Higher spin gravity/symmetry: status report and applications to 3d bosonization duality

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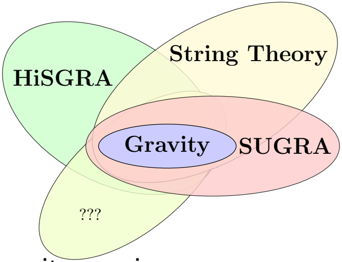
Main Messages

- Higher spin states, $s > 2$, should be important for Quantum Gravity. UV=massless. Can we have higher spin gravities? yes, but very few and they are very peculiar
- There is a class of HiSGRA — Chiral — in $4d$ that can be thought of as a higher spin extension of SDYM and SDGR. It has some stringy features and is UV-finite at least at one loop
- Chiral HiSGRA is related to (Chern-Simons) vector models and is instrumental in attacking the $3d$ bosonization duality
- Higher spin symmetry can be interesting on its own, as an extended conformal symmetry (new Virasoro) and allows us to make general statements about $3d$ bosonization duality
- **plan**: tour over HiSGRA's, applications to $3d$ bosonization and higher spin symmetries



Various examples

- string theory
- divergences in (SU)GRA's
- Quantum Gravity via AdS/CFT

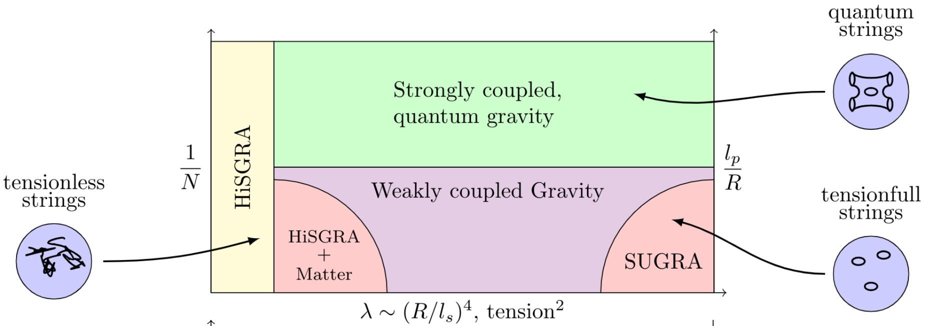
seem to indicate that quantization of gravity requires

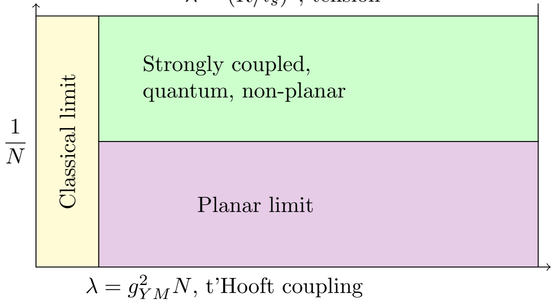
- infinitely many states
- the spectrum is unbounded in spin

HiSGRA is to find the most minimalistic extension of gravity by massless, i.e. gauge, higher spin fields. Vast gauge symmetry should render it finite.

Quantizing Gravity via HiSGRA $\approx$ Constructing Classical HiSGRA

HiSGRA from Tensionless Strings, duals of weakly coupled CFT's





Sundborg; Klebanov, Polyakov; Sezgin, Sundell; Tseytlin; Beisert, Bianchi, Morales, Samtleben; Leigh, Petkou; Gaberdiel, Gopakumar

Why massless higher spins should not exist?

	Flat	**(A)dS**
Global		
Local		

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Global	decoupling of longitudinal modes $\delta\Phi_{a_1\ldots a_s} = \partial_{(a_1}\xi_{a_2\ldots a_s)}$, or tensorial charges $Q_{a_1\ldots a_{s-1}}$ impose ∞-many constraints: $\boldsymbol{S=1}^{**}$ (Weinberg; Coleman, Mandula)	
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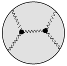
Local	Noether procedure, i.e. $\mathcal{L}=(\partial\Phi)^2+\mathcal{O}(\Phi^3)$, $\delta\Phi=\partial\xi+\mathcal{O}(\Phi,\xi)$ is obstructed at Φ^4 (Bekaert, Boulanger, Leclercq; Roiban, Tseytlin; Ponomarev, E.S., ...)	AdS/CFT gives one-line proof $\Phi^4\sim$  (Erdmenger, Bekaert, {Ponomarev}, (Sleight; Taronna)), which again invalidates Noether procedure at Φ^4

Constraints on (holographic) S-matrix

Asymptotic higher spin symmetries (HSS)

$$\delta\Phi_{\mu_1\dots\mu_s}(x) = \nabla_{(\mu_1}\xi_{\mu_2\dots\mu_s)}$$

seem to completely fix (holographic) S-matrix to be

$$S_{\text{HiSGRA}} = \begin{cases} 1^{***}, & \text{flat space}, \\ \text{free CFT}, & \text{AdS, unbroken HSS}, \\ \textbf{Chern-Simons Matter}, & \textbf{AdS}_4\textbf{, slightly-broken HSS} \end{cases}$$

(Sundborg; Klebanov, Polyakov; Sezgin, Sundell; Leigh, Petkou; Maldacena, Zhiboedov; Giombi, Yin, ...)

Trivial/known S-matrix can still be helpful for QG toy-models

The most interesting applications are to three-dimensional dualities (power of HSS is underexplored)

Both Minkowski and AdS cases reveal certain non-localities to be tamed. HSS mixes ∞ spins and derivatives, invalidating the local QFT approach

Four classes of local HiSGRA in 2022

3d massless and partially-massless (Blencowe, Bergshoeff, Stelle, Campoleoni, Fredenhagen, Pfenninger, Theisen, Henneaux, Rey, Gaberdiel, Gopakumar, Grumiller, Grigoriev, Mkrtchyan, E.S., ...), $S = S_{CS}$ for a higher spin extension of $sl_2 \oplus sl_2$

$$S_{CS} = \int \omega d\omega + \tfrac{2}{3}\omega^3$$

3d conformal (Pope, Townsend; Fradkin, Linetsky; Grigoriev, Lovrekovic, E.S.), $S = S_{CS}$ for higher spin extension of $so(3,2)$

4d conformal (Tseytlin, Segal; Bekaert, Joung, Mourad; McLoughlin, Adamo, Tseytlin), higher spin extension of Weyl gravity, local Weyl symmetry tames non-localities

$$S = \int \sqrt{g}\,(C_{\mu\nu,\lambda\rho})^2 + ... \qquad S \log \Lambda \in \text{Eff.Action}$$

4d massless chiral (Metsaev; Ponomarev, E.S.; Ponomarev; E.S., Tran, Tsulaia; E.S.). The smallest higher spin theory with propagating fields. **This talk!**

The theories avoid all no-go's. Surprisingly, all of them have simple actions and are clearly well-defined, as close to Field Theory as possible

Other ideas and proposals

- **Reconstruction**: invert AdS/CFT
  - Brute force (Bekaert, Erdmenger, Ponomarev, Sleight; Taronna, Sleight)
  - Collective Dipole (Jevicki, Mello Koch et al; Aharony et al)
  - Holographic RG (Leigh et al, Polchinski et al)
- **IKKT matrix model for fuzzy H_4** (Steinacker, Sperling, Fredenhagen, Tran)
- **Formal HiSGRA**: constructing L_∞-extension of HS algebras, i.e. a certain odd Q, $QQ=0$, and write AKSZ sigma model (Barnich, Grigoriev)

$$dW = Q(W)$$

Warning: Boulanger, Kessel, E.S., Taronna

(Vasiliev; E.S., Sharapov, Bekaert, Grigoriev, E.S.; Grigoriev, E.S.; E.S., Sharapov, Tran; Bonezzi, Boulanger, Sezgin, Sundell; Neiman) AdS/CFT: (Sundborg, Sezgin, Sundell, Klebanov, Polyakov, Giombi, Yin, ...) $0.99*$Chiral HiSGRA (E.S., Van Dongen)

Certain things do work, but the general rules are yet to be understood, e.g. non-locality, relation to field theory, quantization, ...

Chiral Higher Spin Gravity

Self-dual Theories

- actions are not real in Minkowski space
- actions are simpler than the complete theories
- integrability, instantons (Penrose, Ward, Atiyah, Hitchin, Drinfeld, Manin; ...)
- **SD theories are consistent truncations**, so anything we can compute will be a legitimate observable in the full theory; **any solution of SD is a solution of the full**; ...
- different expansion schemes: instantons instead of flat, MHV, etc.

In general: amplitudes (MHV, BCFW, double-copy, ...), strings, QFT, Twistors, ... encourage to go outside Minkowski

In higher spins: little explored (Adamo, Hähnel, McLoughlin; Krasnov, E.S., Ponomarev, Tran), can be the only reasonably local theories

Chiral HSGRA in Minkowski

Self-dual Yang-Mills in Lorentzian signature is a useful analogy

- the theory is non-unitary due to the interactions ($\boldsymbol{A_\mu} \rightarrow \boldsymbol{\Phi}^{\pm}$)

$$\mathcal{L}_{\text{YM}} = \mathbf{tr}\, F_{\mu\nu} F^{\mu\nu}$$

$$\wr\wr$$

$$\mathcal{L}_{\text{SDYM}} = \Phi^{-}\Box\Phi^{+} + V^{++-} + V^{--+} + V^{++--}$$

- tree-level amplitudes vanish, $A_{\text{tree}} = 0$
- one-loop amplitudes do not vanish, are rational and coincide with $(++\ldots+)$ of pure QCD
- Yang-Mills Theory as a perturbation of SDYM is a fruitful idea
- integrability, instantons, ...

Chiral HSGRA in Minkowski

Chiral HiSGRA (Metsaev; Ponomarev, E.S.) is a 'higher spin extension' of SDYM/SDGR. It has fields of all spins $s = 0, 1, 2, 3, ...$:

$$\mathcal{L} = \sum_{\lambda} \Phi^{-\lambda} \Box \Phi^{+\lambda} + \sum_{\lambda_i} \frac{\kappa \, l_{\mathrm{Pl}}^{\lambda_1+\lambda_2+\lambda_3-1}}{\Gamma(\lambda_1+\lambda_2+\lambda_3)} V^{\lambda_1,\lambda_2,\lambda_3}$$

light-cone gauge is very close to the spinor-helicity language

$$V^{\lambda_1,\lambda_2,\lambda_3} \sim [12]^{\lambda_1+\lambda_2-\lambda_3}[23]^{\lambda_2+\lambda_3-\lambda_1}[13]^{\lambda_1+\lambda_3-\lambda_2}$$

Locality + Lorentz invariance + genuine higher spin interaction result in a unique completion

This is the smallest higher spin theory and it is unique.
Graviton and scalar field belong to the same multiplet

No UV Divergences! One-loop finiteness

Tree amplitudes vanish. The interactions are naively non-renormalizable, the higher the spin the more derivatives:

$$\boldsymbol{V^{\lambda_1,\lambda_2,\lambda_3} \sim \partial^{|\lambda_1+\lambda_2+\lambda_3|}\Phi^3}$$

but there are **no UV divergences!** (E.S., Tsulaia, Tran). Some loop momenta eventually factor out, just as in $\mathcal{N}=4$ SYM, but ∞-many times.

At one loop we find three factors: (1) SDYM or all-plus 1-loop QCD; (2) higher spin dressing to account for λ_i; (3) total number of d.o.f.:

$$\boldsymbol{A}^{\text{1-loop}}_{\text{Chiral}} = \boldsymbol{A}^{++\ldots+}_{\text{QCD,1-loop}} \times \boldsymbol{D}^{\text{HSG}}_{\boldsymbol{\lambda_1,\ldots,\lambda_n}} \times \sum_{\lambda} 1$$

$\#$ d.o.f.$=\sum_\lambda 1 = 1+2\sum_{\lambda>0} 1 = 1+2\zeta(0)=0$ to comply with no-go's, (Beccaria, Tseytlin) and agrees with many results in AdS, where $\neq 0$

Chiral HSGRA in Minkowski

- **stringy 1**: the spectrum is infinite $s = 0, (1), 2, (3), 4, ...$
- **stringy 2**: admit Chan-Paton factors, $U(N)$, $O(N)$ and $USp(N)$
- **stringy 3**: we have to deal with spin sums $\sum_s$ (worldsheet takes care of this in string theory) and ζ-function helps
- **stringy 4**: the action contains parts of YM and Gravity
- **stringy 5**: higher spin fields soften amplitudes
- consistent with Weinberg etc. $S = 1^{***}$ (in Minkowski)
- gives all-plus QCD or SDYM amplitudes from a gravity

Apart from Minkowski space the theory exists also in (anti)-de Sitter space, where holographic S-matrix turns out to be nontrivial ... and related to Chern-Simons matter theories

Covariantization, SDYM, SDGR

Chiral HiSGRA admits two contractions (Ponomarev) to higher spin extensions of SDYM and SDGR. These HS-SDYM and HS-SDGR can be covariantized (Krasnov, E.S., Tran). New (Hitchin) free action

$$\boldsymbol{S = \int \Psi^{A_1\dots A_{2s}} \wedge H_{A_1A_2} \wedge \nabla \omega_{A_3\dots A_{2s-2}}}$$

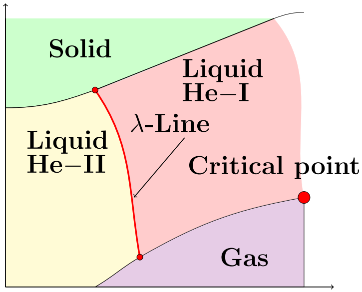
$H^{AB} \equiv e^{A}{}_{C'} \wedge e^{BC'}$. Interactions can be introduced by taking sum over s and by replacing $\nabla\omega$ or both H and $\nabla\omega$ with

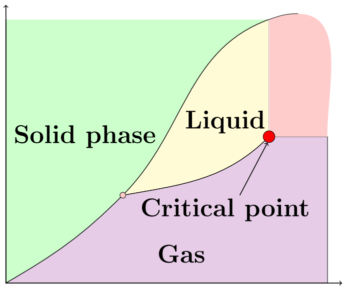
$$F = d\omega + \tfrac{1}{2}[\omega,\omega]$$

where $\omega \equiv \sum_k \omega_{A_1\dots A_k}\, y^{A_1}\dots y^{A_k}$ and the commutator is either due to Yang-Mills groups or due to Poisson bracket on $\mathbb{R}^2$ of $f(y)$, same as $w_{1+\infty}$.

Full covariant form is 0.99 available (E.S., Van Dongen). Twistors (Tran)

Chern-Simons Matter Theories and bosonization duality





In AdS_4/CFT_3 one can do much better — there exists a large class of models, Chern-Simons Matter theories (extends to ABJ(M))

$$\frac{k}{4\pi}S_{CS}(A) + \text{Matter} \begin{cases} (D\phi^i)^2 & \textbf{free boson} \\ (D\phi^i)^2 + g(\phi^i\phi^i)^2 & \textbf{Wilson-Fisher (Ising)} \\ \bar{\psi}\not{D}\psi & \textbf{free fermion} \\ \bar{\psi}\not{D}\psi + g(\bar{\psi}\psi)^2 & \textbf{Gross-Neveu} \end{cases}$$

- describe physics (Ising, quantum Hall, ...), break parity
- two parameters $\lambda = N/k$, $1/N$ (λ continuous for N large)
- exhibit remarkable dualities, e.g. $\boldsymbol{3d}$ **bosonization duality** (Aharony, Alday, Bissi, Giombi, Karch, Maldacena, Minwalla, Prakash, Seiberg, Tong, Witten, Yacobi, Yin, Zhiboedov, ...)

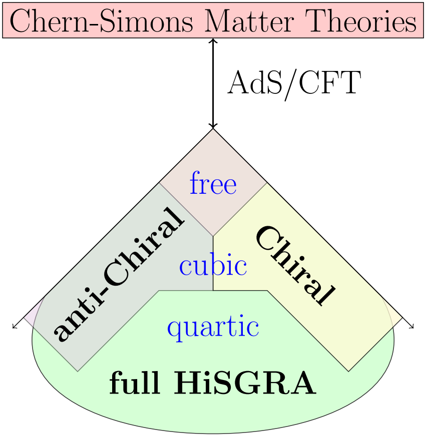
Chern-Simons Matter theories and dualities

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\textbf{free boson} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"'] & \textbf{Gross-Neveu} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"] \\
\textbf{Wilson-Fisher} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] & \textbf{free fermion} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"]
\end{tikzcd}
$$

The simplest gauge-invariant operators are higher spin currents

$$J_s = \phi D ... D\phi \qquad\qquad J_s = \bar{\psi}\gamma D ... D\psi$$

which are AdS/CFT dual to higher spin fields



(anti)-Chiral Theories are rigid, we need to learn how to glue them

gluing depends on one parameter, which is introduced via simple EM-duality rotation $\Phi_{\pm s} \rightarrow e^{\pm i\theta}\Phi_{\pm s}$

gives all 3-point correlators consistent with (Maldacena, Zhiboedov)

Bosonization is manifest! Concrete predictions from HiSGRA.

(anti)-Chiral Theories provide a complete base for 3-pt amplitudes

$$\boldsymbol{V_3 = V_{chiral} \oplus \bar{V}_{chiral} \quad \leftrightarrow \quad \langle JJJ \rangle}$$

Higher spin symmetry and bosonization duality

Unbroken Higher spin symmetry

In free theories we have ∞-many conserved $J_s = \phi\partial...\partial\phi$ tensors.

Free CFT $=$ Associative (higher spin) algebra

Conserved tensor $\rightarrow$ current $\rightarrow$ symmetry $\rightarrow$ invariants$=$correlators.

$$\partial \cdot J_s = 0 \quad \Longrightarrow \quad Q_s = \int J_s \quad \Longrightarrow \quad [Q,Q] = Q \quad \& \quad [Q,J] = J$$

HS-algebra (free boson) $=$ HS-algebra (free fermion) in $3d$.

Correlators are given by invariants (Sundell, Colombo; Didenko, E.S.; ...)

$$\langle J...J\rangle = \mathrm{Tr}(\Psi \star ... \star \Psi) \qquad\qquad \Psi \leftrightarrow J$$

where Ψ are coherent states representing J in the higher spin algebra

$$\langle JJJJ\rangle_{F.B.} \sim \cos(Q^2_{13} + Q^3_{24} + Q^4_{31} + Q^1_{43}) \cos(P_{12}) \cos(P_{23}) \cos(P_{34}) \cos(P_{41}) + ...$$

Slightly-broken Higher spin symmetry is new Virasoro?

In large-N Chern-Simons vector models (e.g. Ising) higher spin symmetry does not disappear completely (Maldacena, Zhiboedov):

$$\partial \cdot J = \frac{1}{N}[JJ] \qquad\qquad [Q, J] = J + \frac{1}{N}[JJ]$$

What is the right math? We should deform the algebra together with its action on the module, so that the currents can 'backreact':

$$\delta_\xi J = l_2(\xi, J) + l_3(\xi, J, J) + \dots\,, \qquad\qquad [\delta_{\xi_1}, \delta_{\xi_2}] = \delta_\xi\,,$$

where $\xi = l_2(\xi_1, \xi_2) + l_3(\xi_1, \xi_2, J) + \dots$. This leads to L_∞-algebra.

Correlators = invariants of L_∞-algebra and are unique (Gerasimenko, Sharapov, E.S.), **which proves $3d$ bosonization duality at least in the large-N**. Without having to compute anything one prediction is

$$\langle J \dots J \rangle = \sum \langle \text{fixed} \rangle_i \times \text{params}$$

Concluding Remarks

- Some HiSGRA do exist, e.g. Chiral HSGRA. It reveals (almost) trivial S-matrix in flat space, but not in AdS
- Chiral HiSGRA is a toy model with stringy features and shows how higher spin fields improve the UV behaviour: **no UV divergences, supersymmetry vs. higher spin symmetry**
- It gives all 3-pt functions in Chern-Simons Matter theories, making new predictions and proves the bosonization duality to this order.
- Higher spin symmetry itself can be useful via L_∞-algebras. Proof of bosonization duality in the large-N. L_∞ in physics
- **Optimistically: HiSGRA can give viable quantum gravity models not free of direct applications to physics**

Thank you for your attention!