Recent Developments in Non-Relativistic String Theory

Eurostrings 2022, Lyon, April 26, 2022 Niels Obers (Nordita & Niels Bohr Institute)



based on work:

2107.006542 (JHEP) (Bidussi,Harmark,Hartong,NO,Oling)
2011.02539 (JHEP) (Harmark,Hartong,NO,Oling)
1907.01663 (JHEP) (Harmark,Hartong,Menculini,NO,Oling)
& 1810.05560 (JHEP) (Harmark,Hartong,Menculini,NO,Yan)
1705.03535 (PRD) (Harmark,Hartong,NO)
& work to appear with Bidussi,Harmark,Hartong,Oling

Non-relativistic physics and cube of physical theories



a third route towards (relativistic) quantum gravity

how does this fit with string theory/holography?

already (classical) non-relativistic gravity (NRG) is more than just Newtonian gravity

Non-Lorentzian geometries

recent progress in understanding non-relativistic corners of: gravity, quantum field theory and string theory:

→ builds on improved understanding of non-Lorentzian geometries
= spacetimes with local symmetries other than Lorentz

NL geometries appear in:

- bdry geometries in non-AdS holography (e.g. Lifshitz flat space)
- covariant formulations of PN approximation in GR
- covariant formulations of non-Lorentzian fluids and CMT systems (FQHE, fractons, ..)
- Horava-Lifshitz gravity, non-relativistic versions of CS, JT
- cosmology, black hole horizons (Carroll)
- double field theory
- non-relativistic corners of String Theory
- near-BPS limits of string theory on AdS5 × S5

Non-perturbative string theory

complete understanding of non-perturbative regime is still lacking despite much progress made in last many decades:

- non-perturbative dualities

Matrix theory:
 infinite boost limit of ST on spacelike circle = DLCQ of ST

can be viewed as ST on light-like circle \rightarrow non-relativistic behavior

• NRST as a novel way to study corners of relativistic string theory

Why non-relativistic (NR) string theory?

- perhaps simpler (UV complete) theory
- non-relativistic gravity via beta functions
- limit of AdS/CFT and novel sigma models
- certain NR strings contained in double field theory
- rich limit of string theory
- what is the landscape of NR string theories ?
 & non-Lorentzian holography (see discussion)

NR strings

NR strings on flat spacetime = Gomis-Ooguri string

Gomis,Ooguri(2000); Danielsson et al.(2000);

 \rightarrow Newton-Cartan geometries when spacetime is curved

Andringa et al (2012), Harmark, Hartong, NO(2017); Bergshoeff, Gomis, Yan(2018); Harmark, Hartong, Menculini, NO, Yan(2018); Gomis, Oh, Yan(2019); Gallegos, Gursoy, Zinnato(2019), Harmark, Hartong, Menculini, NO, Oling(2019); Bergshoeff, Gomis, Rosseel, Simsek, Yan(2019); Kluson (2018/19), Yan (2021), Bergshoeff, Lahnsteiner, Romano, Rosseel, Simsek (2021); Bidussi, Harmark, Hartong, NO, Oling (2021);

also:

- tensionless strings

e.g.Lindstrom,Sundborg,Theodoridis(1991) Bagchi,Gopakumar(2009) Bagchi,Banerjee,Parekh(2019)

- Galilean strings

Battle,Gomis,Not(2016))

- relation to double field theory

Ko,Melby-Thompson,Meyer,Park(2015) Morand,Park(2017);Berman,Blair,Otsuki(2019);Blair(2019)

NR strings (on flat spacetime)

Gomis,Ooguri(2000); Danielsson et al.(2000)

zero Regge slope limit of relativistic string theory in near-critical B-field

$$S = \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \sqrt{h} \left(h^{\alpha\beta} \,\partial_{\alpha} X^{A'} \,\partial_{\beta} X_{A'} + \lambda \,\bar{\mathcal{D}} X + \bar{\lambda} \,\mathcal{D} \overline{X} \right),$$

in conformal gauge:

$$S = \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \left(\partial_{\alpha} X^{A'} \, \partial^{\alpha} X_{A'} + \lambda \, \bar{\partial} X + \bar{\lambda} \, \partial \overline{X} \right),$$

- Galilean invariant dispersion relation
- no massless physical states
- low-energy effective theory described by Newton-like gravity
- all asymptotic states carry non-zero winding along (compact) X_1
- space-time S-matrix with NR symmetry

Gomis/Ooguri NR string lives in flat space



figures from review on NRST Gerben Oling & Ziqi Yan (2202.12698)

Main part of talk:

Q: what is the general target space probed by NR strings ?



Main result

→ find formulation in which geometry contains a 2-form field that couples to tension current and transforming under string Galilei boosts

i.e. 2-form is intrinsic part of the geometry (in parallel with NR particle and its coupling to Newton-Cartan geometry)

- follows from both null reduction & $c \rightarrow$ infinity limit
- geometry arises from gauging novel algebra: F-string Galilei algebra
 - = Inonu-Wigner contraction of

Poincare algebra + syms underlying Kalb-Ramond field

Outline

- intro to Newton-Cartan geometry for particles:
 null reduction and c → infinity limit
- Torsional string Newton-Cartan geometry (TSNC)
 from c → infinity limit
 - preliminaries: KR B-field from string Poincare
 - NR string action with TSNC target space/symmetries
 - Remarks: null reduction; limits vs. expansions; beta-fns
- NR world-sheet models from limits of NR strings & limits of AdS/CFT
- Outlook & discussion

Space-Time symmetries and Geometry

local symmetries of space and time $\leftarrow \rightarrow$ geometry of space and time



Cartan: Galilean $\leftarrow \rightarrow$ Newton-Cartan geometry

[Eisenhart,Trautman,Dautcourt,Kuenzle,Duval,Burdet,Perrin,Gibbons,Horvathy,Julia,Nicolai,...] ..

Positive Curvature

Negative Curvature

Flat Curvature





- geometrize Poisson equation of Newtonian gravity falling observers see Galilean laws of physics

TNC geometry from null reduction

Lorentzian metric with null isometry

$$ds^2 = g_{MN} dX^M dX^N = 2 au_\mu dx^\mu (du - m_
u dx^
u) + h_{\mu
u} dx^\mu dx^
u ,$$

$$\tau_{\mu}h^{\mu\nu} = 0$$

torsional Newton–Cartan (TNC) geometry: τ_{μ} , $h_{\mu\nu}$, m_{μ} ,

local syms:

$$\begin{split} \delta \tau_{\mu} &= \mathcal{L}_{\xi} \tau_{\mu} , \quad \delta h_{\mu\nu} = \mathcal{L}_{\xi} h_{\mu\nu} + \lambda_{\mu} \tau_{\nu} + \lambda_{\nu} \tau_{\mu} ,\\ \delta m_{\mu} &= \mathcal{L}_{\xi} m_{\mu} + \lambda_{\mu} + \partial_{\mu} \sigma , \end{split}$$

 $\begin{array}{ll} \lambda_{\mu} & \text{Galilean (Milne) boosts} \\ \sigma & \text{U(1) (mass) parameter} \end{array}$

torsional Newton-Cartan geometry



no condition on au_{μ}

TNC

Christensen, Hartong, NO, Rollier (2013)

TNC geometry as background geometry for NRFTs

putting relativistic field theory on a curved spacetime

$$\delta S_{\rm rel.matter} \sim \int d^4x \ T_{\mu\nu} \ \delta g^{\mu\nu}$$

• non-relativistic FT naturally couples to torsional Newton-Cartan:



see e.g. Son (2013), Hartong, Kiritsis, NO(2014), Jensen(2014)

Warmup: Non-Relativistic particle from null reduction

null-reduction of relativistic particle

$$S = \int rac{1}{2e} g_{MN} \dot{X}^M \dot{X}^N d\lambda$$
 =

→ reduce on target space with null Killing vector :

probe mass is conserved momentum in null direction: $p_u = m$



• action has TNC local target space symmetries

Other properties

• geodesic equation on flat NC space with:

$$m_t = \Phi_{\text{Newt}} \rightarrow \text{Newton's law}$$

• TNC geometry can also be obtained by gauging Bargmann algebra Andringa,Bergshoeff,Gomis,de Roo (2012)

$$[G_a, P_b] = -\delta_{ab}N$$
, $[G_a, H] = -P_a$ & rotations
mass generator

(just as pseudo-Riemannian geometry follows from gauging Poincare)

NR particle from limit of extremal particle action of charged relativistic particle:

$$S = -mc \int \sqrt{-g_{\mu
u}\dot{x}^{\mu}\dot{x}^{
u}}d\lambda + q \int A_{\mu}\dot{x}^{\mu}d\lambda$$

• time-space split in metric: $g_{\mu
u} = -c^2 T_\mu T_
u + h_{\mu
u}$

expand for large c:

$$S = -mc^2 \int \left[T_{\mu} - \frac{q}{mc^2} A_{\mu} \right] \dot{x}^{\mu} d\lambda + \frac{m}{2} \int \frac{h_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu}}{T_{\rho} \dot{x}^{\rho}} d\lambda + \mathcal{O}(c^{-2})$$

NR particle from limit of extremal particle action of charged relativistic particle:

$$S = -mc \int \sqrt{-g_{\mu
u}\dot{x}^{\mu}\dot{x}^{
u}}d\lambda + q \int A_{\mu}\dot{x}^{\mu}d\lambda$$

• time-space split in metric: $g_{\mu
u} = -c^2 T_\mu T_
u + h_{\mu
u}$

expand for large c:

$$S = -mc^2 \int \left[T_\mu - \frac{q}{mc^2} A_\mu \right] \dot{x}^\mu d\lambda + \frac{m}{2} \int \frac{h_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}{T_\rho \dot{x}^\rho} d\lambda + \mathcal{O}(c^{-2})$$

extremal particle diverge $q = mc^2$. with:

divergent term. cancels with:

$$T_{\mu} = au_{\mu} + rac{1}{2c^2}m_{\mu} \,,$$

 $A_{\mu} = au_{\mu} - rac{1}{2c^2}m_{\mu} \,,$

1

algebra level: IW contraction

$$H = cP_0 + Q \ , \ N = rac{1}{2c^2} \left(cP_0 - Q
ight)$$

energy

mass

Mimic the procedure for strings

```
fundamental strings are extremally charged under B-field
(tension = charge)
```

```
what is the analogue of Poincare x U(1) for strings ?
```

```
→ understand the spacetime symmetry underlying the Kalb-Ramond B-field
```

Kalb-Ramond B-field from string Poincare

metric and B-field symmetries $\bar{\delta}g_{\mu\nu} = \mathcal{L}_{\xi}g_{\mu\nu} , \quad \bar{\delta}B_{\mu\nu} = \mathcal{L}_{\xi}B_{\mu\nu} + 2\partial_{[\mu}\lambda_{\nu]}$

→ can be obtained from string extension of Poincare:
 (with extra set of translational generators (cf. double field theory)

$$\begin{split} [M_{\underline{a}\underline{b}}, M_{\underline{c}\underline{d}}] &= \eta_{\underline{a}\underline{c}} M_{\underline{b}\underline{d}} - \eta_{\underline{b}\underline{c}} M_{\underline{a}\underline{d}} + \eta_{\underline{b}\underline{d}} M_{\underline{a}\underline{c}} - \eta_{\underline{a}\underline{d}} M_{\underline{b}\underline{c}}, \\ [M_{\underline{a}\underline{b}}, P_{\underline{c}}] &= \eta_{\underline{a}\underline{c}} P_{\underline{b}} - \eta_{\underline{b}\underline{c}} P_{\underline{a}}, \\ [M_{\underline{a}\underline{b}}, Q_{\underline{c}}] &= \eta_{\underline{a}\underline{c}} Q_{\underline{b}} - \eta_{\underline{b}\underline{c}} Q_{\underline{a}}, \end{split}$$

Lie algebra valued connection:

$$\mathcal{A}_{\mu} = e^{\underline{a}}_{\mu} P_{\underline{a}} + \frac{1}{2} \omega_{\mu} \frac{ab}{2} M_{\underline{a}\underline{b}} + \pi^{\underline{a}}_{\mu} Q_{\underline{a}},$$

$$g_{\mu\nu} = \eta_{\underline{a}\underline{b}} \, e^{\underline{a}}_{\mu} e^{\underline{b}}_{\nu}. \qquad \qquad B_{\mu\nu} = \eta_{\underline{a}\underline{b}} \, e^{\underline{a}}_{[\mu} \pi^{\underline{b}}_{\nu]}.$$

(see e.g. Ne'eman,Regge/D'Auria,Fre)

NR string action from $c \rightarrow$ infinity limit

$$S = S_{
m NG} + S_{
m WZ},$$

 $S_{
m NG} = -c T_{
m F} \int d^2 \sigma \sqrt{-\det g_{lphaeta}}, \qquad S_{
m WZ} = -c rac{T_{
m F}}{2} \int d^2 \sigma B_{lphaeta} \epsilon^{lphaeta}.$

- use vielbein decomposition of the NSNS target space:

$$g_{\mu\nu} = \eta_{\underline{a}\underline{b}} e^{\underline{a}}_{\mu} e^{\underline{b}}_{\nu} \quad , \qquad B_{\mu\nu} = \eta_{\underline{a}\underline{b}} e^{\underline{a}}_{[\mu} \pi^{\underline{b}}_{\nu]},$$

- split tangent space into A=0,1: longitudinal a=2,..d-1: transverse

$$e^{\underline{a}}_{\mu} = (cE^{A}_{\mu}, e^{a}_{\mu}) \ , \quad \pi^{\underline{a}}_{\mu} = (c\Pi^{A}_{\mu}, \pi^{a}_{\mu}),$$

 reparametrize longitudinal vielbeins:

$$\begin{split} E^{A}_{\mu} &= \tau^{A}_{\mu} + \frac{1}{2c^{2}}\pi^{B}_{\mu}\epsilon_{B}{}^{A}, \\ \Pi^{A}_{\mu} &= \epsilon^{A}{}_{B}\tau^{B}_{\mu} + \frac{1}{2c^{2}}\pi^{A}_{\mu}, \end{split}$$

divergent term in action cancels since F-string is extremal

NRST action on TSNC target space



• weak equivalence principle for NR string

Symmetries of the action

• gauge:
$$\bar{\delta}m_{\mu\nu} = 2\partial_{[\mu}\lambda_{\nu]}.$$

• string Galilean boosts:

$$\bar{\delta}h_{\mu\nu} = -\lambda_{Ab} \left(\tau^A_\mu e^b_\nu + \tau^A_\nu e^b_\mu \right) \quad , \qquad \bar{\delta}m_{\mu\nu} = -2\epsilon_{AB}\lambda^B{}_c\tau^A_{[\mu}e^c_{\nu]}.$$

→ string analogue of the symmetries of NR particle coupling to Newton-Cartan

Question: what is underlying NR symmetry algebra ?

F-string Galilean (FSG) algebra

• decompose string $\underline{a} = (A, a)$ Poincare algebra: longitudinal, transverse

 P_A , Q_A , P_a , Q_a , $J_{AB} = \epsilon_{AB}J = M_{AB}$, $J_{ab} = M_{ab}$, $c G_{Ab} = M_{Ab}$.

 $H_A = c(P_A + Q_B \epsilon^B{}_A)$, $N_A = \frac{1}{2c}(\epsilon_A{}^B P_B + Q_A)$ (basis transformation)

after IW contraction (c \rightarrow infinity):

$$[G_{Ab}, H_C] = \eta_{AC} P_b + \epsilon_{AC} Q_b,$$

$$[G_{Ab}, P_c] = -\delta_{bc} \epsilon_A{}^B N_B,$$

$$[G_{Ab}, Q_c] = -\delta_{bc} N_A,$$

& further commutators involving $SO(1,1) \ge SO(d-2)$ rotations

• symmetry trafos follow from FSG-valued connection:

$$\mathcal{A}_{\mu} = \tau_{\mu}^{A} H_{A} + e_{\mu}^{a} P_{a} + \omega_{\mu} J + \frac{1}{2} \omega_{\mu}{}^{ab} J_{ab} + \omega_{\mu}{}^{Ab} G_{Ab} + \pi_{\mu}^{A} Z_{A} + \pi_{\mu}^{a} Q_{a},$$

Non-relativistic strings from null reduction

- start from Polyakov action (including NSNS) and reduce along null isometry
- implement conservation of string momentum along null isometry using Lagrange multipliers
- go to dual formulation that exchanges the (fixed) momentum along null direction for fixed winding of string along compact dual direction
 - → action of non-relativistic strings moving in torsional string Newton-Cartan target space
 & FSG symmetries can also be derived

Limit vs. I/c expansion

- limit geometry: type I (cancellation of divergent term)
- geometry from expansion: type II (each term in the action generates more gauge fields)

type II studied for:

- NR particle (and coupling to non-relativistic gravity from expanding GR) van den Bleeken (2018), Hansen, Hartong, NO (2019, 2020)
- NR string

Hartong,Have (2021)

spectrum: (compact long. direction) center of mass velocity << c

$$E = \frac{c^2 w R_{\text{eff}}}{\alpha'_{\text{eff}}} + \frac{N_{(0)} + \tilde{N}_{(0)}}{w R_{\text{eff}}} + \frac{\alpha'_{\text{eff}}}{2w R_{\text{eff}}} p_{(0)}^2 + \mathcal{O}(c^{-2})$$

Beta-functions

beta functions/effective spacetime actions for the NR string obtained in various different formulations/using different methods

> Gomis,Oh,Yan(2019) Bergshoeff,Gomis,Rosseel,Simsek,Yan(2019) Yan,Yu(2019) Bergshoeff et al (2021), Yan (2021) Gallegos,Gursoy,Zinnato(2019) Gallegos,Gursoy,Verma,Zinnato(2020

→ describe the dynamics of (versions of) non-relativistic gravity

see also Hansen, Hartong, NO(2019, 2020)

Applications to AdS/CFT

can take a further world-sheet NR limit

-→ new class of sigma models with NR target spacetime that are also non-relativistic on worldsheet:

exhibits 2D GCA (classically)

 $[L_n, L_m] = (n-m)L_{n+m}, \qquad [L_n, M_m] = (n-m)M_{n+m}.$

• NR WS theories directly related to near-BPS limits of AdS/CFT (spin-matrix theory (SMT))

simplest example: LL model appearing from continuum limit of Heisenberg spin chains

Spin Matrix Theory (intermezzo)

Harmark, Orselli (1409)

 SMT limits of AdS/CFT obtained by zooming in to unitarity bounds of N=4 SYM on RxS3:

$$\lambda \to 0$$
 , $\frac{E-Q}{\lambda} = \text{fixed}$

Q = linear sum of Cartan charges of PSU(2,2|4)

 \rightarrow N=4 SYM simplifies and becomes QM theory

- reduces to nearest-neighbor spin chains in planar N limit

Spin Matrix Theory (intermezzo)

SMT limits of AdS/CFT obtained by zooming in to unitarity bounds of N=4 SYM on RxS3:

$$\lambda \to 0$$
 , $\frac{E-Q}{\lambda} = \text{fixed}$

Q = linear sum of Cartan charges of PSU(2,2|4)

Kruczenski (0311)

 \rightarrow N=4 SYM simplifies and becomes QM theory

- reduces to nearest-neighbor spin chains in planar N limit

low energy excitations of spin chains = magnons

$$E - Q = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} - 1$$
 becomes in
SMT limt:
$$H - Q = \frac{g}{2\pi^2} \sin^2 \frac{p}{2}$$
non-relativistic
non-relativistic

- semi-classical limits of spin chains become sigma models: e.g. Landau-Lifshitz model $\mathcal{L}_{\rm LL} = \frac{J}{4\pi} \left| \sin\theta \dot{\phi} - \frac{1}{4} \left((\theta')^2 + \cos^2\theta (\phi')^2 \right) \right|$ for SU(2) sector

Stringy side of SMT gives NR sigma models

- using AdS/CFT dictionary: SMT (near-BPS) limit can be formulated as limit of type IIB string theory on AdS5xS5
 - correspond to non-relativistic world-sheet string theories !
- → LL model (and generalizations for other near-BPS sectors) is example of our novel class of non-relativistic worldsheet STs with a Newton-Cartan type target spacetime

- one of target space dimensions = position along the spin chain (zero momentum because of cyclicity of trace)

- strongly suggests: bulk description of SMT is a type of NR gravity

 new class of flat-fluxed backgrounds obtained recently: analogue of flat Minkowski space using Penrose type limits
 natural starting point to quantize the theory

Outlook (non-relativistic string theory)

- open strings and branes:
 - non-relativistic open string sector and DBI actions Gomis,Yan,Yu (2020)
 - connection to NR D/M-branes

Kluson/Blair,Gallegos,Zinnato (2021)/Ebert,Sun,Yan(2021)

- strings/branes as background solutions Bergshoeff,Lahnsteiner,Romano,Rosseel(2022)
- generalize procedure to non-relativistic limit of extremal p-branes TSNC analogue for p-branes (incl. D/M) Bidussi,Harmark,Hartong,NO,Oling (in progress
- SUSY generalization of stringy Poincare (include RR fields) NR limit & relations to DFT/exceptional FT
 non-perturbative dualities in NR string theory
- connection to integrable models

Gomis,Gomis,Kamimura(2005)/Roychowdhury(2019/ Fontanella,NietoGarcia,Torielli(2021),Fontanello,van Tongeren(2022)

Outlook (non-relativistic worldsheet models)

- further study (quantization) of NR world-sheet theories
 role of GCA
- Hamiltonian analysis (including for non-relativistic world-sheet models) Kluson (2021), Bidussi, Harmark, Hartong, NO, Oling (to appear)
- obtain beta functions for NR worldsheet models
- connection with explicit construction of SMT using classical reduction of N=4 SYM & suitable quantization method Harmark,Wintergerst (2019),Baiguera,Harmark,Wintergerst(2020)

Harmark, Wintergerst (2019), Baiguera, Harmark, Wintergerst (2020) Baiguera, Harmark, Lei, Wintergerst (2020)

• Carrollian (small speed of light) gravity and strings....

Duality web of `non-Lorentzian' string theories?

web of decoupled non-gravitational theories

(`open string sector')



web of non-Lorentzian gravitational
string theories
 (`closed string sector'_)



back to 2000s...

self-contained corners of ST w. own geometrynew window on non-perturbative effects ?

Non-AdS holography & NR holography



Routes towards non-Lorentzian holography

I. branes

- Dp-branes as probes of TSNC geometry
- Dp-branes as backgrounds solutions of NR (super) gravity actions

Can one find decoupling limits giving rise to avatars of AdS/CFT ?

II. limits of AdS/CFT

- SMT-limit dual to NR world-sheet theories
- study quantization
- Hamiltonian analysis
- beta-functions

→ tractable limit of AdS/CFT, finite N, simpler moduli space/genus expansion ?