D-instanton Effects in Type IIB String Theory

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April 28, 2022

hep-th/2205.xxxxx [Agmon, BB, Cho, Rodriguez,Yin]

Introduction

There's a well-known formalism for computing closed string scattering amplitudes in a perturbative expansion in the string coupling.

Sphere $2 \to 2$ scattering amplitude of gravitons

$$\mathcal{A}^{\text{tree}}_{2\to 2} = -\frac{1}{64} K(e_i, p_i)\kappa^2 \frac{\Gamma(-\frac{s}{4})\Gamma(-\frac{t}{4})\Gamma(-\frac{u}{4})}{\Gamma(\frac{s}{4}+1)\Gamma(\frac{t}{4}+1)\Gamma(\frac{u}{4}+1)}$$

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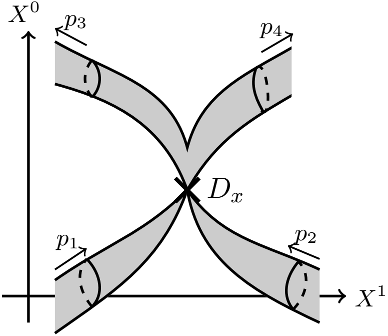
Today I will present a formalism to compute *non-perturbative* contributions to closed string scattering amplitudes, and I will apply it to type IIB string theory.

$$\mathcal{A}^{(1,0)}_{2\to 2} = K(e_i, p_i)\frac{4\pi^8}{\tau_2^{\frac{7}{2}}} e^{2\pi i \tau}\left(1 - \frac{1}{2\pi\tau_2}\left[s\psi\left(1-\frac{s}{4}\right) + t\psi\left(1-\frac{t}{4}\right) + u\psi\left(1-\frac{u}{4}\right)\right]\right)$$

$\tau = \tau_1 + i\tau_2$ is the axion-dilaton field of type IIB string theory ($\tau_2 \sim 1/g_s \sim 1/\kappa$).

$\psi(x)$ is the digamma function ($\psi(x) \equiv \Gamma'(x)/\Gamma(x)$)

These contributions arise via a new class of worldsheet diagrams with D-instanton boundary conditions.



Groundwork laid out by `[Polchinski'94]`. Applied at leading order by `[Green,Gutperle'97]`.

See also `[BB,Rodriguez,Yin'19]` for recent developments in the context of $2d$ string theory, and also `[Sen'19-'21]` for further understanding using SFT.

Write $2 \to 2$ amplitude as (in Planck units)

$$\mathcal{A}_{2\to 2} = \mathcal{A}_{2\to 2}^{\text{SUGRA}} M(\underline{s}, \underline{t}, \tau, \bar{\tau}) \qquad \left(\underline{s} = s\tau_2^{-\frac{1}{2}} \right)$$

Low energy expansion:

$$\begin{aligned} M(\underline{s}, \underline{t}, \tau, \bar{\tau}) =& \underline{stu} \Big[\frac{1}{\underline{stu}} + f_0(\tau, \bar{\tau}) + f_4(\tau, \bar{\tau})(\underline{s}^2 + \underline{t}^2 + \underline{u}^2) \\ & + f_6(\tau, \bar{\tau})(\underline{s}^3 + \underline{t}^3 + \underline{u}^3) + f_8(\tau, \bar{\tau})(\underline{s}^4 + \underline{t}^4 + \underline{u}^4) + \cdots \Big] \end{aligned}$$

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$f_k(\tau, \bar{\tau})$ captures the $D^k R^4$ graviton vertex in the low-energy (quantum) effective action of type IIB string theory. $f_k(\tau, \bar{\tau})$ are invariant under $SL(2, \mathbb{Z})$.

$$\tau \to \frac{a\tau + b}{c\tau + d}, \qquad ad - bc = 1, \qquad a, b, c, d \in \mathbb{Z}$$

Supersymmetry and $SL(2,\mathbb{Z})$ are enough to completely fix $f_0(\tau,\bar\tau)$, $f_4(\tau,\bar\tau)$, $f_6(\tau,\bar\tau)$ (R^4, D^4R^4, D^6R^4)
[D'Hoker,Green,Gutperle,Sethi,Vanhove,Wang,Wen,Yin,...]

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\begin{aligned}
f_0(\tau,\bar\tau) &= \frac{\zeta(3)}{32}\tau_2^{\frac{3}{2}} + \cdots + e^{2\pi i\tau}\left(\frac{\pi}{16} + \cdots\right) + \cdots \\
f_4(\tau,\bar\tau) &= \frac{\zeta(5)}{1024}\tau_2^{\frac{5}{2}} + \cdots + e^{2\pi i\tau}\left(\frac{\zeta(2)}{128} + \cdots\right) + \cdots \\
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Amplitude admits the expansion:

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&\quad + \sum_{n,m=0}^{\infty} e^{2\pi i(n\tau - m\bar\tau)}\tau_2^{N(n,m)} \sum_{L=0}^{\infty} \tau_2^{-L}\mathcal{A}_{2\to 2;L}^{\text{inst},(n,m)} \qquad (n>0 \text{ or } m>0)
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Q: Can we obtain these non-perturbative contributions from a worldsheet formalism?

D-instanton calculus

Non-perturbative processes are mediated by D-instantons

Boundary conditions that are localized in (Euclidean) spacetime - "$D(-1)$-branes"

$$\begin{aligned} X^\mu\big|_{\partial\Sigma} &= x^\mu, \\ \psi^\mu + \widetilde{\psi}^\mu\Big|_{\partial\Sigma} &= 0, \qquad\qquad \mu = 0, ..., 9 \end{aligned}$$

In Type IIB, have $\frac{1}{2}$-BPS D- and $\bar{D}$-instanton, oppositely charged under RR axion τ_1.

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SFT approach: write string field theory action for open+closed strings. Integrate out open string fields to obtain an effective action for the closed strings only (special care must be taken in integrating over zero modes of the D-instanton) `[Sen'20]`

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Worldsheet approach: extend the rules of worldsheet perturbation theory to include worldsheet diagrams with boundaries. Diagrams with holes are the "effective vertices" obtained after integrating out open strings (simpler, but some numerical constants are ambiguous)

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D-instanton boundary condition does not preserve translation invariance.
Must allow for *disconnected* worldsheet diagrams

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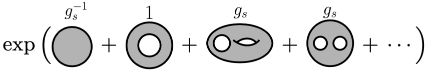

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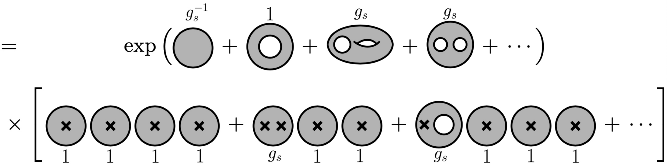

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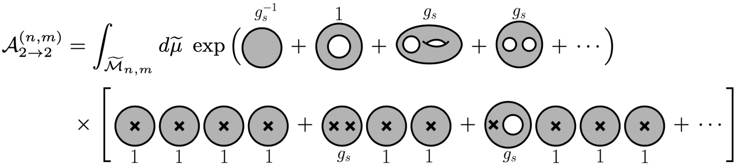

$\widetilde{\mathcal{M}}_{n,m}$ is the moduli space of collective coordinates for a (n,m) D-instanton, and $d\widetilde{\mu}$ its measure.

This expansion is of the form

$$\sum_{n,m=0}^{\infty} e^{2\pi i(n\tau - m\bar{\tau})} \tau_2^{N(n,m)} \sum_{L=0}^{\infty} \tau_2^{-L} \mathcal{A}_{2\to2;L}^{\mathrm{inst},(n,m)} \qquad (n > 0 \text{ or } m > 0)$$

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(1,0) D-instanton

- NS sector zero modes: position x^μ of D-instanton
- R sector zero modes: fermionic collective coordinates θ_α associated to spacetime supercharges Q^α broken by the D-instanton

In worldsheet diagrams, the dependence on θ^α comes from the insertion of the R sector boundary deformation $\theta_\alpha Q^\alpha$

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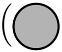
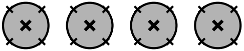
(1,0) D-instanton contribution at leading open string loop order:
`[Green,Gutperle'97]`

$\mathcal{A}^{(1,0)}_{2\to 2;L=0} = \mathcal{N}_D \exp\Big($  $+$  $\Big) \int d^{10}x\, d^{16}\theta$ 

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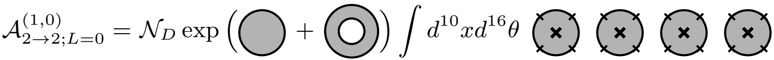

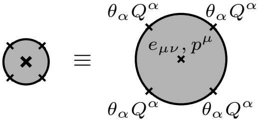

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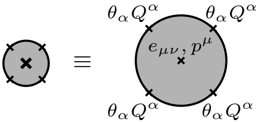
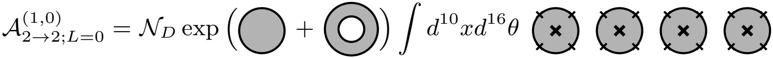
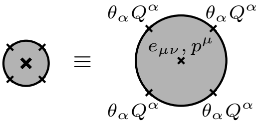
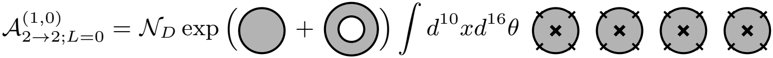
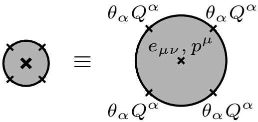


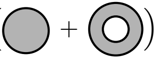
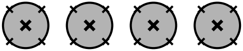






$$= e^{2\pi i\tau}\mathcal{N}_D \int d^{10}x\ e^{i\sum_i p_i\cdot x} K(e_i, p_i)\left(A^{D_2}_{1-\mathrm{pt}}\right)^4$$



$$\mathcal{A}^{(1,0)}_{2\to 2; L=0} = \mathcal{N}_D \exp \Big($$  $$\Big) \int d^{10}x d^{16}\theta$$ 

$$= e^{2\pi i \tau} \mathcal{N}_D \int d^{10}x \; e^{i\sum_i p_i \cdot x} K(e_i, p_i) \left(A^{D_2}_{1-\mathrm{pt}} \right)^4$$

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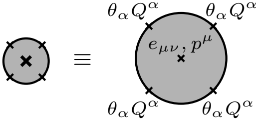
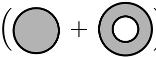
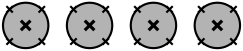


$\mathcal{A}^{(1,0)}_{2\to 2; L=0} = \mathcal{N}_D \exp\Big($  $\Big) \int d^{10}x d^{16}\theta$ 

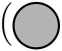
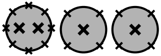
$$= e^{2\pi i\tau}\mathcal{N}_D \int d^{10}x \; e^{i\sum_i p_i \cdot x} K(e_i, p_i) \left(A^{D_2}_{1-\mathrm{pt}}\right)^4$$

$$= i(2\pi)^{10}\delta^{(10)}\left(\sum_i p_i\right) K(e_i, p_i)\mathcal{N}_D e^{2\pi i\tau}$$

Only contributes to R^4 vertex of low-energy quantum effective action.

D-instanton calculus

Now consider $\mathcal{A}^{(1,0)}_{2\to 2;L=1}$

$\mathcal{A}^{(1,0)}_{2\to 2;L=1} = \mathcal{N}_D \exp\Big($  $+$  $\Big) \int d^{10}x\, d^{16}\theta \Big[$  $+$ permutations $\Big]$

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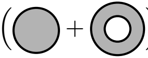
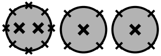


$$\Bigg)\int d^{10}x d^{16}\theta \Bigg[$$



$$+ \text{permutations}\Bigg]$$

$$= e^{2\pi i\tau}\mathcal{N}_D \int d^{10}x \; e^{i\sum_i p_i\cdot x} K(e_i, p_i)\left[-\frac{s}{2\tau_2}\psi\left(1-\frac{s}{4}\right) + (t,u)\right]\left(A^{D_2}_{1-\mathrm{pt}}\right)^2$$

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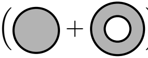

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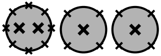

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Contributes to all vertices $D^k R^4$ in the low-energy effective action!

Low-energy expansion of $\mathcal{A}^{(1,0)}_{2\to 2;L=0} + \mathcal{A}^{(1,0)}_{2\to 2;L=1}$:

$$M^{(1,0)}_{L=0,1}(\underline{s},\underline{t},\tau,\bar{\tau}) = e^{2\pi i\tau}\,\frac{\pi}{16}\left[1 - \frac{\underline{s}\tau_2^{\frac{1}{2}}}{2\pi\tau_2}\psi\left(1 - \frac{\underline{s}\tau_2^{\frac{1}{2}}}{4}\right) + (\underline{t},\underline{u})\right]$$

$$= e^{2\pi i\tau}\left(\frac{\pi}{16} + \frac{\zeta(2)}{128}(\underline{s}^2 + \underline{t}^2 + \underline{u}^2) + \frac{\zeta(3)}{2^9}\tau_2^{\frac{1}{2}}(\underline{s}^3 + \underline{t}^3 + \underline{u}^3) + \cdots\right)$$

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Overall constant $\mathcal{N}_D$ has been fixed by:

- SFT approach `[Sen'21]`
- $SL(2,\mathbb{Z})$ duality `[Green,Gutperle'97]`

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Low-energy expansion of $\mathcal{A}^{(1,0)}_{2\to 2;L=0} + \mathcal{A}^{(1,0)}_{2\to 2;L=1}$:

$$M^{(1,0)}_{L=0,1}(\underline{s},\underline{t},\tau,\bar{\tau}) = e^{2\pi i\tau}\frac{\pi}{16}\left[1 - \frac{\underline{s}\tau_2^{\frac{1}{2}}}{2\pi\tau_2}\psi\left(1-\frac{\underline{s}\tau_2^{\frac{1}{2}}}{4}\right) + (\underline{t},\underline{u})\right]$$

$$= e^{2\pi i\tau}\left(\frac{\pi}{16} + \frac{\zeta(2)}{128}(\underline{s}^2+\underline{t}^2+\underline{u}^2) + \frac{\zeta(3)}{2^9}\tau_2^{\frac{1}{2}}(\underline{s}^3+\underline{t}^3+\underline{u}^3)+\cdots\right)$$

Overall constant $\mathcal{N}_D$ has been fixed by:

- SFT approach `[Sen'21]`
- $SL(2,\mathbb{Z})$ duality `[Green,Gutperle'97]`

$$f_0(\tau,\bar{\tau}) = \frac{\zeta(3)}{32}\tau_2^{\frac{3}{2}} + \cdots + e^{2\pi i\tau}\left(\frac{\pi}{16}+\cdots\right)$$

$$f_4(\tau,\bar{\tau}) = \frac{\zeta(5)}{1024}\tau_2^{\frac{5}{2}} + \cdots + e^{2\pi i\tau}\left(\frac{\zeta(2)}{128}+\cdots\right)$$

$$f_6(\tau,\bar{\tau}) = \frac{\zeta(3)^2}{3\cdot 2^{11}}\tau_2^{3} + \cdots + e^{2\pi i\tau}\left(\frac{\zeta(3)}{2^9}\tau_2^{\frac{1}{2}}+\cdots\right)$$

Perfect agreement with all predictions from susy+$SL(2,\mathbb{Z})$-duality!

Summary

I discussed a worldsheet formalism for computing non-perturbative scattering amplitudes of closed strings and applied it to type IIB string theory. We obtained the leading D-instanton correction to all vertices $D^k R^4$ of the low-energy effective action.

Very general formalism, can be applied to any string model with a worldsheet description (see the next talks!)
`[BB,Rodriguez,Yin'19;Alexandrov,Sen,Stefanski'21;...]`

In many ways simpler than perturbative scattering amplitudes. We have also computed $2 \to N$ maximal-R-symmetry-violating amplitude at leading non-perturbative order.

Also computed $e^{-4\pi\tau_2}$ contribution to $D^6 R^4$ vertex of low-energy effective action, coming from a D-$\bar{D}$-instanton pair. Result is in perfect agreement with expectation from $f_6(\tau, \bar{\tau})$.

Future directions

- Multiple D-instantons: have to deal with singularities in moduli space. These are associated to open strings going on-shell `[Sen'21;BB,Rodriguez,Yin'19]`

- Application to open string scattering amplitudes

- (1,0) D-instanton contribution to R^4 vertex at subleading order in the open string loop expansion

- Input to S-matrix bootstrap `[Guerrieri,Penedones,Vieira'21]`

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Thank you for listening!