Liouville gravity as dilaton gravity

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Based on arXiv:2006.07072 with G.J. Turiaci
arXiv:2109.07770 with Y. Fan

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Negative cosmological constant $\Lambda = -2 \rightarrow \mathrm{AdS}_2$ space

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Features:

- Appears as near-horizon theory of near-extremal higher-dimensional black holes
- Describes low-energy sector of all (known) SYK-like models
- With suitable boundary conditions, reduces to Schwarzian action Almheiri-Polchinski '15, Jensen '16, Maldacena-Stanford-Yang '16, Engelsöy-TM-Verlinde '16
- **Exactly solvable to large extent**

Review - JT disk partition function

Exact quantum solution of amplitudes: $\int \frac{[\mathcal{D}g_{\mu\nu}][\mathcal{D}\Phi]}{\mathrm{Vol(Diff)}} \dots e^{-S_{\mathrm{JT}}}$

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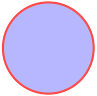
$Z(\beta) =$  $= \int_0^{+\infty} dk (k \sinh 2\pi k) e^{-\beta k^2}$

Energy variable $E = k^2, \quad \rho(E) = \sinh 2\pi\sqrt{E}$

AdS_2 Euclidean holographic boundary with length $= \beta$, filled in by path integrating over all metrics and dilatons

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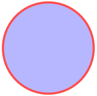
$Z(\beta) =$  $= \int_0^{+\infty} dk(k \sinh 2\pi k) e^{-\beta k^2}$

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Thermodynamic limit (saddle):

$\rho(E) \sim e^{2\pi\sqrt{E}} \Rightarrow \sqrt{E} = \frac{\pi}{\beta}$

Matches with black hole first law (mass vs Hawking temperature) for classical AdS_2 JT black hole solution: Almheiri-Polchinski '15

$ds^2 = -(r^2 - r_h^2)dt^2 + \frac{dr^2}{r^2 - r_h^2}, \quad \Phi(r) = r$

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Boundary two-point function (of some boundary CFT matter operators $\mathcal{O}^h(\tau)$ coupled to gravity):

Bagrets-Altland-Kamenev '16, '17, TM-Turiaci-Verlinde '17, '18, Blommaert-TM-Verschelde '18, Yang '18, Kitaev-Suh '18, '19, Iliesiu-Pufu-Verlinde-Wang '19 ...

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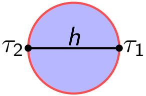

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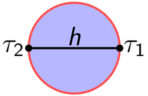

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$$\int_0^{+\infty} dk_1 (k_1 \sinh 2\pi k_1) \int_0^{+\infty} dk_2 (k_2 \sinh 2\pi k_2)\, e^{-\tau k_1^2 - (\beta-\tau) k_2^2} \underbrace{\frac{\Gamma\left(h \pm ik_1 \pm ik_2\right)}{\Gamma(2h)}}_{\text{vertex functions}}$$

$(\tau = \tau_2 - \tau_1)$

$\pm$ sign convention $=$ take product of all cases

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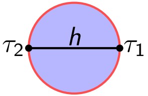

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Generalization to multi-point functions and OTOCs possible

TM-Turiaci-Verlinde '17, '18

JT gravity: summary

Importance of these calculations for gravity:

- Boundary correlators: interpretation in terms of gravitational physics (shockwaves, complexity = volume, bulk entanglement entropy, spectral occupation and Unruh heat bath physics for bulk operators ...) Maldacena-Stanford-Yang '16, Lam-TM-Turiaci-Verlinde '18, Yang '18, TM '19, Blommaert-TM-Verschelde '19, '20 ...

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GOAL: discuss 2d Liouville gravity amplitudes in same language
$\rightarrow$ interpret as specific model of dilaton gravity

Liouville gravity: definition

Liouville gravity = non-critical string = 2d matter CFT coupled to gravity, or critical string with a 2d Liouville + matter + ghost CFT

Polyakov '81, David '88, Distler-Kawai '89 . . .

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- S_{gh} is usual bc-ghost theory with $c_{\mathrm{gh}} = -26$

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$\longrightarrow$ Reinterpret worldsheet topology as the 2d Euclidean spacetime of a gravitational model

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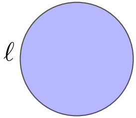
- Matter $+$ ghost: vacuum brane boundary (Dirichlet)

Disk partition function



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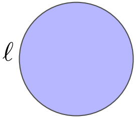

Fixed length amplitude:

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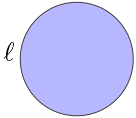

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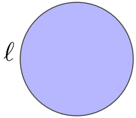

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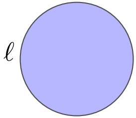

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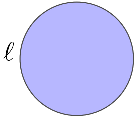

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  $\to$ holographic UV/IR connection: not aAdS like JT gravity

Boundary two-point function

Boundary tachyon vertex operator: $\mathcal{B}_\beta = c\,\Phi_M e^{\beta\phi}$
$\longrightarrow$ Matter primary Φ_M gravitationally dressed by Liouville operator $e^{\beta\phi}$, and gauge-fixed

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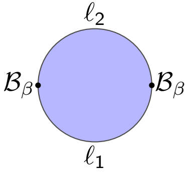

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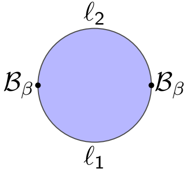

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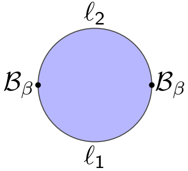

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Technicality: Liouville piece $= \left\langle e^{\beta_1 \phi}(x) e^{\beta_2 \phi}(0) \right\rangle \sim \frac{\delta(\beta_1 - \beta_2)}{|x|^{2\Delta_\beta}} \mathcal{A}_{\beta_1,\beta_2}$

$\longrightarrow$ $\delta(0)$ cancelled by modding out CKG

$\longrightarrow$ Matter + ghost cancels out worldsheet coordinate dependence

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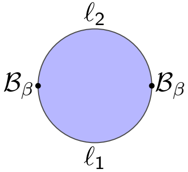

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Technicality: Liouville piece $= \left\langle e^{\beta_1\phi}(x) e^{\beta_2\phi}(0) \right\rangle \sim \frac{\delta(\beta_1-\beta_2)}{|x|^{2\Delta_\beta}} \mathcal{A}_{\beta_1,\beta_2}$

$\longrightarrow$ $\delta(0)$ cancelled by modding out CKG

$\longrightarrow$ Matter + ghost cancels out worldsheet coordinate dependence

JT limit ($b \to 0$, $\beta_M = bh$): $S_b(bx) \sim \Gamma(x)$:

$$\int_0^{+\infty} dk_1\, dk_2\, (k_1 \sinh 2\pi k_1)\,(k_2 \sinh 2\pi k_2)\, e^{-k_1^2 \ell_{\rm JT1}}\, e^{-k_2^2 \ell_{\rm JT2}}\, \frac{\Gamma(h \pm i k_1 \pm i k_2)}{\Gamma(2h)}$$

Group theory interpretation of JT gravity

Example: JT boundary two-point function:

$$\int dk_1 (k_1 \sinh 2\pi k_1) \int dk_2 (k_2 \sinh 2\pi k_2)\, e^{-\tau k_1^2 - (\beta-\tau) k_2^2}\, \frac{\Gamma\left(h \pm i k_1 \pm i k_2\right)}{\Gamma(2h)}$$

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Explanation: 1^{st} order $\mathrm{SL}(2,\mathbb{R})$ BF formulation of JT gravity

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Liouville gravity amplitudes arise from (modification of) $\mathcal{U}_q(\mathfrak{sl}(2,\mathbb{R}))$, $q = e^{\pi i b^2}$ ($b \to 0 \equiv q \to 1$ is undeformed algebra)

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Parabolic matrix element of $\mathcal{U}_q(\mathfrak{sl}(2,\mathbb{R}))$ $R^{\epsilon}_{s,00}(x)$ Kharchev et al. '01:

$e^{\pi i 2sx} \int_{-\infty}^{+\infty} \frac{d\zeta}{(2\pi b)^{-2i\zeta/b - 2is/b}} S_b(-i\zeta) S_b(-i2s - i\zeta) e^{-\pi i \epsilon(\zeta^2 + 2s\zeta)} e^{2\pi i \zeta x}$

Leads to correct vertex function as 3j-symbol with two such insertions and one discrete rep insertion: TM-Turiaci '20, Fan-TM '21

$$\int_{-\infty}^{+\infty} dx\, R^{\epsilon}_{s_1,00}(x) R^{\epsilon *}_{s_2,00}(x) e^{2\beta_M \pi x} \sim \frac{S_b(\beta_M \pm is_1 \pm is_2)}{S_b(2\beta_M)}$$

? **Explanation** by some q-BF formulation of Liouville gravity?
$\longrightarrow$ Open problem, we will present a different gauge theory perspective further on

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Extension: $\mathcal{N} = 1$ Liouville supergravity analogously has $\mathcal{U}_q(\mathfrak{osp}(1|2, \mathbb{R}))$ quantum supergroup structure Fan-TM '21
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computed using representation theory of quantum supergroup

Liouville gravity as a dilaton gravity model

Rewrite Liouville gravity as a dilaton gravity model with specific potential Seiberg-Stanford (unpublished), TM-Turiaci '20

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Assumption: Describe matter as timelike Liouville:

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$S_L[\phi] = \frac{1}{4\pi} \int_\Sigma \left[(\hat{\nabla}\phi)^2 + 4\pi\mu e^{2b\phi} \right]$

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Setting $ds^2 = e^{2\rho} dz d\bar{z}$ ($R \sim e^{-2\rho}\partial\bar{\partial}\rho$) $\rightarrow$ first term $= \int \sqrt{g} R \Phi$

Second term $= W(\Phi) \sim \sinh\left(2\pi b^2 \Phi\right)$

The limit for small b gives back JT gravity

$\Rightarrow$ **Liouville gravity = sinh dilaton gravity**

Classical dilaton gravity and asymptotics (1)

Let's now investigate holography in sinh dilaton gravity

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Up to bulk diffeo's, the general classical solution can be written as:

$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)}, \quad \Phi(r) = r$ with $A(r) = \int_{r_h}^{r} dr' W(r')$

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For $r \rightarrow \infty$, $R \sim e^{2\pi b^2 r}$ $\rightarrow$ Curvature singularity at the boundary

q-deformed holography?

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Boundary behavior of fields:

Recall our field redefinition: $\phi = b^{-1}\rho - b\pi\Phi$ and $\chi = b^{-1}\rho + b\pi\Phi$

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- $e^{b\chi}\big|_{\partial} \sim \lim_{r\to+\infty} e^{2\pi b^2 r} \rightarrow +\infty \ \Rightarrow$ Matter sector needs to be described by identity brane (i.e. ZZ) boundary condition, which was used in the Liouville gravity calculations indeed

Poisson sigma model description (1)

How do we see the quantum group structure within this action?

First-order form of 2d dilaton gravity:

$$S = \frac{1}{2}\int d^2x\sqrt{-g}\,(\Phi R + W(\Phi)) =$$
$$\int \left[\Phi\, d\omega + \frac{1}{4}W(\Phi)\epsilon^{ab}e_a \wedge e_b + X^a(de_a + \epsilon_a{}^b\omega \wedge e_b)\right]$$

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$$S = \int \left(A_i \wedge dX^i - \frac{1}{2}A_i \wedge A_j P^{ji}(X)\right)$$

where A_i is gauge connection, and X^i, $i = 1..m$ coordinatize a m-dimensional Poisson manifold target space:

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Identifying $A_i = (e_0, e_1, \omega)$ and $X^i = (X^0, X^1, \Phi)$

$\rightarrow$ 2d dilaton gravity is a special case of the PS model:

$$\left\{X^0, X^1\right\}_{\text{PB}} = \frac{W(X^2)}{2}, \qquad \left\{X^a, X^2\right\}_{\text{PB}} = \epsilon^a{}_b X^b$$

Poisson sigma model description (1)

How do we see the quantum group structure within this action?

First-order form of 2d dilaton gravity:

$$S = \frac{1}{2}\int d^2x\sqrt{-g}\,(\Phi R + W(\Phi)) =$$
$$\int \left[\Phi\, d\omega + \frac{1}{4}W(\Phi)\epsilon^{ab}e_a \wedge e_b + X^a(de_a + \epsilon_a{}^b\omega \wedge e_b)\right]$$

The general Poisson sigma model is of the form: Ikeda '93, Schaller-Strobl '94

$$S = \int \left(A_i \wedge dX^i - \frac{1}{2}A_i \wedge A_j P^{ji}(X)\right)$$

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or with lightcone coordinates $E^\pm \equiv -X^0 \pm X^1$ and $H \equiv X^2$:

$$\{H, E^\pm\}_{\text{PB}} = \pm E^\pm, \qquad \{E^+, E^-\}_{\text{PB}} = W(H)$$

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charge algebra: see e.g. Cattaneo-Felder '01

$$\Rightarrow \boxed{\left\{ Q^i, Q^j \right\} = P^{ij}(Q)}$$

$\rightarrow$ same as original Poisson algebra, but now realized as a canonical phase-space algebra

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Upon quantization: $\left[\hat{Q}^i, \hat{Q}^j\right] \stackrel{?}{=} i\hbar \hat{P}^{ij}(\hat{Q})$

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- new: For prepotential $u(H) \sim \sinh 2\pi b^2 H$ in $\mathcal{N} = 1$ dilaton supergravity, the charge algebra becomes the q-deformed algebra $U_q(\mathfrak{osp}(1|2,\mathbb{R}))$

Deformation of JT and bigger picture (1)

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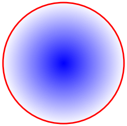
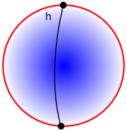
Geometric picture: With boundary asymptotics of classical $\Phi = r$
$\Rightarrow$ Does not modify $r \rightarrow +\infty$ asymptotics of JT gravity:



Similar argument works when boundary operator insertions are present:



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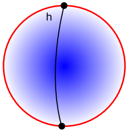

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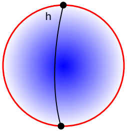

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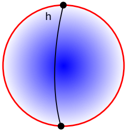

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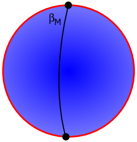

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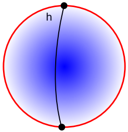

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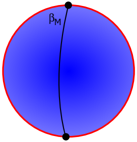

Suggests classification of dilaton gravity models in different classes depending on the asymptotics of the dilaton potential

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Thank you!