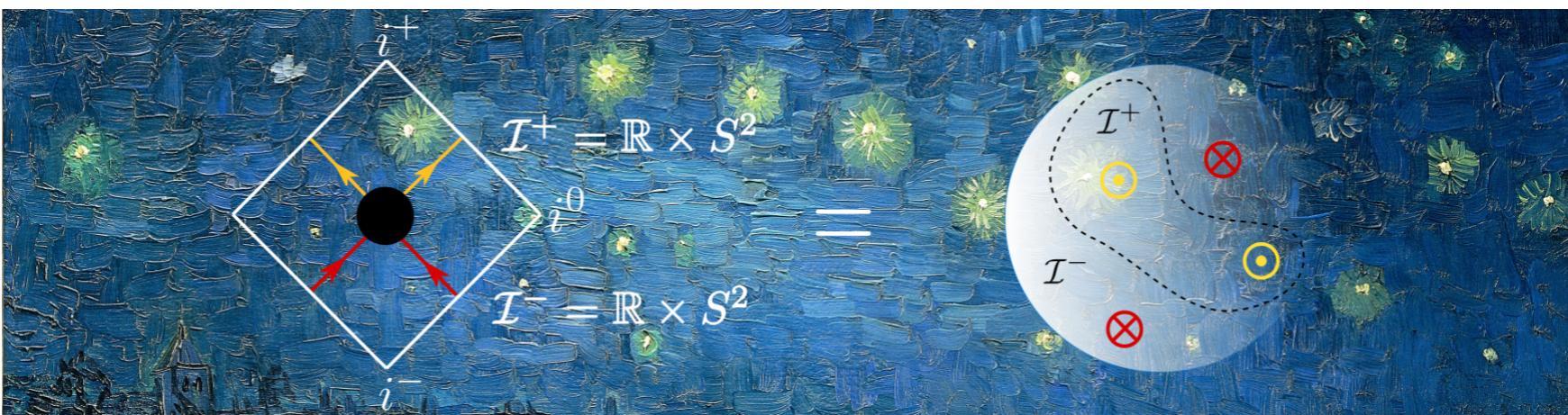




INSTITUT
POLYTECHNIQUE
DE PARIS



CELESTIAL AMPLITUDES



ANDREA PUHM

EUROSTRINGS 2022, 26 APRIL @ ENS LYON



European Research Council

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Scattering amplitudes

learn about a thing by throwing something at it

Surprising simplicity and hidden structure



search for new conceptual formulations and gateway to new physics.

Profusion of fascinating results in scattering amplitudes from
on-shell methods in a *translation* invariant basis.

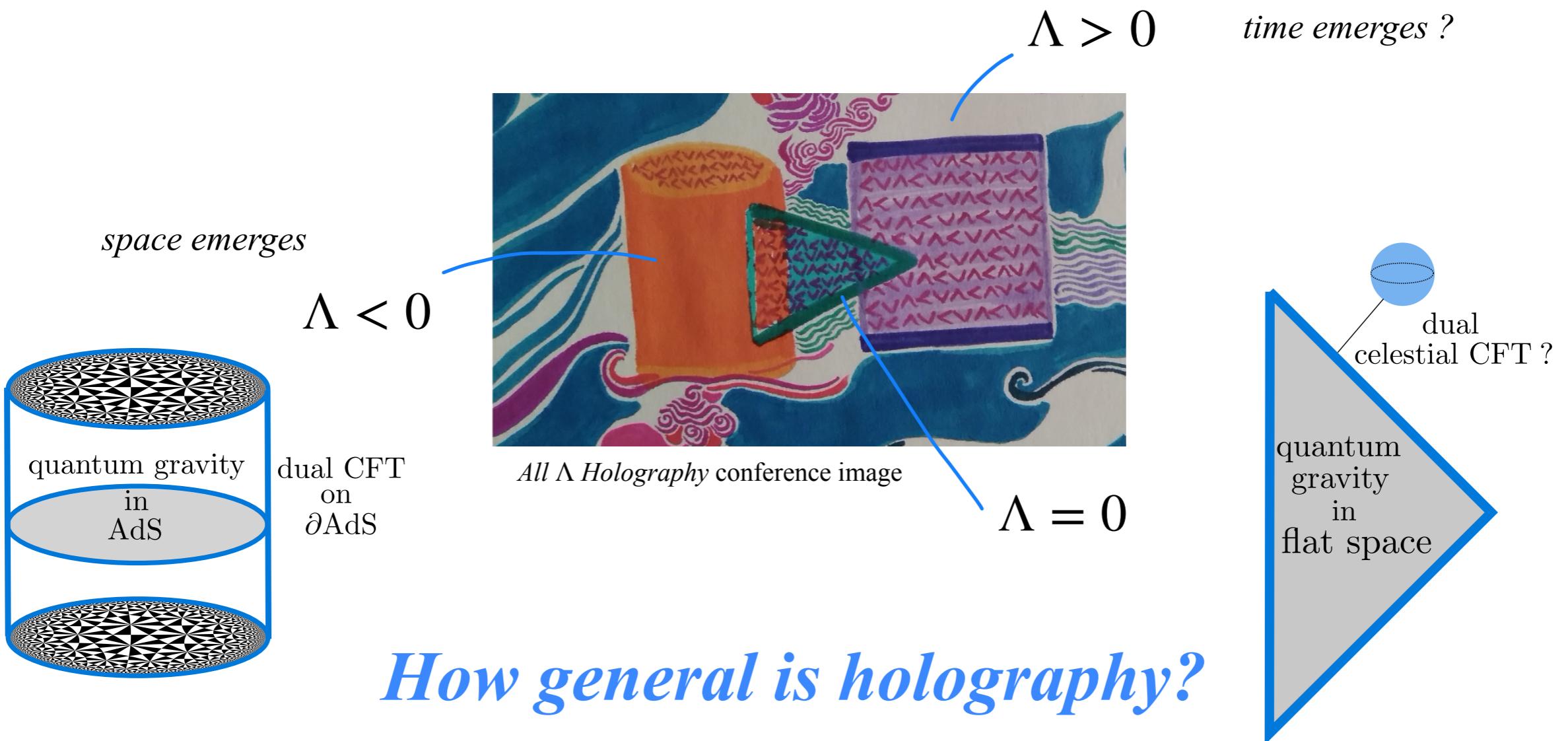
Novel insights from making *Lorentz* symmetry manifest:

celestial amplitudes

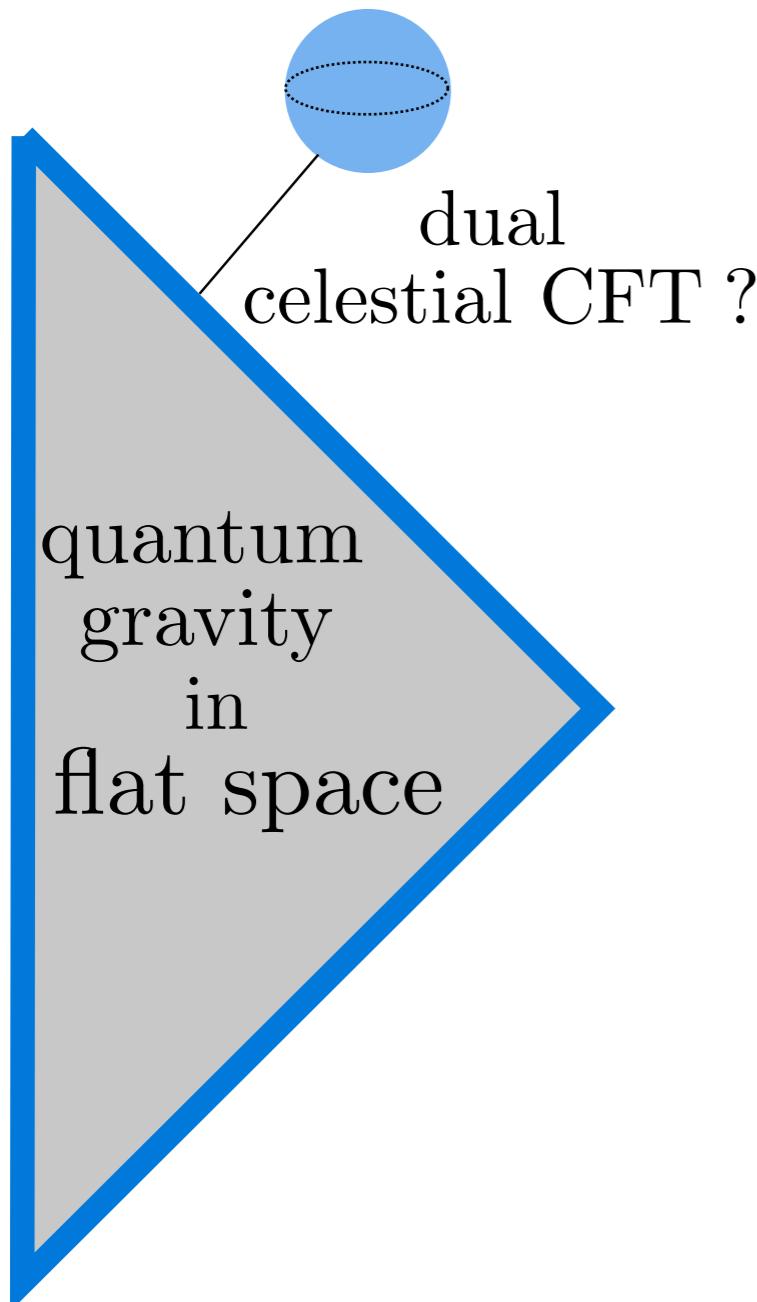
Holography principle

learn about a thing by studying the 'dual' thing

*Duality between quantum gravity on a manifold and
a field theory on its boundary.*

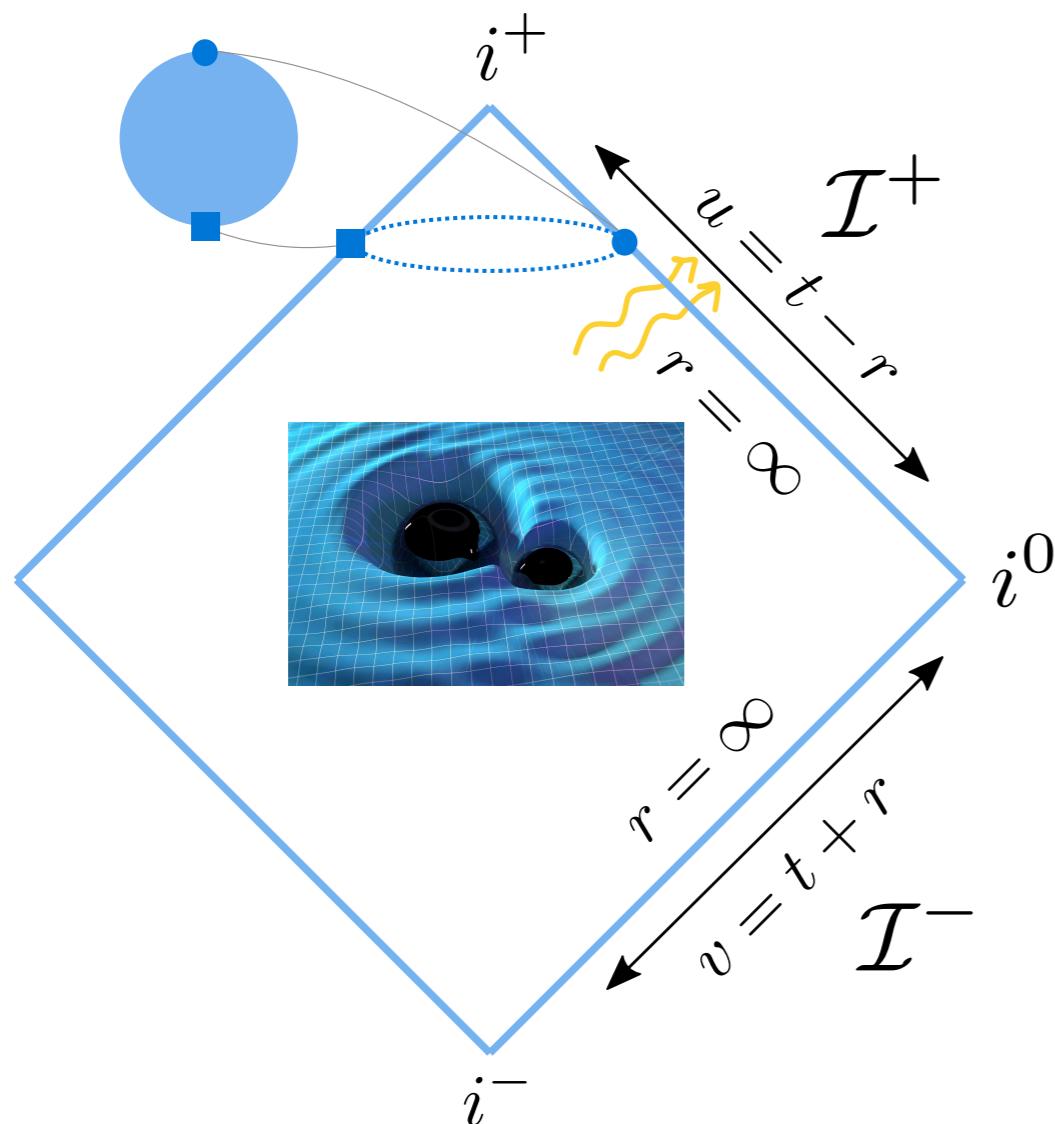


Flat space holography: why?



- Asymptotically flat space is good approximation to many processes in the universe.
- Non-perturbative definition of quantum gravity.
- Black hole information and microscopic entropy.
- New insights into (bulk) QFT scattering via (boundary) CFT inspired methods.

Asymptotically flat space



$$ds^2 = -(1 + \dots)du^2 - (2 + \dots)dudr + (\dots)dudx^A + (r^2\gamma_{AB} + rC_{AB} + \dots)dx^A dx^B$$

angular momentum

mass

gravitational waves

Quantum gravity: metric fluctuates
 \Rightarrow flat $+ \frac{1}{r}$ corrections

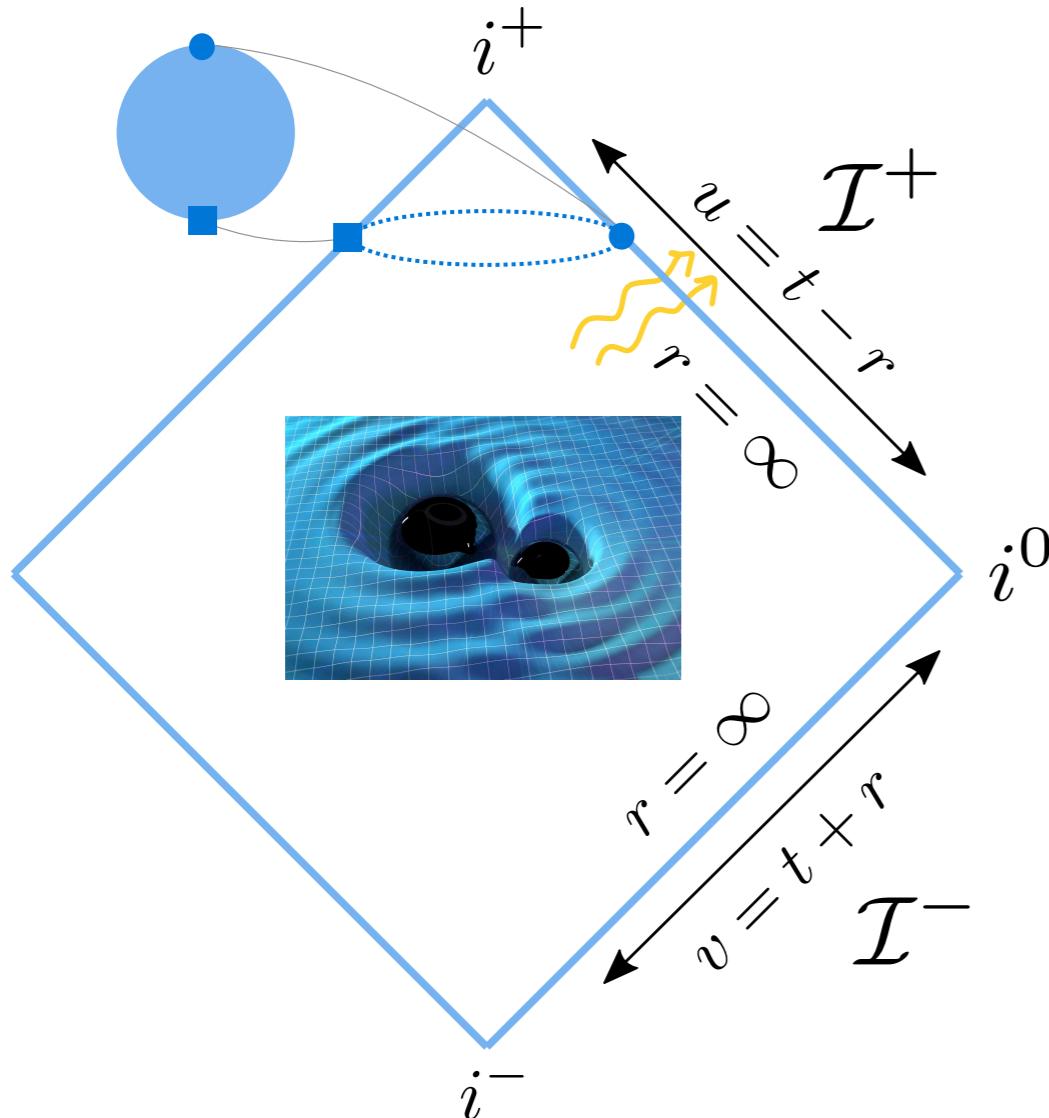
Radiative data at future null infinity \mathcal{J}^+ :

$$\{m, N_A, C_{AB}\}$$

Bondi mass aspect | angular momentum aspect gravitational data/shear
 \Rightarrow Bondi news

$$N_{AB} = \partial_u C_{AB}$$

Flat space holography: how?



Challenges:

- asymptotic boundary is **null** (Carrollian structure)
(as opposed to timelike in AdS)
- asymptotic boundary is "leaky" (non-zero news)
(as opposed to "box" in AdS)
- Poincaré group enhanced to ∞ dim **BMS** group
translations → **supertranslations**
rotations & boosts → **superrotations**
- Soft gravitons, memory, IR divergences...

Celestial holography

*Quantum gravity in
 $D = d + 2$ dimensional
asymptotically flat bulk*



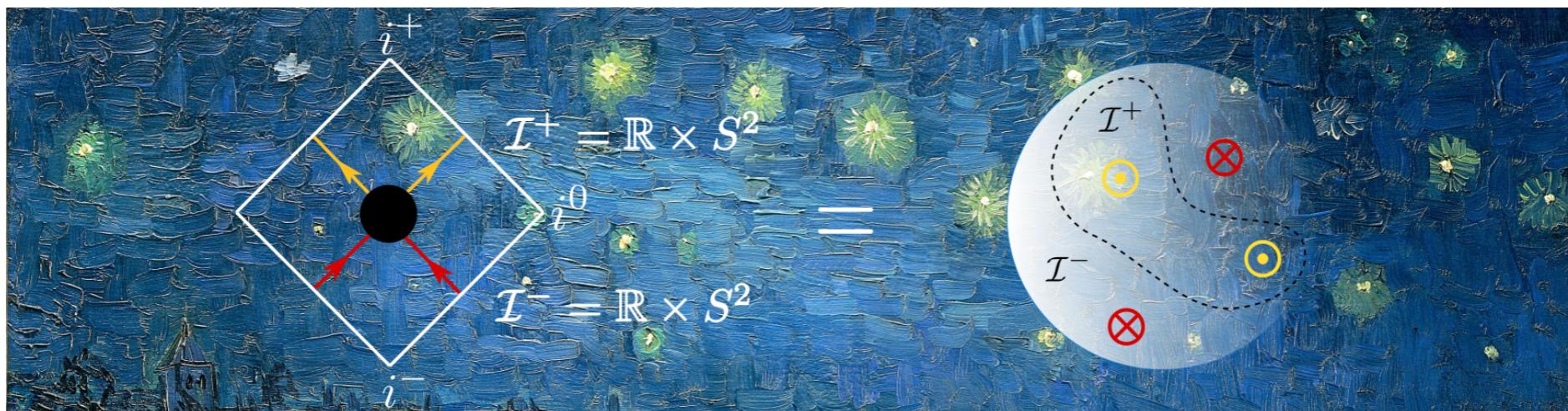
*"Celestial CFT" on
 S^d @ null boundary*

exploits:

*Lorentz group in
 $D=d+2$ dimensions*



*Euclidean conformal
group in d dimensions*



S-matrix as celestial amplitude:

$$\text{boost} \langle \text{out} | \mathcal{S} | \text{in} \rangle_{\text{boost}} = \langle \mathcal{O}_{\Delta_1, J_1}^{\epsilon_1}(x_1) \dots \mathcal{O}_{\Delta_n, J_n}^{\epsilon_n}(x_n) \rangle_{\text{celestial CFT}}$$

Plan

- Asymptotically flat space and the S-matrix
memory effects | soft theorems | asymptotic symmetries
- S-matrix as celestial amplitude
celestial CFT operators & conformal primary wavefunctions
- Universal aspects of celestial amplitudes
analytic structure, celestial OPEs, hidden properties
- Celestial symmetries
asymptotic symmetries from celestial currents, conformally soft theorems, conformal multiplets, ∞ dimensional holographic symmetry algebras
- Summary & outlook

S-matrix

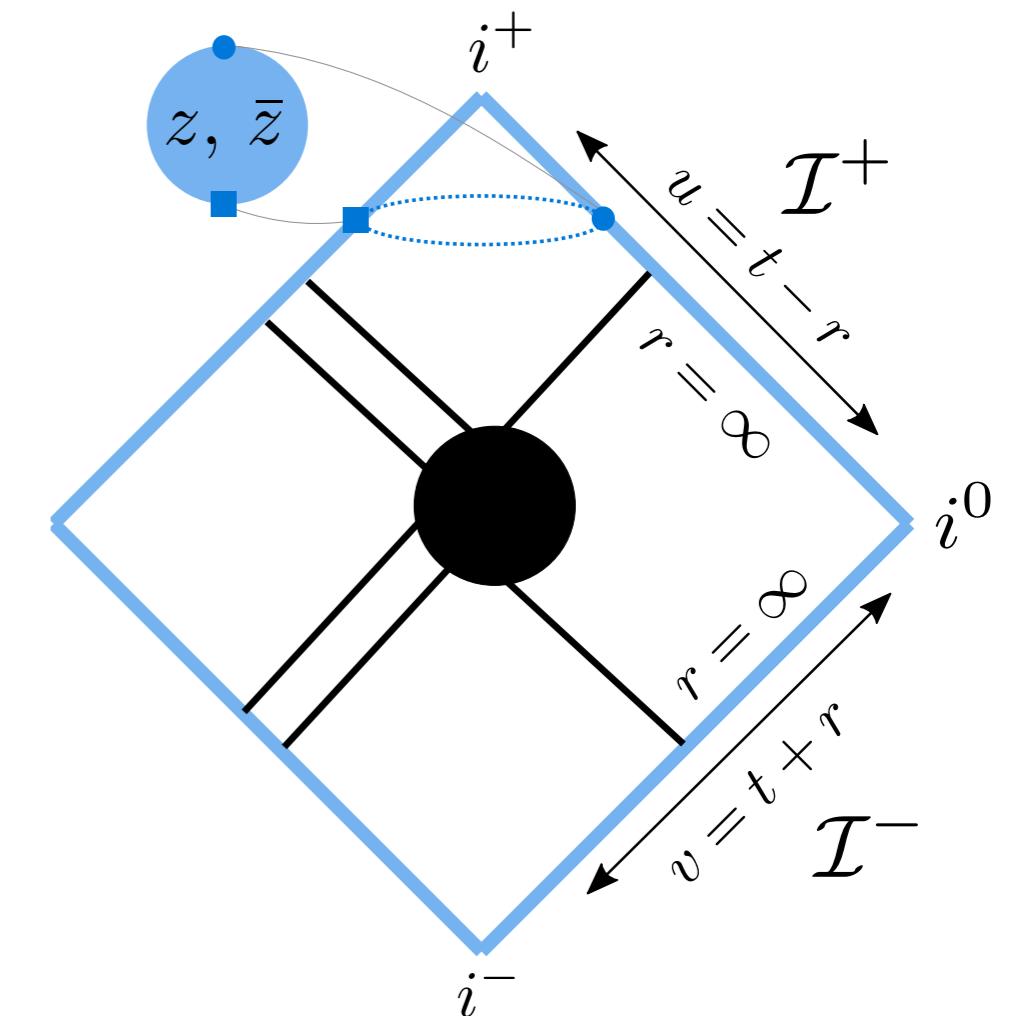
basic observable in quantum gravity in asymptotically flat space

Holographic flavor

Initial and final states for massless particles prepared in terms of data at past and future null infinity \mathcal{I}^- and \mathcal{I}^+ :

$$p_i^\mu = \epsilon_i \omega_i q_i^\mu(z_i, \bar{z}_i)$$

energy ω_i , direction z_i
sign ϵ_i for in (-) and out (+)



S-matrix

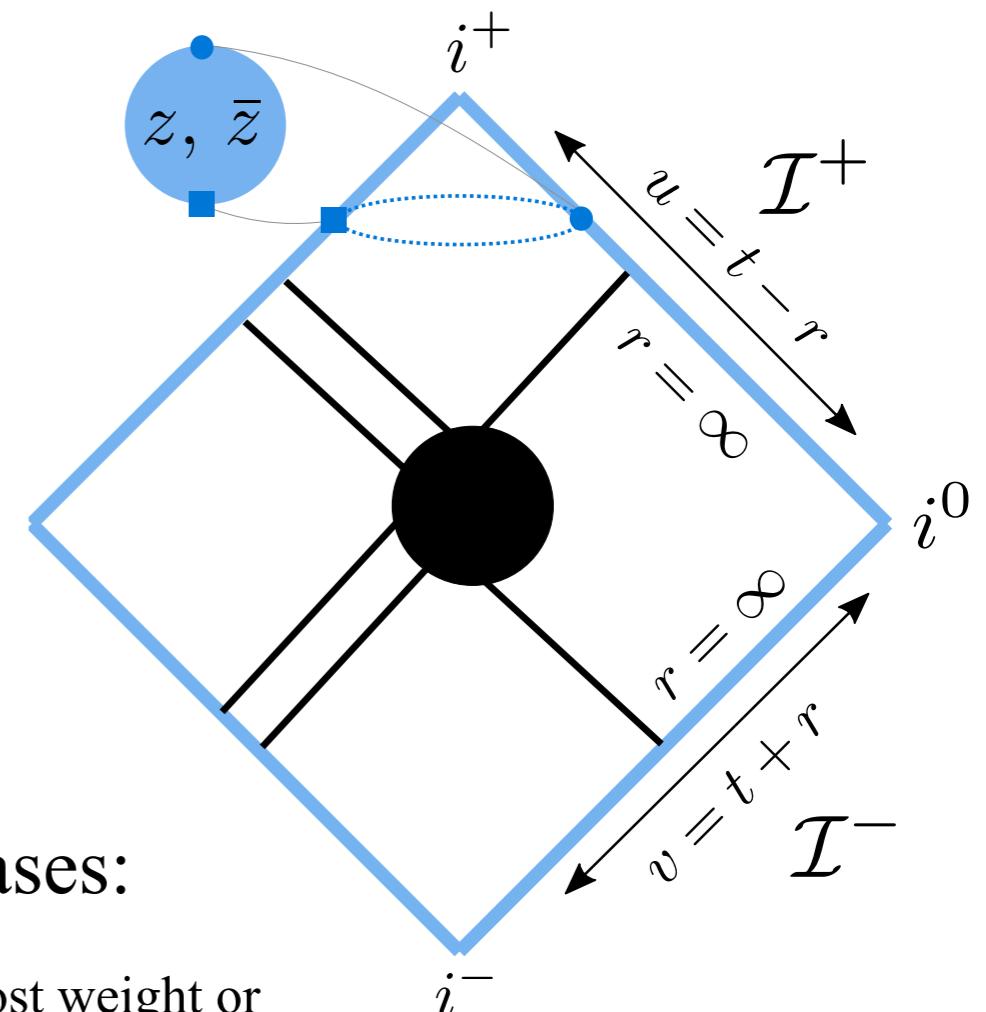
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Asymptotic data can be presented in different bases:

Fourier transform

$$\text{null time} \quad \mathcal{F} \quad \mathcal{M}$$

$$u \longleftrightarrow \omega \longleftrightarrow \Delta$$

boost weight or
Rindler energy

Mellin transform

$$\hat{f}(u) = \int_{-\infty}^{+\infty} d\omega e^{i\omega u} f(\omega)$$

$$\int_0^\infty d\omega \omega^{\Delta-1} f(\omega) = \phi(\Delta)$$

Each basis highlights distinct aspects of physics.

Extrapolate dictionary

Operators living in 3-dimensional space $b \times \mathbb{CP}^1$ where $b \in \{\omega, u, \Delta\}$
defined via boundary limit of bulk operators:

[Donnay,Pasterski,AP]

1. spin- s bulk operator $\hat{O}^s(X)$
2. wavefunction $\Phi_R^s(X; b, z, \bar{z})$ satisfying the free spin- s eom transforming under appropriate bulk (s) and boundary (R) reps of Poincare
3. inner product $(.,.)_\Sigma$ for single particle wavefunction defined on a Cauchy slice Σ , from symplectic product $\Omega(.,.) = i(.,.*)_\Sigma$

we can construct the boundary operator:

$$\mathcal{O}_R^\epsilon(b, z, \bar{z}) = i(\hat{O}^s(X), \Phi_R^s(X_{-\epsilon}, z, \bar{z})^*)_\Sigma$$

incoming vs outgoing modes selected by $X_\epsilon^0 = X^0 \mp i\epsilon$

Extrapolate dictionary

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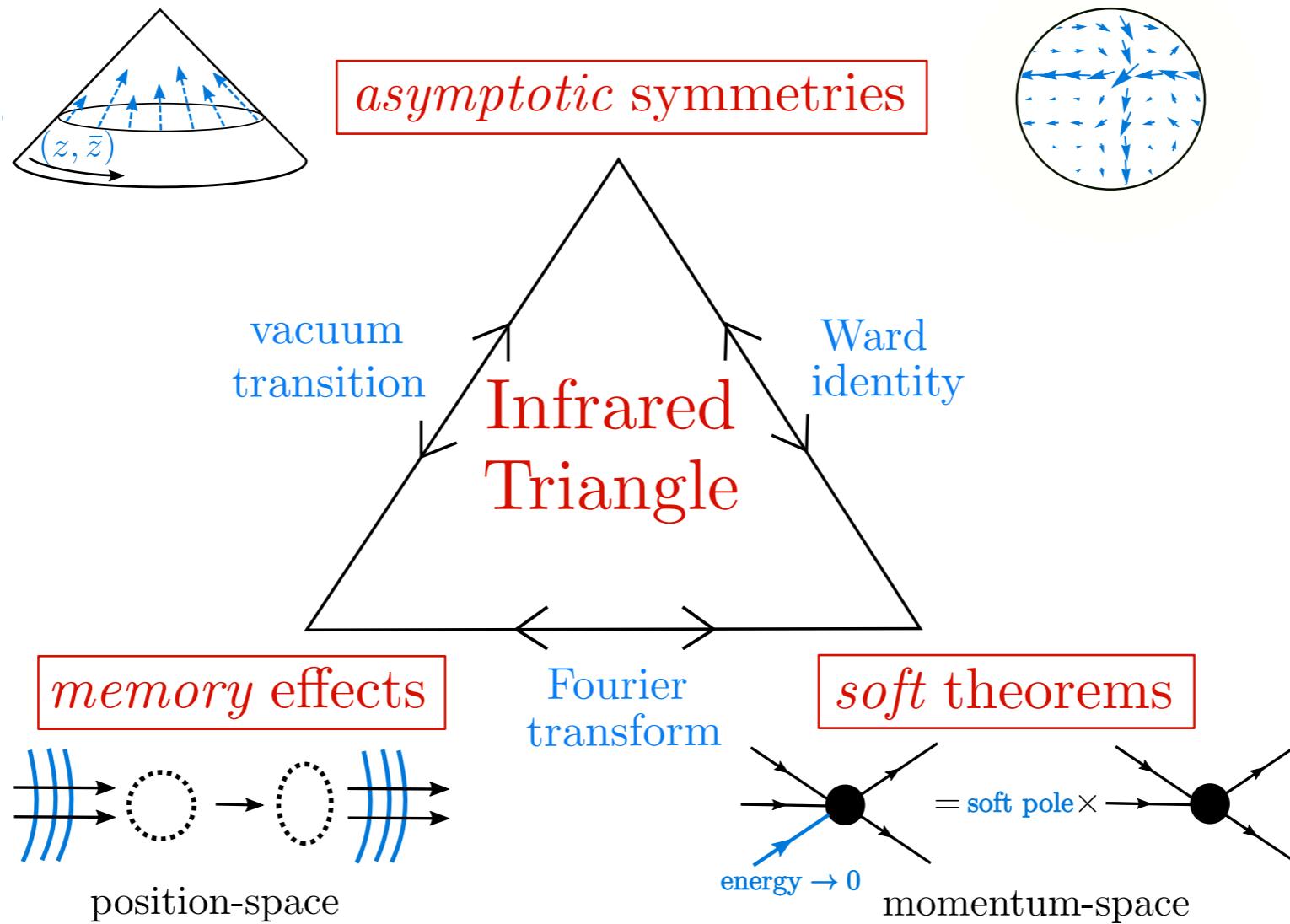
$$\Omega(\Phi, \Phi')_{\Sigma_0} = \int_{\Sigma_0} d^3X^i \Phi \overleftrightarrow{\partial}_{X^0} \Phi' = i(\Phi, \Phi'^*)_{KG}$$

we can construct the boundary operator:

$$\mathcal{O}_R^\epsilon(b, z, \bar{z}) = i(\hat{O}^s(X), \Phi_R^s(X_{-\epsilon}, z, \bar{z})^*)_\Sigma$$

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Infrared aspects of gravity



Memory effects and soft theorems are related by a basis change:

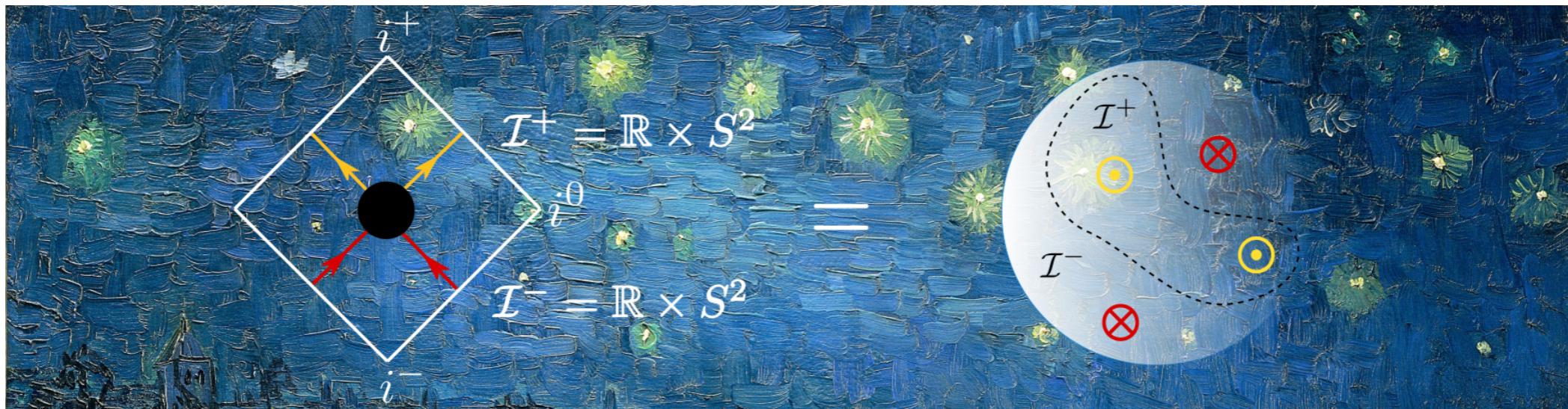
$$u \xleftrightarrow{\mathcal{F}} \omega$$

\Rightarrow uncover potentially observational signatures of soft physics

S-matrix as celestial amplitude

What are all the symmetries of nature?

Aim of celestial holographer: make max # of symmetries manifest to determine constraints on consistent S-matrix $\Rightarrow \Delta$ basis!



$$\omega \xleftarrow{\mathcal{M}} \Delta \xrightarrow{\mathcal{M}}$$

$$\langle p_1^{out} \dots p_n^{out} | \mathcal{S} | p_1^{in} \dots p_n^{in} \rangle$$

$$\langle \mathcal{O}_{\Delta_1, J_1}^{\epsilon_1}(z_1, \bar{z}_1) \dots \mathcal{O}_{\Delta_n, J_n}^{\epsilon_n}(z_n, \bar{z}_n) \rangle$$

\Rightarrow uncover hidden structures / symmetries of amplitudes

Three bases for scattering

position

momentum

boost

$$A(u) \xleftrightarrow{\mathcal{F}} A(\omega) \xleftrightarrow{\mathcal{M}} A(\Delta)$$

Three bases for scattering

position

momentum

boost

$$A(u) \xleftrightarrow{\mathcal{F}} A(\omega) \xleftrightarrow{\mathcal{M}} A(\Delta)$$

memory effects

$$\int_{-\infty}^{+\infty} du n \cdot \mathcal{J} = 0$$

constraint



Three bases for scattering

position

momentum

boost

$$A(u) \xleftrightarrow{\mathcal{F}} A(\omega) \xleftrightarrow{\mathcal{M}} A(\Delta)$$

memory effects

soft theorems

$$\int_{-\infty}^{+\infty} du n \cdot \mathcal{J} = 0$$

constraint

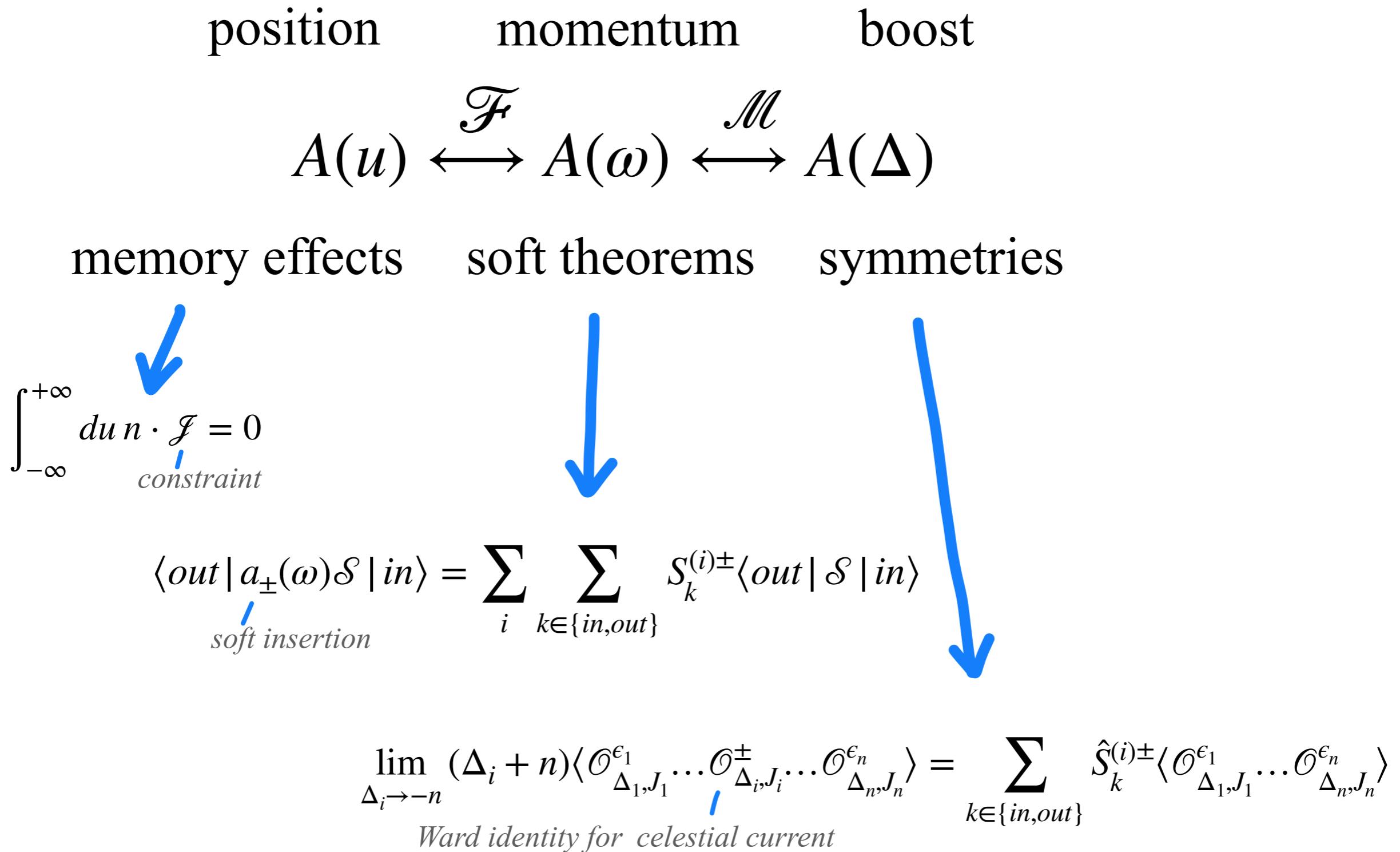
$$\langle out | a_{\pm}(\omega) \mathcal{S} | in \rangle = \sum_i \sum_{k \in \{in, out\}} S_k^{(i)\pm} \langle out | \mathcal{S} | in \rangle$$

soft insertion

$$\frac{1}{\omega} \quad \omega^0 \quad \omega^1 \quad \dots$$

i=0 1 2 ...

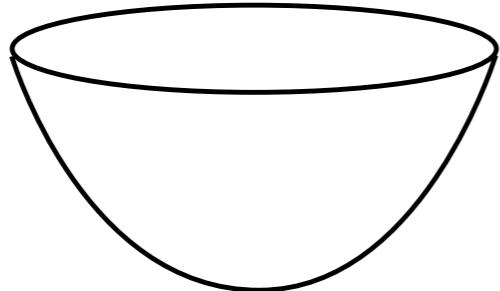
Three bases for scattering



Celestial basis

For S-matrix elements we specify **on-shell momenta** for *in* and *out* states:

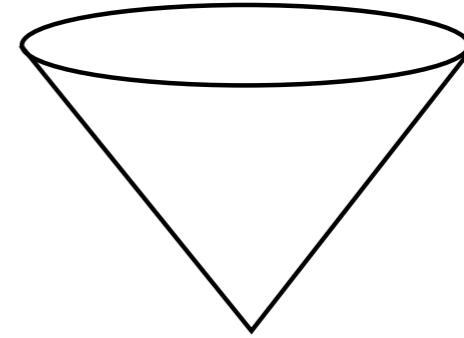
massive particles



$$p^2 = -m^2$$

$$p^\mu = \frac{m}{2y} (1 + y^2 + w\bar{w}, w + \bar{w}, i(\bar{w} - w), 1 - y^2 - w\bar{w})$$

massless particles



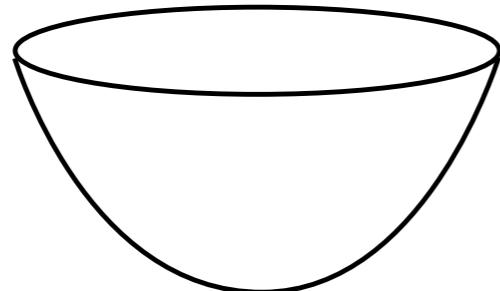
$$p^2 = 0$$

$$p^\mu = \omega (1 + z\bar{z}, z + \bar{z}, i(\bar{z} - z), 1 - z\bar{z})$$

Celestial basis

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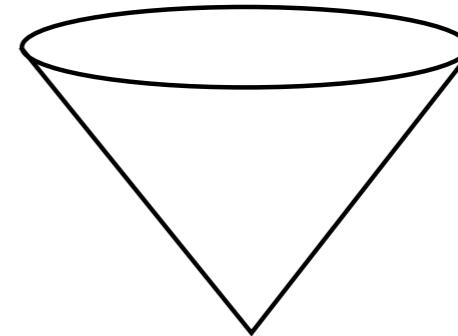
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massless particles



$$p^2 = 0$$

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Holographic map:

[de Boer,Solodukhin]
[Pasterski,Shao,Strominger]
[Cheung,de la Fuente,Sundrum]

$$\int_0^\infty \frac{dy}{y^3} \int d^2w G_\Delta(y, w, \bar{w}; z, \bar{z})(.)$$

bulk-to-boundary
propagator

$$\int_0^\infty \frac{d\omega}{\omega} \omega^\Delta(.)$$

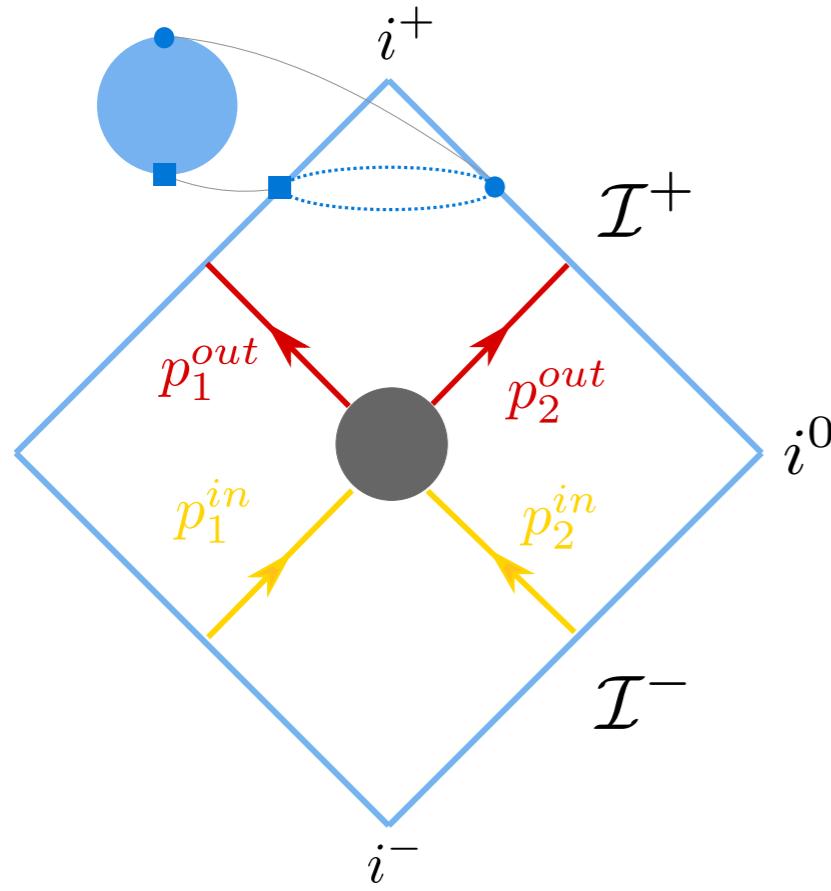
Mellin transform

For CFT correlators we specify the **conformal dimension Δ** of the operators and the point (z, \bar{z}) on S^2 where they are inserted.

Celestial amplitude

For massless scattering the map is a Mellin transform:

[de Boer,Solodukhin]
 [Pasterski,Shao,Strominger]
 [Cheung,de la Fuente,Sundrum]

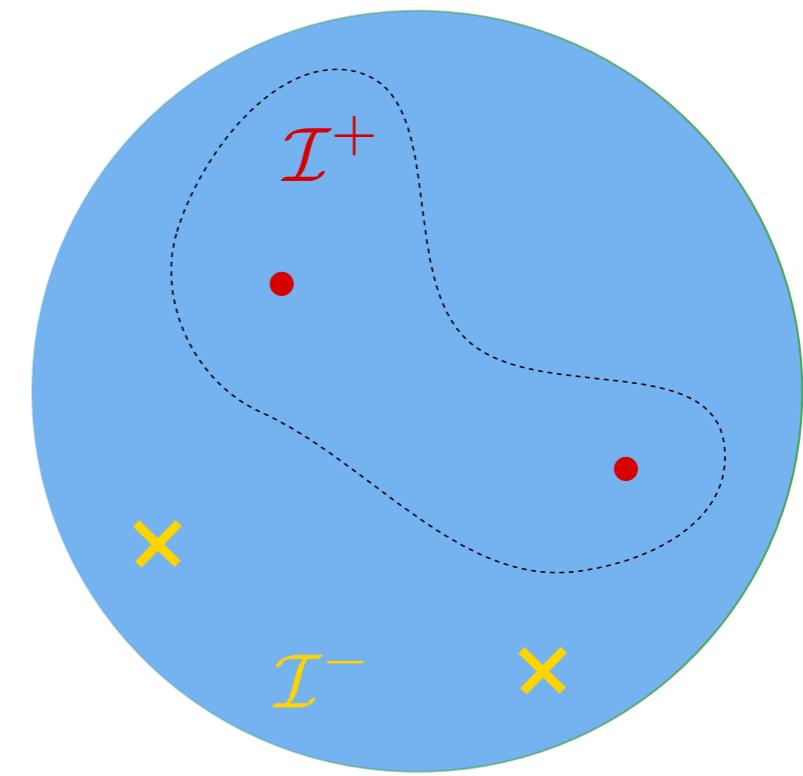


momentum-space amplitude

$$\mathcal{A}_n(\omega_i, \ell_i, z_i, \bar{z}_i) \equiv \langle out | \mathcal{S} | in \rangle$$

manifest translation symmetry

$$\int_0^\infty d\omega \omega^{\Delta-1}(.)$$



celestial amplitude

$$\mathcal{M}_n(\Delta_i, J_i, z_i, \bar{z}_i) \equiv \langle \prod_{i=1}^n \mathcal{O}_{\Delta_i, J_i}^{\epsilon_i}(z_i, \bar{z}_i) \rangle$$

manifest conformal symmetry

Celestial Operators

$$\mathcal{O}_{\Delta,J}^{\pm}(z, \bar{z}) = i(\hat{O}^s(X), \Phi_{\Delta,J}(X_{\mp}; z, \bar{z})^*)_{\Sigma}$$

[Donnay,Pasterki,AP]

Conformal primary wavefunction:

[Pasterki,Shao,Strominger]

$$\Phi_{\Delta,J}^s\left(\Lambda^{\mu}_{\nu}X^{\nu}; \frac{az+b}{cz+d}, \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}\right) = (cz+d)^{\Delta+J}(\bar{c}\bar{z}+\bar{d})^{\Delta-J} \underbrace{D_s(\Lambda)}_{3+1\text{D spin-}s\text{ representation of the Lorentz algebra}} \Phi_{\Delta,J}^s(X^{\mu}; z, \bar{z})$$

3+1D spin- s representation of the Lorentz algebra

bulk point $X^{\mu} \mapsto \Lambda^{\mu}_{\nu}X^{\nu}$

Lorentz transformation

boundary point $z \mapsto \frac{az+b}{cz+d}$ $\bar{z} \mapsto \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}$

$$ad - bc = 1 = \bar{a}\bar{d} - \bar{b}\bar{c}$$

conformal transformation

4D spin- s field under Lorentz transformations

2D conformal primary with conformal dimension Δ and spin J

Conformal primary wavefunctions

Mellin transform of plane wave:

$$\phi_{\Delta}(X_{\pm}^{\mu}; w, \bar{w}) = \int_0^{\infty} d\omega \omega^{\Delta-1} e^{\pm i\omega q \cdot X_{\pm}} = \frac{(\mp i)^{\Delta} \Gamma(\Delta)}{(-q \cdot X_{\pm})^{\Delta}}$$

\uparrow
 $p^{\mu} = \pm \omega q^{\mu}$

polarizations \leftarrow

$$\sqrt{2}\epsilon_{+}^{\mu} = \partial_z q^{\mu}$$
$$\sqrt{2}\epsilon_{-}^{\mu} = \partial_{\bar{z}} q^{\mu}$$
$$q^{\mu} = (1 + z\bar{z}, z + \bar{z}, i(\bar{z} - z), 1 - z\bar{z})$$

[Pasterki, Shao]

Conformal primary wavefunctions

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 $p^\mu = \pm \omega q^\mu$

$\sqrt{2}\epsilon_+^\mu = \partial_z q^\mu$ polarizations
 $\sqrt{2}\epsilon_-^\mu = \partial_{\bar{z}} q^\mu$ \leftarrow
 $q^\mu = (1 + z\bar{z}, z + \bar{z}, i(\bar{z} - z), 1 - z\bar{z})$

[Pasterki, Shao]

[Pasterki, AP]

Spinning conformal primaries:

$$\varphi_\Delta = \frac{1}{(-q \cdot X)^\Delta} \quad \&$$

$$m^\mu = \epsilon_+^\mu + \frac{\epsilon_+ \cdot X}{(-q \cdot X)} q^\mu \quad J = +1$$

$$\bar{m}^\mu = \epsilon_-^\mu + \frac{\epsilon_- \cdot X}{(-q \cdot X)} q^\mu \quad J = -1$$

$w/ \quad \& \quad \Delta = 0$

$$A_{\Delta, J=+1; \mu} = m_\mu \varphi_\Delta$$

$$A_{\Delta, J=-1; \mu} = \bar{m}_\mu \varphi_\Delta$$

$$h_{\Delta, J=+2; \mu\nu} = m_\mu m_\nu \varphi_\Delta$$

$$h_{\Delta, J=-2; \mu\nu} = \bar{m}_\mu \bar{m}_\nu \varphi_\Delta$$

Spectrum

$$\int \frac{d\omega}{\omega} \omega^\Delta \quad \text{what is } \Delta? \quad \Delta \in \mathbb{R}, \mathbb{Z}, \mathbb{C}, \text{free parameter?}$$

Conformal primary wavefunctions $\Phi_{\Delta,J}(X; z, \bar{z})$ [Pasterki,Shao]

form a δ -fct normalizable basis when the conformal dimension is
 $\Delta \in 1 + i\mathbb{R}$... principal continuous series of the $SL(2, \mathbb{C})$ Lorentz group.

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$$\text{→ } \widetilde{\mathcal{O}}_{2-\Delta,-J}(z, \bar{z}) = \frac{k_{\Delta,J}}{2\pi} \int d^2 z' \frac{\mathcal{O}_{\Delta,J}(z', \bar{z}')}{(z - z')^{2-\Delta-J} (\bar{z} - \bar{z}')^{2-\Delta+J}}$$

Conformal primary *shadow* wavefunctions $\widetilde{\Phi}_{2-\Delta,-J}(X; z, \bar{z})$

form equally good δ -fct normalizable basis when $\Delta \in 1 + i\mathbb{R}$.

Spectrum

$$\int \frac{d\omega}{\omega} \omega^\Delta \quad \text{what is } \Delta? \quad \Delta \in \mathbb{R}, \mathbb{Z}, \mathbb{C}, \text{free parameter?}$$

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Conformal primary *shadow* wavefunctions $\widetilde{\Phi}_{2-\Delta,-J}(X; z, \bar{z})$

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Asymptotic symmetries and *conformally* soft factorization of celestial amplitudes for $\Delta \in 1 - \mathbb{Z}_{\geq 0}$ ($s \in \mathbb{Z}$)!

Obtained by analytic continuation off the principal series.

[Donnay,Pasterski,AP]

Analytic structure I

(focus on massless four-point scattering of scalars)

All information about the scattering is encoded in the analytic structure of celestial amplitudes as functions of **net boost weight β** dual to the center of mass energy and a **conformally invariant cross ratio on the sphere z** related to the bulk scattering angle:

$$\mathcal{A}_4(p_1, p_2, p_3, p_4) = A(s, t) \delta^{(4)}(p_1 + p_2 + p_3 + p_4)$$

[Stieberger,Taylor]

$$\mathcal{M}_4(\Delta_i, z_i, \bar{z}_i) = K(\Delta_i, z_i, \bar{z}_i) \int_0^\infty d\omega \omega^{\beta-1} A(\omega^2, -\omega^2 z) \delta(z - \bar{z})$$

[Arkani-Hamed,Pate,Raclariu,Strominger]

conformally covariant structure +
universal conformally invariant function

$$\beta \equiv \sum_{i=1}^4 \Delta_i$$

$$z \equiv -\frac{t}{s} = \frac{z_{13}z_{24}}{z_{12}z_{34}}$$

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↓ \mathcal{M}
 ↘
 conformally covariant structure +
 universal conformally invariant function

$$\beta \equiv \sum_{i=1}^4 \Delta_i$$

$$z \equiv -\frac{t}{s} = \frac{z_{13}z_{24}}{z_{12}z_{34}}$$

Anti-Wilsonian paradigm: *integral is over all center of mass energies!*

Thus momentum-space amplitudes need to have sufficiently soft behavior at high energies for the celestial amplitudes to be well-defined.

Analytic structure II

Low point celestial amplitudes have kinematic singularities:

$$\mathcal{A}_n(p_j^\mu) = A(p_j^\mu) \delta^{(4)}\left(\sum_j p_j^\mu\right)$$

[Pasterski,Shao,Strominger]
[Schreiber,Volovich,Zlotnikov]

Analytic structure II

Low point celestial amplitudes have kinematic singularities:

n Mellin integrals:

$$\prod_{j=1}^n \int_0^\infty \frac{d\omega_j}{\omega_j} \omega_j^{\Delta_j}(\cdot) = \int_0^\infty d\Omega \Omega^{\sum_i \Delta_i - 1} \prod_{j=1}^n \int_0^1 d\sigma_j \sigma_j^{\Delta_j - 1} \delta(\sum_i \sigma_i - 1)(\cdot)$$

$\Omega \equiv \sum_j \omega_j$ $\sigma_j = \frac{\omega_j}{\Omega}$

A_n(p_j^μ) = A(p_j^μ) δ⁽⁴⁾($\sum_j p_j^\mu$ *)*

n < 5: $\mathcal{M}_n(\Delta_j; z_j, \bar{z}_j)$ is singular

[Pasterski,Shao,Strominger]
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Analytic structure II

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\uparrow \uparrow

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n < 5: $\mathcal{M}_n(\Delta_j; z_j, \bar{z}_j)$ is singular

[Pasterski,Shao,Strominger]
[Schreiber,Volovich,Zlotnikov]

Recent work δ -fct \mapsto power-law using shadow & twistor transforms

[Fan,Fotopoulos,Stieberger,Taylor,Zhu]
[Crawley,Miller,Narayanan,Strominger]
[Sharma]
[He,Lippstreu,Spradlin,Srikant,Volovich]

and by computing celestial amplitudes on non-trivial backgrounds.

relates celestial amplitudes to AdS Witten diagrams

[Fan,Fotopoulos,Stieberger,Taylor,Zhu]

[Casali,Melton,Strominger]

Gravity = Gauge Theory²

[Bern,Carrasco,Johansson]

manifest translation symmetry

gluon amplitude

$$\mathcal{A}_n^{YM} = \delta^{(4)}\left(\sum_{i=1}^n p_i^\mu\right) \sum_{\gamma \in \Gamma} \frac{c_\gamma n_\gamma}{\Pi_\gamma}$$

color kinematic
propagator

double copy \downarrow

$$c \mapsto n$$

kinematic functions

$$\mathcal{A}_n^G = \delta^{(4)}\left(\sum_{i=1}^n p_i^\mu\right) \sum_{\gamma \in \Gamma} \frac{n_\gamma^2}{\Pi_\gamma}$$

$n(\omega)$

graviton amplitude

Gravity = Gauge Theory²

[Bern,Carrasco,Johansson]

manifest translation symmetry

$$\mathcal{M}(.) = \int_0^\infty \frac{d\omega}{\omega} \omega^\Delta(.)$$

[Casali,AP]

manifest conformal symmetry

gluon amplitude

$$\mathcal{A}_n^{YM} = e^{ip \cdot X} \delta^{(4)}\left(\sum_{i=1}^n p_i^\mu\right) \sum_{\gamma \in \Gamma} \frac{c_\gamma n_\gamma}{\prod_\gamma \text{propagator}}$$

double copy

$$c \mapsto n$$

$$\mathcal{A}_n^G = \delta^{(4)}\left(\sum_{i=1}^n p_i^\mu\right) \sum_{\gamma \in \Gamma} \frac{n_\gamma^2}{\prod_\gamma}$$

graviton amplitude

| | | |
|------------------------------------|-----------|---|
| kinematic functions $n(\omega)$ | \mapsto | kinematic operators $\mathcal{N}(e^{\partial_\Delta})$ |
|------------------------------------|-----------|---|

celestial gluon amplitude

$$\mathcal{M}_n^{YM} = \sum_{\gamma \in \Gamma} c_\gamma \mathcal{N}_\gamma \mathcal{M}_n^{\text{scalar}}$$

celestial double copy

$$\mathcal{M}_n^G = \sum_{\gamma \in \Gamma} \mathcal{N}_\gamma^2 \mathcal{M}_n^{\text{scalar}}$$

celestial graviton amplitude

\Rightarrow Operator-valued *celestial double copy*

Gravity = Gauge Theory²

[Bern,Carrasco,Johansson]

manifest translation symmetry

$$\mathcal{M}(.) = \int_0^\infty \frac{d\omega}{\omega} \omega^\Delta(.)$$

[Casali,AP]

manifest conformal symmetry

gluon amplitude

$$\mathcal{A}_n^{YM} = e^{ip \cdot X} \delta^{(4)}\left(\sum_{i=1}^n p_i^\mu\right) \sum_{\gamma \in \Gamma} \frac{c_\gamma n_\gamma}{\prod_\gamma \text{propagator}}$$

double copy

$$c \mapsto n$$

$$\mathcal{A}_n^G = \delta^{(4)}\left(\sum_{i=1}^n p_i^\mu\right) \sum_{\gamma \in \Gamma} \frac{n_\gamma^2}{\prod_\gamma}$$

graviton amplitude

| | | |
|------------------------------------|-----------|---|
| kinematic functions $n(\omega)$ | \mapsto | kinematic operators $\mathcal{N}(e^{\partial_\Delta})$ |
|------------------------------------|-----------|---|

celestial gluon amplitude

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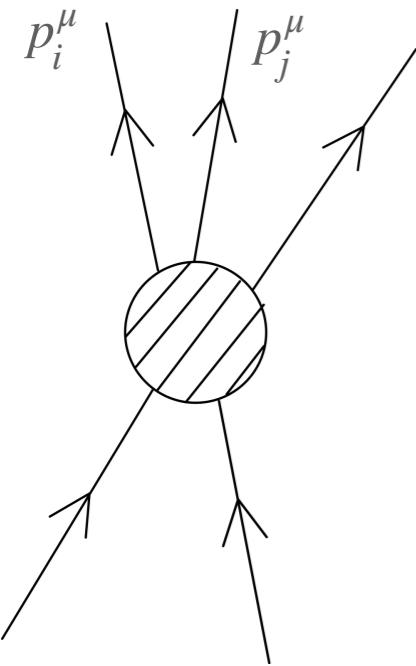
\Rightarrow Operator-valued *celestial double copy*

Path towards double copy in curved space operator-valued?

E.g. operator-valued double copy for ambitwistor strings in AdS

[Roehrig,Skinner]
[Eberhardt,Komatsu,Mizera]

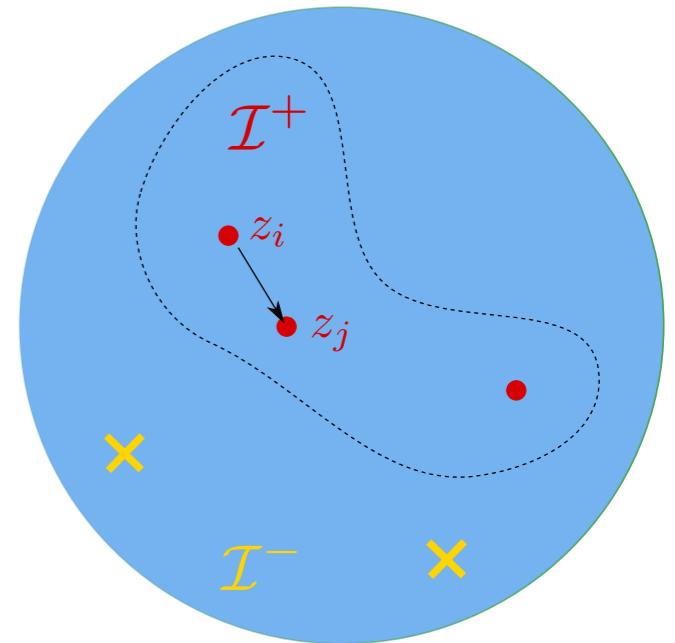
OPEs from collinear limits I



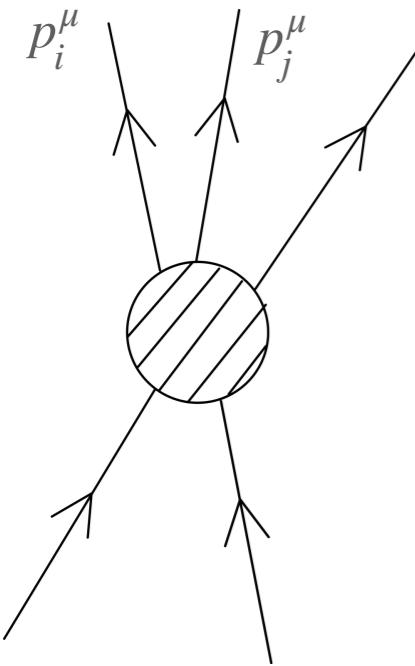
The collinear limit of 4D momentum space scattering amplitudes extracts the singularities in the 2D celestial operator product expansion (OPE):

$$p_i^\mu | | p_j^\mu \quad \leftrightarrow \quad z_{ij} \equiv z_i - z_j \rightarrow 0$$
$$p_i^\mu = \epsilon_i \omega_i(1 + z_i \bar{z}_i, z_i + \bar{z}_i, i(\bar{z}_i - z_i), 1 - z_i \bar{z}_i)$$

'holomorphic' limit



OPEs from collinear limits I

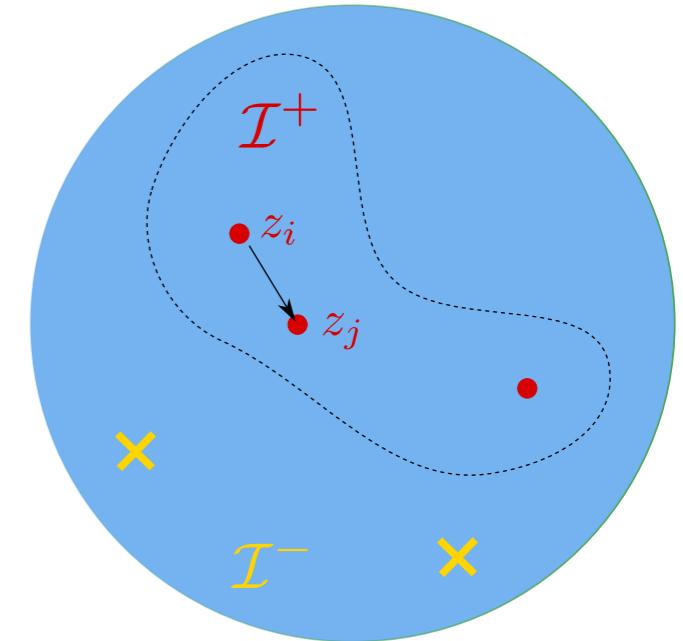


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'holomorphic' limit



Collinear divergences from "splitting" of massless virtual particle into collinear pair

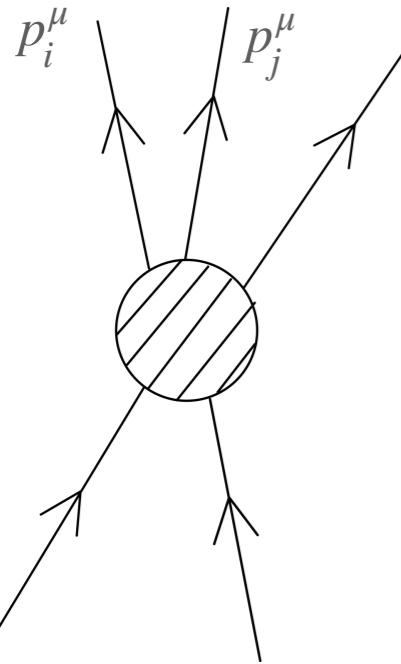
$$\frac{1}{(p_i + p_j)^2} \sim \frac{1}{p_i \cdot p_j}$$

universal form extracted from 3-pt vertices

$$\mathcal{A}_{\ell_1, \dots, \ell_n}(p_1, \dots, p_n) \xrightarrow{z_i \rightarrow z_j} \sum_{\ell \in \pm s} \text{Split}_{\ell_i \ell_j}^\ell(p_i, p_j) \mathcal{A}_{\ell_1 \dots \ell \dots \ell_n}(p_1, \dots, P, \dots, p_n)$$

combined momentum of collinear pair: $P^\mu = p_i^\mu + p_j^\mu$ $\omega_p = \omega_i + \omega_j$

OPEs from collinear limits II



The collinear limit of 4D momentum space scattering amplitudes extracts the singularities in the 2D celestial operator product expansion (OPE):

Mellin transform and
change variables

$$\omega_i = t\omega_P \quad \omega_j = (1-t)\omega_P$$

only t-dependence from splitting factor:

$$B(x, y) = \int_0^1 dt t^{x-1} (1-t)^{y-1} = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

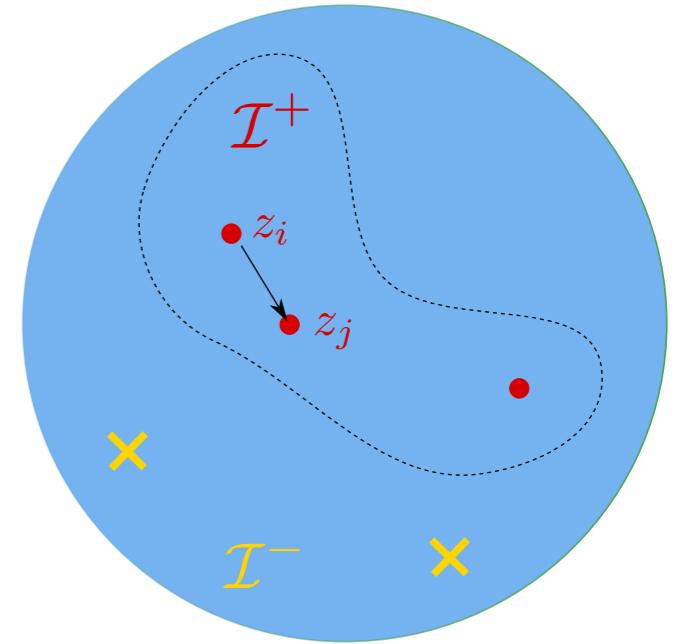
[Fan,Fotopoulos,Taylor]
[Fotopoulos,Stieberger,Taylor,Zhu]

$$\text{Split}_{11}^1(p_i, p_j) = \frac{1}{z_{ij}} \frac{\omega_P}{\omega_i \omega_j}$$

$$\mathcal{O}_{\Delta_i+1}^a(z_i, \bar{z}_i) \mathcal{O}_{\Delta_j+1}^b(z_j, \bar{z}_j) \sim - \frac{i f_c^{ab}}{z_{ij}} B(\Delta_i - 1, \Delta_j - 1) \mathcal{O}_{\Delta_i + \Delta_j - 1, +1}^c(z_j, \bar{z}_j)$$

$$\text{Split}_{22}^2(p_i, p_j) = - \frac{\kappa}{2} \frac{\bar{z}_{ij}}{z_{ij}} \frac{\omega_P^2}{\omega_i \omega_j}$$

$$\mathcal{O}_{\Delta_i+2}(z_i, \bar{z}_i) \mathcal{O}_{\Delta_j+2}(z_j, \bar{z}_j) \sim - \frac{\kappa}{2} \frac{\bar{z}_{ij}}{z_{ij}} B(\Delta_i - 1, \Delta_j - 1) \mathcal{O}_{\Delta_i + \Delta_j, +2}^c(z_j, \bar{z}_j)$$



Celestial OPEs also from ambitwistor worldsheet.

[Adamo,Bu,Casali,Sharma]

Poincaré symmetry

Lorentz transformations: $L_0, L_{\pm 1}, \bar{L}_0, \bar{L}_{\pm 1}$

$$(h, \bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$
$$[L_m, \mathcal{O}_{h,\bar{h}}^\epsilon(z, \bar{z})] = z^m \left((m+1)h + z\partial_z \right) \mathcal{O}_{h,\bar{h}}^\epsilon(z, \bar{z})$$

Lorentz invariance of celestial amplitudes \rightarrow global conformal Ward identity:

$$\sum_{j=1}^n L_m^{(j)} \mathcal{M}_n = 0$$

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Lorentz invariance of celestial amplitudes \rightarrow **global conformal Ward identity:**

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Translations: $\mathcal{P}^\mu = \pm q^\mu e^{\partial_\Delta}$ **weight-shifting operator** [Stieberger,Taylor]

$$= \pm (1 + z\bar{z}, z + \bar{z}, i(\bar{z} - z), 1 - z\bar{z}) e^{(\partial_h + \partial_{\bar{h}})/2}$$

$$[P_{k,l}, \mathcal{O}_{h,\bar{h}}^\epsilon(z, \bar{z})] = \epsilon z^{k+\frac{1}{2}} \bar{z}^{l+\frac{1}{2}} \mathcal{O}_{h+\frac{1}{2}, \bar{h}+\frac{1}{2}}^\epsilon(z, \bar{z})$$

$$P_{\mp\frac{1}{2}, \mp\frac{1}{2}} = \frac{1}{2}(\mathcal{P}^0 \pm \mathcal{P}^3)$$

$$P_{\pm\frac{1}{2}, \mp\frac{1}{2}} = \frac{1}{2}(\mathcal{P}^1 \pm i\mathcal{P}^2)$$

Translation invariance \rightarrow relates celestial amplitudes with **shifted weights**:

$$\sum_{j=1}^n P_\mu^{(j)} \mathcal{M}_n = 0$$

Poincaré → BMS

$$[L_m, L_n] = (m - n)L_{m+n} \quad [L_m, P_{k,l}] = (\frac{m}{2} - k)P_{k+m,l} \quad [P_{k,l}, P_{k',l'}] = 0$$

$$\begin{array}{c} m, n \in \{-1, 0, +1\} \\ k, l = \pm \frac{1}{2} \end{array} \qquad \rightarrow \qquad \begin{array}{c} m, n \in \mathbb{Z} \\ k, l \in \frac{1}{2}\mathbb{Z} \end{array}$$

BMS supertranslations and superrotations:

$$\xi^\mu|_{\mathcal{J}^+} = \left(f(x^A) + \frac{u}{2} D_A Y^A(x^A) \right) \partial_u + Y^A(x^A) \partial_A$$

[Bondi, van der Burg, Metzner][Sachs]
[de Boer, Solodukhin]
[Barnich, Troessaert]
[Campiglia, Laddha]

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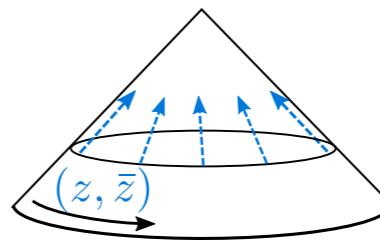
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Translations: $f(x^A)$ e.g. $\xi_{f=1}|_{\mathcal{J}^+} = \partial_u$

Lorentz: $Y^A(x^A)$ global CKVs



4+6=10 Poincare generators

Poincaré → BMS

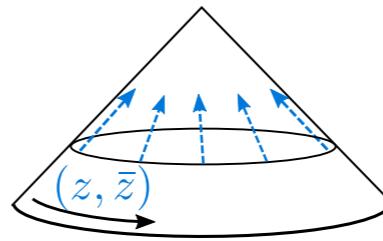
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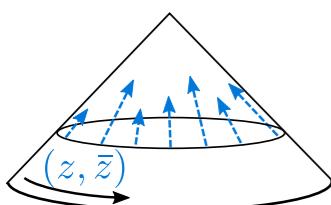
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Lorentz: $Y^A(x^A)$ global CKVs



extended
generalized

$\text{BMS} = \text{Supertranslations} \ltimes \text{Lorentz}$

4+6=10 Poincare generators



∞ generators!

$\text{BMS} = \text{Supertranslations} \ltimes \text{Superrotations}$

Perspective from spatial infinity: talk by M. Henneaux.

$Y^A(x^A)$ local CKVs
arbitrary \rightarrow Virasoro
Diff(S^2)

Conformal Goldstones

$$\mathcal{O}_{\Delta,J} = \Omega(\hat{O}, \Phi_{\Delta,J})_{\Sigma \rightarrow \mathcal{J}}$$

canonical charge for asymptotic symmetry
for values of Δ for which $\Phi_{\Delta,J}$ is pure gauge

Pure gauge wavefunctions:

$$A_{\Delta=1,J;\mu}^G = \nabla_\mu \Lambda_{gauge}$$

[Donnay,AP,Strominger]
[Donnay,Pasterski,AP]

$$h_{\Delta=1,J;\mu\nu}^G = \nabla_{(\mu} \xi_{\nu)}^1 = \nabla_\mu \nabla_\nu \Lambda_{gravity}$$

$$h_{\Delta=0,J;\mu\nu}^G = \nabla_{(\mu} \xi_{\nu)}^0$$

$$\tilde{h}_{\Delta=2,J;\mu\nu}^G = \nabla_{(\mu} \xi_{\nu)}^2$$

Conformal Goldstones

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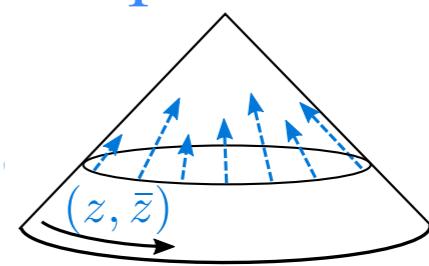
canonical charge for asymptotic symmetry
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Pure gauge wavefunctions:

$$A_{\Delta=1,J;\mu}^G = \nabla_\mu \Lambda_{gauge} \quad \rightarrow \text{large gauge transformation}$$

[Donnay,AP,Strominger]
[Donnay,Pasterski,AP]

$$h_{\Delta=1,J;\mu\nu}^G = \nabla_{(\mu} \xi_{\nu)}^1 = \nabla_\mu \nabla_\nu \Lambda_{gravity} \quad \rightarrow \text{BMS supertranslation}$$



[Campiglia,Laddha]

$\text{Diff}(S^2)$

Virasoro

[Barnich,Troessaert]

$$h_{\Delta=0,J;\mu\nu}^G = \nabla_{(\mu} \xi_{\nu)}^0 \quad \rightarrow$$

superrotation

(related by shadow transform)

[Donnay,Pasterski,AP]

$$\tilde{h}_{\Delta=2,J;\mu\nu}^G = \nabla_{(\mu} \xi_{\nu)}^2$$

Asymptotic symmetries

4D (asymptotic) symmetries generated by 2D celestial currents!

$$\curvearrowleft \mathcal{O}_{\Delta,J} = \Omega(\hat{O}, \Phi_{\Delta,J})_{\Sigma \rightarrow \mathcal{F}}$$

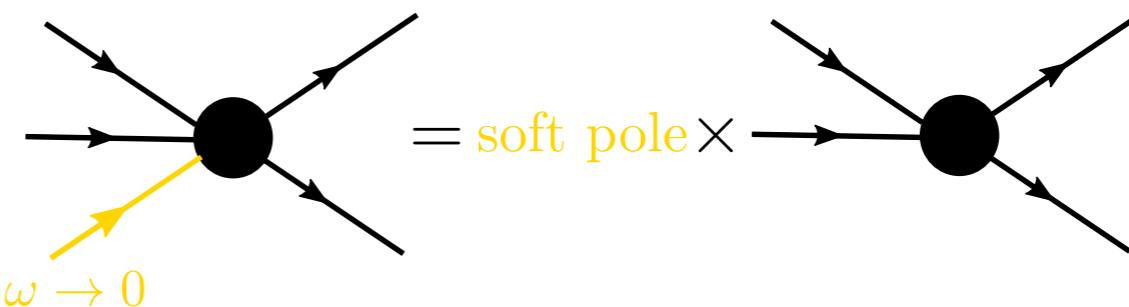
asymptotic symmetry generator $\stackrel{\uparrow}{\text{pure gauge}} \quad \Delta \in \frac{1}{2}\mathbb{Z}$

| $s = J $ | Δ | $\tilde{\Delta} = 2 - \Delta$ | energetically soft pole | celestial current | asymptotic symmetry |
|---------------|---------------|-------------------------------|-------------------------|---------------------------------------|---------------------|
| 1 | 1 | 1 | ω^{-1} | \mathcal{J} | large U(1) |
| $\frac{3}{2}$ | $\frac{1}{2}$ | $\frac{3}{2}$ | $\omega^{-\frac{1}{2}}$ | \mathcal{S} & $\tilde{\mathcal{S}}$ | large SUSY |
| 2 | 1 | 1 | ω^{-1} | \mathcal{P} | supertranslation |
| 2 | 0 | 2 | ω^0 | \mathcal{T} & $\tilde{\mathcal{T}}$ | superrotation |

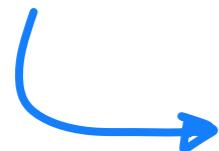
2D stress tensor! [Kapec,Mitra,Raclariu,Strominger]

Conformally soft photon

- * Leading soft photon:

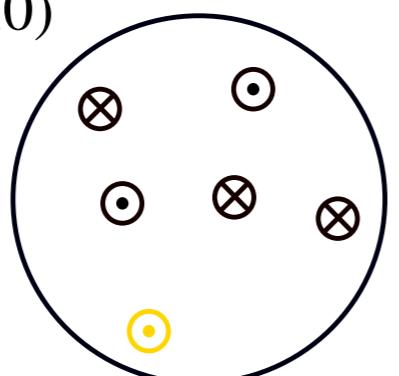


$$\lim_{\Delta \rightarrow 1} (\Delta - 1) \mathcal{M}_n(\Delta, J = +1, z, \bar{z}; \Delta_i, z_i, \bar{z}_i) = \sum_{k=1}^n \frac{Q_k}{z - z_k} \mathcal{M}_n(\Delta_i, z_i, \bar{z}_i)$$

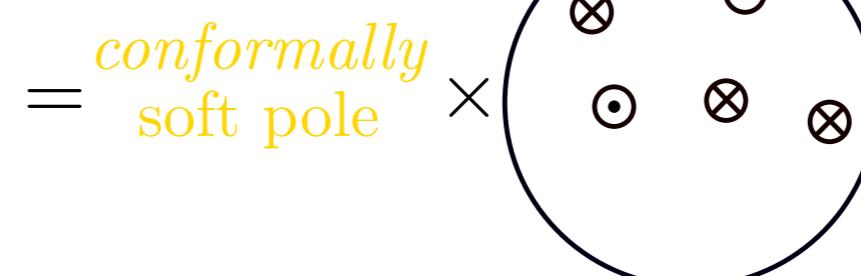


$$\langle \mathcal{J}_z \mathcal{O}_{\Delta_1, J_1} \dots \mathcal{O}_{\Delta_n, J_n} \rangle = \sum_{k=1}^n \frac{Q_k}{z - z_k} \langle \mathcal{O}_{\Delta_1, J_1} \dots \mathcal{O}_{\Delta_n, J_n} \rangle$$

$(h, \bar{h}) = (1, 0)$



conformal weight
 $h \rightarrow 0$



= conformally
soft pole \times

[Fan,Fotopoulos,Taylor]
[Nandan,Schreiber,Volovich,Zlotnikov]
[Pate,Strominger,Raclariu]
[Adamo,Mason,Sharma]

Celestial analogue of 4D soft theorems \Rightarrow 2D Ward identities

Conformally soft graviton

- Leading soft graviton:

[Strominger] [He,Lysov,Mitra,Strominger]
[Adamo,Mason,Scharma] [AP] [Guevara]

$$\lim_{\Delta \rightarrow 1} (\Delta - 1) \mathcal{M}_n(\Delta, J = +2, z, \bar{z}; \Delta_i, z_i, \bar{z}_i) = -\frac{\kappa}{2} \sum_{k=1}^n \frac{\bar{z} - \bar{z}_k}{z - z_k} \mathcal{M}_n(\Delta_k + 1, z_k, \bar{z}_k)$$

$$\text{Circlearrowright}^{\partial \bar{z}} \langle \mathcal{P}_z \mathcal{O}_{\Delta_1, J_1}^{\epsilon_1} \dots \mathcal{O}_{\Delta_n, J_n}^{\epsilon_n} \rangle = -\frac{\kappa}{2} \sum_{k=1}^n \frac{1}{z - z_k} \langle \mathcal{O}_{\Delta_1, J_1}^{\epsilon_1} \dots \mathcal{O}_{\Delta_k+1, J_k}^+ \dots \mathcal{O}_{\Delta_n, J_n}^{\epsilon_n} \rangle$$

shifted weights

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shifted weights



- Sub-leading soft graviton:

shadow of $\Delta = 0$:

$$\langle \mathcal{T}_{zz} \mathcal{O}_{\Delta_1, J_1}^{\epsilon_1} \dots \mathcal{O}_{\Delta_n, J_n}^{\epsilon_n} \rangle = \sum_{k=1}^n \left[\frac{h_j}{(z - z_j)^2} + \frac{\partial_{w_j}}{z - z_j} \right] \langle \mathcal{O}_{\Delta_1, J_1}^{\epsilon_1} \dots \mathcal{O}_{\Delta_n, J_n}^{\epsilon_n} \rangle$$

diagonal in boost basis



2D stress tensor

Enhancement of global to local conformal supports prospect of celestial CFT.

[Barnich,Troessaert]

[Kapec,Mitra,Raclariu,Strominger]
 [Adamo,Mason,Scharma] [Guevara]

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$$\lim_{\Delta \rightarrow 1} (\Delta - 1) \mathcal{M}_n(\Delta, J = +2, z, \bar{z}; \Delta_i, z_i, \bar{z}_i) = -\frac{\kappa}{2} \sum_{k=1}^n \frac{\bar{z} - \bar{z}_k}{z - z_k} \mathcal{M}_n(\Delta_k + 1, z_k, \bar{z}_k)$$

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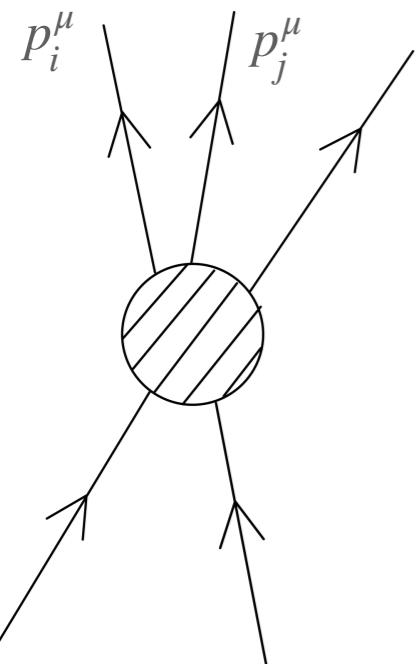
2D stress tensor

Enhancement of global to local conformal supports prospect of celestial CFT.

[Barnich,Troessaert]

Celestial analogue of 4D soft theorems \Rightarrow 2D Ward identities

OPEs from symmetry



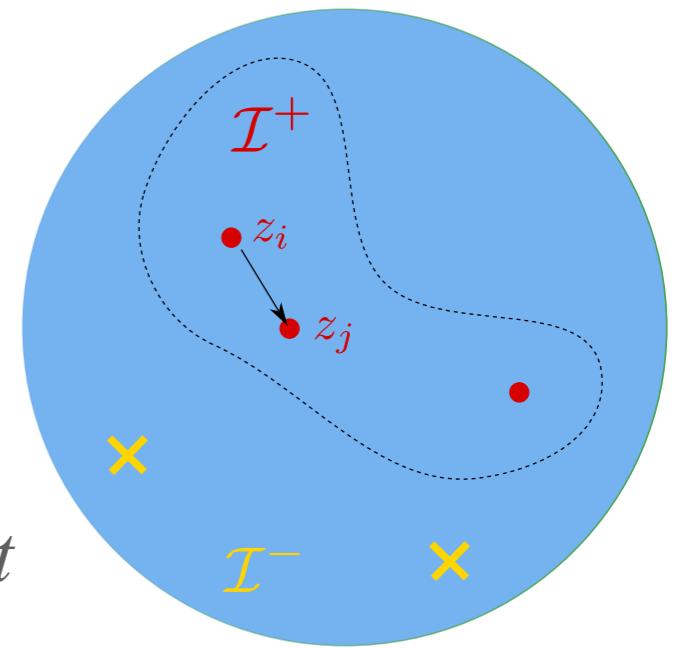
The 2D celestial operator product expansion (OPE) coefficients can be extracted from symmetry.

$$p_i^\mu \parallel p_j^\mu \quad \leftrightarrow \quad z_{ij} \equiv z_i - z_j \rightarrow 0$$

$$p_i^\mu = e_i \omega_i(1 + z_i \bar{z}_i, z_i + \bar{z}_i, i(\bar{z}_i - z_i), 1 - z_i \bar{z}_i)$$

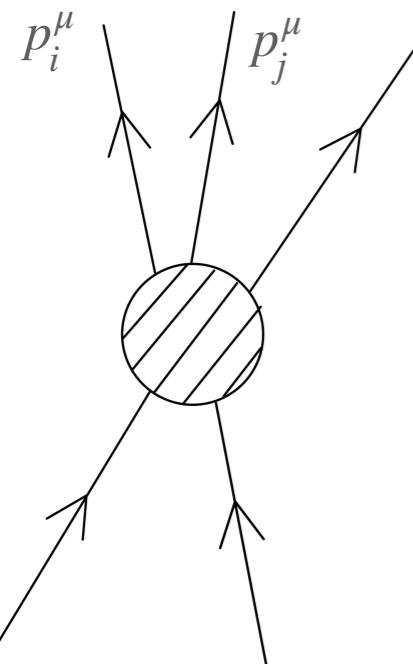
'holomorphic' limit

OPE ansatz
(for gravity): $\mathcal{O}_{\Delta_i+2}(z_i, \bar{z}_i) \mathcal{O}_{\Delta_j+2}(z_j, \bar{z}_j) \sim \frac{\bar{z}_{ij}}{z_{ij}} C(\Delta_i, \Delta_j) \mathcal{O}_{\Delta_i+\Delta_j+2}(z_j, \bar{z}_j)$



[Pate, Raclariu, Strominger, Yuan]

OPEs from symmetry



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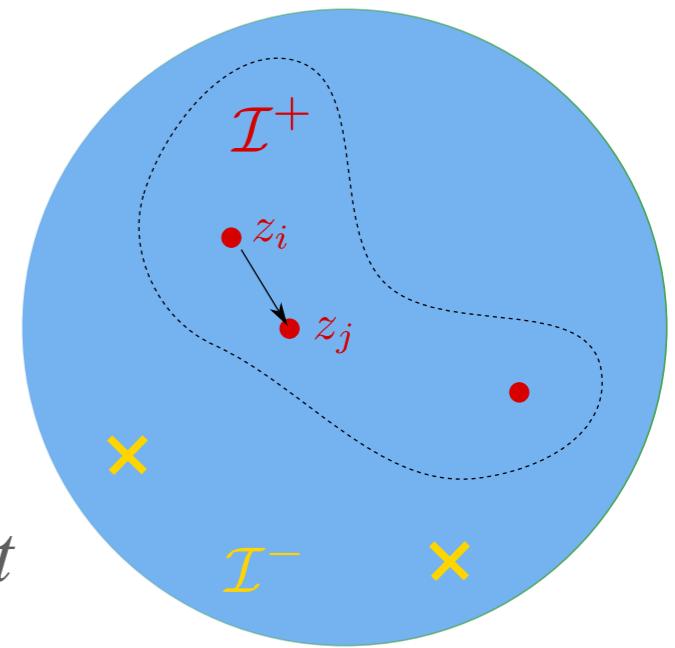
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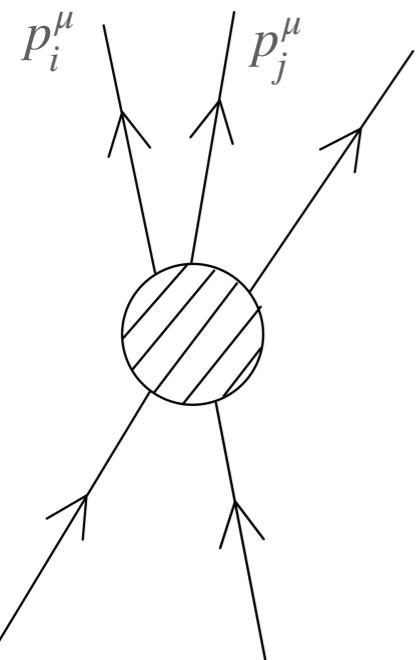
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[Pate, Raclariu, Strominger, Yuan]

- Translation invariance: $C(\Delta_i, \Delta_j) = C(\Delta_i + 1, \Delta_j) + C(\Delta_i, \Delta_j + 1)$



OPEs from symmetry

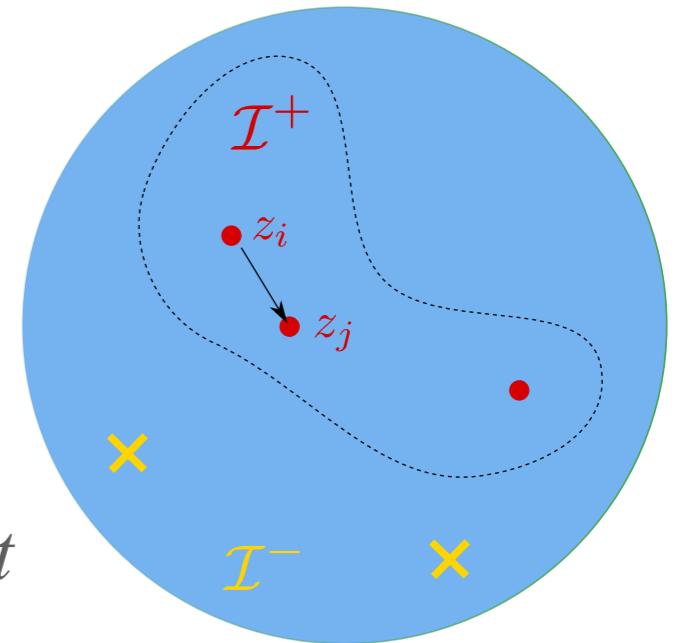


The 2D celestial operator product expansion (OPE) coefficients can be extracted from symmetry.

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'holomorphic' limit

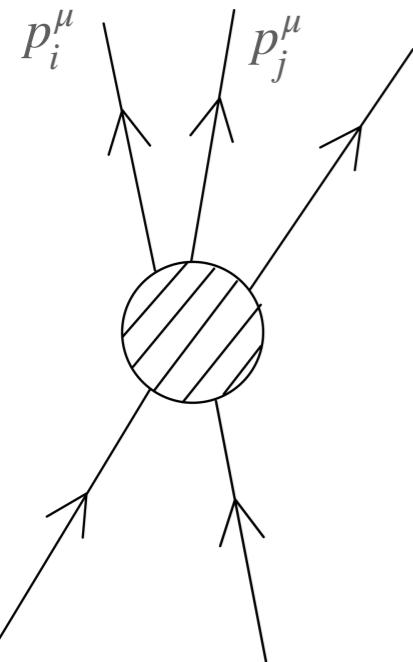


OPE ansatz
(for gravity): $\mathcal{O}_{\Delta_i+2}(z_i, \bar{z}_i) \mathcal{O}_{\Delta_j+2}(z_j, \bar{z}_j) \sim \frac{\bar{z}_{ij}}{z_{ij}} C(\Delta_i, \Delta_j) \mathcal{O}_{\Delta_i+\Delta_j+2}(z_j, \bar{z}_j)$

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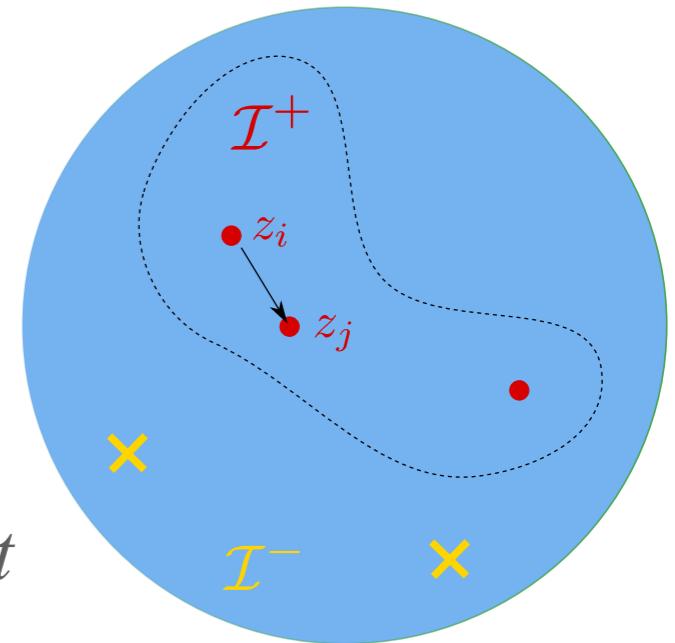


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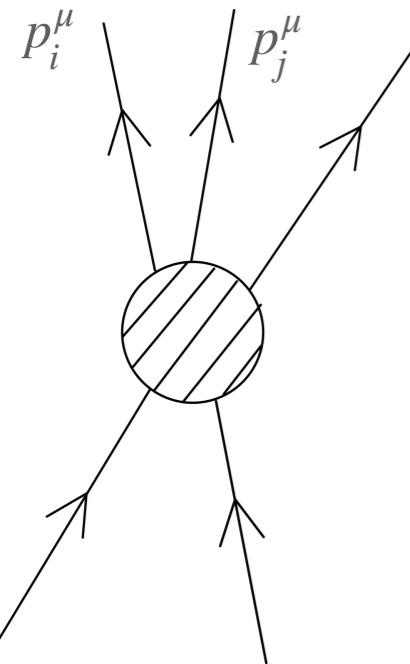


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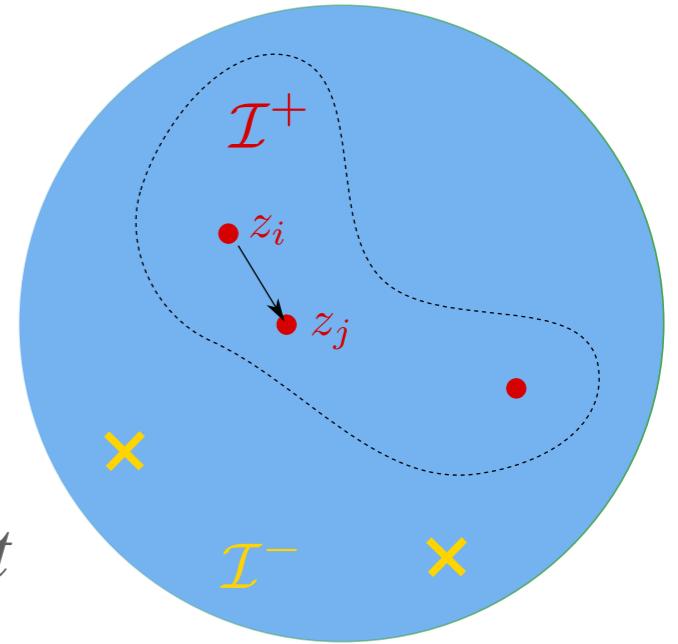


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celestial OPE
of gravitons:

$$\mathcal{O}_{\Delta_i+2}(z_i) \mathcal{O}_{\Delta_j+2}(z_j) \sim -\frac{\kappa}{2} \frac{\bar{z}_{ij}}{z_{ij}} B(\Delta_i - 1, \Delta_j - 1) \mathcal{O}_{\Delta_i+\Delta_j+2}(z_j)$$

Streamlined OPE derivation with descendants from Poincaré symmetry.

[Himwich,Pate,Singh]

Conformally soft symmetries

Celestial operator:

$$\mathcal{O}_{\Delta,J}(z, \bar{z}) = \int_0^\infty d\omega \omega^{\Delta-1} \mathcal{O}_{\ell=J}(\omega, z, \bar{z})$$

Expand for $\omega \ll \omega_*$:

$$\mathcal{O}_J(\omega, z, \bar{z}) = \sum_k \omega^k \mathcal{O}_{J,k}(z, \bar{z})$$

Conformally soft limit:

$$\lim_{\Delta \rightarrow -n} (\Delta + n) \mathcal{O}_{\Delta,J}(z, \bar{z}) = \lim_{\Delta \rightarrow -n} (\Delta + n) \sum_k \int_0^{\omega_*} d\omega \omega^{\Delta+k-1} \mathcal{O}_{J,k} = \mathcal{O}_{J,n}(z, \bar{z})$$

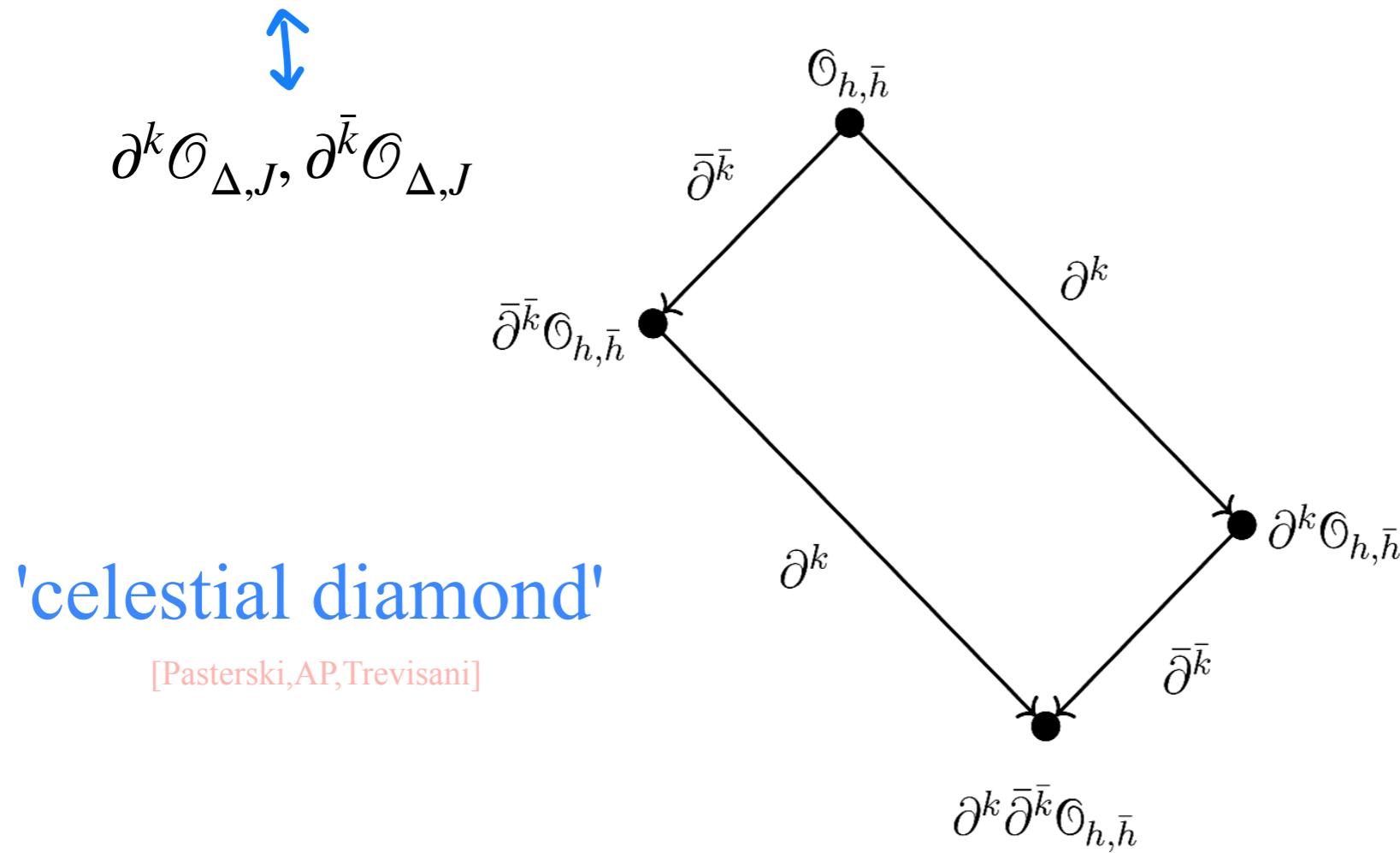
picks out $\mathcal{O}(\omega^n)$ term in expansion around $\omega = 0$. $n = -1, 0, 1, \dots$

Conformal multiplets in CCFT

$\mathcal{O}_{\Delta,J}(0,0)$ creates state $|h, \bar{h}\rangle$ with weights $h = \frac{1}{2}(\Delta + J)$, $\bar{h} = \frac{1}{2}(\Delta - J)$.

For operators with $h = \frac{1-k}{2}$, $\bar{h} = \frac{1-\bar{k}}{2}$ for $k, \bar{k} \in \mathbb{Z}_{>0}$ primary descendants

$L_{-1}^k \mathcal{O}_{\Delta,J}, \bar{L}_{-1}^{\bar{k}} \mathcal{O}_{\Delta,J}$ decouple in correlation functions \rightarrow conservation law.

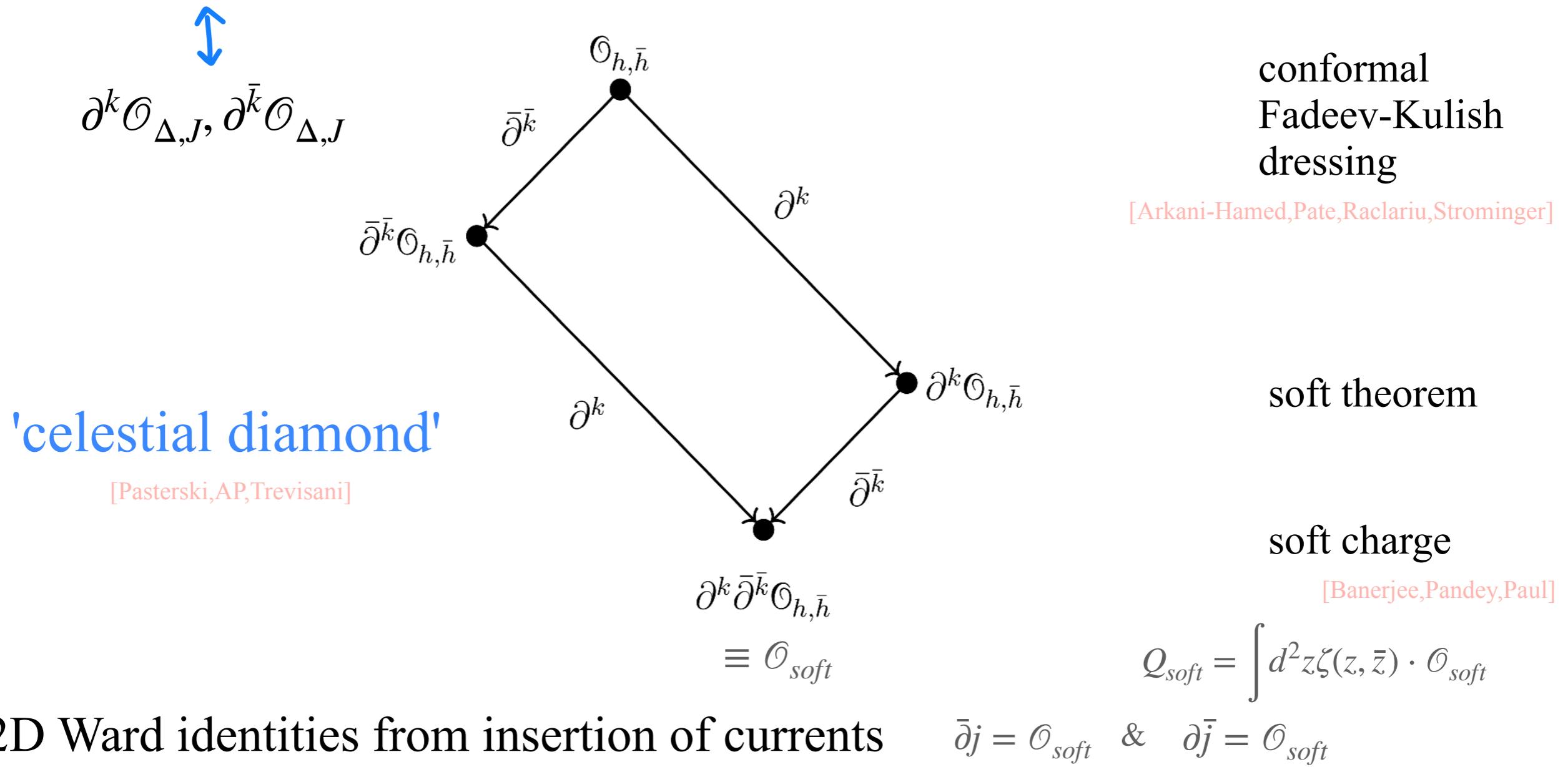


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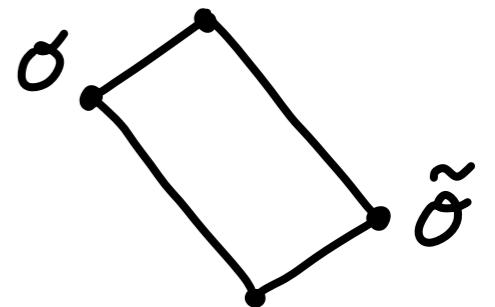
Celestial diamonds

Three types of primary descendants in celestial CFT:

- $1 - s < \Delta < 1 + s$: **Goldstones**

Asymptotic symmetry currents from pure gauge wavefunctions.

[Donnay,AP,Strominger] [Donnay,Pasterski,AP]



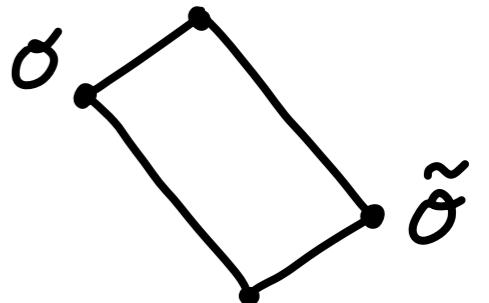
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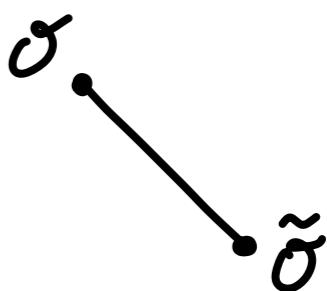
- $\Delta = 1 - s$: **Goldstone-like 'Goldilocks'**

Degenerate diamond: primary with $\Delta = 1 - s$ descends to primary with $\Delta' = 1 + s$. Non-obvious asymptotic symmetry interpretation.

[Campiglia,Laddha] [Freidel,Prancetti,Raclariu] [Donnay,Pasterski,AP]

Important for constraining celestial OPEs

[Pate,Raclariu,Strominger,Yuan]



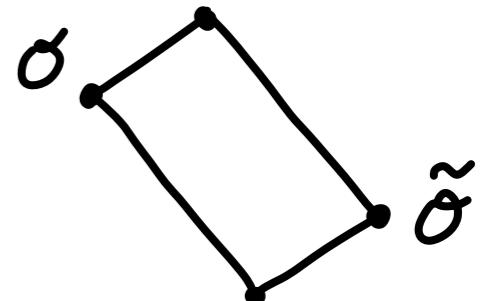
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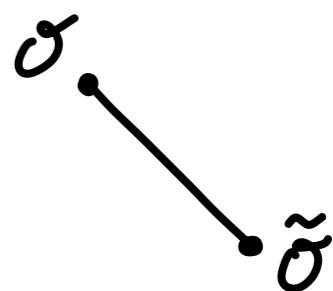


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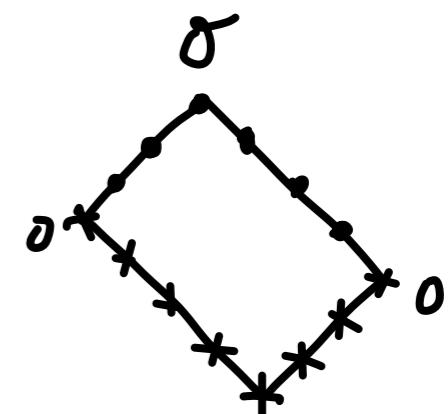


- $\Delta < 1 - s$: **Goldstone-like tower**

Level- n descendant of $\Delta = 1 - s - n$ primary is naively 0.

Holographic symmetry algebras ($w_{1+\infty}$) highly non-trivial!

[Guevara,Himwich,Pate,Strominger] [Strominger]



Holographic symmetry algebras

The tower of celestial symmetry generators in gauge theory and gravity

$$R^\Delta = \lim_{\epsilon \rightarrow 0} \epsilon \mathcal{O}_{\Delta+\epsilon, \pm 1}$$
$$\Delta = 1, 0, -1, \dots$$

$$H^\Delta = \lim_{\epsilon \rightarrow 0} \epsilon \mathcal{O}_{\Delta+\epsilon, \pm 2}$$
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form closed algebra!

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Compute the commutators of modes in truncated antiholomorphic expansion

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from the OPE of conformally soft currents and after rescaling:

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wedge algebra of $w_{1+\infty}$ [Pope]

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wedge algebra of $w_{1+\infty}$ [Pope]

Derivable from asymptotic Einstein equations. [Freidel,Pranzetti,Raclariu]

Exact in self-dual gravity. [Ball,Narayanan,Salzer,Strominger]

$w_{1+\infty}$ from twistor space. [Adamo,Mason,Sharma]

Relation to twisted holography. [Costello,Paquette]

Celestial Triangle

Asymptotic symmetries related to memory effects (IR Triangle).

Their identification can be obscured (subleading soft modes:
divergent gauge parameter or no obvious pure gauge mode).

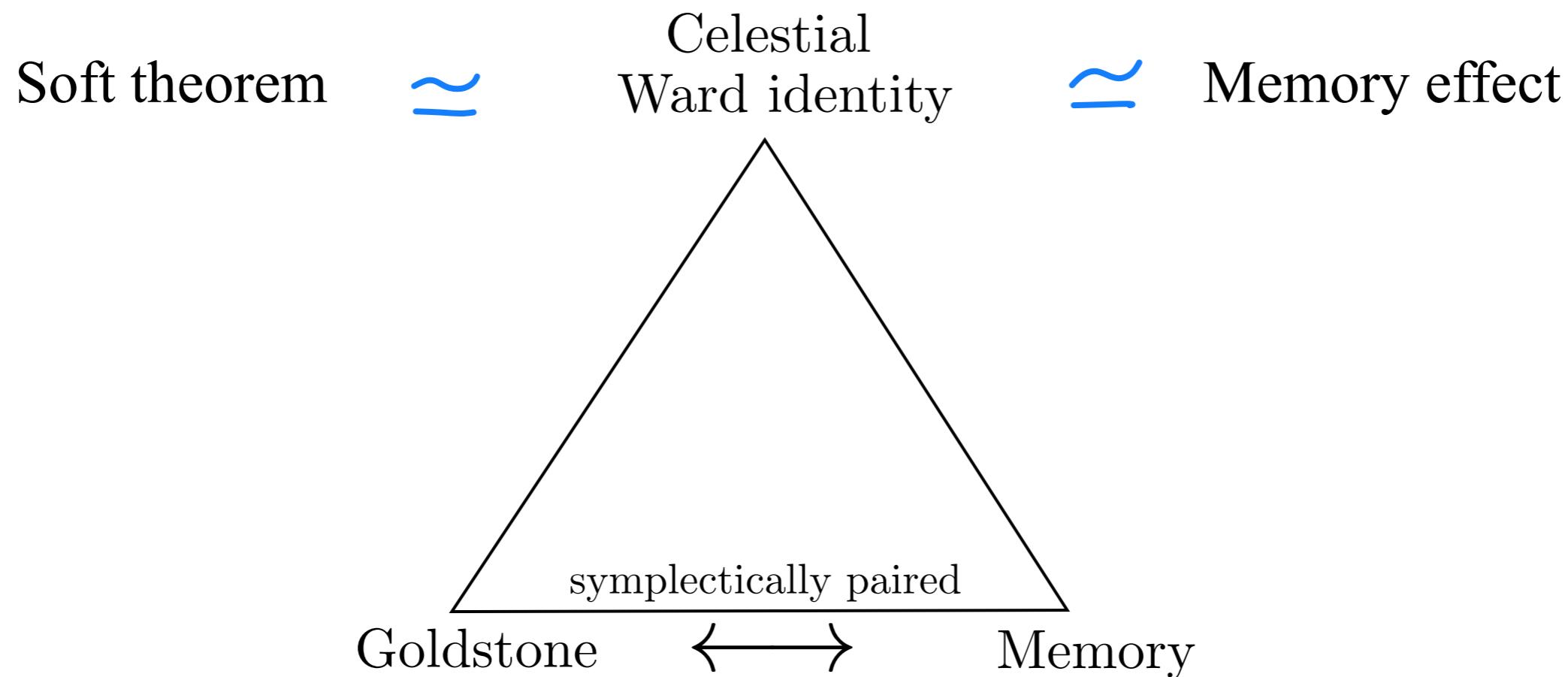
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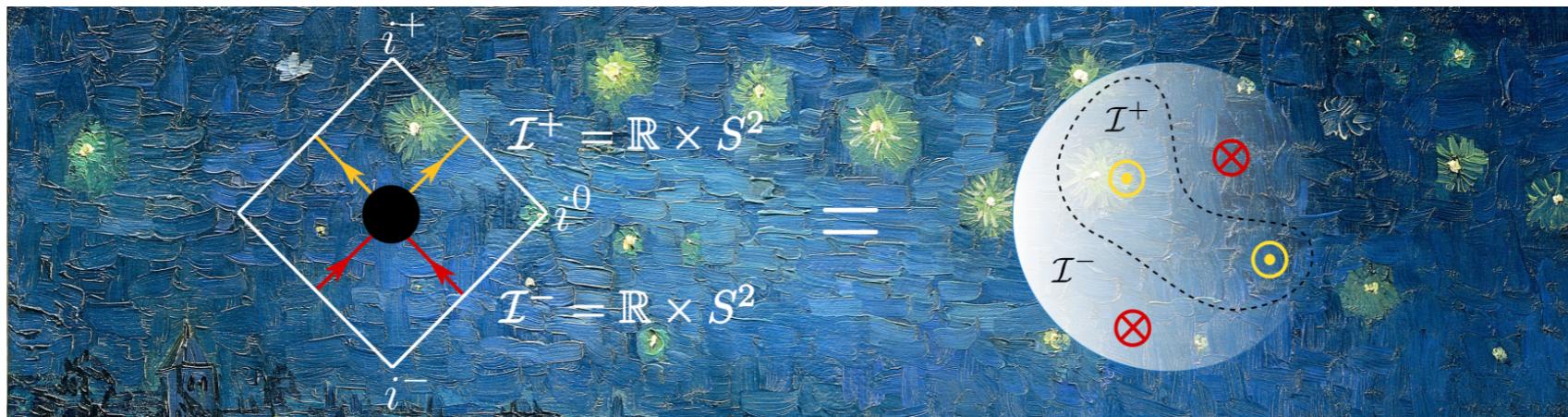
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Basis-independent statement: symplectically paired Goldstone
& Memory *like* modes and a Ward identity for their insertions. [Donnay,AP,Strominger]
[Donnay,Pasterski,AP]



Summary

S-matrix as celestial amplitude:



- reorganizes (conformally) soft and collinear (OPE) behavior
- probes all energy scales → sensitive to deep UV!
- reveals ∞ tower of *conformally soft symmetries*

BMS only \subset symmetries of Nature!

- reveals new structures/properties of amplitudes?

*New framework for tackling quantum
gravity in asymptotically flat spacetimes.*

outlook

flat holo
from
AdS/CFT?

celestial
 $D \leftrightarrow D - 2$

Carrollian
 $D \leftrightarrow D - 1$

Twisted
holography

IR divergences
& dressing

Celestial
holography

physical
spectrum

$$\Delta \in 1 + i\mathbb{R}$$

$$\Delta \in \frac{1}{2}\mathbb{Z}$$

bulk unitarity,
 locality,
 causality
 on bdy?

Quantization

$$\text{natural 2D} \\ L_n^\dagger = L_{-n}$$

$$\text{natural 4D} \\ L_n^\dagger = -\bar{L}_n$$

Hilbert space

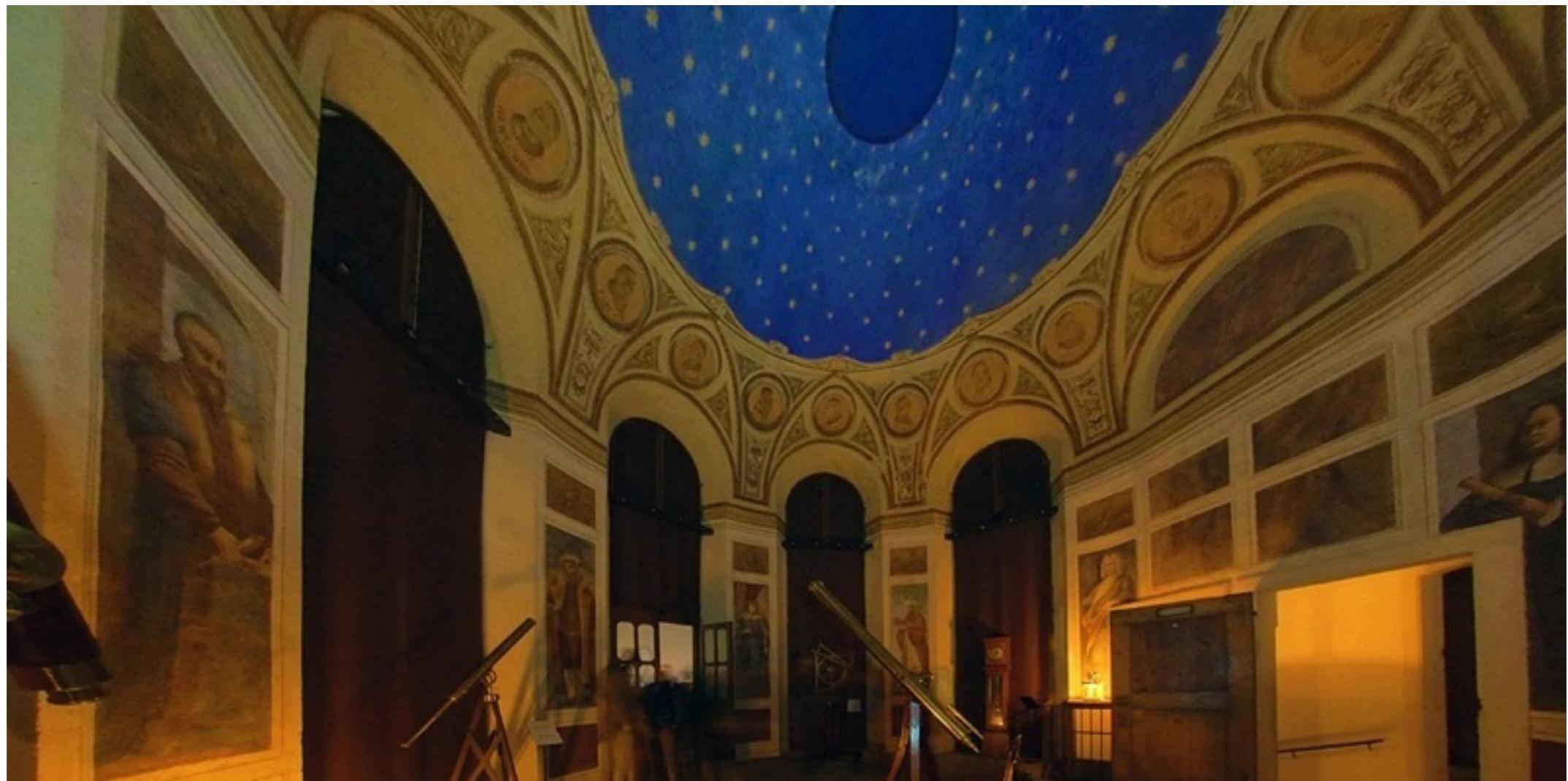
What are all the
symmetries?

extension to
massive particles

black holes

$w_{1+\infty}$ beyond tree-level, self-dual gravity, single helicity?

Exploration of celestial territory has only begun!



Thank you!