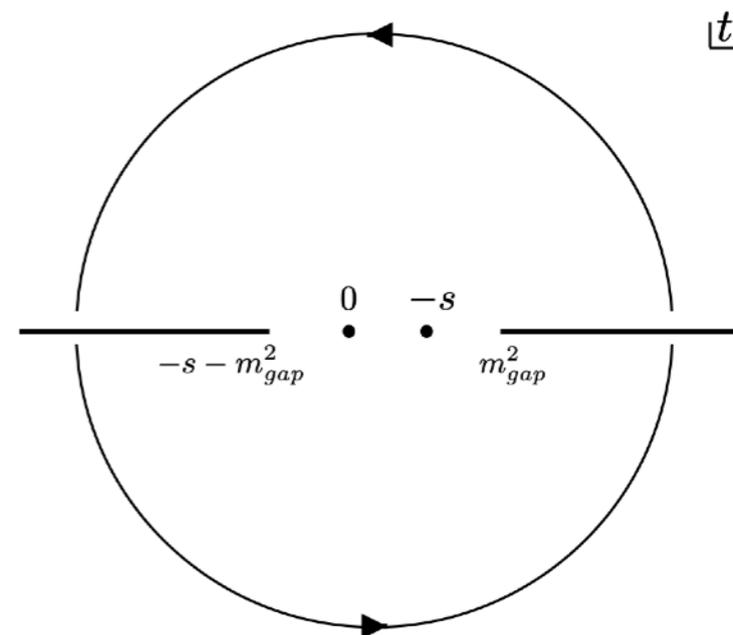
$$\Lambda < 0$$

AdS/CFT
(conformal bootstrap)



$$\Lambda = 0$$

S-matrix theory
(dispersion relations)



$$\Lambda > 0$$

?

$$\Lambda < 0$$

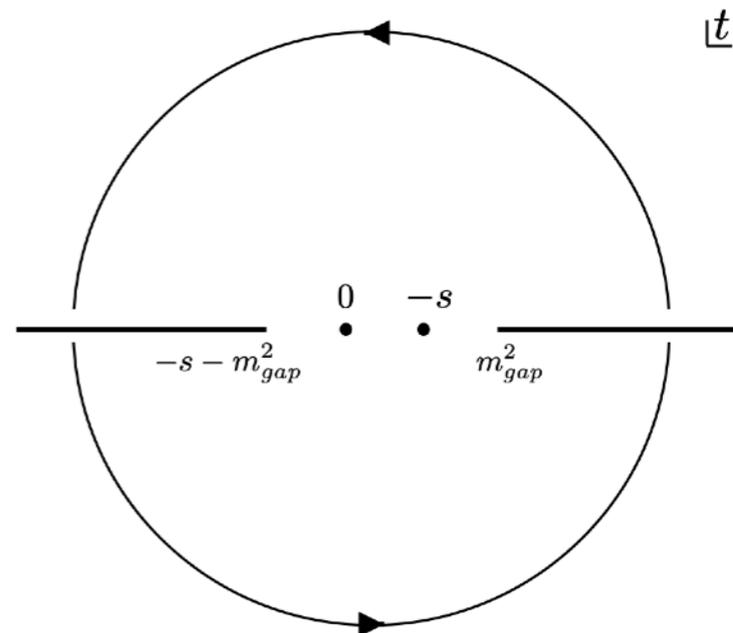
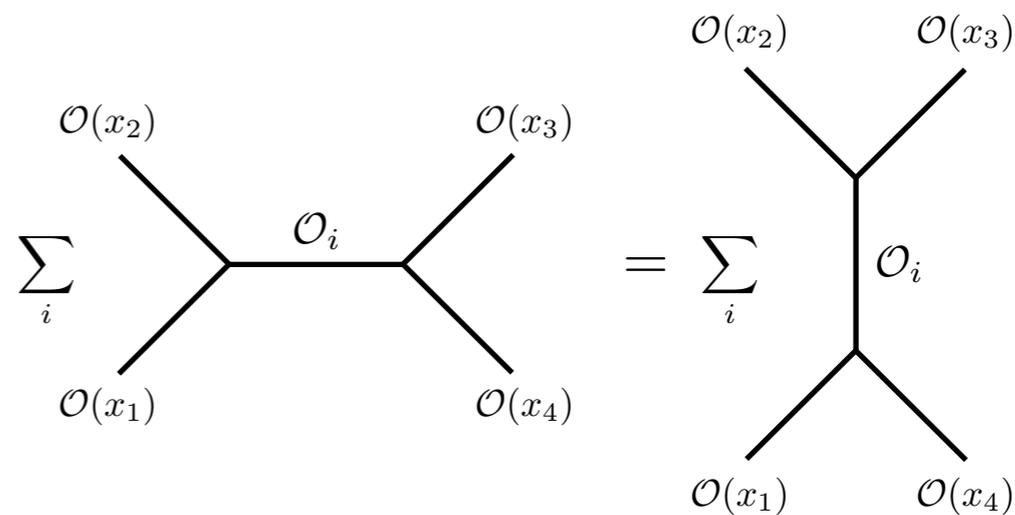
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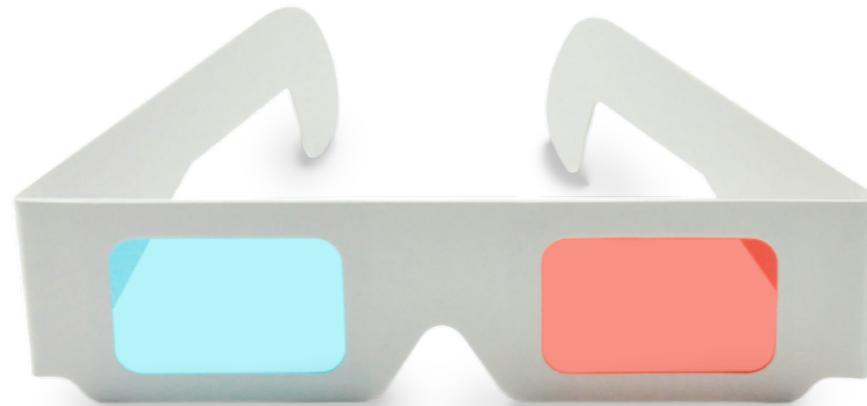
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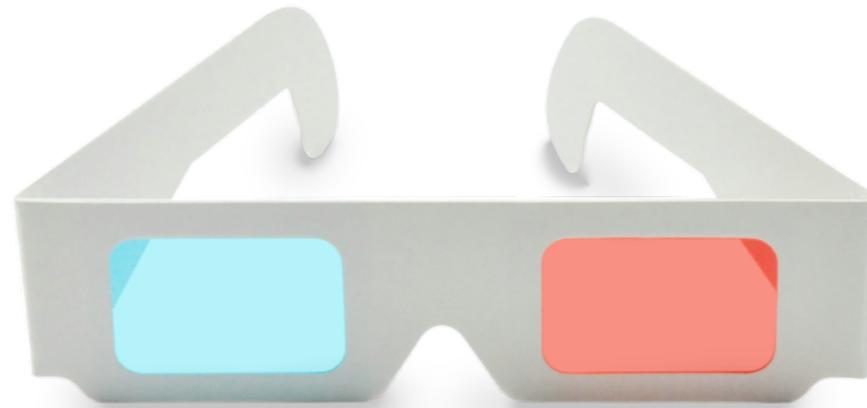
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Please put on your $d > 3$ bootstrap glasses now

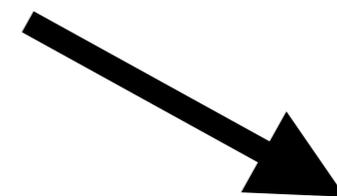
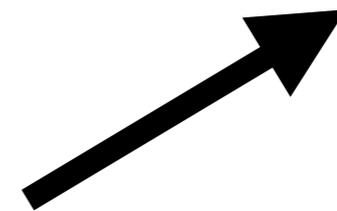
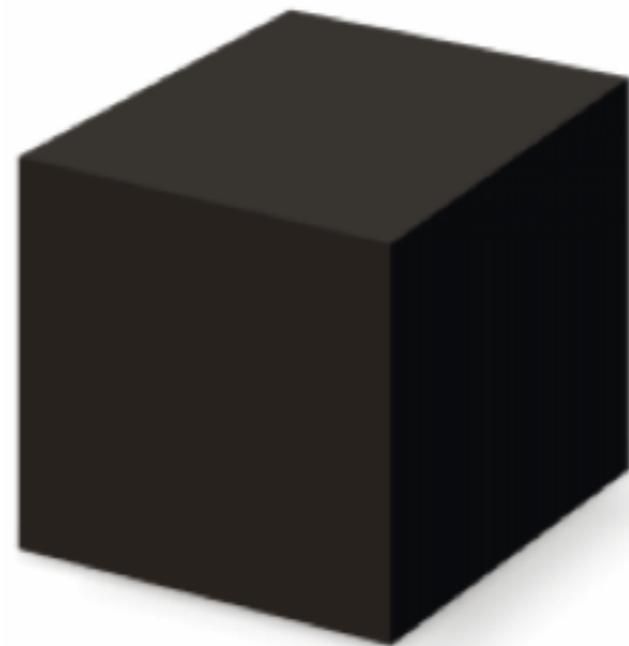
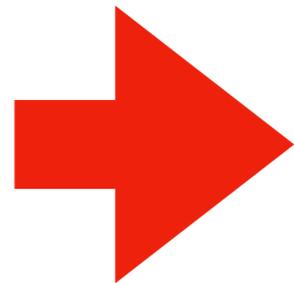


Please put on your $d > 3$ bootstrap glasses now



NO

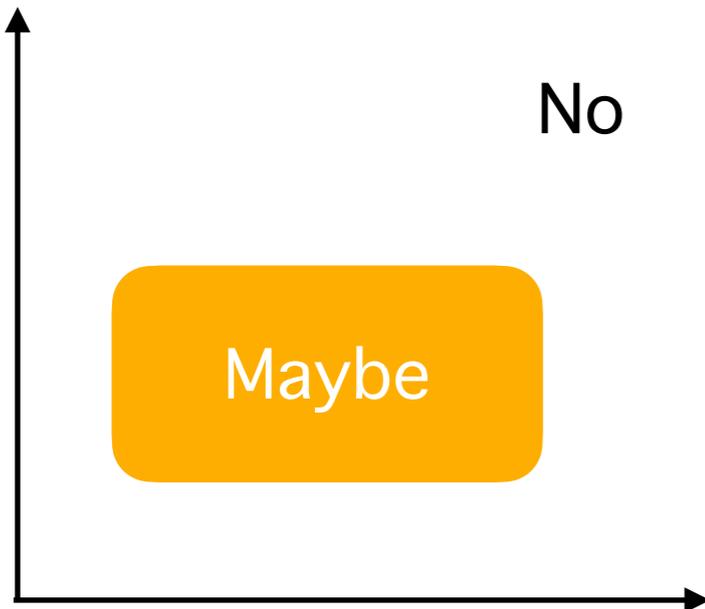
Tentative
EFT



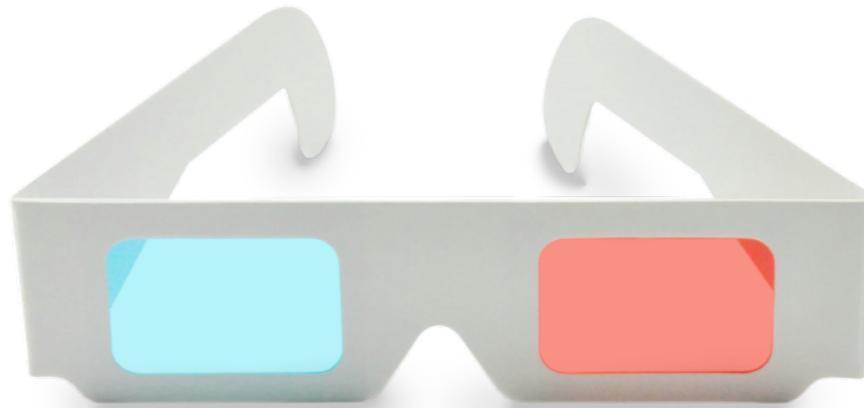
MAYBE

No

Maybe

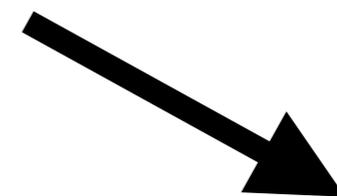
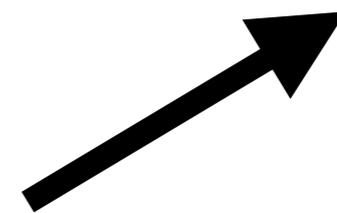
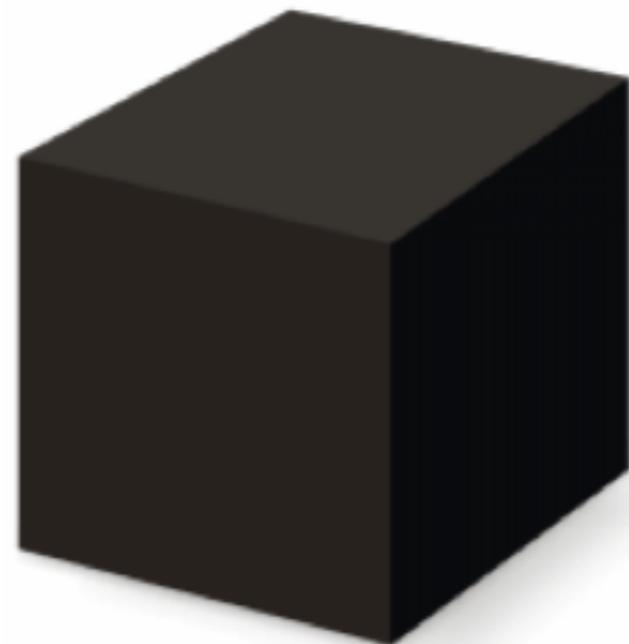
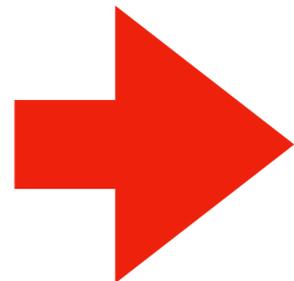


Please put on your $d > 3$ bootstrap glasses now



NO

Tentative
EFT



MAYBE

No

Maybe

“One is never sure to have completely exploited the axioms of QFT.”

A. Martin

[cf. talk by Córdova]

Weak coupling

Gravity is weakly coupled

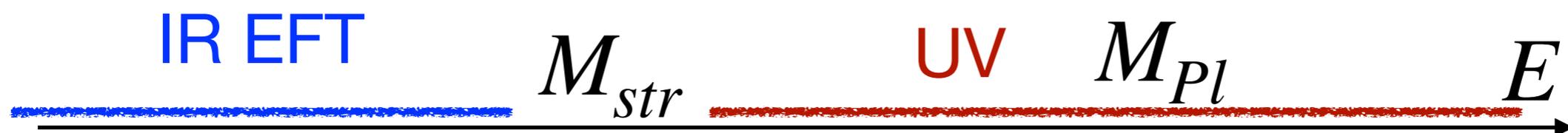
$$M_{\text{Pl}} = 10^{19} \text{ GeV}$$

In this talk I will effectively neglect graviton loops.

Assume that the low-energy theory admits another scale, **the string scale**, such that

$$M_{\text{str}} \ll M_{\text{Pl}}$$

The low-energy EFT works all the way to $E \lesssim M_{\text{str}}$.

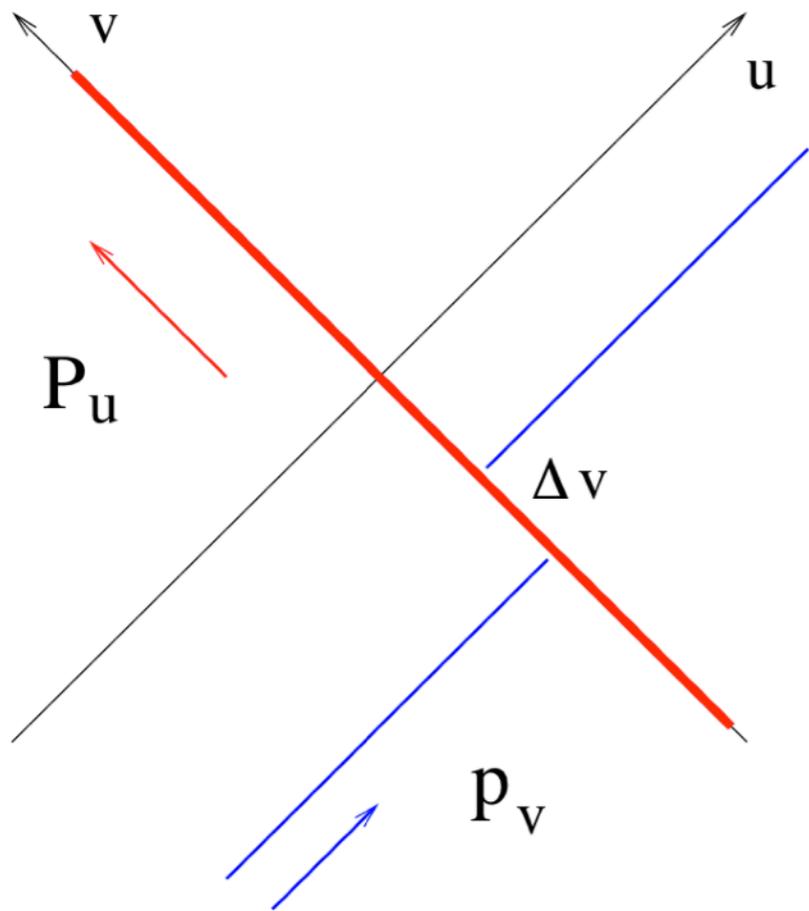


Why should you care about EFT bounds?

Let us try to discover string theory...

Thought experiment

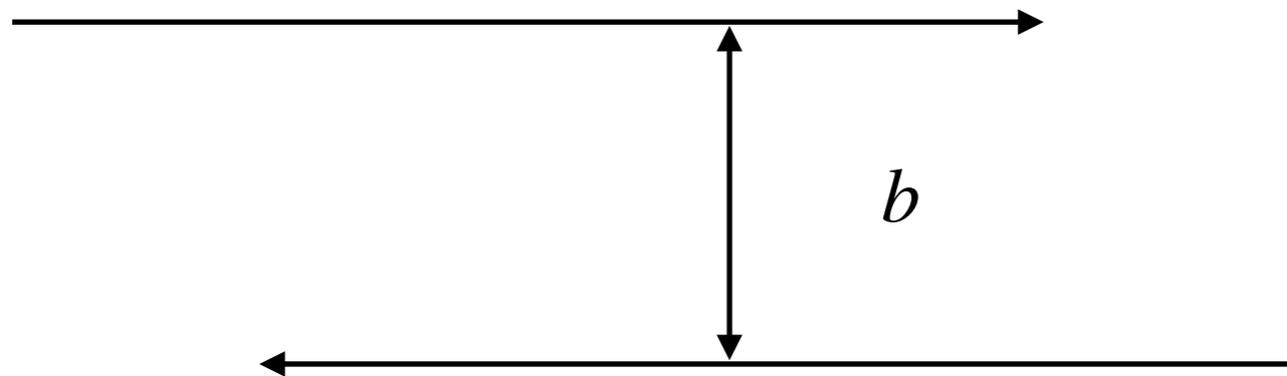
$$S = M_{Pl}^2 \int d^4x \sqrt{-g} (R + \cancel{g_2 R^2} + g_3 R^3 + \dots)$$



By scattering polarized graviton we find:

$$\Delta v = \Delta t_{GR} \left(1 \pm \frac{g_3}{b^4 \log b} \right)$$

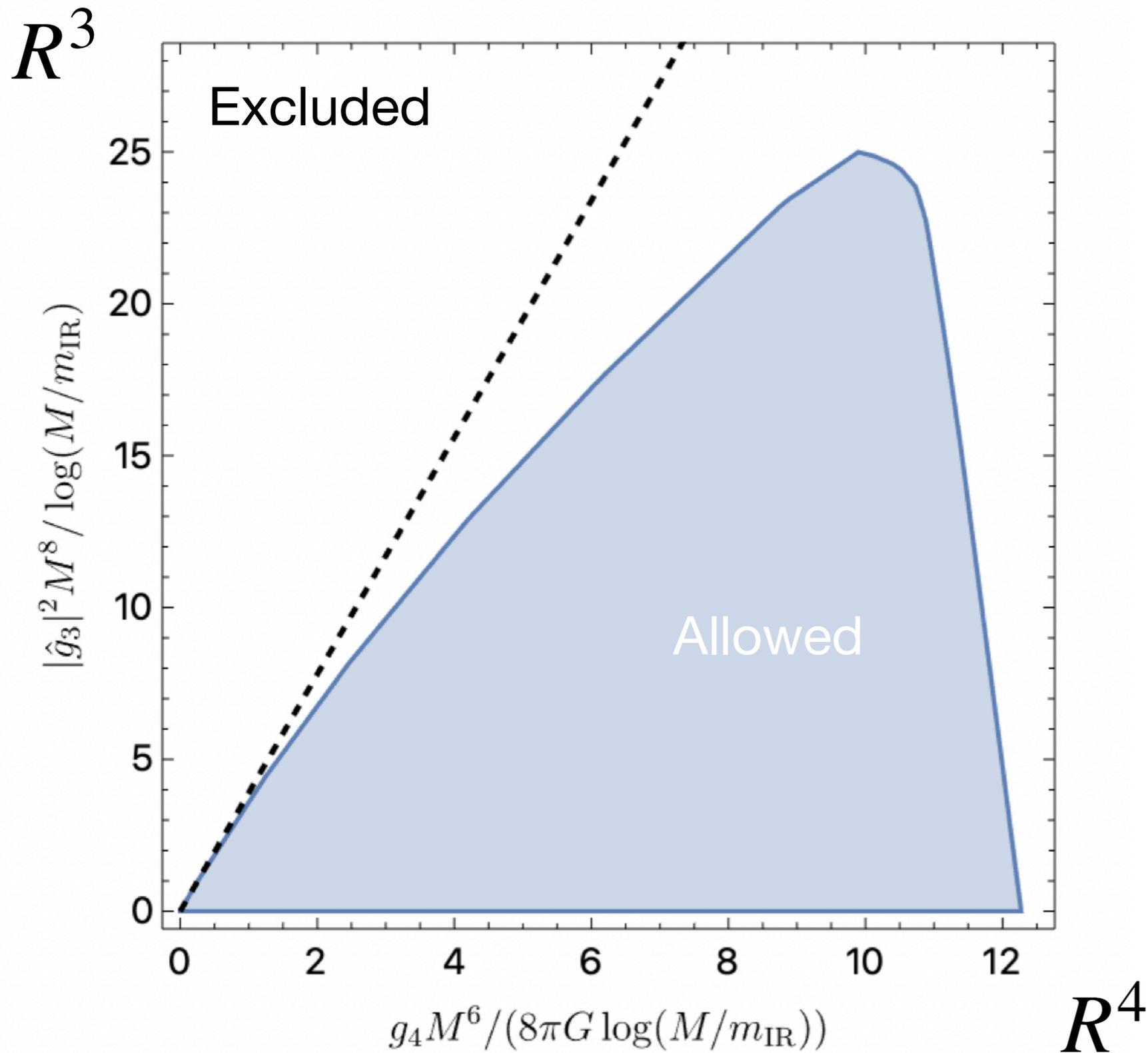
Only higher spin particles can fix the problem!



$$g_3 \sim \frac{1}{M_{str}^4}$$

Sharp bounds!

[Caron-Huot Li Parra-Martinez Simmons-Duffin '22]



AdS₅/CFT₄ :

$$\left| \frac{a - c}{c} \right| \leq \frac{\#}{\Delta_{\text{gap}}^2}$$

Bound on local growth

In the same thought experiment **causality** implies that scattering amplitudes never never never grow faster than s^2

(locally 2-2 ampl.) $T(s, t) \lesssim s^2$

[Camanho Edelman Maldacena AZ '14]
[Maldacena Shenker Stanford '15]
[Häring AZ '22]

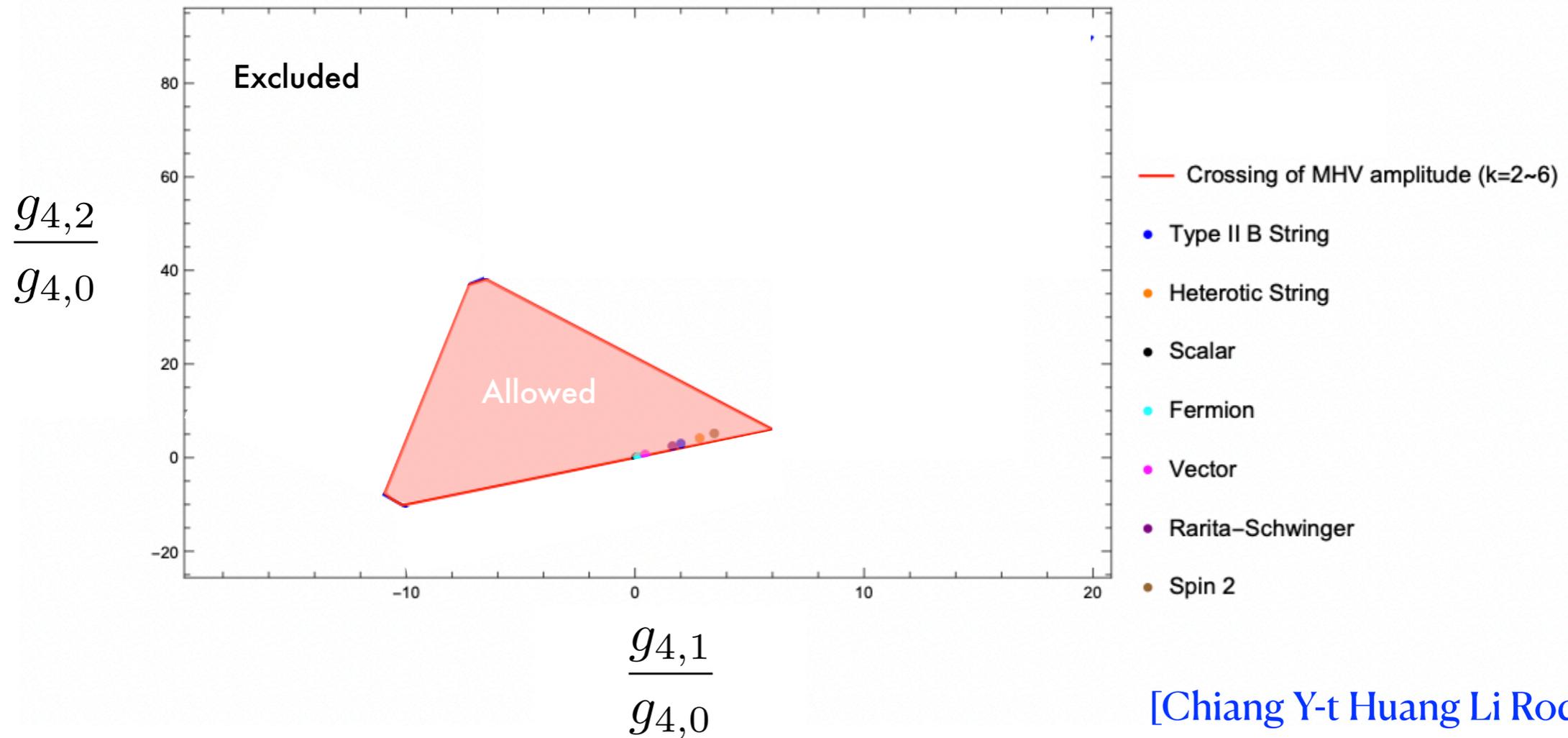
Higher-order Wilson coefficients never dominate

$$T(s, t) \simeq \sum_n g_n s^n$$

they live inside $O(1)$ **EFT**hedron.

[Tolley Wang Zhou '20]
[Caron-Huot van Duong '20]
[Arkani-Hamed T-C Huang Y-t Huang '20]

Which EFTs can be UV completed?



[Chiang Y-t Huang Li Rodina Weng]

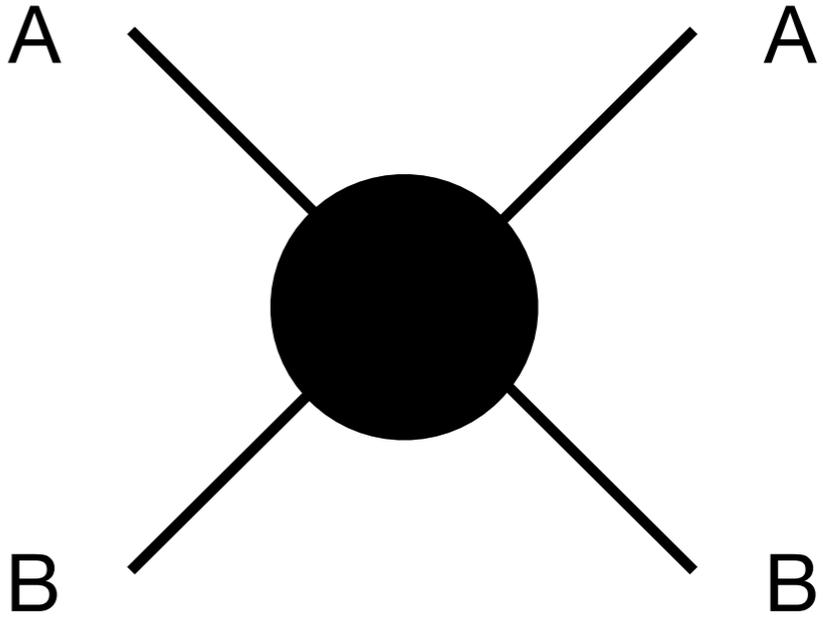
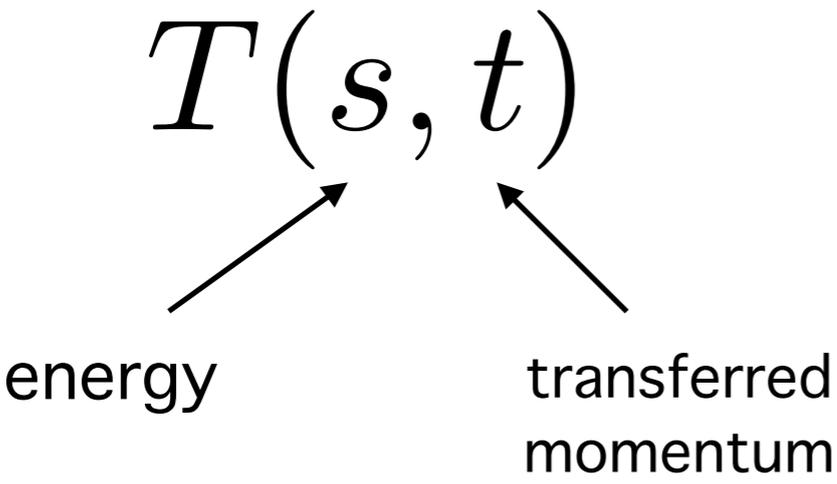
What justifies the traditional EFT view?

Dimensional analysis is a consequence of the basic principles applied at all scales.

[Book material]

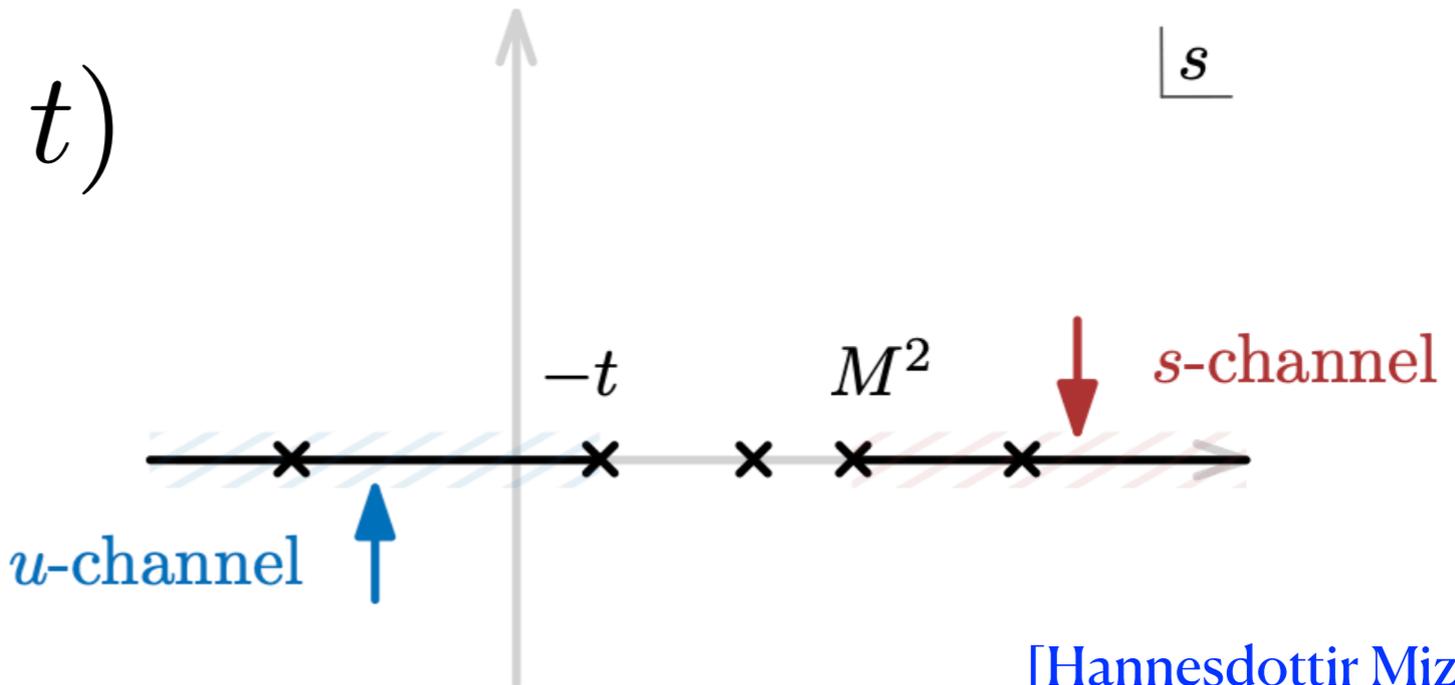
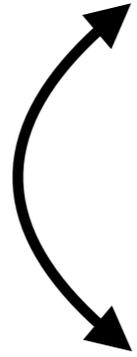
Let's start with some basics

It all started with the discovery of the complex plane



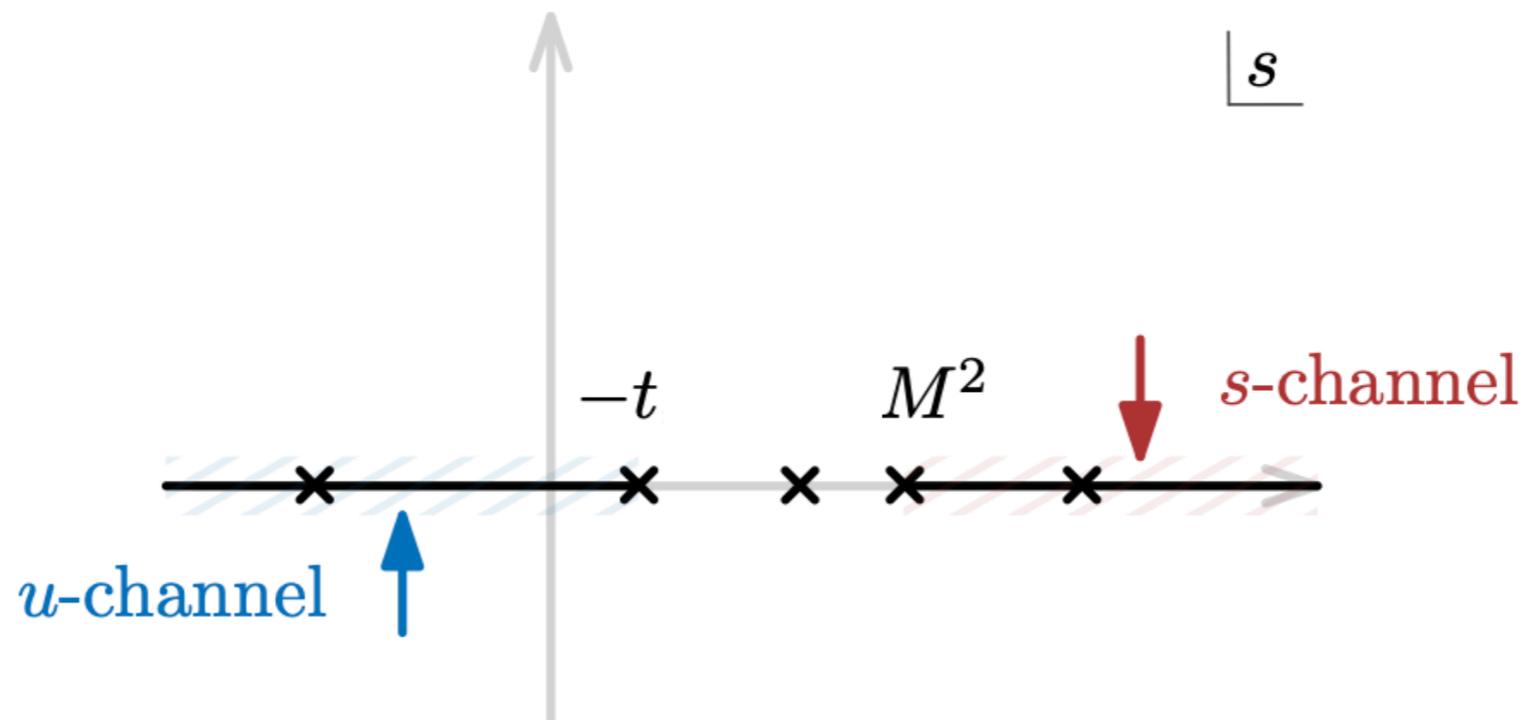
Analyticity (causality):

$$T(s^*, t) = T^*(s, t)$$



[Hannesdottir Mizera '22]

Unitarity ($SS^\dagger = 1$):



PWE:
$$T(s, t) = s^{\frac{4-d}{2}} \sum_{J=0}^{\infty} n_J^{(d)} f_J(s) P_J^{(d)}(\cos \theta)$$

$$2 \geq \text{Im} f_J(s) \geq |f_J(s)|^2 \geq 0$$

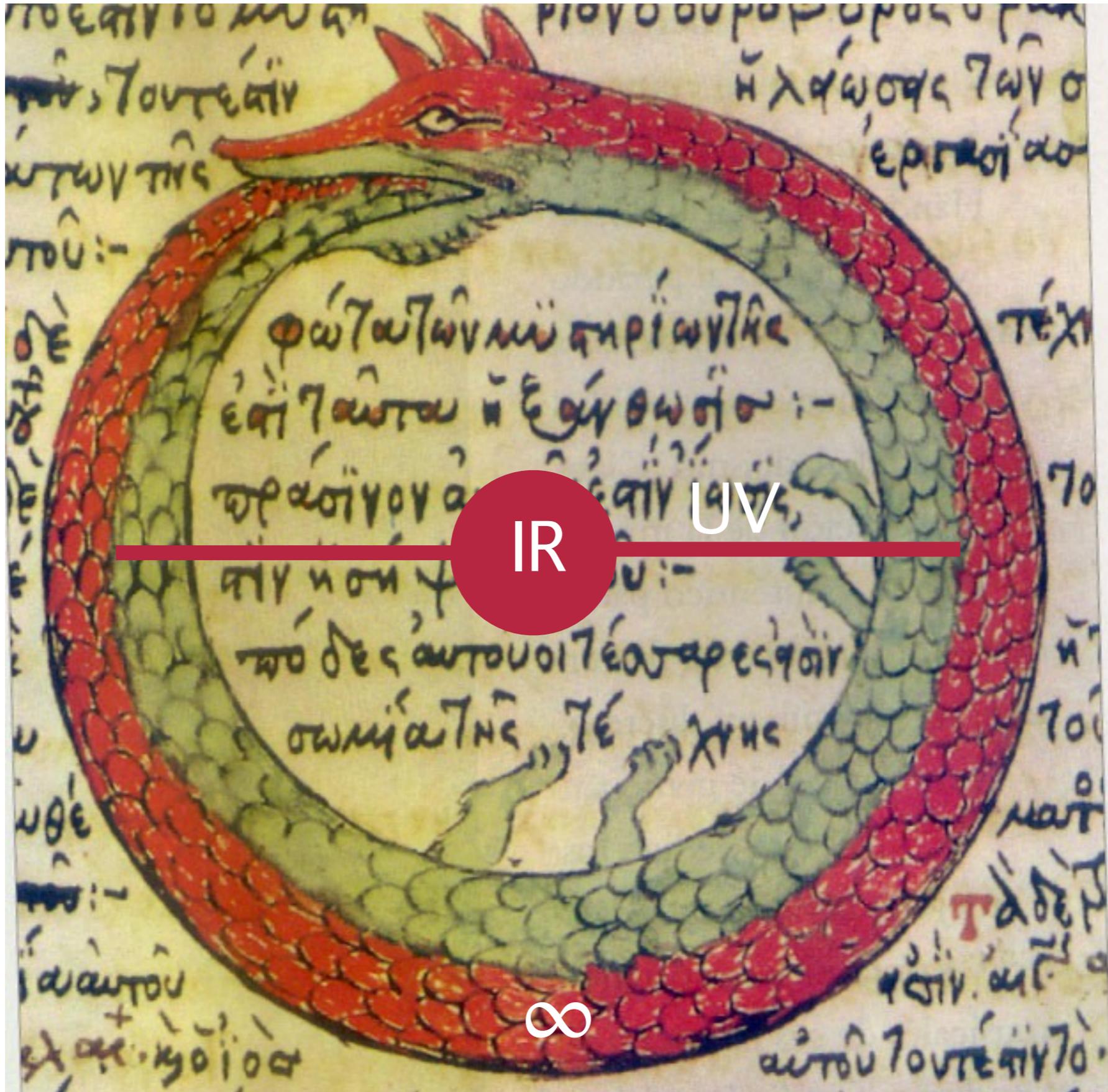
nonperturbative
unitarity

perturbative
unitarity

Crossing:
$$T_{AB \rightarrow AB}(-s - t - i\epsilon, t) = T_{A\bar{B} \rightarrow A\bar{B}}(s + i\epsilon, t)$$

UV and IR are linked via the Regge limit

S



IR

UV

∞

Regge boundedness

The link between the IR and UV is provided by the bound on the Regge limit

$$\lim_{|s| \rightarrow \infty} \frac{T(s, t)}{s^2} = 0, \quad t < 0$$

High energy \neq Short distance

What the Regge bound is **NOT**: murky black hole physics, mysterious quantum gravity, etc.

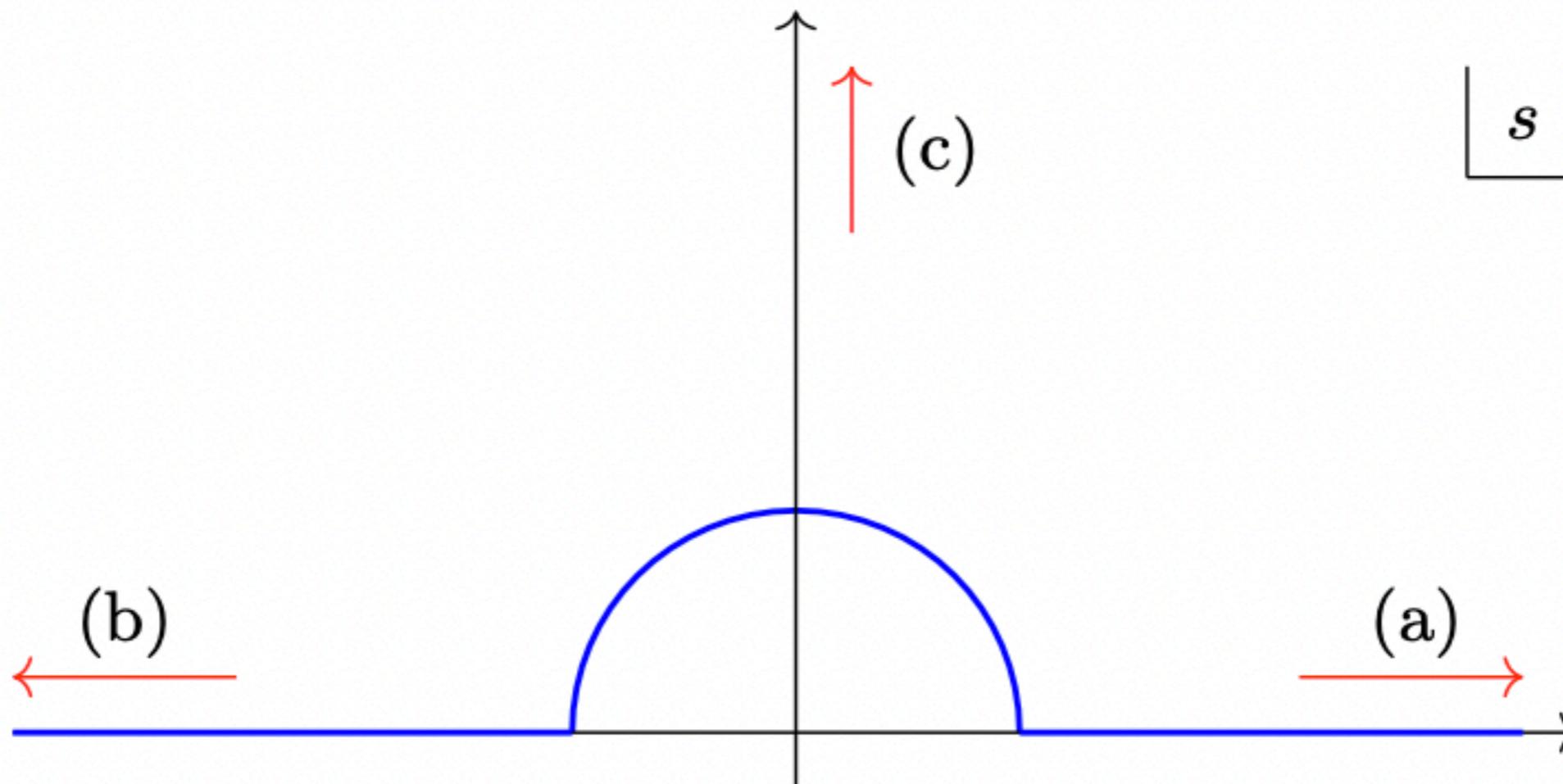
The bound is controlled by simple semi-classical physics!

$$g_{\text{eff}} = G_N s |t|^{\frac{d-4}{2}} \sim \frac{G_N s}{b^{d-4}}$$

Regge boundedness

Subexponentiality
Analyticity

maximum modulus principle
(Pragmen-Landelof)

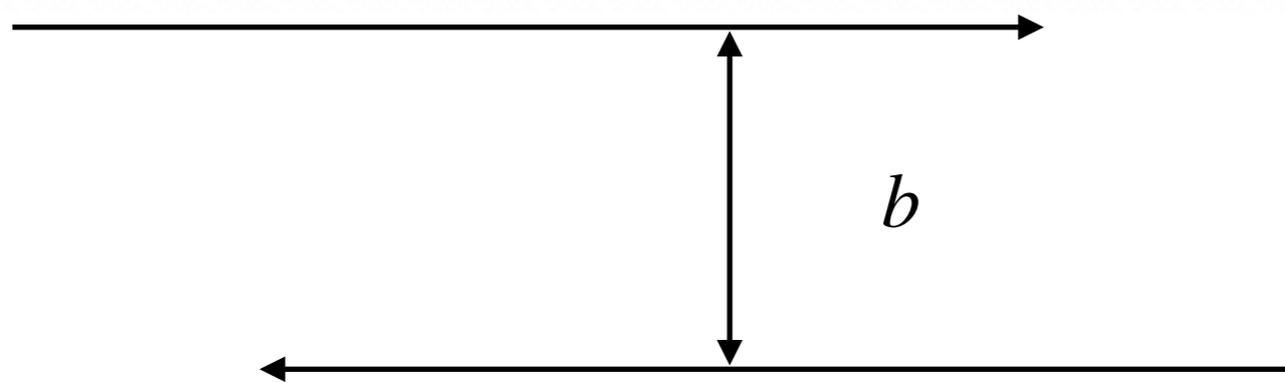
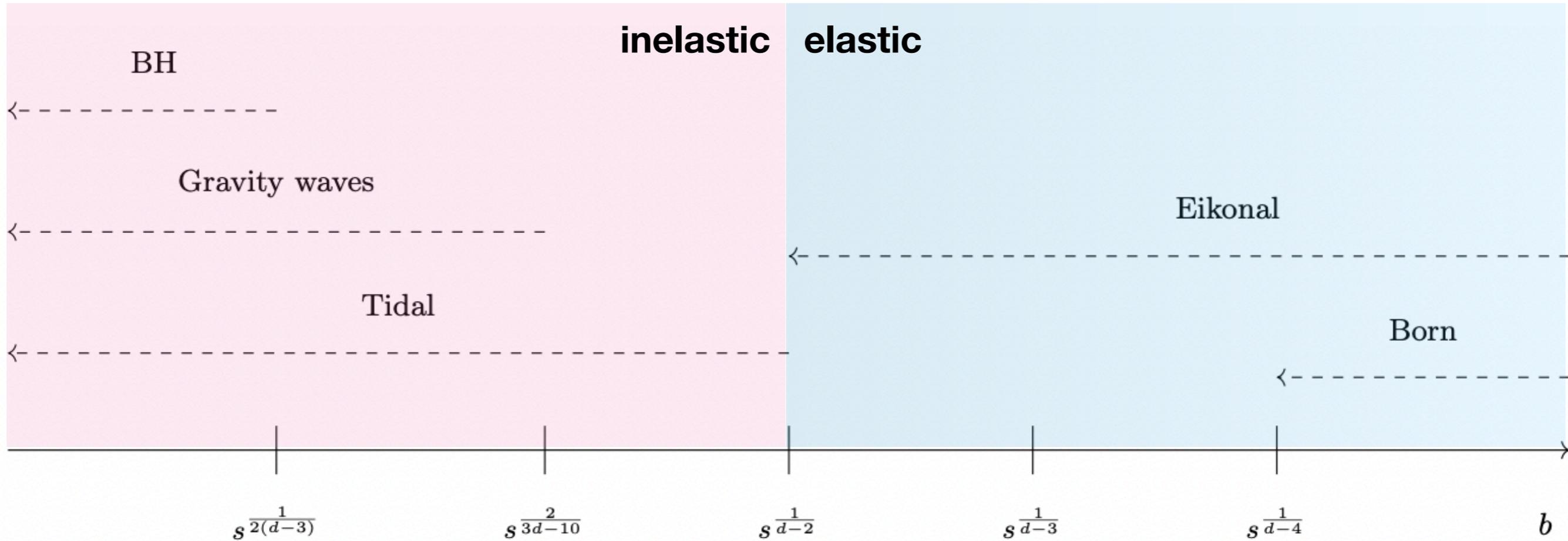


Unitarity+crossing+GR

Unitarity+GR

Impact parameter space

[’t Hooft, Amati Ciafaloni Veneziano, Verlinde Verlinde, Kabat Ortiz, ...]



$$T(s, t) = 2is \int d^{d-2} \vec{b} e^{-i\vec{q}\vec{b}} \left(1 - e^{2i\delta(s, |\vec{b}|)} \right)$$

Eikonal+tidal:

$$T(s, t) \lesssim s^{2 - \frac{d-4}{2(d-3)}}$$

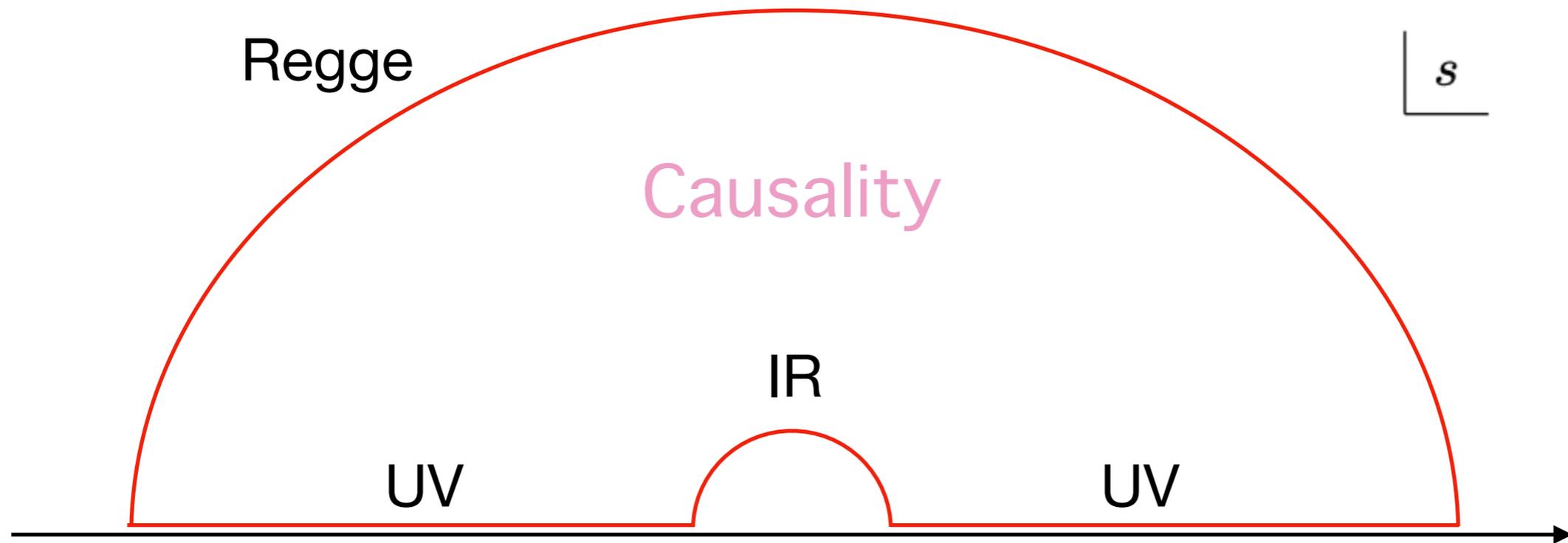
Dispersive sum rules

Dispersive sum rule (aka Cauchy theorem)

$$\text{IR} + \text{UV} =$$



Bound on
Regge

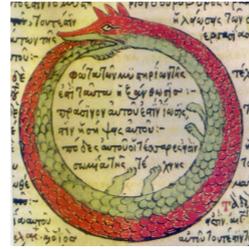


- assumptions in flat space
- rigorous in AdS

$$\oint \frac{ds}{2\pi i} f(s, t) T(s, t) = 0$$

Dispersive sum rules

$$IR + UV =$$



Bound on
Regge

- forward scattering

[Adams Arkani-Hamed Dubovsky Nicolis Rattazzi '06] [Bellazzini de Rham Melville Tolley Zhou ...]

-
- non-forward scattering, crossing

[2019-now]

[Arkani-Hamed T-C Huang Y-t Huang][Tolley Wang Zhou] [Caron-Huot van Duong] [Sinha Zahed '20]

- graviton pole [Caron-Huot Mazáč Rastelli Simmons-Duffin '21]

- spinning particles

[Arkani-Hamed T-C Huang Y-t Huang] [Bern, Kosmopolous, AZ] [Henriksson McPeak Russo Vichi]

[Chiang Y-t Huang Li Rodina Weng] [Caron-Huot Li Parra-Martinez Simmons-Duffin '22]

- AdS

[Carmi Caron-Huot] [Mazáč, Mazáč Paulos, Mazáč Rastelli Zhou] [Penedones Silva AZ '19]

[Caron-Huot Mazáč Rastelli Simmons-Duffin '21]

- quantum corrections

[Bellazzini Mirò Rattazzi Riembau Riva '20][Bellazzini Riembau Riva '21]

- nonperturbative unitarity

[Guerrieri Penedones Vieira, Paulos van Rees] [Chiang Y-t Huang Li Rodina Weng '22]



Let's get some bounds

UV casts a shadow on the IR through consistency



EFThedron

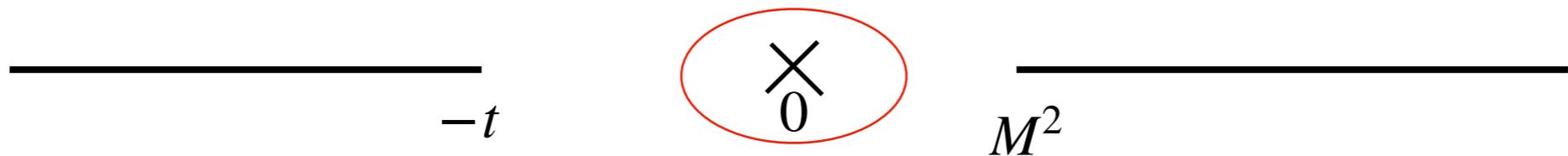
Wilson coefficients

Consider next the following low-energy parameterisation

$$T_{\phi\phi\rightarrow\phi\phi}^{\text{IR}}(s, t) = \sum_{k, q} g_{k, q} s^{k-q} t^q$$

$$g_{k, q} = \frac{1}{2\pi i} \frac{\partial_t^q}{q!} \oint \frac{ds}{s^{k+1}} T_{\phi\phi\rightarrow\phi\phi}(s, t) \Big|_{t=0}$$

⌊S

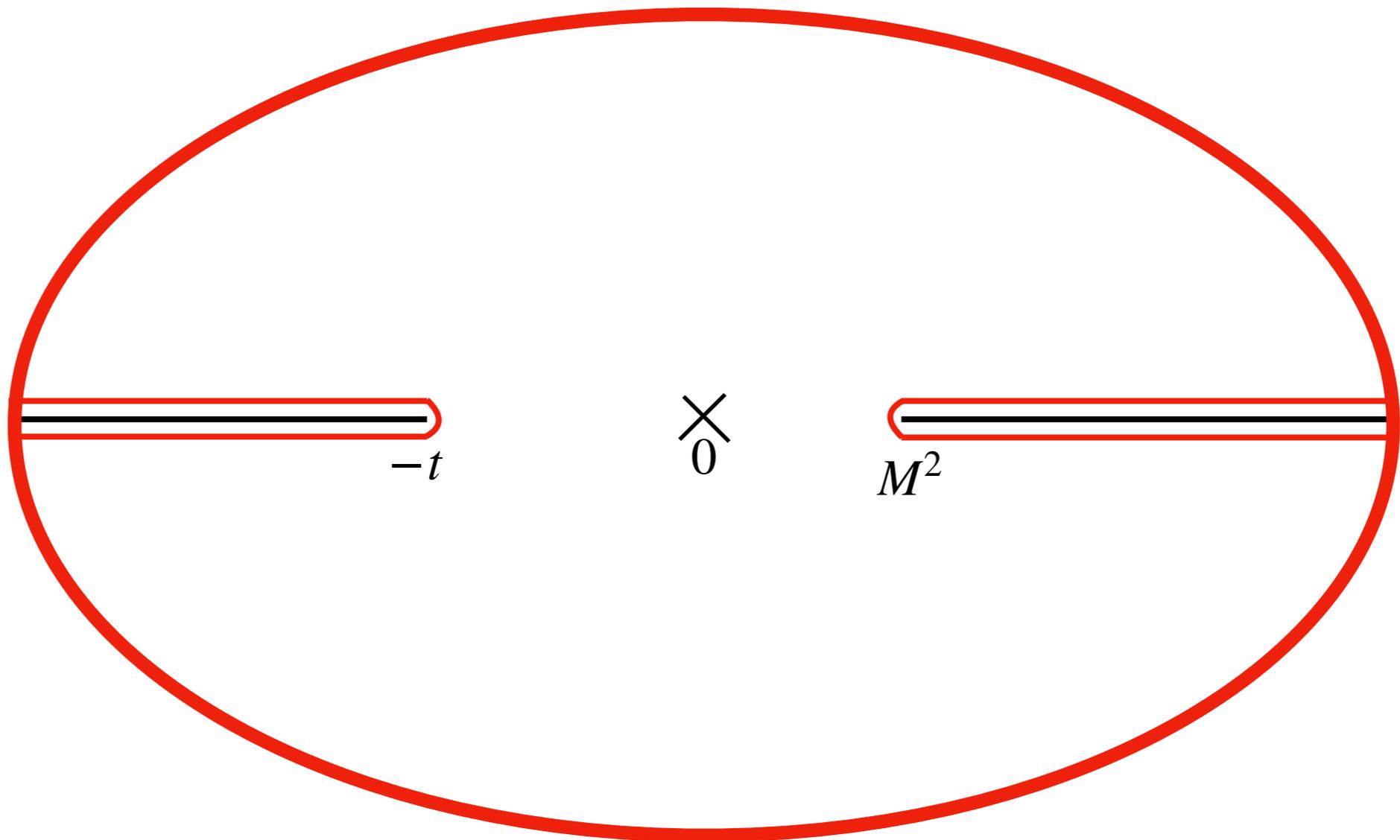


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$\lfloor S$

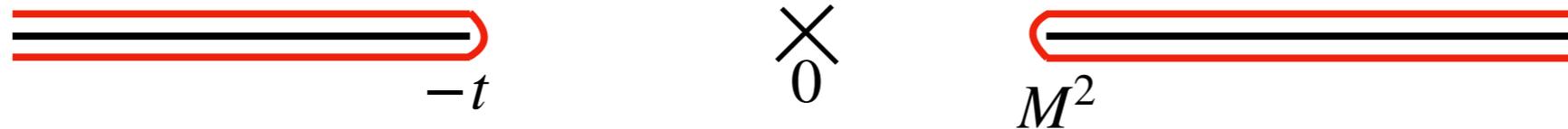
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⌊S



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By deforming the contour and dropping the arc at infinity we get

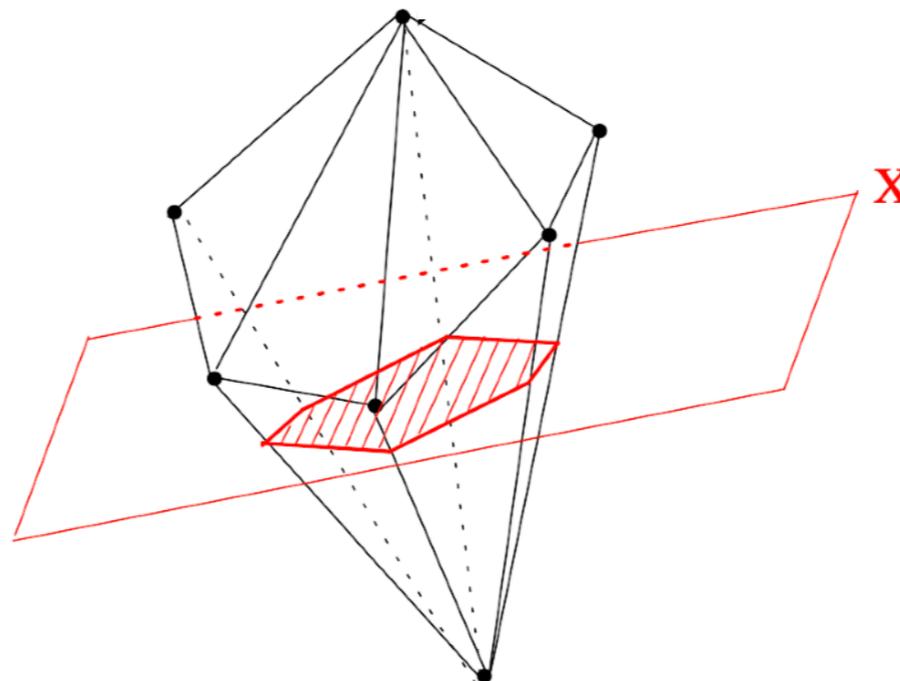
$$g_{k, q} = \int_{M^2} \frac{dm^2}{\pi} \frac{1}{(m^2)^{k+1}} \left(\sum_{J=0}^{\infty} \frac{1 + (-1)^J}{2} \rho_J^{++}(m^2) P_{++}^q(\mathcal{J}^2) + \sum_{J=0}^{\infty} \rho_J^{+-}(m^2) P_{+-}^{k, q}(\mathcal{J}^2) \right)$$

dispersive coupling

$$\mathcal{J}^2 = J(J + d - 3)$$

Unitarity:

$$2 \geq \rho_J(m^2) \geq 0$$



convex hull

EFT-hedron

Performing a linear transformation on the space of coefficients we can get to the following basis

2d moment problem:
$$a_{k,q} = \sum_{J=0}^{\infty} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) \frac{1}{m^{2k+2}} \mathcal{J}^{2q}$$

(energy, spin)

Moment matrix:

$$\forall v \quad v^T M(a) v \geq 0 : \quad M(a) = \begin{pmatrix} a_{0,0} & a_{1,0} & a_{0,1} & a_{2,0} & & \\ a_{1,0} & a_{2,0} & a_{1,1} & a_{3,0} & & \\ a_{0,1} & a_{1,1} & a_{0,2} & a_{2,1} & \cdots & \\ a_{2,0} & a_{3,0} & a_{2,1} & a_{4,0} & & \\ & & \vdots & & \ddots & \end{pmatrix} \succcurlyeq 0$$

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Crossing symmetry is an extra linear constraint among a's.

Example: 1d moment problem

$$H_2 = \begin{pmatrix} a_0 & a_1 \\ a_1 & a_2 \end{pmatrix} \geq 0 \quad \Rightarrow \quad a_0 \geq 0 \text{ and } a_0 a_2 - a_1^2 \geq 0$$

The 1d truncated moment problem

$$H_3 = \begin{pmatrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & \mathbf{a_3} \\ a_2 & \mathbf{a_3} & \mathbf{a_4} \end{pmatrix} \geq 0 \quad \forall a_3, a_4$$

[Bellazzini Mirò Rattazzi Riembau Riva '20]

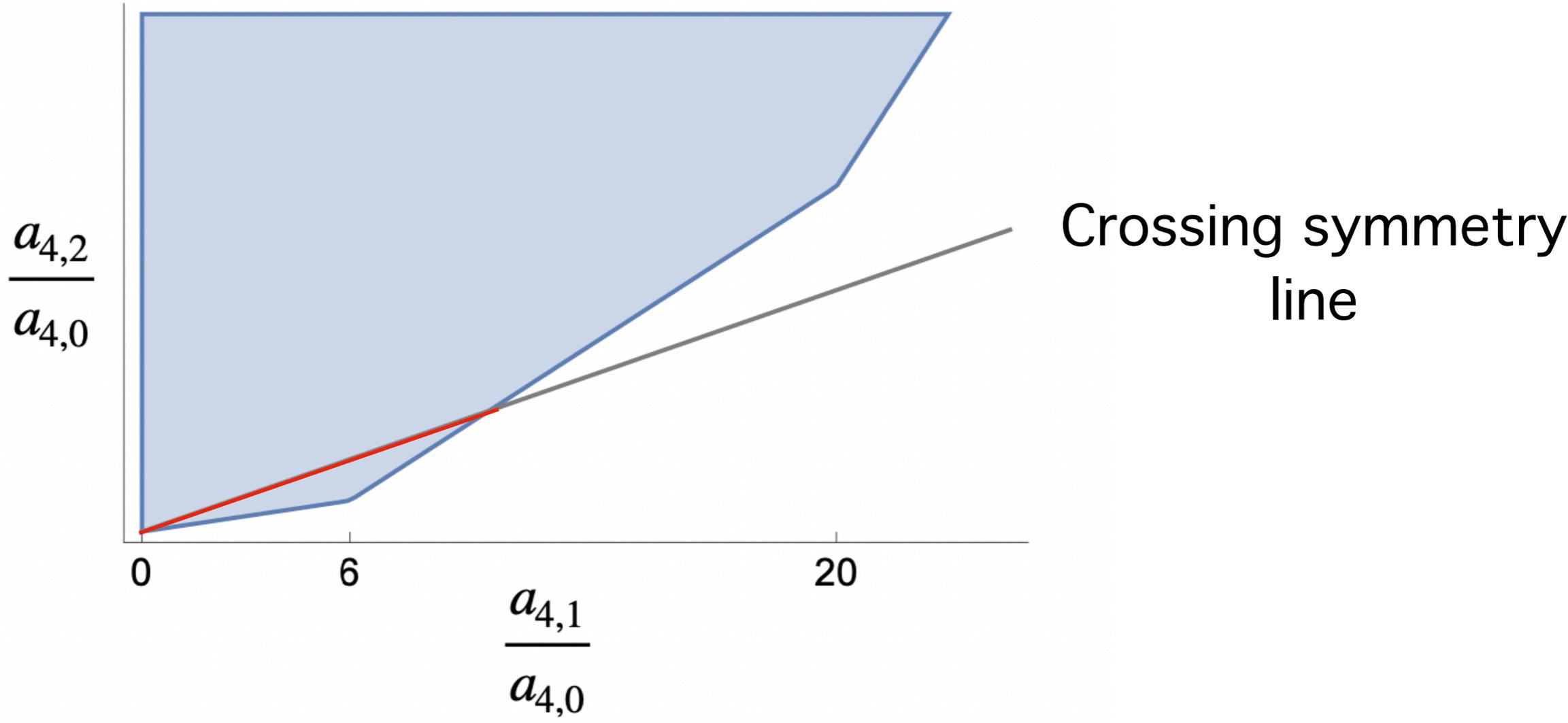
For the 2d truncated moment problem the general solution is not known. Experimentally, bounds converge quickly.

Here, we only used positivity $\rho_J(m^2) \geq 0$. Generalizes to

$$2 \geq \rho_J(m^2) \geq 0$$

[Chiang Y-t Huang Li Rodina Weng '22]

Example: extra crossing



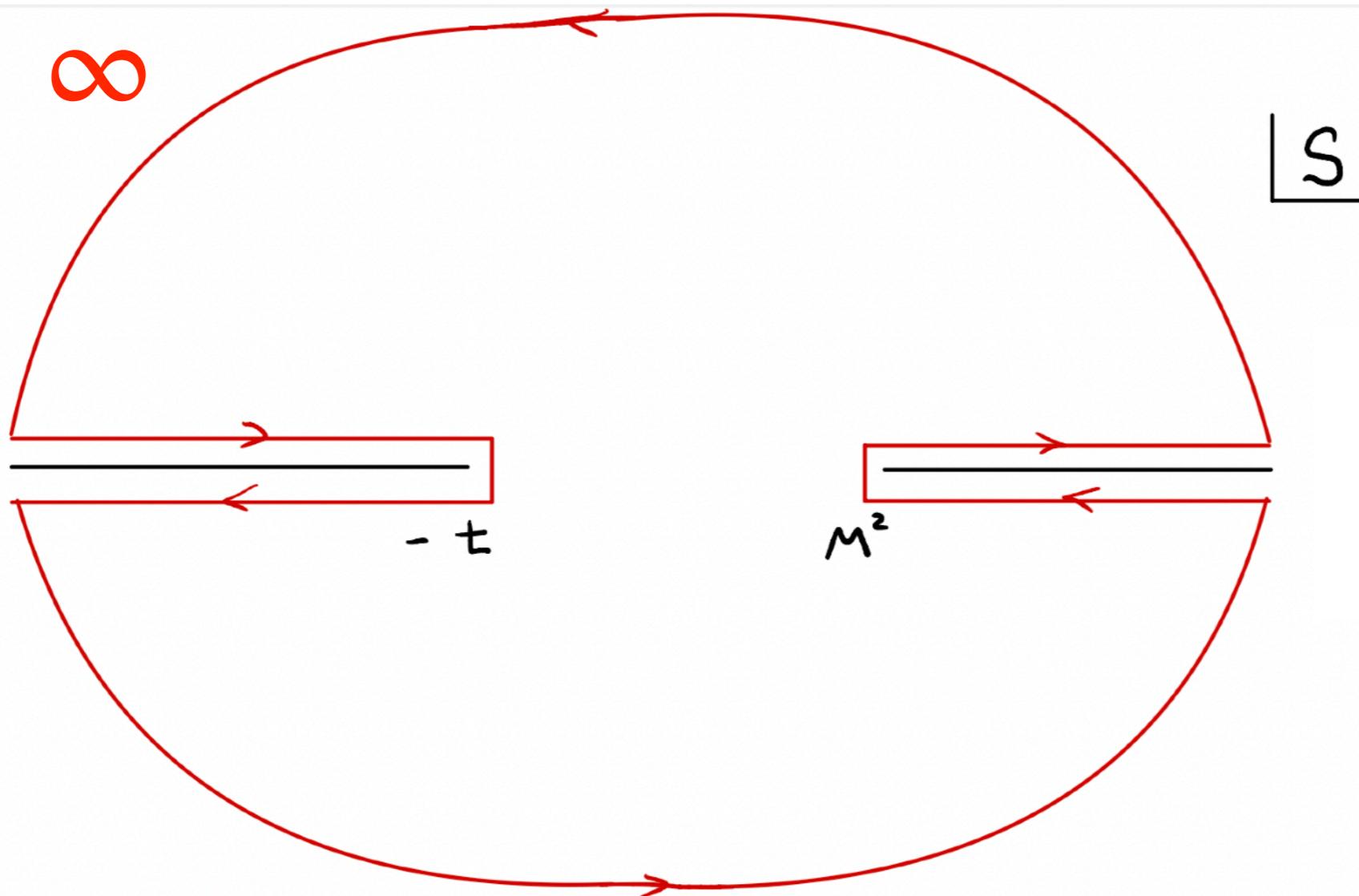
Crossing leads to two-sided bounds!

Dispersive sum rules (identical scalars, no gravity)

[Caron-Huot van Duong '20]

Consider instead the following set of sum rules

$$B_k(t) \equiv \oint_{\infty} \frac{ds}{2\pi i} \frac{1}{s} \frac{T(s, t)}{[s(s+t)]^{k/2}} = 0, \quad (t < 0, k = 2, 4, 6, \dots)$$

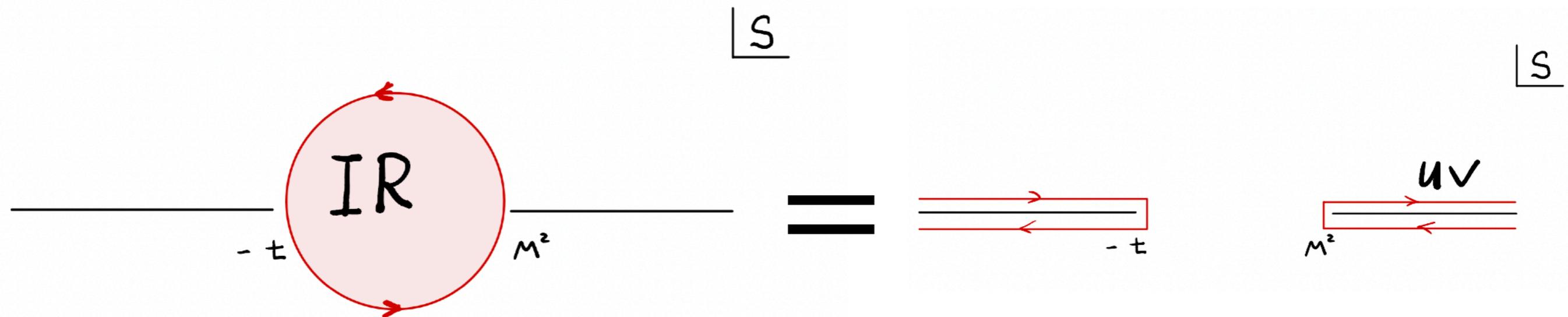


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IR


UV

Dispersive sum rules (identical scalars, no gravity)

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$$B_2 : \quad 2g_2 - g_3t + 8g_4t^2 + \dots = \sum_{J \text{ even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) F_2(J, m^2, t)$$

$$B_4 : \quad \underbrace{4g_4 + \dots}_{\text{IR}} = \sum_{J \text{ even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) \underbrace{F_4(J, m^2, t)}_{\text{UV}}$$

Dispersive sum rules (identical scalars, no gravity)

[Caron-Huot van Duong '20]

Consider instead the following set of sum rules

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Null constraint:

$$\partial_t^2 B_2(t) - 4B_4(t) \Big|_{t=0} : \quad \sum_{J \text{ even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) \frac{\mathcal{J}^2 (2\mathcal{J}^2 - (5d - 4))}{m^8} = 0$$

Linear optimization problem

The EFT space can be then carved out as follows

$$v(m^2, J) \equiv (g_2(m^2, J), M^2 g_3(m^2, J), n_4(m^2, J), n_5(m^2, J), \dots).$$

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Task

maximize: A

subject to: $0 \leq (-A, 1, c_4, c_5, \dots) \cdot v(m^2, J) \quad \forall m \geq M, \forall J = 0, 2, 4, \dots$

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$$0 \leq -Ag_2 + M^2 g_3 + 0$$

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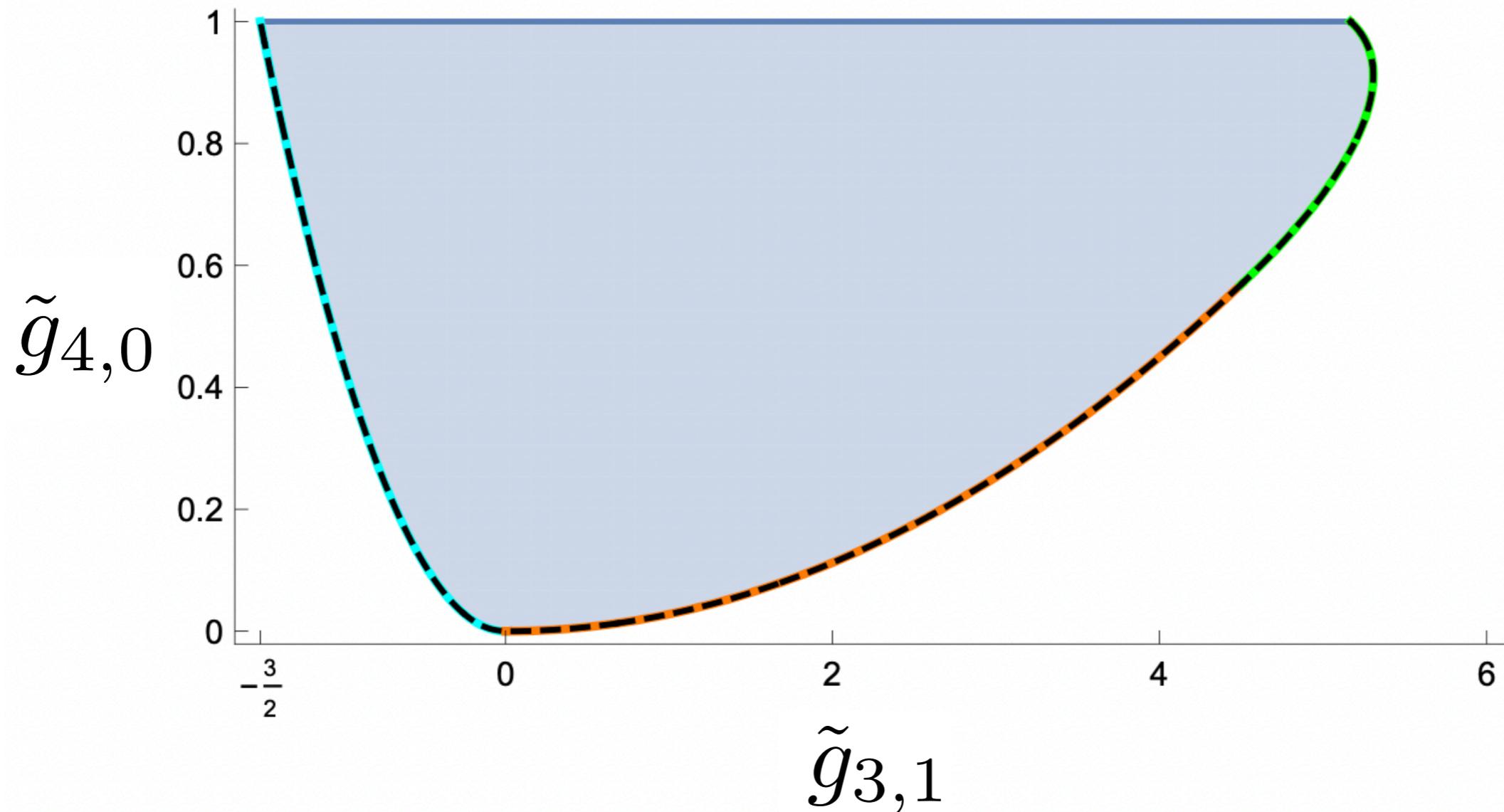
$$\tilde{g}_n = \frac{g_n M^{2(n-2)}}{g_2}$$

One gets two-sided bounds

$$A \leq \tilde{g}_3 \leq B$$

Two-sided bounds

$$\tilde{g}_{n,q} = \frac{g_{n,q} M^{2(n-2)}}{g_2}$$



[Tolley Wang Zhou]

[Caron-Huot van Duong]

[Chiang Y-t Huang Li Rodina Weng]

Further developments

Treatment of the graviton pole

[Caron-Huot Mazáč Rastelli Simmons-Duffin '21]

Certain sum rules cannot be expanded around the origin $t=0$

$$B_2 : \quad -\frac{8\pi G_N}{t} + 2g_2 - g_3 t + 8g_4 t^2 + \dots = \sum_{J \text{ even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) F_2(J, m^2, t)$$

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The resolution is simply to consider smeared amplitudes

$$\int_0^M dq \psi(q) T(s, -q^2) \quad \psi(q) \text{ polynomial}$$

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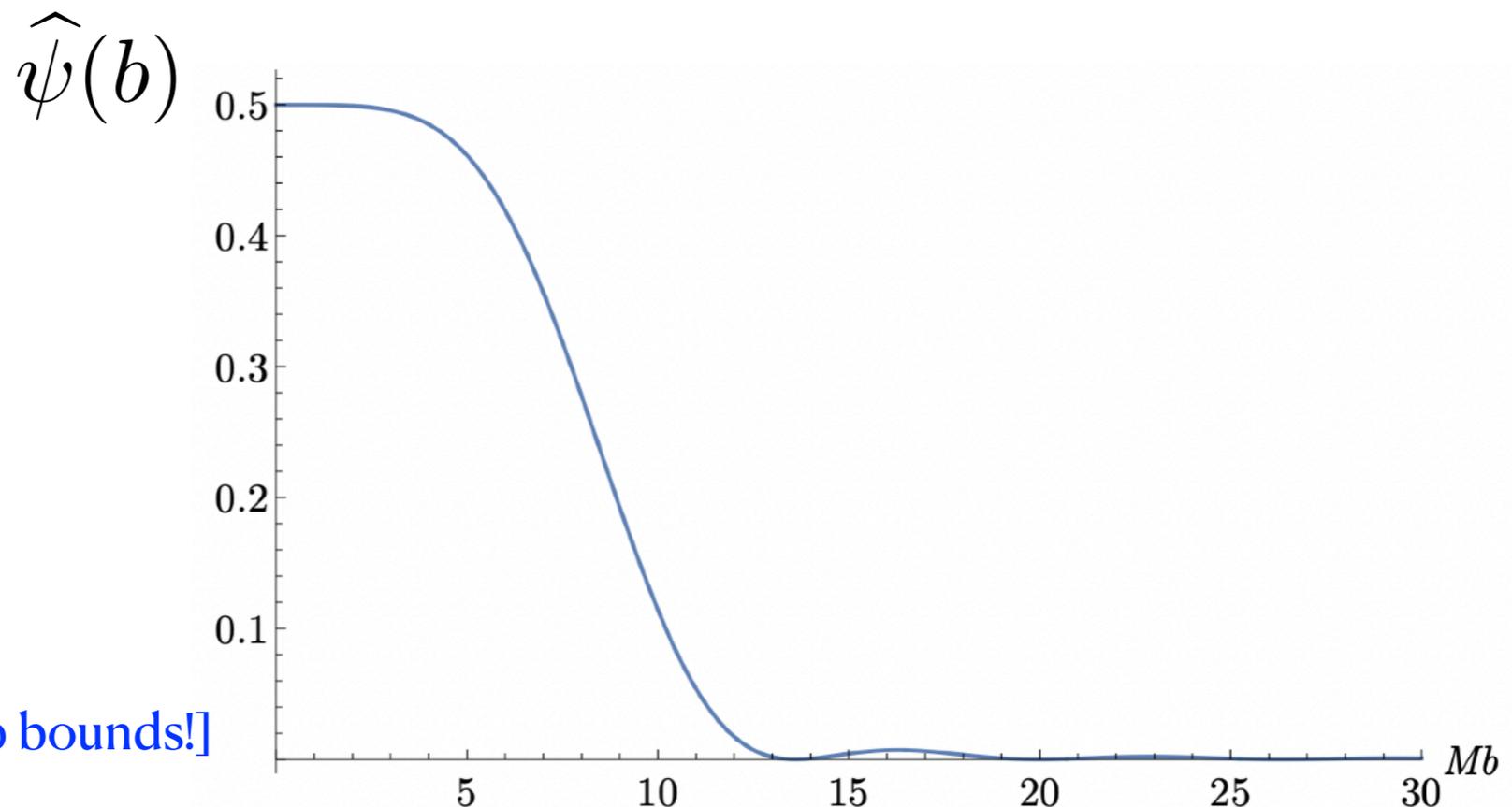
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Physically

$$b \lesssim \frac{1}{M}$$



[Thought experiment with sharp bounds!]

CRG conjecture

[Chowdhury Gadde Gopalka Halder Janagal Minwalla '19]

[Chandokar Chowdhury Kundu Minwalla '21]

[Häring AZ '22]

(locally)

$$T(s, t) \lesssim s^2, \quad t \leq 0$$

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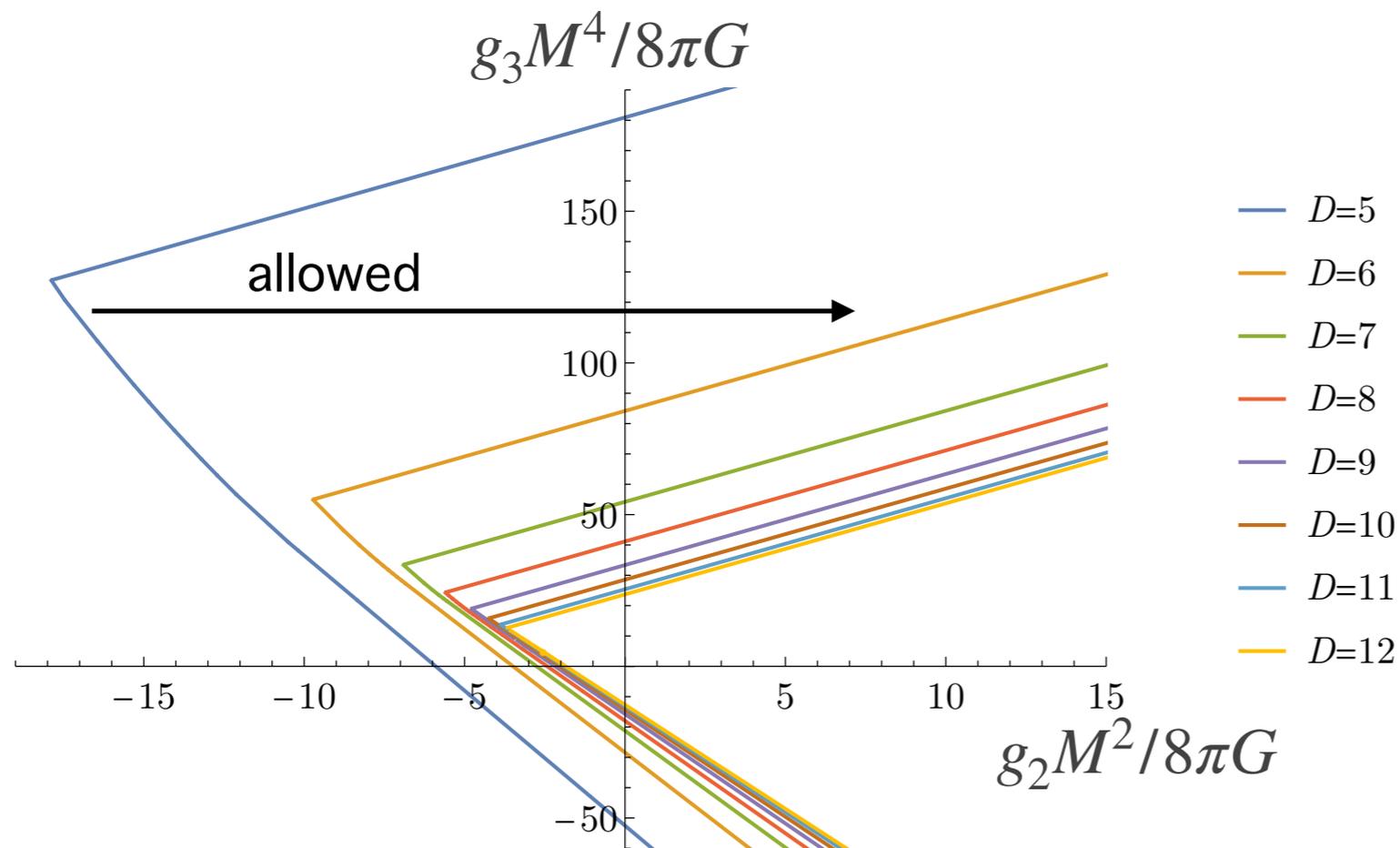
[Chandokar Chowdhury Kundu Minwalla '21]

[Häring AZ '22]

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EFT bounds with gravity



$$g_2 (\partial\phi)^4$$

$$\text{QFT} : \quad g_2 \geq 0$$

[Adams et al '06]

[Caron-Huot Mazáč Rastelli Simmons-Duffin '21]

Superconvergence

[Kologlu Kravchuk Simmons-Duffin AZ]

[Bern Kosmopolous AZ]

Consider next scattering of particles with spin, e.g. gravitons

$$T(1^+, 2^-, 3^+, 4^-) = \langle 24 \rangle^4 [13]^4 f(s, t) \quad f(s, t) = f(t, s)$$

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The Regge bound is still

$$\lim_{s \rightarrow \infty} \frac{T(1^+, 2^-, 3^+, 4^-)}{s^2} = 0$$

$$\Rightarrow \lim_{s \rightarrow \infty} s^2 f(s, t) = 0$$

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At low energies

$$f(s, u) = \frac{8\pi G_N}{stu} + |\beta_{R^3}|^2 \frac{su}{t} - |\beta_\phi|^2 \frac{1}{t} + \dots$$

Superconvergence: external spin makes couplings dispersive

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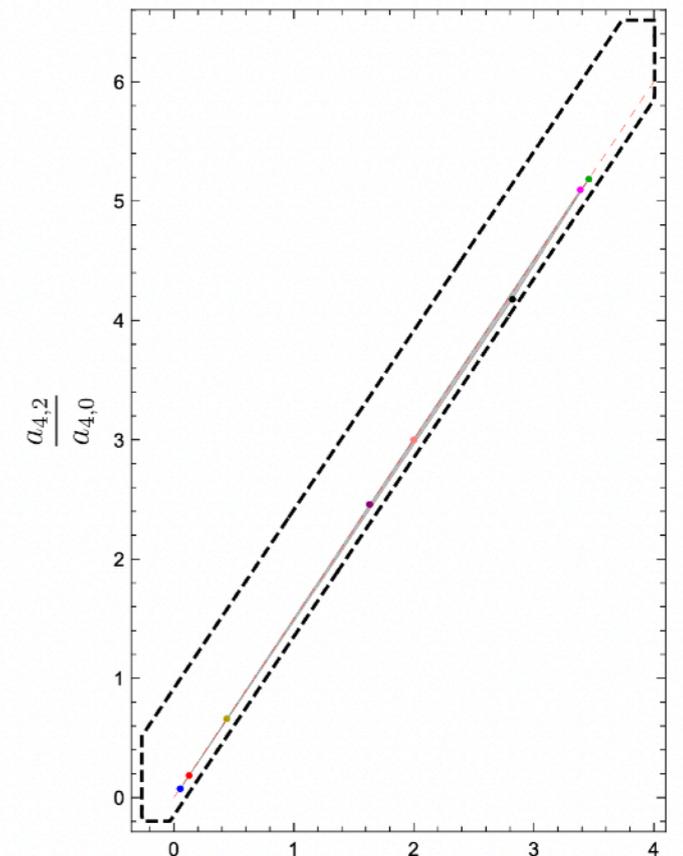
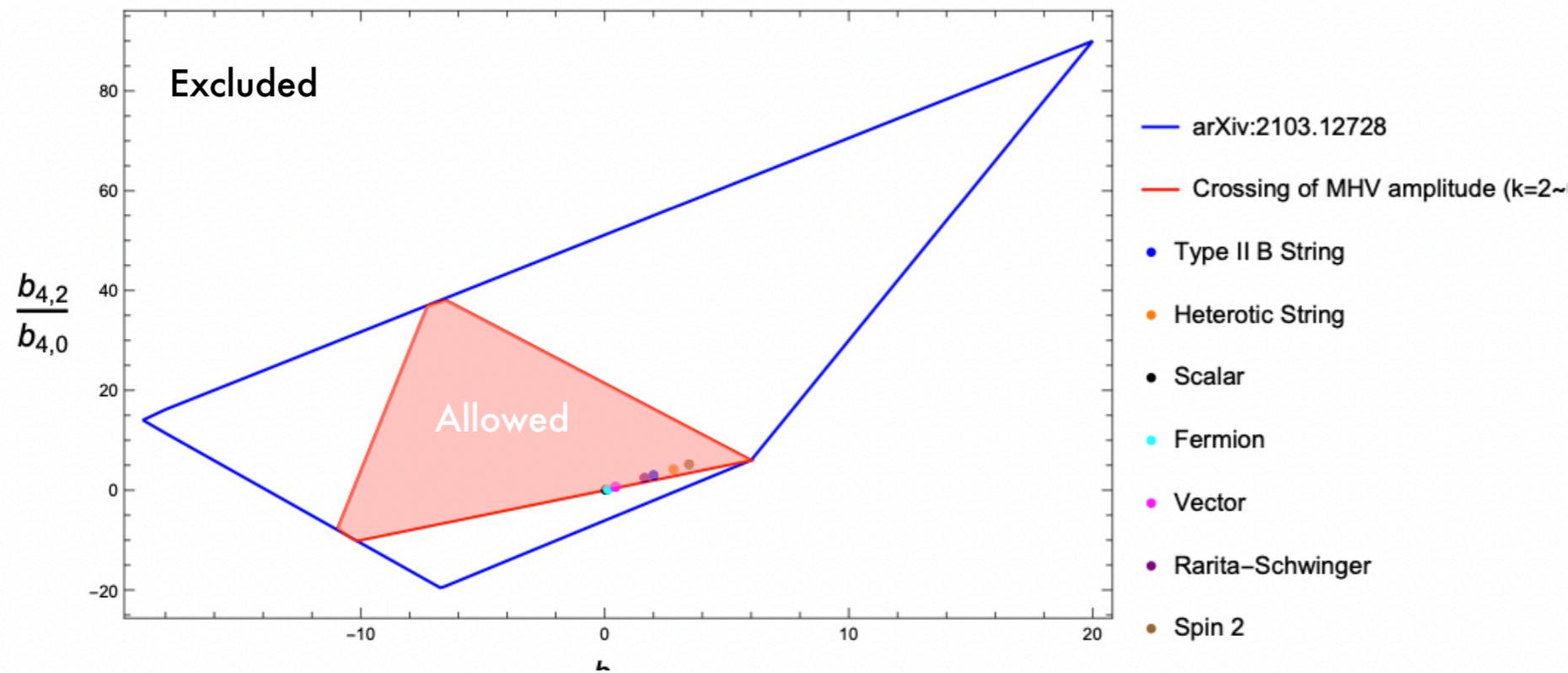
Superconvergence: external spin makes couplings dispersive

$$G_N = \int UV \quad \text{as seen by gravitons}$$

Low spin dominance

It is interesting to fill in the theory space with known theories:

- tree-level string theories
- one-loop matter



$$\text{Low-spin dominance } (\alpha\text{-factor}) : \quad \langle \rho_4^{+-} \rangle_k \geq \alpha \langle \rho_{J>4}^{+-} \rangle_k, \quad \langle \rho_0^{++} \rangle_k \geq \alpha \langle \rho_{J>0}^{++} \rangle_k.$$

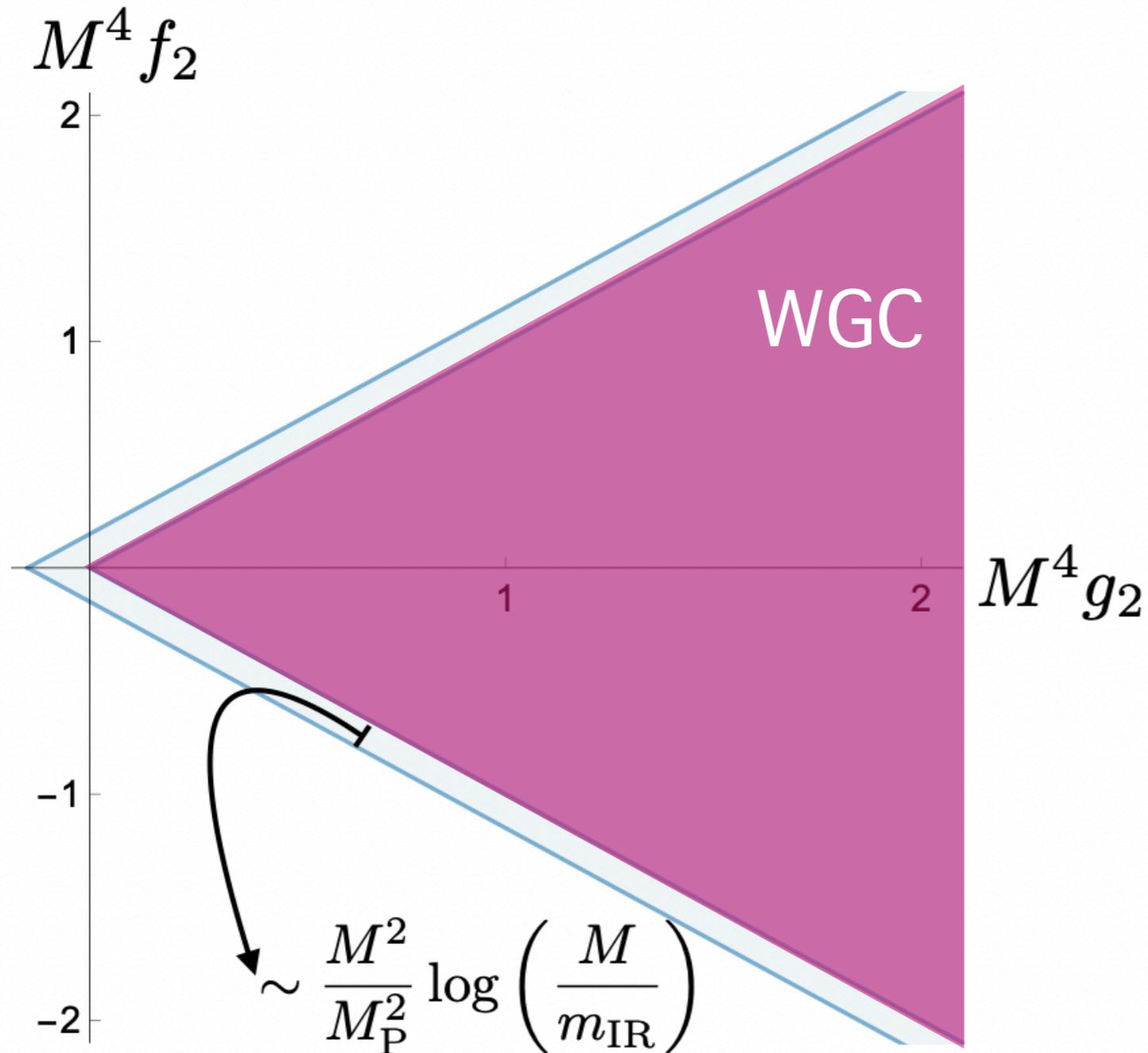
Infinitely many bounds are controlled by the lightest lowest spin exchanged particle.

(Related to species bound?)

Black hole WGC

[Henriksson McPeak Russo Vichi '22]

$$\mathcal{L} = \sqrt{-g} \left(\frac{M_{\text{P}}^2}{2} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 + \alpha_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \beta W_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \dots \right).$$



$$\frac{Q}{M} \geq \frac{Q}{M} \Big|_{\text{extr}} :$$

$$16\alpha_{1,2} = g_2 \pm f_2$$

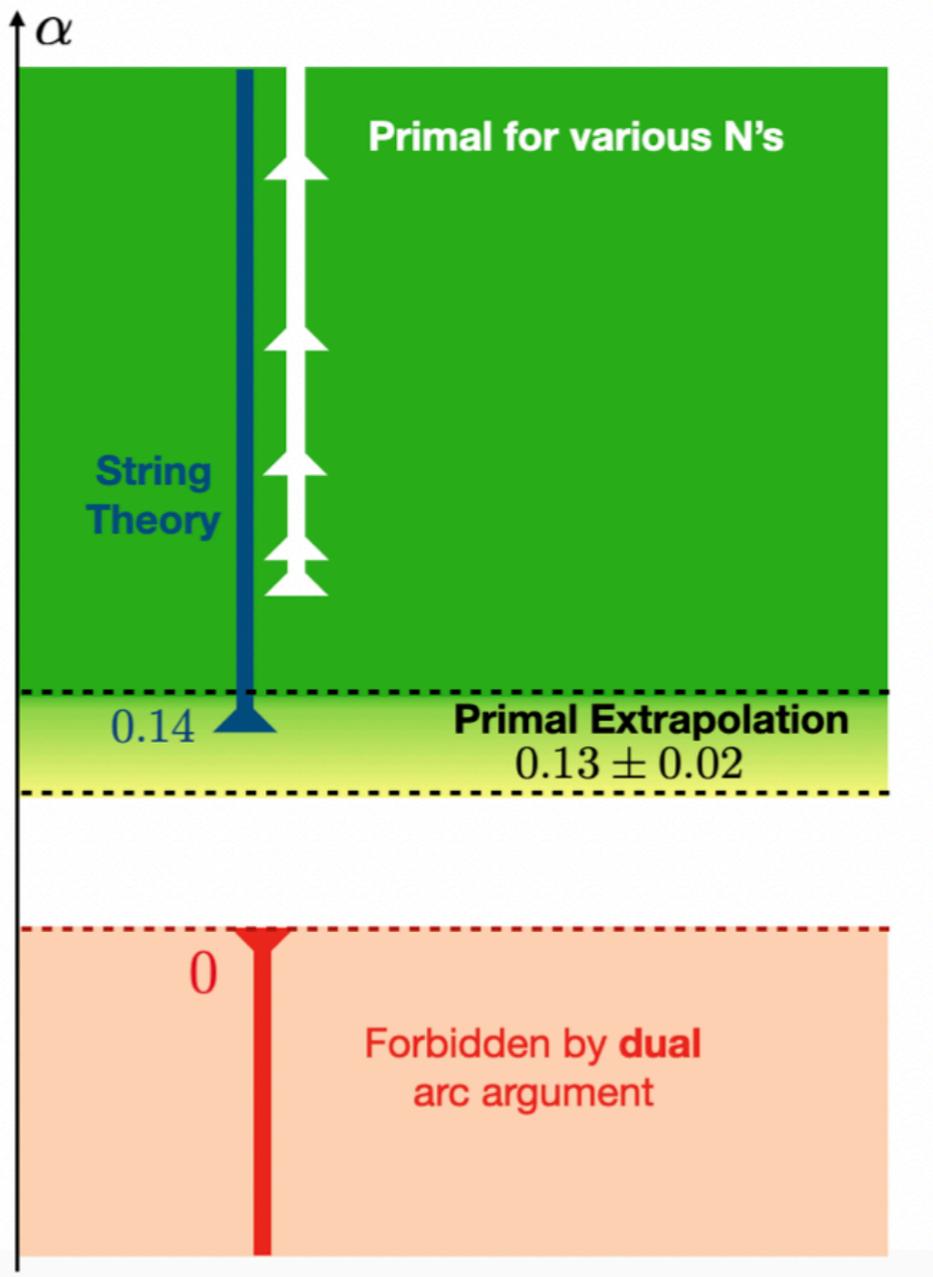
$$g_2 \geq |f_2| \geq 0$$

10d maximal sugra: graviton scattering

Scalars from the graviton multiplet

$$T_{\text{Sugra}} = \frac{8\pi G_N}{stu} + g_0 + \dots$$

$$g_0 \sim \frac{\alpha}{M_{\text{Pl}}^6}$$



weak coupling

Weakly coupled EFT

$$0 \leq g_0 \leq 3.000 \frac{8\pi G_N}{M_{\text{str}}^6}$$

Type II:

$$\frac{g_0 M_{\text{str}}^6}{8\pi G_N} = 2\zeta(3) \simeq 2.404$$

[Guerrieri Penedones Vieira '21]

[Caron-Huot Mazáč Rastelli Simmons-Duffin '21]

AdS generalization

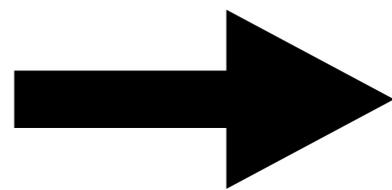
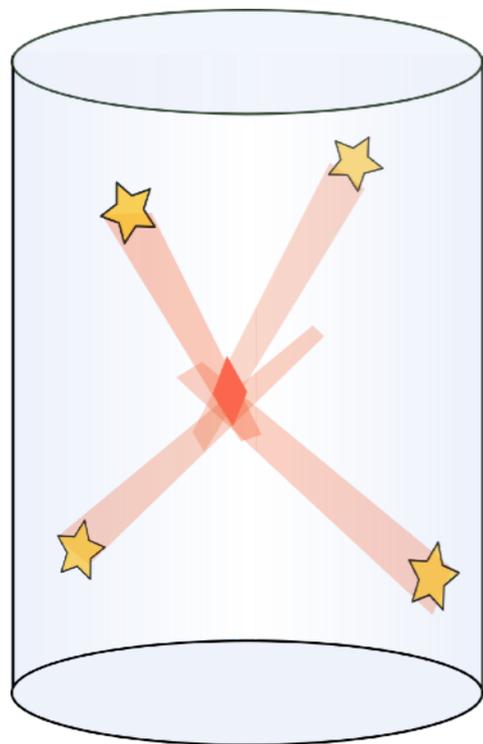
The same techniques apply in AdS but kinematics is more complicated: CFT dispersion relations.

[Carmi Caron-Huot, Mazáč, Mazáč Paulos, Mazáč Rastelli Zhou, Penedones Silva AZ]

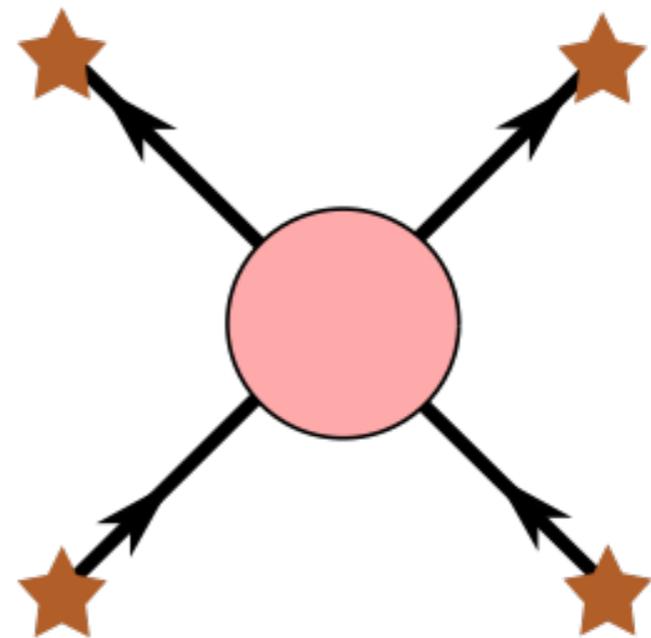
Formulated simplest in Mellin space

$$T(s, t) \rightarrow M(s, t)$$

Dispersive sum rules and bounds match the flat space ones as expected



$$R_{\text{AdS}} \rightarrow \infty$$



[Caron-Huot Mazáč Rastelli Simmons-Duffin]

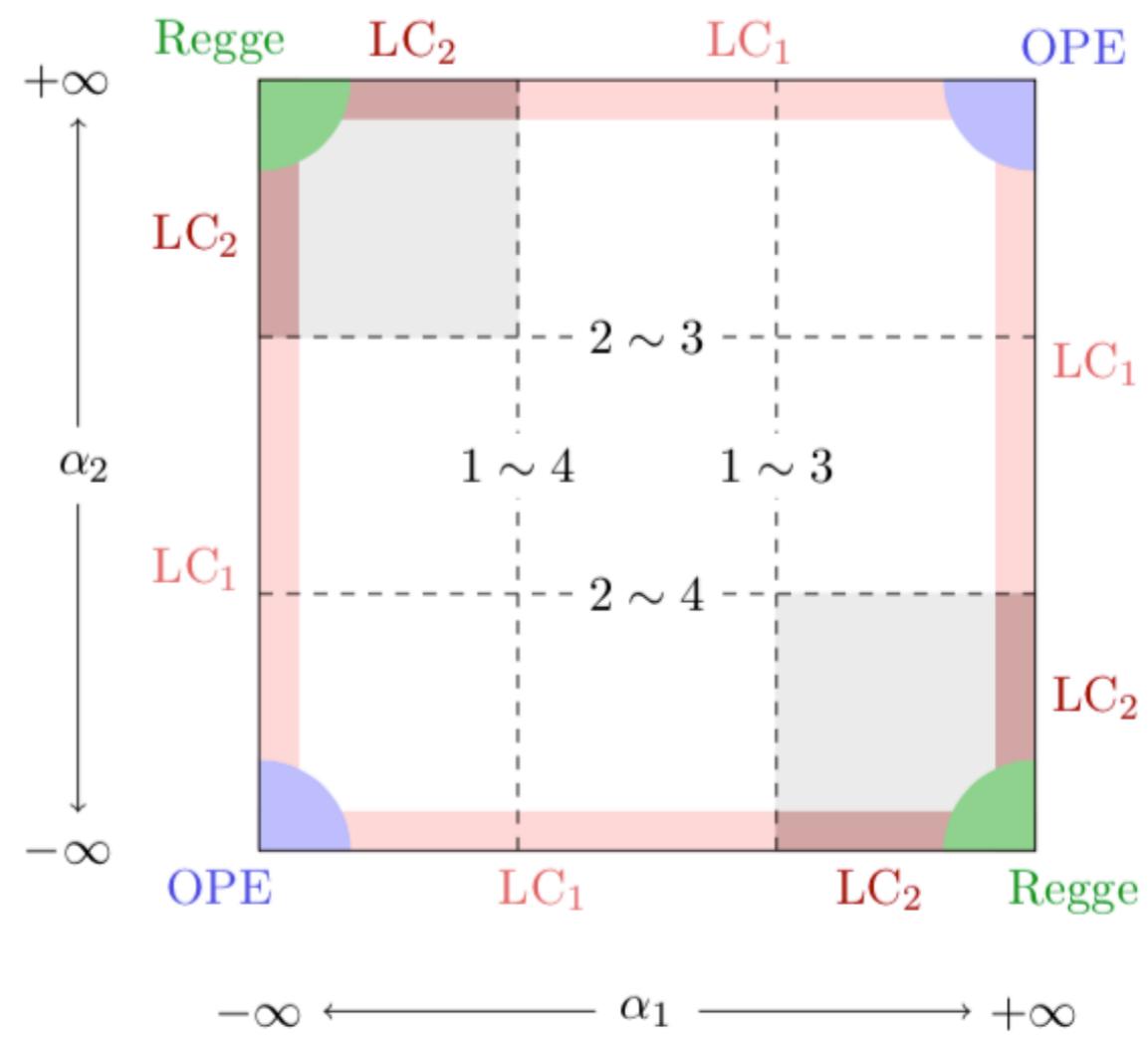
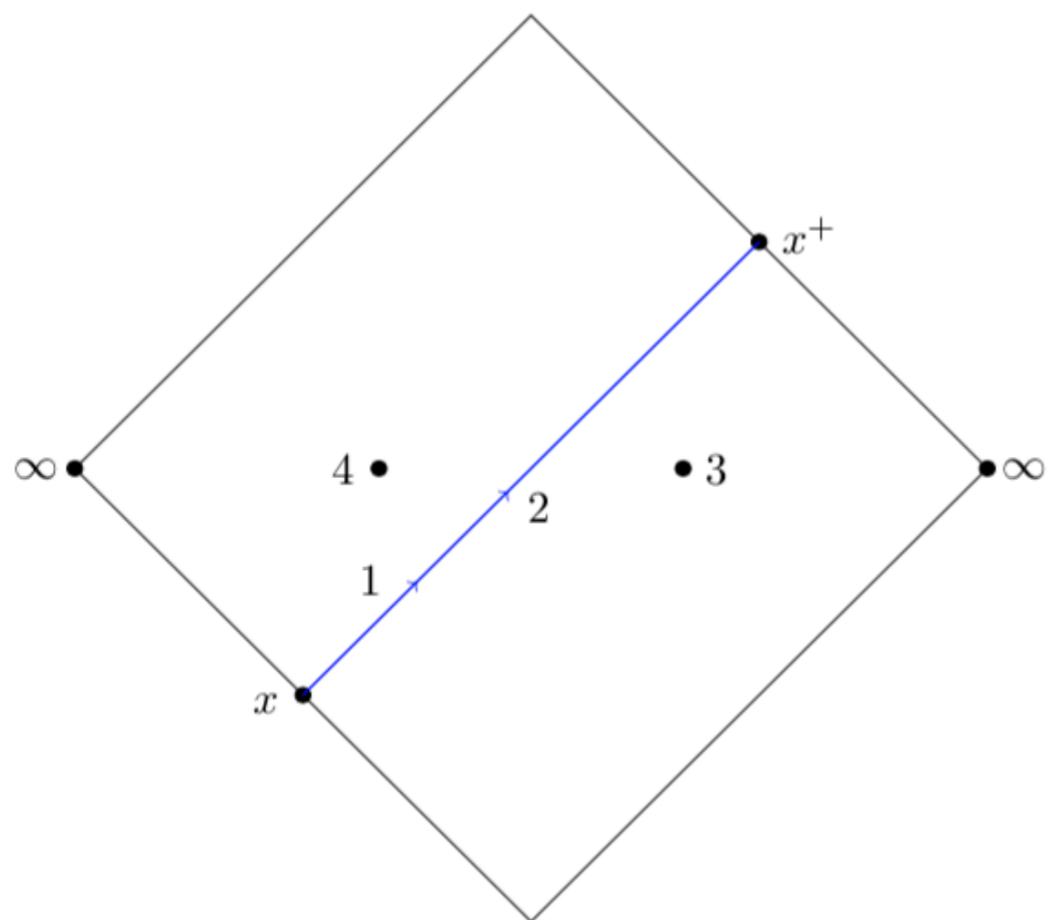
AdS causality

Causality is easy to state

$$[\mathcal{O}_1(x_1), \mathcal{O}_2(x_2)] = 0, \quad (x_1 - x_2)^2 \text{ space-like}$$

What is relevant here is instead commutativity of light-ray operators

$$[\mathbf{L}[\mathcal{O}_1](\vec{x}_1), \mathbf{L}[\mathcal{O}_2](\vec{x}_2)] = 0$$



Bound on Regge!

[Caron-Huot]

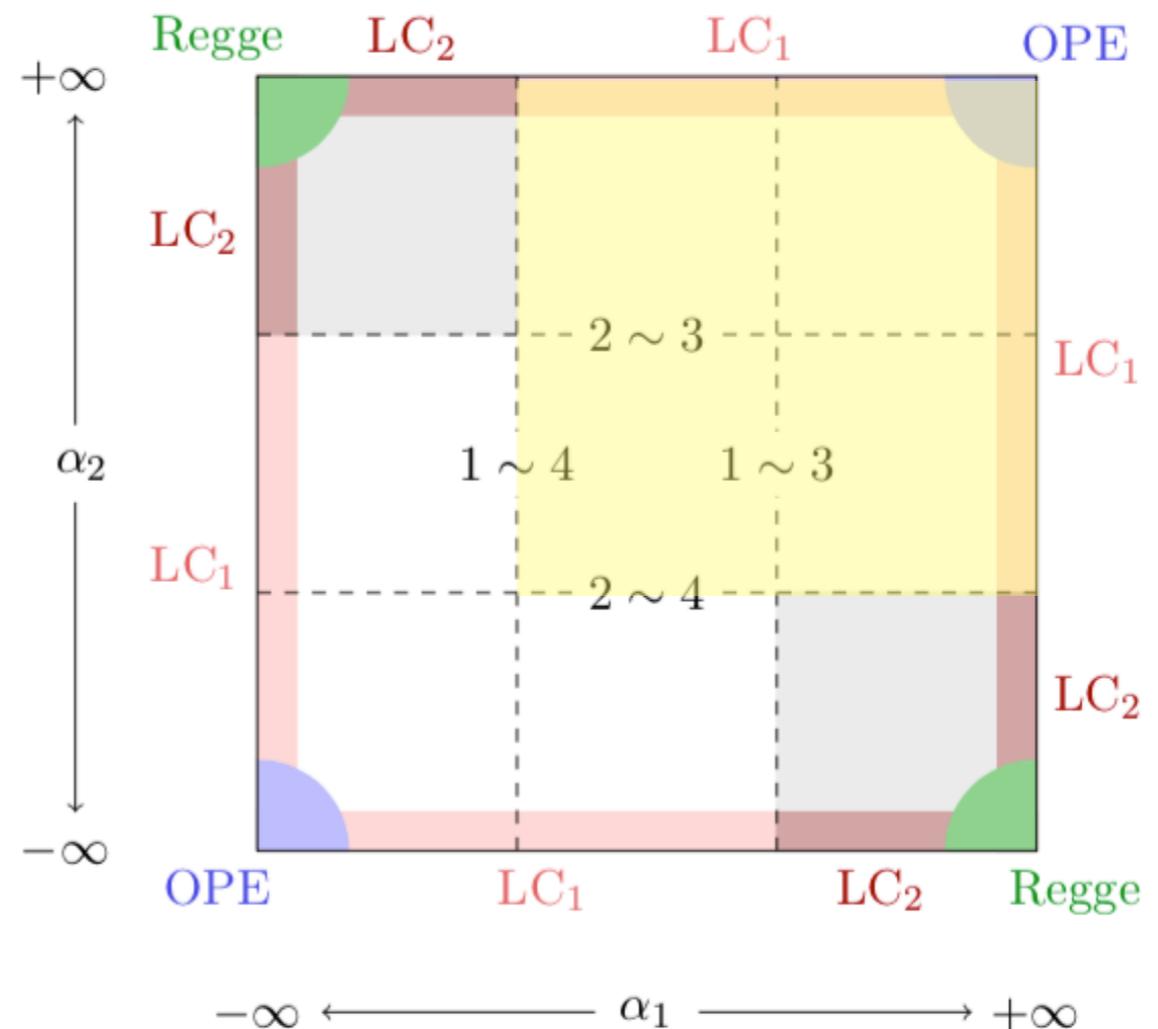
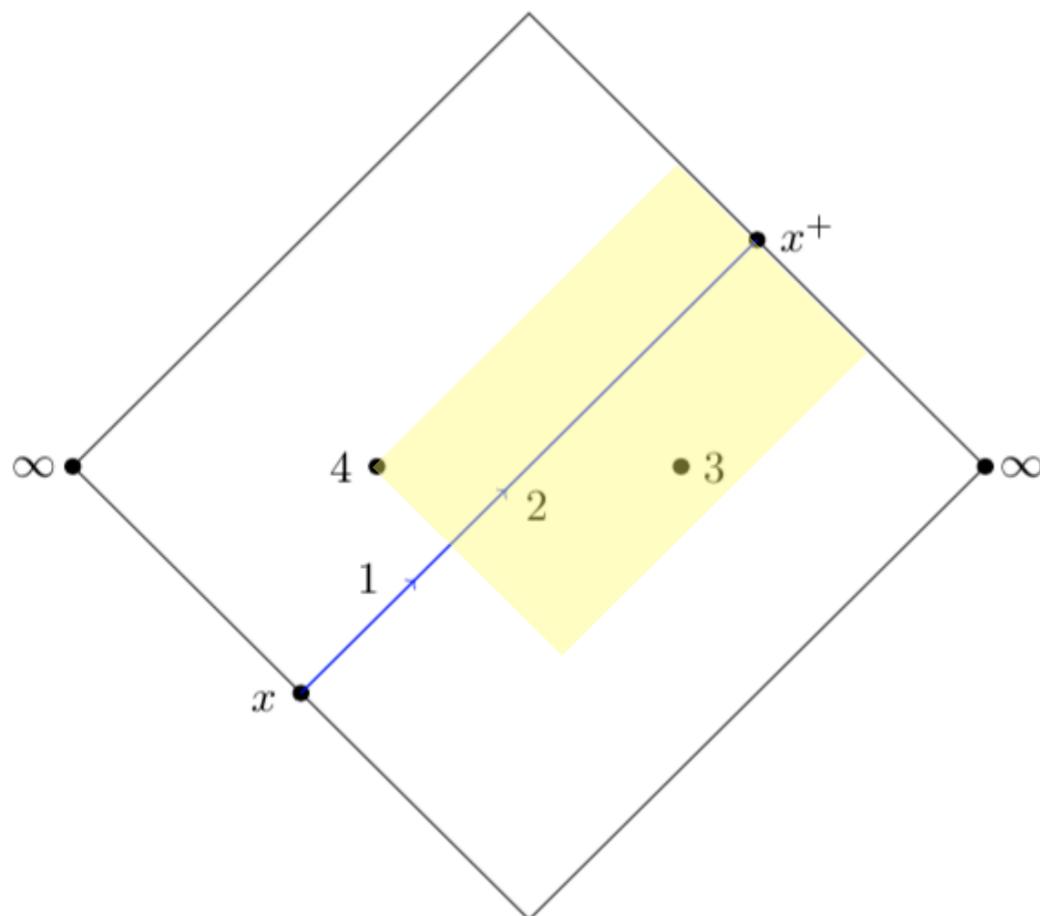
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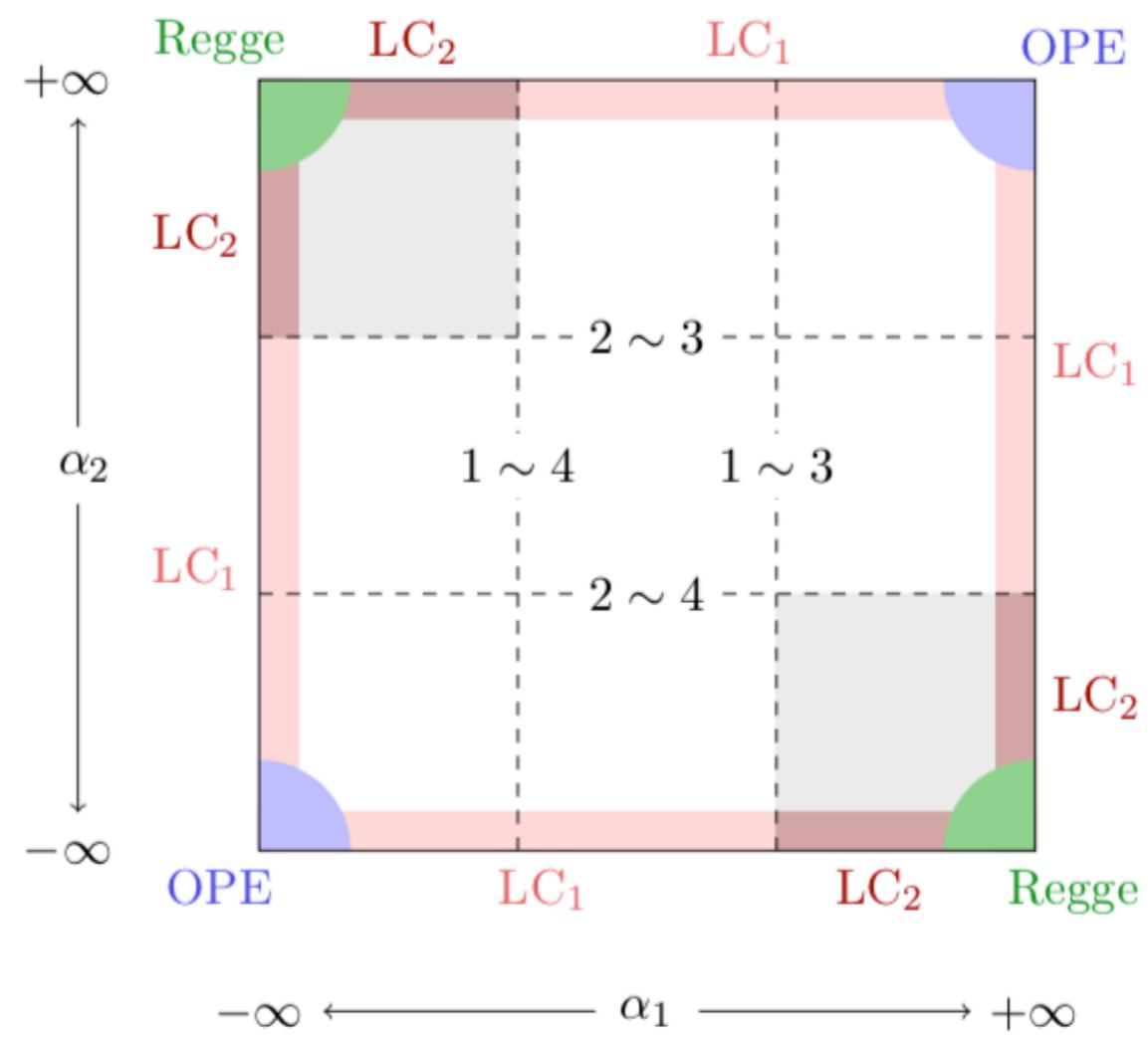
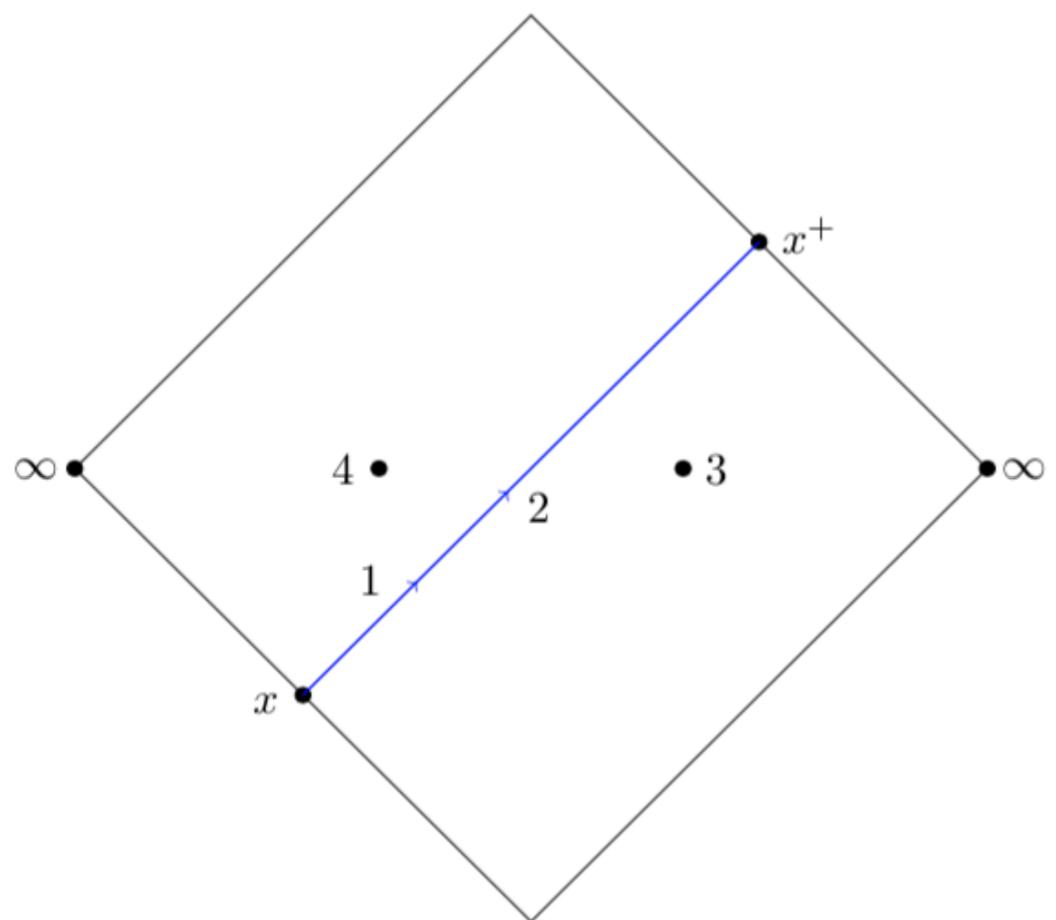
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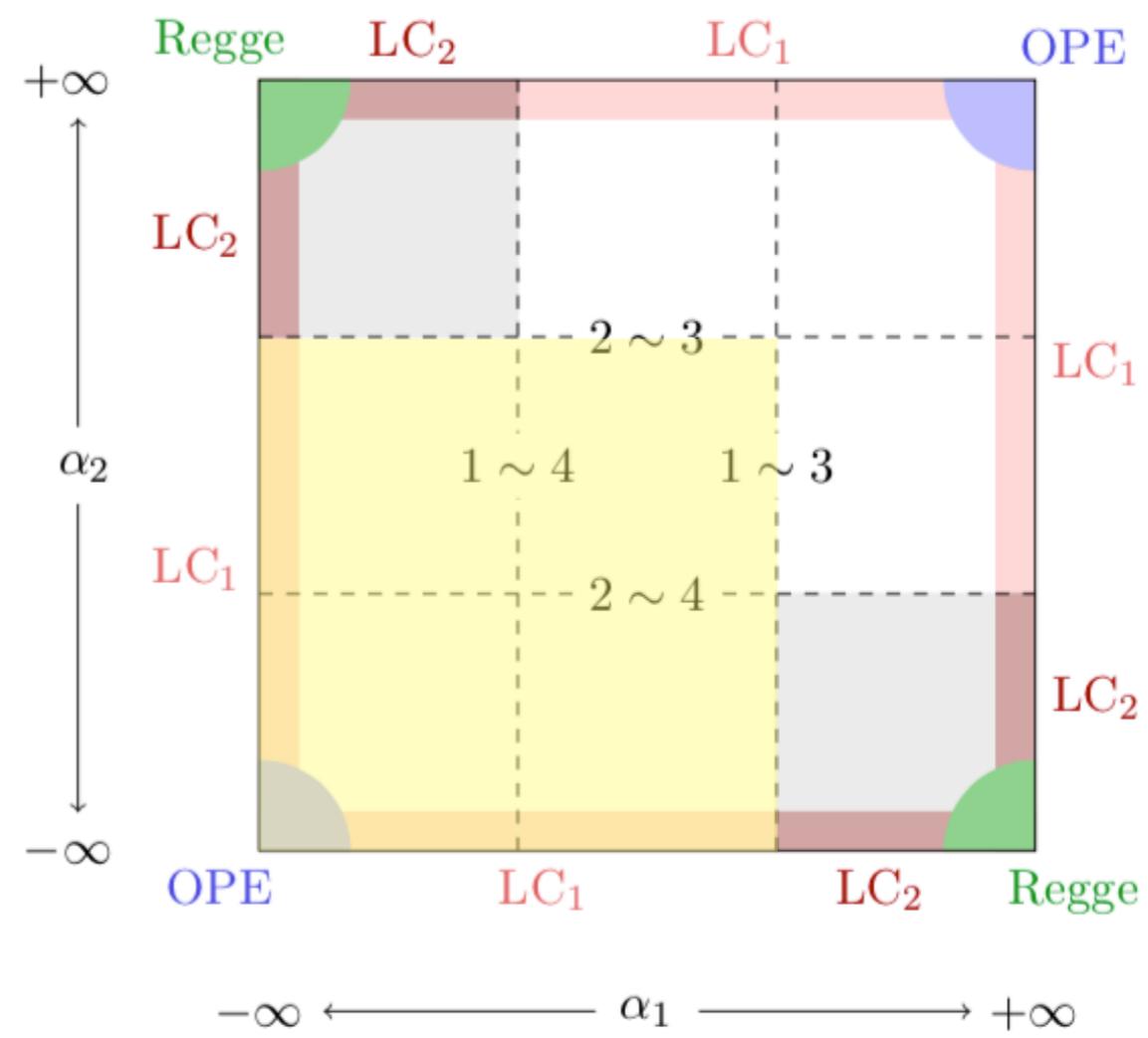
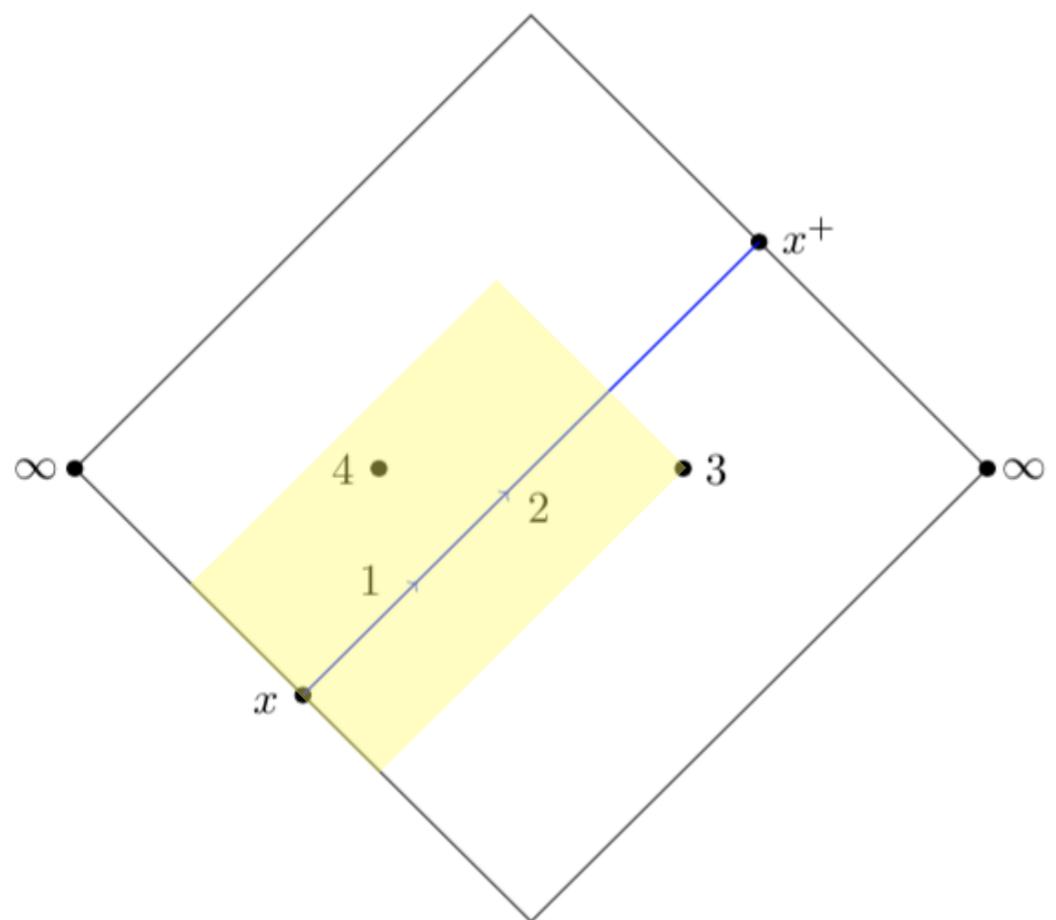
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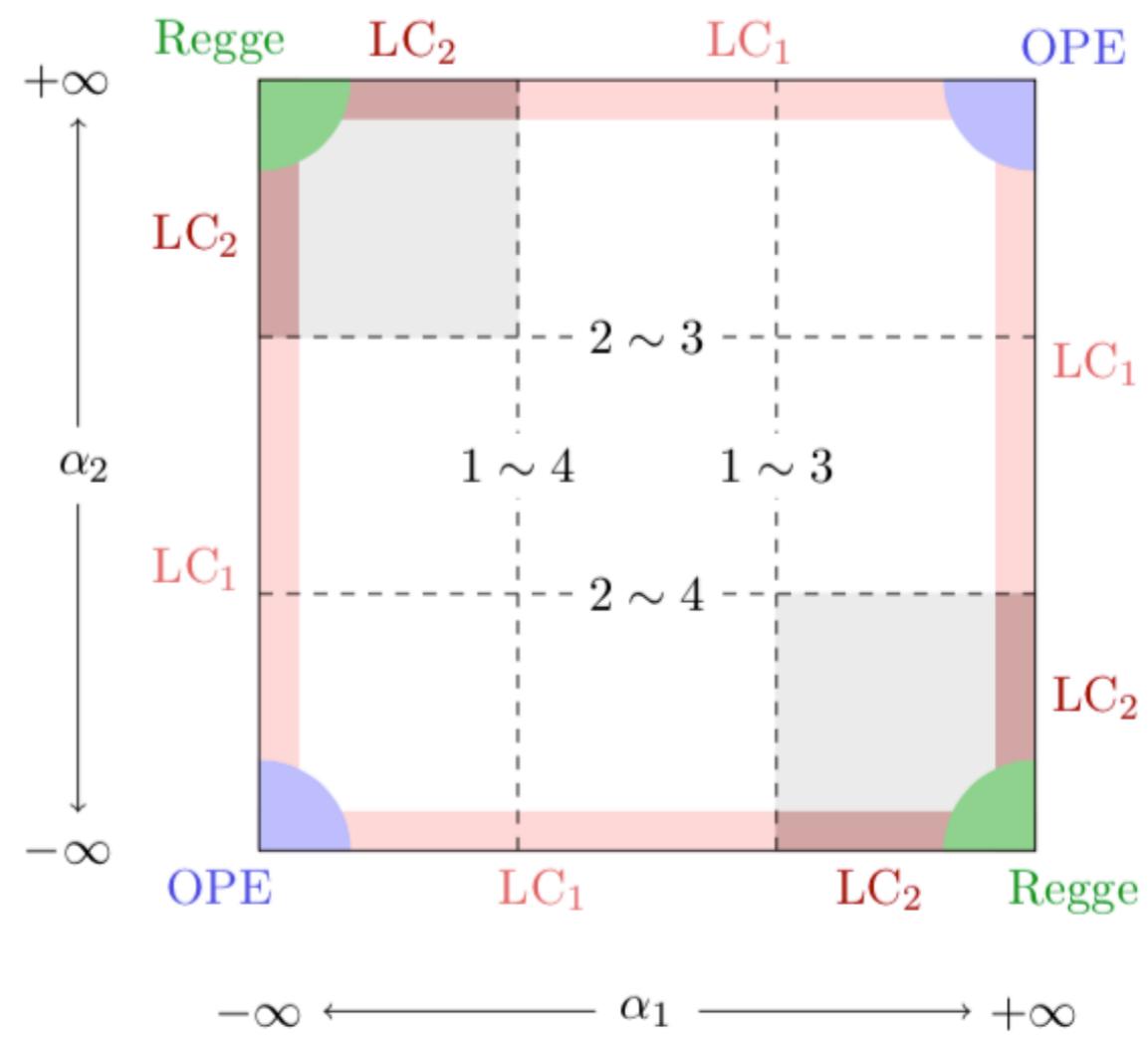
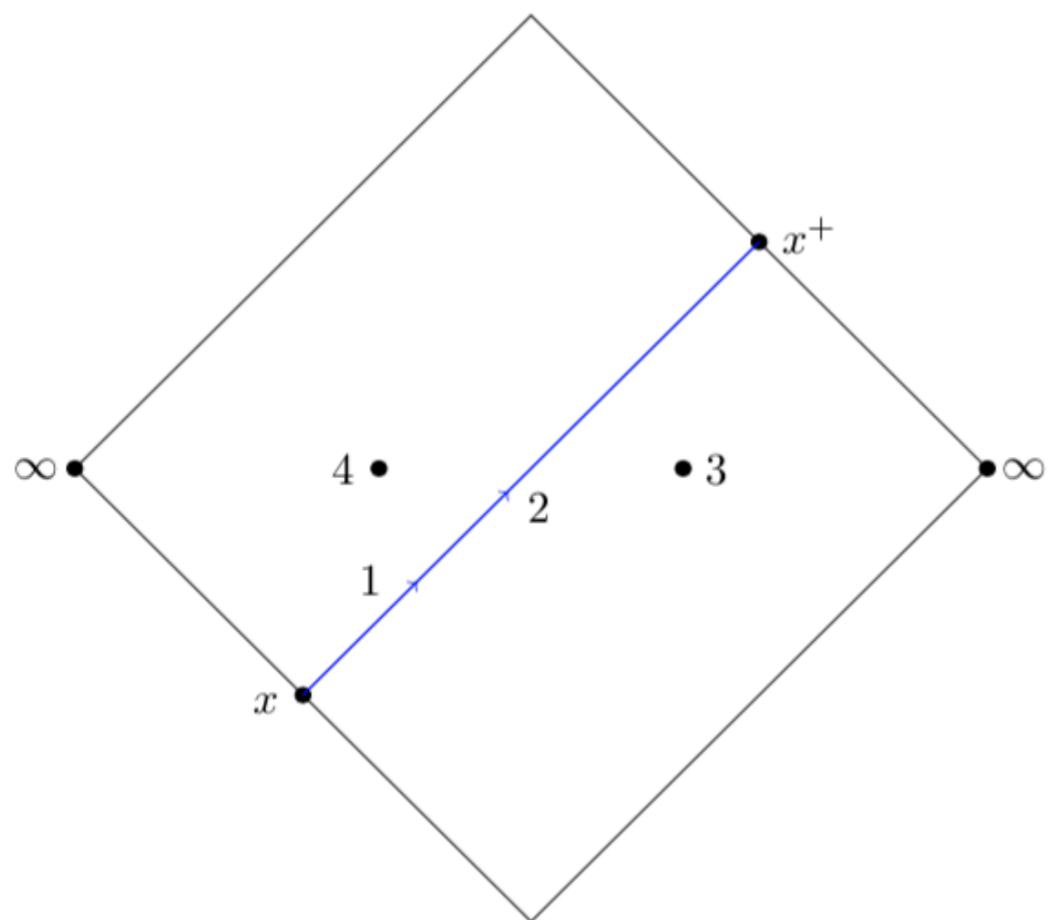
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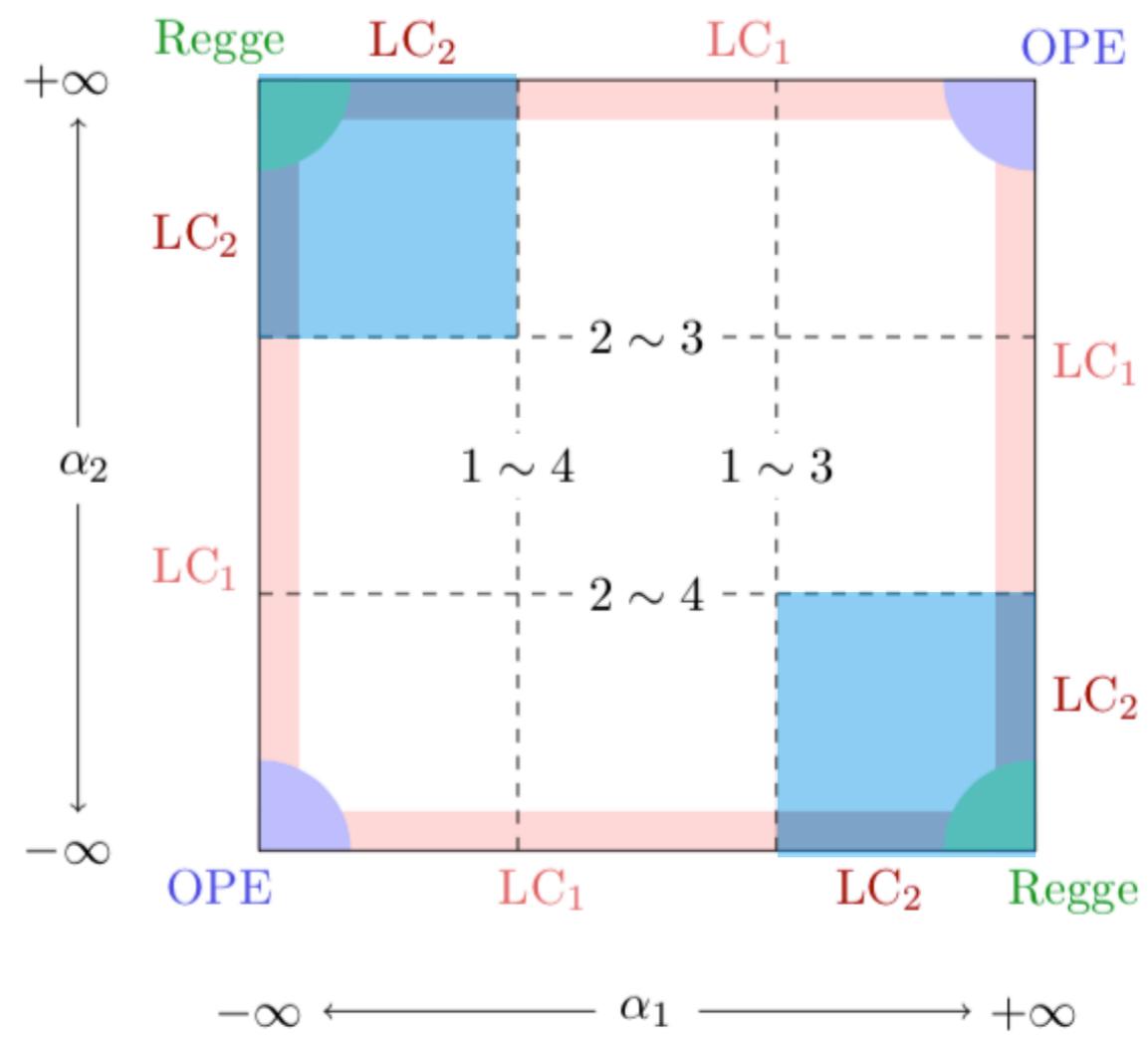
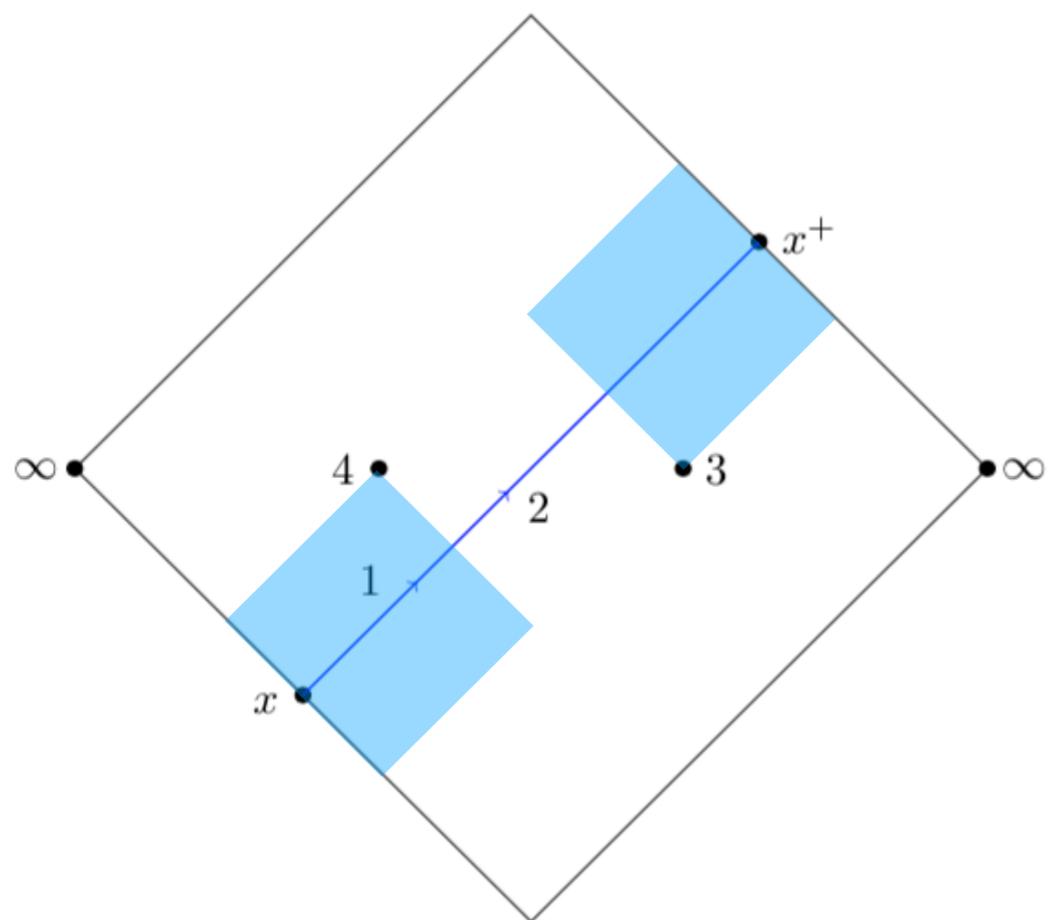
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Bound on Regge!

[Caron-Huot]

| | flat space | AdS |
|------------------------|----------------------------------|--|
| amplitude | \mathcal{M} | \mathcal{G} |
| discontinuity | $\text{Im}[\mathcal{M}]$ | $\text{dDisc}[\mathcal{G}]$ |
| energy | $s = m^2$ | $m^2 = (\Delta - J - d + 1)(\Delta + J - 1)$ |
| angular momentum | J | J |
| impact parameter | $b = \frac{2J}{m}$ | $\beta = \cosh^{-1}(\eta_{\text{AdS}}) = \log \frac{\Delta + J - 1}{\Delta - J - d + 1}$ |
| transverse momentum | $u = -p^2$ | $u = -\nu^2$ |
| Regge limit | $m \gg 1$ at fixed b | $m \gg 1$ at fixed β |
| bulk-point limit | $m \gg 1$ at fixed J | $m \gg 1$ at fixed J |
| number of subtractions | k | k |
| dispersive sum rule | $\mathcal{C}_{k,u}$ | $\mathcal{C}_{k,\nu}$ |
| improved sum rule | $\mathcal{C}_{k,u}^{\text{imp}}$ | $\mathcal{C}_{k,\nu}^{\text{imp}}$ |

Double-trace operators are suppressed in $\text{dDisc}[\mathcal{G}]$.

[cf Dymarsky, Kos, Kravchuk, Poland, Simmons-Duffin '17]

IR divergencies in 4d

[Caron-Huot Mazáč Rastelli Simmons-Duffin]

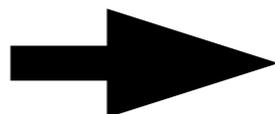
In four dimensions when taking the flat space limit IR divergencies manifest themselves through the **logarithmic** dependence on the AdS radius

$$T(s, t) = \dots + g_2(s^2 + t^2 + u^2) + g_3(stu) + \dots$$

$$-10.4g_2 - 48.4 \frac{8\pi G}{M^2} \log(0.23 MR_{\text{AdS}}) \leq g_3 M^2 \leq 3g_2 + 207 \frac{8\pi G}{M^2} \log(0.11 MR_{\text{AdS}}).$$

A natural hypothesis is to set $R_{\text{AdS}} \rightarrow L_{\text{exp}}$.

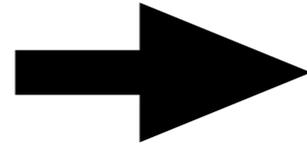
Democratic Box Hypothesis: All boxes are created equal.
(for the EFT bounds)



Uplift to dS.

Summary

Bootstrap (**causality!**)



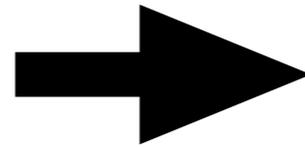
Traditional EFT with
sharp bounds

Experiments + basic principles =?

(colliders, gravity waves, black holes...)

Summary

Bootstrap (**causality!**)



Traditional EFT with
sharp bounds

Experiments + basic principles =?

(colliders, gravity waves, black holes...)

- Positivity vs non-perturbative bootstrap (loops)

[Bellazzini Mirò Rattazzi Riembau Riva] [Guerrieri Penedones Vieira, Paulos van Rees]
[Chiang Y-t Huang Li Rodina Weng]

- Higher-point functions, multiple correlators, spin, gauge theories, solvable theories

[Guerrieri Penedones Vieira] [Albert Rastelli]

- Causality in a finite volume (IR divergencies)

[Chen de Rham Margalit Tolley Zhang] [Prabhu Satishchandran Wald]

- Flat space limit of AdS/CFT

[Cordova He Paulos] [Komatsu Paulos van Rees Zhao]

- Include BH production, gravity waves, stringy effects [Häring AZ]

- Deriving swampland conjectures from bootstrap?

Bootstrap dream

String theory is the only way to make quantum gravity weakly coupled.



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Strings are similar to the Higgs boson with the difference that they allow to make higher spin particles massive.



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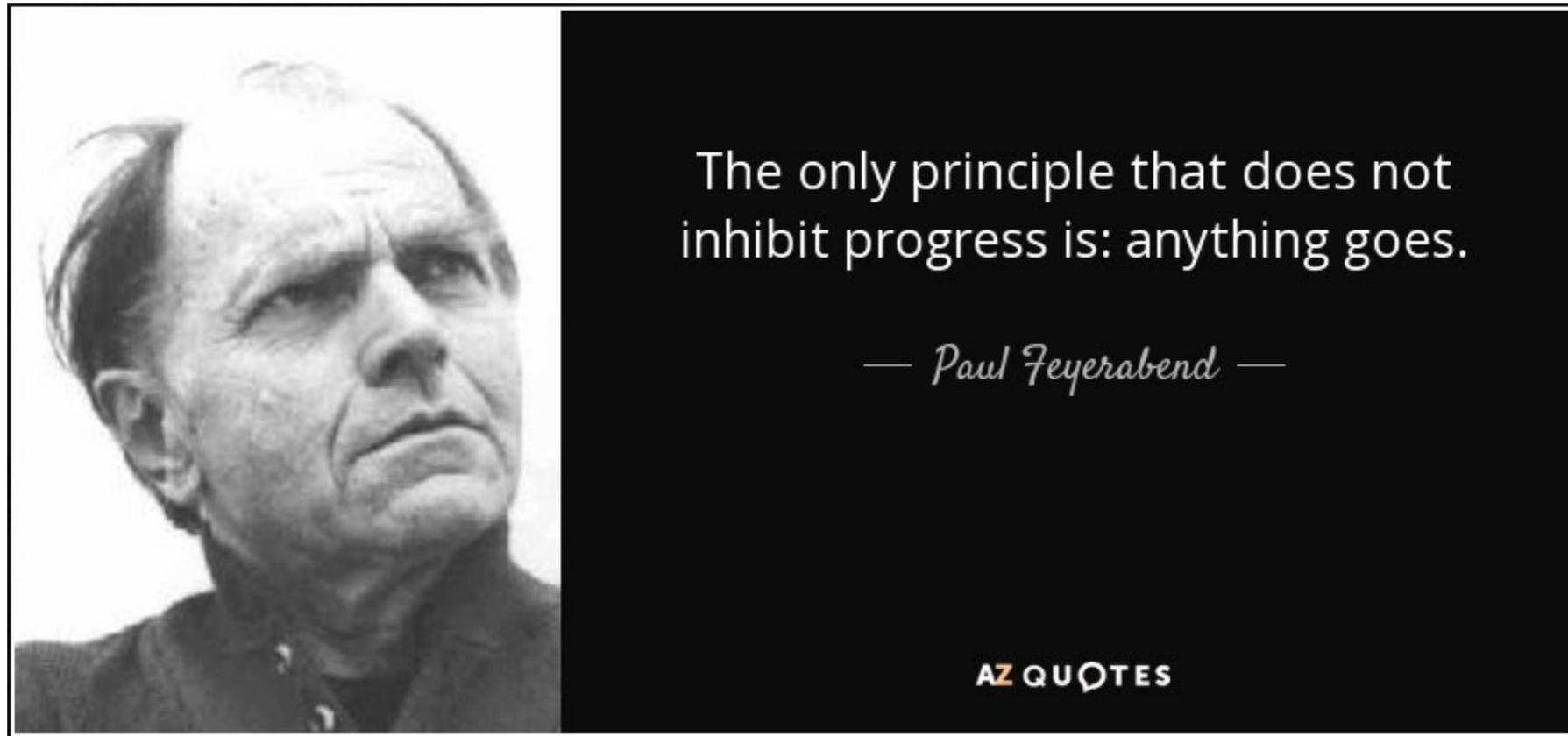
Thank you!



Acknowledgments

I am grateful to the participants of the S-matrix bootstrap workshop for many stimulating discussions on related topics.

Wilsonian anarchism=Acausality



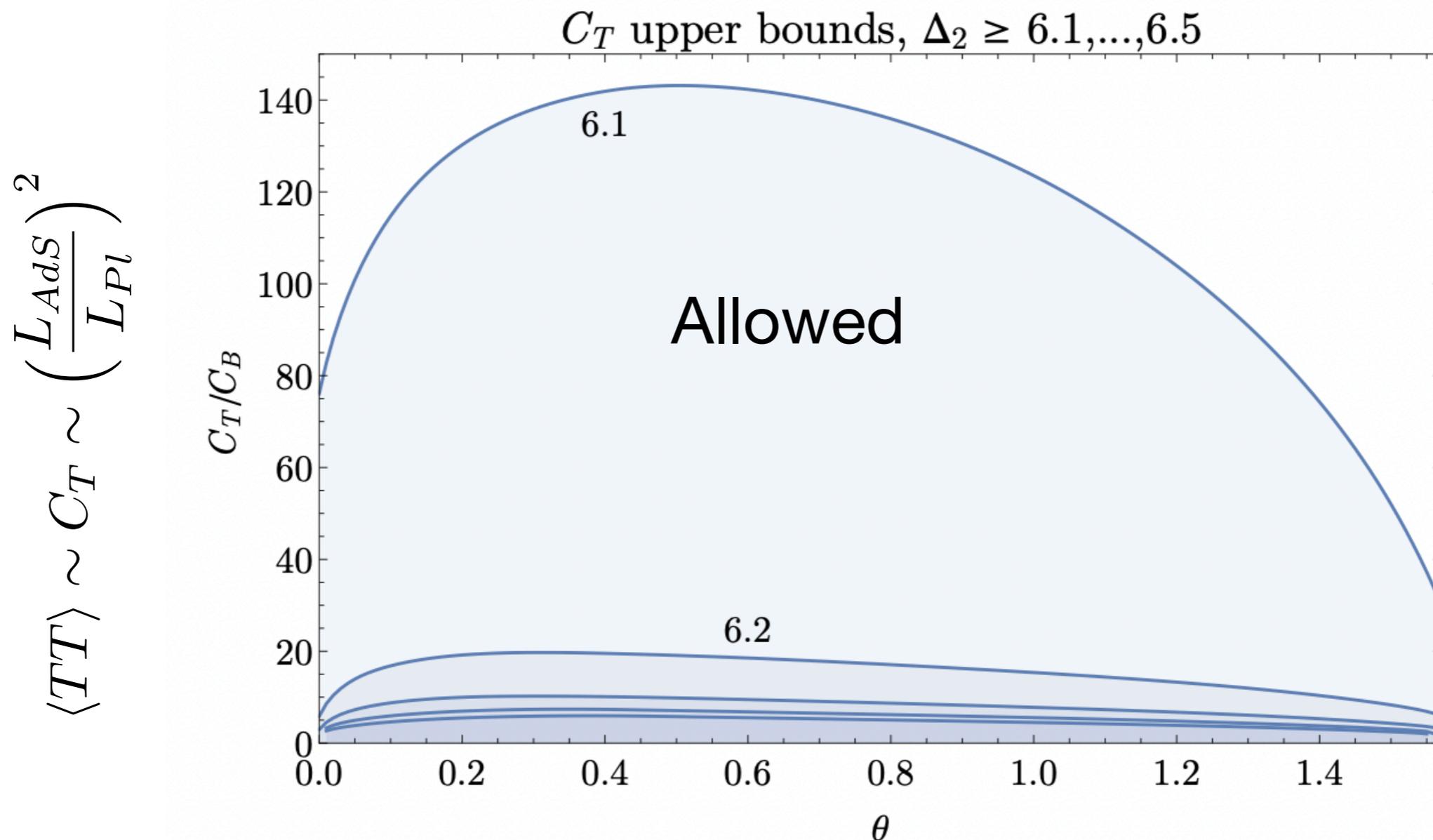
Not anything goes in causal, unitary EFTs.

5 years ago: 3d CFT

[Dymarsky, Kos, Kravchuk, Poland, Simmons-Duffin '17]

Scattering of gravitons in AdS

$\langle TTTT \rangle$



Bound on the lightest spin two operator: $\Delta_{T_{\mu\sigma}T^{\sigma\nu}} = 6 + O\left(\frac{1}{C_T}\right)$

Signal model

$$f_{\text{out}}(t) = \int_{-\infty}^{\infty} dt' S(t - t') f_{\text{in}}(t')$$

causality

$$f_{\text{in}}(t < 0) = 0 \rightarrow f_{\text{out}}(t < 0) = 0$$

- analyticity
- subexponentiality

unitarity

$$\|f_{\text{out}}\| \leq \|f_{\text{in}}\|$$

- Regge bound

$$|S(\omega)| \leq 1, \quad \text{Im}\omega > 0$$