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Large N topologically twisted indices, holography, and black holes

Valentin Reys

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Based on [arXiv:2203.14981] and work in progress
with **Nikolay Bobev** & **Junho Hong**.

Motivation

- AdS/CFT provides a gauge theory description of string/M-theory on asymptotically locally AdS backgrounds:

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- $Z_{\mathsf{CFT}} = Z_{\mathsf{string}}$ is meant to be valid beyond the planar limit!
- Supersymmetric localization can be used to compute susy observables in SCFTs exactly.
- Provides a new handle on AdS vacua of string/M-theory with non-trivial fluxes, including AdS black holes.

Outline

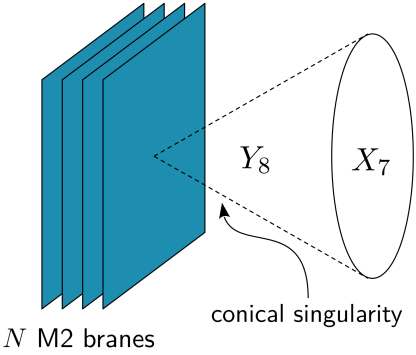
1. ABJM theory on S^3
2. ABJM theory on $S^1 \times \Sigma_{\mathfrak{g}}$
3. Holography and black holes
4. Summary and outlook

$\text{AdS}_4/\text{CFT}_3$ dual pairs from M2 branes

- Consider 3d SCFTs with $\mathcal{N} \geq 2$ decribing the low energy limit of N M2 branes probing a conical singularity.

Superconformal Chern-Simons theories coupled to matter can be used to describe M-theory on $\text{AdS}_4 \times X_7$ backgrounds.

Focus on $X_7 = S^7/\mathbb{Z}_k$ corresponding to ABJM theory (ask me about other X_7).



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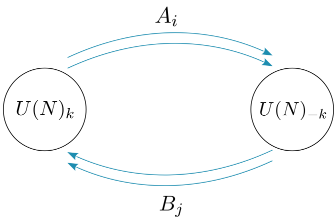
ABJM theory

- $U(N)_k \times U(N)_{-k}$ with two pairs of bifundamental chirals and superpotential $W = \text{Tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1)$.



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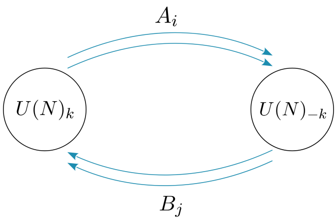

- For $k > 2$, $\mathcal{N} = 6$ susy and $SU(4)_R \times U(1)_T$ global symmetry.
- The partition function on S^3 can be computed by localization and yields a matrix model. [Kapustin,Willett,Yaakov'09]
- $Z_{S^3}(N,k)$ can be studied at large N and fixed k.
  [Herzog,Klebanov,Pufu,Tesileanu'10; Mariño,Putrov'11]

The sphere partition function

- The perturbative part is an Airy function:

[Mariño,Putrov'11; Fuji,Hirano,Moriyama'11]

$$Z_{S^3}(N,k) = e^{\mathcal{A}(k)}\, C^{-\frac{1}{3}}\, \mathrm{Ai}\big[C^{-\frac{1}{3}}(N - B)\big] + \mathcal{O}(e^{-\sqrt{N}})\,,$$

where $C = \frac{2}{\pi^2 k}$ and $B = \frac{k}{24} + \frac{1}{3k}$ controls a shift in N.
The function $\mathcal{A}(k)$ is also known in closed form.

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- Systematic large N expansion of the free energy $F_{S^3} = -\log Z_{S^3}$,

$$F_{S^3} = \frac{2}{3\sqrt{C}} N^{\frac{3}{2}} - \frac{B}{\sqrt{C}} N^{\frac{1}{2}} + \frac{1}{4} \log N - \mathcal{A}(k) + \frac{1}{4}\log\frac{32}{k} + \mathcal{O}(N^{-\frac{1}{2}})\,.$$

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- In the bulk, reproduced by two-derivative and four-derivative supergravity regularized on-shell actions and loop corrections.

  [Emparan,Johnson,Myers'99; Bobev,Charles,Hristov,VR'21]

  [Bhattacharyya,Grassi,Mariño,Sen'12]

An Airy tale

- Deformations that preserve susy and break conformal invariance:
  - Put the theory on squashed S_b^3 with $U(1) \times U(1)$ isometry.
  - Turn on real masses $m_{1,2,3}$ in the Cartan of the flavor group.

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- Conjecture: the partition function is again an Airy function!

[Bobev,Hong,VR'22; Hristov'22]

$$Z_{S_b^3}(N,k,\Delta) = e^{\mathcal{A}(k,b,\Delta)} C_\Delta^{-\frac{1}{3}} \mathrm{Ai}\left[C_\Delta^{-\frac{1}{3}}(N - B_\Delta)\right] + \mathcal{O}(e^{-\sqrt{N}})$$

$$C_\Delta = \frac{2}{\pi^2 k} \frac{(b+b^{-1})^{-4}}{\prod_a \Delta_a}, \quad B_\Delta = \frac{k}{24} - \frac{\sum_a \Delta_a^{-1}}{12k} + \frac{1 - \frac{1}{4}\sum_a \Delta_a^2}{3k(b+b^{-1})^2 \prod_a \Delta_a}.$$

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- Written in terms of parameters $\Delta_a(b, m_i)$ such that $\sum_{a=1}^4 \Delta_a = 2$. Setting $b=1$ and $\Delta_a = \frac{1}{2}$ recovers the round sphere above.

Evidence and predictions

- In agreement with known results in various limits for (b, m_i).

[Nosaka‘15; Hatsuda‘16; Chester,Kalloor,Sharon‘21]

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  [Closset,Dumitrescu,Festuccia,Komargodski'12]

$$F_{S_b^3} = \frac{\pi\sqrt{2k}}{12}(b+b^{-1})^2 N^{\frac{3}{2}} - \frac{\pi\sqrt{2k}}{12}\left(\frac{k^2-16}{16k}(b+b^{-1})^2 + \frac{6}{k}\right) N^{\frac{1}{2}} + \ldots$$

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- Leading and subleading terms obtained in supergravity from two-derivative and four-derivative regularized on-shell actions.
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- Airy encodes integrated correlation functions of ABJM on S^3
  $\rightarrow$ can give access to graviton scattering in M-theory.

  [Chester,Pufu,Yin'18; Binder,Chester,Pufu'18]

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- The TTI is a partition function of 3d $\mathcal{N} = 2$ SCFTs on $S^1 \times \Sigma_{\mathfrak{g}}$.
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- Localization reduces the path-integral to a matrix model.

[Benini,Zaffaroni‘15; Closset,Kim‘16]

- The result depends on (N, k) and on four magnetic charges $\mathfrak{n}_a$ and four electric chemical potentials Δ_a (mixed ensemble). Satisfy susy constraints $\sum_{a=1}^{4} \mathfrak{n}_a = 2(1 - \mathfrak{g})$ and $\sum_{a=1}^{4} \Delta_a = 2$.

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- Picking up residues, the TTI can be recast as a sum over solutions to a complicated set of transcendental equations that resemble Bethe Ansatz Equations (BAE).
- Strategy: use the large N solution of the BAE as a starting point to numerically evaluate the TTI to high order in $1/N$.

[Benini,Hristov,Zaffaroni'15; Liu,Pando Zayas,Rathee,Zhao'17]

The large N expansion of the TTI

- The numerics are very precise and allow us to propose an analytic expression! It is naturally written in terms of a shifted N,

$$\hat{N}_\Delta = N - \frac{k}{24} + \frac{1}{12k} \sum_{a=1}^{4} \Delta_a^{-1} \, .$$

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- The TTI to all orders in a $1/N$ expansion:

$$F_{S^1\times\Sigma_{\mathfrak{g}}} = \frac{\pi\sqrt{2k\prod_a\Delta_a}}{3}\sum_{a=1}^{4}\frac{\mathfrak{n}_a}{\Delta_a}\Big(\hat{N}_\Delta^{\frac{3}{2}} - \frac{d_a}{k}\,\hat{N}_\Delta^{\frac{1}{2}}\Big) + \frac{1-\mathfrak{g}}{2}\log\hat{N}_\Delta + f(k,\Delta,\mathfrak{n}) + \mathcal{O}(e^{-\sqrt{N}})$$

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- Expand at large N to recover the $N^{\frac{3}{2}}$, $N^{\frac{1}{2}}$ and $\log N$ terms.

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- Susy Euclidean "black saddle" solutions in the 4d STU model that uplift to 11d with 4-form flux, [Bobev,Charles,Min'20] [Azizi,Godazgar,Godazgar,Pope'16]

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- For some values of parameters, black saddles have a Lorentzian interpretation as static BPS black holes with $\mathrm{AdS}_2 \times \Sigma_{\mathfrak{g}}$ NHG.

  [Gauntlett,Kim,Pakis,Waldram'01; Cacciatori,Klemm'09]

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- Our proposed analytic TTI encodes their quantum entropy to all orders in the parameter N of the microscopic theory!

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- Understand the results in terms of the relation $Z_{S^3} = \langle \mathcal{F} \rangle_{S^1 \times S^2}$? Study other 3-manifolds (e.g. Seifert)? [Closset,Kim,Willett'17]
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- Analytic derivation of the TTI from the matrix model?
- Highlighted available match for leading, subleading, and log terms from supergravity. Can we go beyond?
- Localization in supergravity looks very promising...
  [Dabholkar,Drukker,Gomes'14; Hristov,Lodato,VR'17-18; Hristov'21-22]

Thank you for your attention!

The TTI of various 3d SCFTs

- 3d $\mathcal{N} = 3$ SCFT with $X_7 = N^{0,1,0}$: $\hat{N}_{\Delta_a = \frac{1}{2}} = N + \frac{k}{12} + \frac{1}{3k}$ and

$$\frac{F_{S^1\times\Sigma_{\mathfrak{g}}}}{1-\mathfrak{g}} = \frac{4\pi\sqrt{k}}{3\sqrt{3}}\left(\hat{N}^{\frac{3}{2}} - \left(\frac{k}{4} + \frac{5}{4k}\right)\hat{N}^{\frac{1}{2}}\right) + \frac{1}{2}\log\hat{N} + f_0(k)\,.$$

- 3d $\mathcal{N} = 2$ SCFT with $X_7 = V^{5,2}$: $\hat{N}_{\Delta_a = \frac{1}{2}} = N + \frac{k}{6} + \frac{1}{4k}$ and

$$\frac{F_{S^1\times\Sigma_{\mathfrak{g}}}}{1-\mathfrak{g}} = \frac{16\pi\sqrt{k}}{27}\left(\hat{N}^{\frac{3}{2}} - \left(\frac{9k}{16} + \frac{27}{16k}\right)\hat{N}^{\frac{1}{2}}\right) + \frac{1}{2}\log\hat{N} + f_0(k)\,.$$

- 3d $\mathcal{N} = 2$ SCFT with $X_7 = Q^{1,1,1}$: $\hat{N}_{\Delta_a = \frac{1}{2}} = N + \frac{k}{6}$ and

$$\frac{F_{S^1\times\Sigma_{\mathfrak{g}}}}{1-\mathfrak{g}} = \frac{4\pi\sqrt{k}}{3\sqrt{3}}\left(\hat{N}^{\frac{3}{2}} - \left(\frac{k}{4} + \frac{3}{4k}\right)\hat{N}^{\frac{1}{2}}\right) + \frac{1}{2}\log\hat{N} + f_0(k)\,.$$

The universal black hole

- Susy Euclidean solution of 4d $\mathcal{N}=2$ gauged supergravity:

[Romans'92; Bobev,Charles,Min'20]

$$ds_4^2 = U(r)d\tau^2 + \frac{dr^2}{U(r)} + r^2 ds^2_{\Sigma_{\mathfrak{g}}}\,, \quad F = \frac{q}{r^2}d\tau\wedge dr - \frac{\kappa}{g}\mathrm{vol}(\Sigma_{\mathfrak{g}})\,,$$

$$U(r) = \left(\sqrt{2}gr + \frac{\kappa}{2\sqrt{2}gr}\right)^2 - \frac{q^2}{8r^2}\,, \qquad \kappa = \{1,0,-1\}\,.$$

Smooth for $g|q| > \kappa$ with Euclidean periodicity $\beta_\tau = \frac{\pi\sqrt{g|q|-\kappa}}{g^2|q|}$.

- Regular Lorentzian black hole obtained by taking $|q|\to 0$. Only exists for $\kappa=-1$, i.e. hyperbolic horizon.

- Euclidean regularized on-shell action I is independent of β_τ. For the Lorentzian black hole, leads to the usual

$$S_{\mathrm{BH}} = -I\,.$$

The universal TTI

- Our proposed TTI with $\mathfrak{n}_a = \frac{1-\mathfrak{g}}{2}$ and $\Delta_a = \frac{1}{2}$ accounts for the entropy of the black hole to all orders in the $1/N$ expansion!

$$\frac{F_{S^1\times\Sigma_{\mathfrak{g}}}}{1-\mathfrak{g}} = \frac{\pi\sqrt{2k}}{3}\left(N^{\frac{3}{2}} - \frac{k^2+32}{16k}N^{\frac{1}{2}}\right) + \frac{1}{2}\log N + \mathcal{O}(N^0)\,.$$

- Leading term reproduced from two-derivative supergravity regularized on-shell action. [Benini,Hristov,Zaffaroni'15] [Azzurli,Bobev,Crichigno,Min,Zaffaroni'17]
- Subleading term from four-derivative supergravity regularized on-shell action. [Bobev,Charles,Hristov,VR'21]
- Log term from one-loop contributions of the KK modes in 11d. [Liu,Pando Zayas,Rathee,Zhao'17]