D-branes in $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ at $k = 1$ and their holographic duals

J. Vošmera

arxiv:2110.05509 (with M. Gaberdiel and B. Knighton)

Institut für Theoretische Physik, ETH Zürich

Eurostrings 2022

Goal

AdS/CFT correspondence is a proposed strong/weak duality between:

- a theory of quantum gravity in d dimensions
- a gauge theory in $d-1$ dimensions

[Maldacena '97]

Goal

AdS/CFT correspondence is a proposed strong/weak duality between:

- a theory of quantum gravity in d dimensions
- a gauge theory in $d-1$ dimensions

[Maldacena '97]

$\mathrm{AdS}_3 \times \mathrm{S}_3 \times \mathbb{T}^4$ at $k=1$ provides setting where amplitudes and correlators are exactly computable on both sides, thus giving a perturbative proof of the duality.
[Eberhardt, Gaberdiel, Gopakumar, Knighton, Dei, ...]

Goal

AdS/CFT correspondence is a proposed strong/weak duality between:

- a theory of quantum gravity in d dimensions
- a gauge theory in $d-1$ dimensions

[Maldacena '97]

AdS$_3$ $\times$ S$_3$ $\times$ $\mathbb{T}^4$ at $k=1$ provides setting where amplitudes and correlators are exactly computable on both sides, thus giving a perturbative proof of the duality.
[Eberhardt, Gaberdiel, Gopakumar, Knighton, Dei, ...]

Can we extend the duality to cover D-branes in the bulk?

Part I: Introduction and motivation

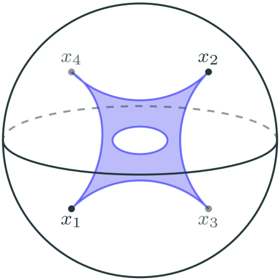
AdS/CFT holographic dictionary

On-shell closed string states in AdS $\iff$ single-trace operators in the CFT

AdS/CFT holographic dictionary

On-shell closed string states in AdS $\Longleftrightarrow$ single-trace operators in the CFT

String amplitudes in AdS $\Longleftrightarrow$ Correlators in the CFT

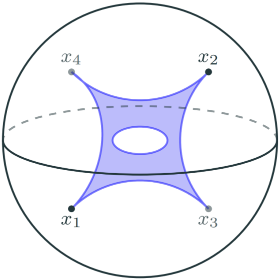


[figures by Bob Knighton]

AdS/CFT holographic dictionary

On-shell closed string states in AdS $\iff$ single-trace operators in the CFT

String amplitudes in AdS $\iff$ Correlators in the CFT



[figures by Bob Knighton]

Genus expansion of amps in AdS $\iff$ loop exp. of CFT correlators

$$\sum_{\text{genus}} g_{\mathsf{s}}^{2g-2} \int_{\mathcal{M}_{g,n}} \mathcal{O}_{\text{string},g,n} = \sum_{\ell} N^{2-2\ell} \mathcal{O}_{\text{CFT},\ell,n}$$

$\implies$ $g_{\mathsf{s}} \sim 1/N$, duality holds order-by-order in g_{s}

$\mathsf{AdS}_3/\mathsf{CFT}_2$ duality

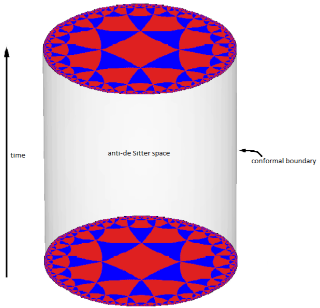
Consider a superposition of N 1-branes and k 5-branes on a $\mathbb{R}^{1,4} \times \mathrm{S}^1 \times \mathbb{T}^4$

AdS$_3$/CFT$_2$ duality

Consider a superposition of N 1-branes and k 5-branes on a $\mathbb{R}^{1,4} \times \mathrm{S}^1 \times \mathbb{T}^4$

Branes backreact on the bulk to produce non-trivial geometry

bulk theory: superstring on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$

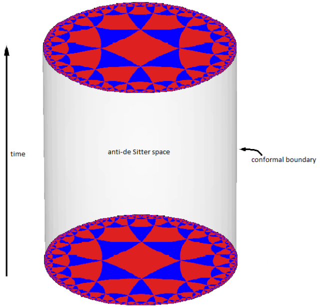


$\mathrm{AdS}_3/\mathrm{CFT}_2$ duality

Consider a superposition of N 1-branes and k 5-branes on a $\mathbb{R}^{1,4} \times \mathrm{S}^1 \times \mathbb{T}^4$

Branes backreact on the bulk to produce non-trivial geometry

bulk theory: superstring on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$



1-branes viewed as SYM instantons within the 5-branes [Seiberg, Witten '99]

2d CFT: sigma-model on the (resolved) ADHM moduli space

A free-field miracle

(Almost) free-field point:

$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv (\mathbb{T}^4)^{\otimes N}/S_N \quad \Longrightarrow \quad \text{symmetric-product orbifold CFT}$$

A free-field miracle

(Almost) free-field point:

$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv (\mathbb{T}^4)^{\otimes N}/S_N \quad \Longrightarrow \quad \text{symmetric-product orbifold CFT}$$

Dual to $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ with $k = 1$ NS5-branes! [Eberhardt, Gaberdiel, Gopakumar '18]

A free-field miracle

(Almost) free-field point:

$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv (\mathbb{T}^4)^{\otimes N}/S_N \qquad \Longrightarrow \qquad \text{symmetric-product orbifold CFT}$$

Dual to $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ with $k = 1$ NS5-branes! [Eberhardt, Gaberdiel, Gopakumar '18]

$\rightarrow$ exact worldsheet description (in hybrid formalism)

$$\mathfrak{psu}(1,1|2)_1 \text{ super-WZW } \oplus \text{top. twisted } \mathbb{T}^4 \oplus \text{ghosts}$$

A free-field miracle

(Almost) free-field point:

$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv (\mathbb{T}^4)^{\otimes N}/S_N \quad \Longrightarrow \quad \text{symmetric-product orbifold CFT}$$

Dual to $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ with $k = 1$ NS5-branes! [Eberhardt, Gaberdiel, Gopakumar '18]

$\rightarrow$ exact worldsheet description (in hybrid formalism)

$$\mathfrak{psu}(1,1|2)_1 \text{ super-WZW } \oplus \text{top. twisted } \mathbb{T}^4 \oplus \text{ghosts}$$

$\rightarrow$ free-field realisation

$$\mathfrak{psu}(1,1|2)_1 \sim 4 \text{ symplectic bosons} \oplus 4 \text{ real fermions}$$

A free-field miracle

(Almost) free-field point:

$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv (\mathbb{T}^4)^{\otimes N}/S_N \quad \Longrightarrow \quad \text{symmetric-product orbifold CFT}$$

Dual to $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ with $k = 1$ NS5-branes! [Eberhardt, Gaberdiel, Gopakumar '18]

$\rightarrow$ exact worldsheet description (in hybrid formalism)

$$\mathfrak{psu}(1,1|2)_1 \text{ super-WZW } \oplus \text{top. twisted } \mathbb{T}^4 \oplus \text{ghosts}$$

$\rightarrow$ free-field realisation

$$\mathfrak{psu}(1,1|2)_1 \sim 4 \text{ symplectic bosons} \oplus 4 \text{ real fermions}$$

$\rightarrow$ can compute spectra and all correlators on both sides!
[Dei, Eberhardt, Gaberdiel, Gopakumar, Knighton]

Our objectives

Can we construct D-branes in this setup?

$\rightarrow$ boundary states in $\mathfrak{psu}(1,1|2)_1$

Our objectives

Can we construct D-branes in this setup?

$\rightarrow$ boundary states in $\mathfrak{psu}(1,1|2)_1$

Can we match them to some dual objects in the $\mathrm{Sym}(\mathbb{T}^4)$ CFT?

$\rightarrow$ boundary states? defects?

Part II: Closed strings on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ at $k = 1$: a review

$\mathfrak{psu}(1,1|2)_{k=1}$ superalgebra and its free-field realisation

Maximal bosonic subalgebra

$$\underbrace{\mathfrak{sl}(2;\mathbb{R})_1 \quad \{J^a\}}_{\mathrm{AdS}_3} \qquad \oplus \qquad \underbrace{\mathfrak{su}(2)_1 \quad \{K^a\}}_{\mathrm{S}^3}$$

$\mathfrak{psu}(1,1|2)_{k=1}$ superalgebra and its free-field realisation

Maximal bosonic subalgebra

$$\underbrace{\mathfrak{sl}(2;\mathbb{R})_1 \quad \{J^a\}}_{\mathrm{AdS}_3} \qquad \oplus \qquad \underbrace{\mathfrak{su}(2)_1 \quad \{K^a\}}_{\mathrm{S}^3}$$

Supercurrents $S^{\alpha\beta\gamma}$ in $(\mathbf{2},\mathbf{2})$

$\mathfrak{psu}(1,1|2)_{k=1}$ superalgebra and its free-field realisation

Maximal bosonic subalgebra

$$\underbrace{\mathfrak{sl}(2;\mathbb{R})_1 \quad \{J^a\}}_{\mathrm{AdS}_3} \qquad \oplus \qquad \underbrace{\mathfrak{su}(2)_1 \quad \{K^a\}}_{\mathrm{S}^3}$$

Supercurrents $S^{\alpha\beta\gamma}$ in $(\mathbf{2},\mathbf{2})$

Can construct $\{J^a, K^a, S^{\alpha\beta\gamma}\}$ as bilinears in terms of 2 pairs of symplectic bosons and complex fermions ($\alpha, \beta = \pm$)

$$\xi^\alpha(z)\,\eta^\beta(w) \sim \frac{\varepsilon^{\alpha\beta}}{z-w}\ , \qquad\qquad \psi^\alpha(z)\chi^\beta(w) \sim \frac{\varepsilon^{\alpha\beta}}{z-w}$$

Representations of $\mathfrak{psu}(1,1|2)_1$

At $k = 1$ only the short supermultiplets relevant

$$\mathcal{F}_\lambda : \qquad (\mathcal{C}^1_{\lambda+\frac{1}{2}}, \mathbf{1}) \qquad (\mathcal{C}^{\frac{1}{2}}_{\lambda}, \mathbf{2}) \qquad (\mathcal{C}^0_{\lambda+\frac{1}{2}}, \mathbf{1})$$

Representations of $\mathfrak{psu}(1,1|2)_1$

At $k = 1$ only the short supermultiplets relevant

$$\mathcal{F}_\lambda : \quad (\mathcal{C}^1_{\lambda+\frac{1}{2}}, \mathbf{1}) \quad (\mathcal{C}^{\frac{1}{2}}_\lambda, \mathbf{2}) \quad (\mathcal{C}^0_{\lambda+\frac{1}{2}}, \mathbf{1})$$

where

$\mathcal{C}^j_\lambda$: cts reps of $\mathfrak{sl}(2;\mathbb{R})$, $j \in \mathbb{R} \cup (\frac{1}{2} + i\mathbb{R})$
quadratic Casimir $\mathcal{C}^{\mathfrak{sl}(2;\mathbb{R})} = -j(j-1)$
$\lambda \in [0,1) \cong \mathbb{R}/\mathbb{Z}$ the fractional part of J^3_0 eigenvalues

$\mathbf{k}$: $\mathfrak{su}(2)$ reps

Representations of $\mathfrak{psu}(1,1|2)_1$

At $k = 1$ only the short supermultiplets relevant

$$\mathcal{F}_\lambda : \qquad (\mathcal{C}^1_{\lambda+\frac{1}{2}}, \mathbf{1}) \qquad (\mathcal{C}^{\frac{1}{2}}_\lambda, \mathbf{2}) \qquad (\mathcal{C}^0_{\lambda+\frac{1}{2}}, \mathbf{1})$$

where

$\mathcal{C}^j_\lambda$: cts reps of $\mathfrak{sl}(2;\mathbb{R})$, $j \in \mathbb{R} \cup (\frac{1}{2} + i\mathbb{R})$
quadratic Casimir $\mathcal{C}^{\mathfrak{sl}(2;\mathbb{R})} = -j(j-1)$
$\lambda \in [0,1) \cong \mathbb{R}/\mathbb{Z}$ the fractional part of J^3_0 eigenvalues

$\mathbf{k}$: $\mathfrak{su}(2)$ reps

$\rightarrow$ full affine reps generated by acting with negative modes of $\mathfrak{psu}(1,1|2)_1$

Representations of $\mathfrak{psu}(1,1|2)_1$

At $k=1$ only the short supermultiplets relevant

$$\mathcal{F}_\lambda : \quad (\mathcal{C}^1_{\lambda+\frac{1}{2}}, \mathbf{1}) \quad (\mathcal{C}^{\frac{1}{2}}_{\lambda}, \mathbf{2}) \quad (\mathcal{C}^0_{\lambda+\frac{1}{2}}, \mathbf{1})$$

where

$\mathcal{C}^j_\lambda$: cts reps of $\mathfrak{sl}(2;\mathbb{R})$, $j \in \mathbb{R} \cup (\frac{1}{2} + i\mathbb{R})$
quadratic Casimir $\mathcal{C}^{\mathfrak{sl}(2;\mathbb{R})} = -j(j-1)$
$\lambda \in [0,1) \cong \mathbb{R}/\mathbb{Z}$ the fractional part of J^3_0 eigenvalues

$\mathbf{k}$: $\mathfrak{su}(2)$ reps

$\rightarrow$ full affine reps generated by acting with negative modes of $\mathfrak{psu}(1,1|2)_1$

Modular invariant bulk CFT spectrum

$$\mathcal{H} = \bigoplus_{w\in\mathbb{Z}} \int_{\lambda\in[0,1)} d\lambda \; \sigma^w(\mathcal{F}_\lambda) \otimes \overline{\sigma^w(\mathcal{F}_\lambda)}$$

$\rightarrow$ $\sigma^w(\mathcal{F}_\lambda)$ spectrally flowed reps (w-times wound long strings)

Worldsheet partition function

The total worldsheet partition function

$$Z_{\mathfrak{psu}(1,1|2)_1}\ Z_{\mathsf{gh}}\ Z_{\mathbb{T}^4}\ =\ \frac{1}{2}\sum_{r,w\in\mathbb{Z}}\delta^2(t-w\tau-r)|q|^{w^2}Z_{\mathbb{T}^4}(t;\tau)$$

where

τ	...	worldsheet-torus modulus
t	...	spacetime-torus modulus ($\mathfrak{sl}(2;\mathbb{R})_1$ chemical potential)

$\rightarrow$ worldsheet holomorphically covers spacetime!

Worldsheet partition function

The total worldsheet partition function

$$Z_{\mathfrak{psu}(1,1|2)_1}\ Z_{\mathsf{gh}}\ Z_{\mathbb{T}^4} \ = \ \frac{1}{2}\sum_{r,w\in\mathbb{Z}} \delta^2(t-w\tau-r)|q|^{w^2} Z_{\mathbb{T}^4}(t;\tau)$$

where

τ	...	worldsheet-torus modulus
t	...	spacetime-torus modulus ($\mathfrak{sl}(2;\mathbb{R})_1$ chemical potential)

$\rightarrow$ worldsheet holomorphically covers spacetime!

Impose on-shell condition & level-matching, end up with on-shell partition function (up to spin structures)

$$Z_{\mathsf{string}}(t) = \sum_{w=1}^{\infty} x^{\frac{w}{4}}\,\bar{x}^{\frac{w}{4}}\,Z_{\mathbb{T}^4}(0;\tfrac{t}{w})$$

Worldsheet partition function

The total worldsheet partition function

$$Z_{\mathfrak{psu}(1,1|2)_1}\ Z_{\text{gh}}\ Z_{\mathbb{T}^4}\ =\ \frac{1}{2}\sum_{r,w\in\mathbb{Z}}\delta^2(t-w\tau-r)|q|^{w^2}Z_{\mathbb{T}^4}(t;\tau)$$

where

τ ... worldsheet-torus modulus

t ... spacetime-torus modulus ($\mathfrak{sl}(2;\mathbb{R})_1$ chemical potential)

$\rightarrow$ worldsheet holomorphically covers spacetime!

Impose on-shell condition & level-matching, end up with on-shell partition function (up to spin structures)

$$Z_{\text{string}}(t)=\sum_{w=1}^{\infty}x^{\frac{w}{4}}\,\bar{x}^{\frac{w}{4}}\,Z_{\mathbb{T}^4}(0;\tfrac{t}{w})$$

$\rightarrow$ single-particle partition function of $\mathrm{Sym}(\mathbb{T}^4)$

Worldsheet partition function

The total worldsheet partition function

$$Z_{\mathfrak{psu}(1,1|2)_1}\ Z_{\text{gh}}\ Z_{\mathbb{T}^4} \ = \ \frac{1}{2}\sum_{r,w\in\mathbb{Z}} \delta^2(t - w\tau - r)|q|^{w^2} Z_{\mathbb{T}^4}(t;\tau)$$

where

τ	...	worldsheet-torus modulus
t	...	spacetime-torus modulus ($\mathfrak{sl}(2;\mathbb{R})_1$ chemical potential)

$\rightarrow$ worldsheet holomorphically covers spacetime!

Impose on-shell condition & level-matching, end up with on-shell partition function (up to spin structures)

$$Z_{\text{string}}(t) = \sum_{w=1}^{\infty} x^{\frac{w}{4}}\,\bar{x}^{\frac{w}{4}}\, Z_{\mathbb{T}^4}(0; \tfrac{t}{w})$$

$\rightarrow$ single-particle partition function of $\mathrm{Sym}(\mathbb{T}^4)$

$\rightarrow$ on-shell w-wound strings in $\mathrm{AdS}_3 \Longleftrightarrow w$-cycle twisted states in $\mathrm{Sym}(\mathbb{T}^4)$

On-shell vertex operators and amplitudes

On-shell states given by vertex ops ($J_0^3, J_0^\pm \to$ global spacetime conf. algebra) [Maldacena, Ooguri '00]

$$V^w_{m,j}(x,z) = e^{-xJ_0^+} V^w_{m,j}(z) e^{+xJ_0^+} \quad \dots \quad x \in \partial \mathrm{AdS}_3$$

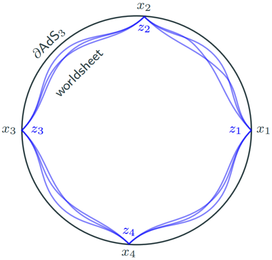
On-shell vertex operators and amplitudes

On-shell states given by vertex ops ($J_0^3, J_0^\pm \to$ global spacetime conf. algebra) [Maldacena, Ooguri '00]

$$V^w_{m,j}(x,z) = e^{-xJ_0^+} V^w_{m,j}(z) e^{+xJ_0^+} \quad \ldots \quad x \in \partial\mathrm{AdS}_3$$

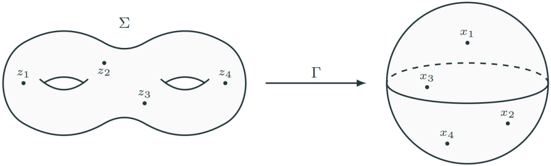
String theory n-point, g-loop amplitude (hybrid-formalism PCO insertions W)

$$\mathcal{A}_{g,n}(x_1,\ldots,x_n) = \int_{\mathcal{M}_{g,n}} \left\langle \prod_{a=1}^{n+2g-2} W(u_a) \prod_{i=1}^{n} V^{w_i}_{m_i,j_i}(x_i,z_i) \right\rangle$$



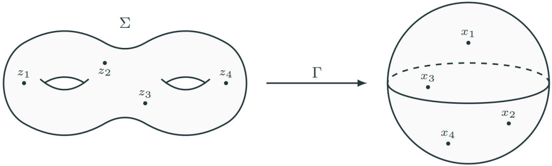
Worldsheet localisation

For tensionless $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$, the $\mathcal{M}_{g,n}$ integral localises at isolated points in $\mathcal{M}_{g,n}$ where $\exists$ a holomorphic covering map $\Gamma : \Sigma_{g,n} \longrightarrow \partial\mathrm{AdS}_3 \cong \mathrm{S}^2$



Worldsheet localisation

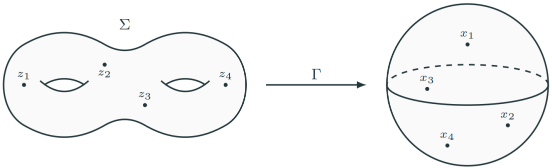
For tensionless $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$, the $\mathcal{M}_{g,n}$ integral localises at isolated points in $\mathcal{M}_{g,n}$ where $\exists$ a holomorphic covering map $\Gamma : \Sigma_{g,n} \longrightarrow \partial\mathrm{AdS}_3 \cong \mathrm{S}^2$



$\Longrightarrow$ Lunin-Mathur for S^2 correlators of w-cycle twisted states in $\mathrm{Sym}_N(\mathbb{T}^4)$

Worldsheet localisation

For tensionless $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$, the $\mathcal{M}_{g,n}$ integral localises at isolated points in $\mathcal{M}_{g,n}$ where $\exists$ a holomorphic covering map $\Gamma : \Sigma_{g,n} \longrightarrow \partial\mathrm{AdS}_3 \cong \mathrm{S}^2$

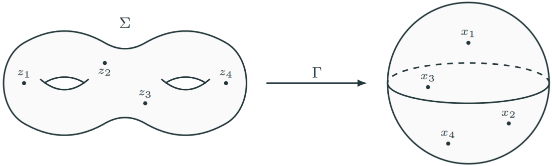


$\implies$ Lunin-Mathur for S^2 correlators of w-cycle twisted states in $\mathrm{Sym}_N(\mathbb{T}^4)$

How?

Worldsheet localisation

For tensionless $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$, the $\mathcal{M}_{g,n}$ integral localises at isolated points in $\mathcal{M}_{g,n}$ where $\exists$ a holomorphic covering map $\Gamma : \Sigma_{g,n} \longrightarrow \partial\mathrm{AdS}_3 \cong \mathrm{S}^2$



$\Longrightarrow$ Lunin-Mathur for S^2 correlators of w-cycle twisted states in $\mathrm{Sym}_N(\mathbb{T}^4)$

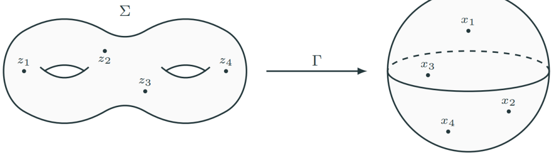
How?

$\rightarrow$ define meromorphic functions

$$\omega^{\pm}(z) = \left\langle \xi^{\pm}(z) \prod_{a=1}^{n+2g-2} W(u_a) \prod_{i=1}^{n} V^{w_i}_{m_i,j_i}(x_i, z_i) \right\rangle$$

Worldsheet localisation

For tensionless $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$, the $\mathcal{M}_{g,n}$ integral localises at isolated points in $\mathcal{M}_{g,n}$ where $\exists$ a holomorphic covering map $\Gamma : \Sigma_{g,n} \longrightarrow \partial\mathrm{AdS}_3 \cong \mathrm{S}^2$



$\Longrightarrow$ Lunin-Mathur for S^2 correlators of w-cycle twisted states in $\mathrm{Sym}_N(\mathbb{T}^4)$

How?

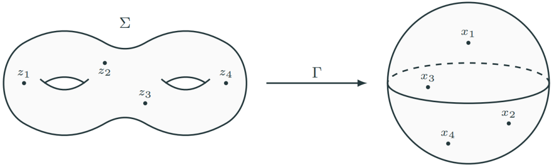
$\rightarrow$ define meromorphic functions

$$\omega^{\pm}(z) = \left\langle \xi^{\pm}(z) \prod_{a=1}^{n+2g-2} W(u_a) \prod_{i=1}^{n} V^{w_i}_{m_i,j_i}(x_i, z_i) \right\rangle$$

$\rightarrow$ analytic properties imply that $\Gamma(z) \equiv -\omega^-(z)/\omega^+(z)$ is the holomorphic covering map $\Sigma_{g,n} \longrightarrow \partial\mathrm{AdS}_3 \cong \mathrm{S}^2$! with ramification indices w_i, if it exists

Worldsheet localisation

For tensionless $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$, the $\mathcal{M}_{g,n}$ integral localises at isolated points in $\mathcal{M}_{g,n}$ where $\exists$ a holomorphic covering map $\Gamma : \Sigma_{g,n} \longrightarrow \partial\mathrm{AdS}_3 \cong \mathrm{S}^2$



$\Longrightarrow$ Lunin-Mathur for S^2 correlators of w-cycle twisted states in $\mathrm{Sym}_N(\mathbb{T}^4)$

How?

$\rightarrow$ define meromorphic functions

$$\omega^{\pm}(z) = \left\langle \xi^{\pm}(z) \prod_{a=1}^{n+2g-2} W(u_a) \prod_{i=1}^{n} V^{w_i}_{m_i,j_i}(x_i, z_i) \right\rangle$$

$\rightarrow$ analytic properties imply that $\Gamma(z) \equiv -\omega^-(z)/\omega^+(z)$ is the holomorphic covering map $\Sigma_{g,n} \longrightarrow \partial\mathrm{AdS}_3 \cong \mathrm{S}^2$! with ramification indices w_i, if it exists

$\rightarrow$ if Γ does not exist, then need to have $\omega^+(z) = 0 \implies$ vanishing amplitude!

Part III: D-branes in AdS_3 and boundary states of the symmetric orbifold

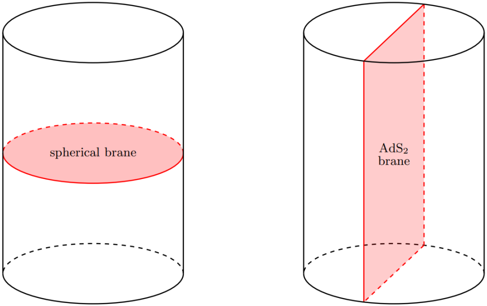
Symmetry-preserving D-branes in AdS_3

D-branes in WZW models $\Longleftrightarrow$ (twined) conjugacy classes on the group manifold

Symmetry-preserving D-branes in AdS_3

D-branes in WZW models $\Longleftrightarrow$ (twined) conjugacy classes on the group manifold

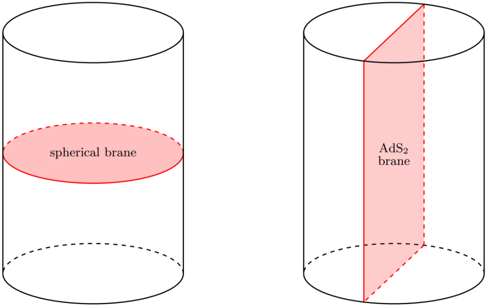
Two inequivalent D-branes preserving the $\mathfrak{sl}(2;\mathbb{R})_1$ subalgebra of $\mathfrak{psu}(1,1|2)_1$:
[Bachas, Petropoulos '00; Ponsot, Schomerus '01; Ooguri, Lee, Park '01]



Symmetry-preserving D-branes in AdS_3

D-branes in WZW models $\Longleftrightarrow$ (twined) conjugacy classes on the group manifold

Two inequivalent D-branes preserving the $\mathfrak{sl}(2;\mathbb{R})_1$ subalgebra of $\mathfrak{psu}(1,1|2)_1$:
[Bachas, Petropoulos '00; Ponsot, Schomerus '01; Ooguri, Lee, Park '01]

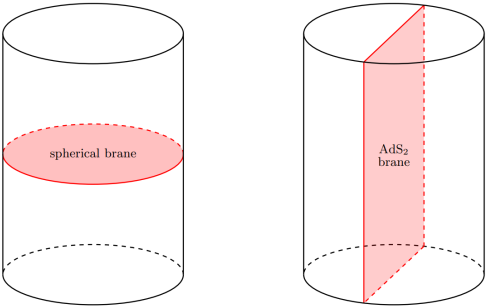


$\rightarrow$ spherical branes: instantonic H_2 planes in AdS_3 (but S^2 in EAdS_3)

Symmetry-preserving D-branes in AdS_3

D-branes in WZW models $\Longleftrightarrow$ (twined) conjugacy classes on the group manifold

Two inequivalent D-branes preserving the $\mathfrak{sl}(2;\mathbb{R})_1$ subalgebra of $\mathfrak{psu}(1,1|2)_1$:
[Bachas, Petropoulos '00; Ponsot, Schomerus '01; Ooguri, Lee, Park '01]



$\rightarrow$ spherical branes: instantonic H_2 planes in AdS_3 (but S^2 in EAdS_3)

$\rightarrow$ AdS_2 branes: D-strings stretched between antipodal points on $\partial\mathrm{AdS}_3$

Boundary states for $\mathfrak{psu}(1,1|2)_1$: spherical D-branes

Ishibashi states $|w,\lambda\rangle\rangle$ satisfy

$$\begin{aligned}(J^3_n - \bar{J}^3_{-n})|w,\lambda\rangle\rangle &= 0\ ,\\ (J^{\pm}_n + \bar{J}^{\mp}_{-n})|w,\lambda\rangle\rangle &= 0\ ,\end{aligned}$$

Boundary states for $\mathfrak{psu}(1,1|2)_1$: spherical D-branes

Ishibashi states $|w,\lambda\rangle\rangle$ satisfy

$$\begin{aligned}(J^3_n - \bar{J}^3_{-n})|w,\lambda\rangle\rangle &= 0 \ , \\ (J^{\pm}_n + \bar{J}^{\mp}_{-n})|w,\lambda\rangle\rangle &= 0 \ ,\end{aligned}$$

$\rightarrow$ compatible with all $\lambda \in [0,1)$ and spectral flow

$$|w,\lambda\rangle\rangle = \sigma^w(|0,\lambda\rangle\rangle) \quad \text{for all } w \in \mathbb{Z}$$

Boundary states for $\mathfrak{psu}(1,1|2)_1$: spherical D-branes

Ishibashi states $|w,\lambda\rangle\rangle$ satisfy

$$(J^3_n - \bar{J}^3_{-n})|w,\lambda\rangle\rangle = 0 \; , \\ (J^\pm_n + \bar{J}^\mp_{-n})|w,\lambda\rangle\rangle = 0 \; ,$$

$\rightarrow$ compatible with all $\lambda \in [0,1)$ and spectral flow

$$|w,\lambda\rangle\rangle = \sigma^w(|0,\lambda\rangle\rangle) \quad \text{for all } w \in \mathbb{Z}$$

Full boundary states

$$\| W, \Lambda\rangle\rangle = \sum_{w\in\mathbb{Z}} \int_0^1 d\lambda \, e^{2\pi i [w(\Lambda - \frac{1}{2}) + (\lambda - \frac{1}{2})W]} |w,\lambda\rangle\rangle$$

Boundary states for $\mathfrak{psu}(1,1|2)_1$: spherical D-branes

Ishibashi states $|w,\lambda\rangle\rangle$ satisfy

$$(J^3_n - \bar{J}^3_{-n})|w,\lambda\rangle\rangle = 0 \ ,$$
$$(J^\pm_n + \bar{J}^\mp_{-n})|w,\lambda\rangle\rangle = 0 \ ,$$

$\to$ compatible with all $\lambda \in [0,1)$ and spectral flow

$$|w,\lambda\rangle\rangle = \sigma^w(|0,\lambda\rangle\rangle) \quad \text{for all } w \in \mathbb{Z}$$

Full boundary states

$$\|W,\Lambda\rangle\rangle = \sum_{w\in\mathbb{Z}} \int_0^1 d\lambda\, e^{2\pi i[w(\Lambda-\frac{1}{2})+(\lambda-\frac{1}{2})W]}|w,\lambda\rangle\rangle$$

$\to$ W: integer shift along AdS_3 time direction

$\to$ Λ: angular Wilson line

Cylinder amplitude for spherical branes

Worldsheet boundary state

$$\|W,\Lambda,u\rangle\rangle \equiv \underbrace{\|W,\Lambda\rangle\rangle}_{\mathfrak{psu}(1,1|2)_1} \; \underbrace{\|u\rangle\rangle}_{\mathbb{T}^4} \; \underbrace{\|\mathsf{gh}\rangle\rangle}_{\rho\sigma\ \text{ghosts}}$$

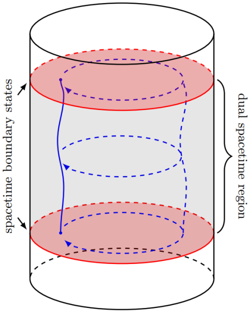
Cylinder amplitude for spherical branes

Worldsheet boundary state

$$\|W,\Lambda,u\rangle\!\rangle \equiv \underbrace{\|W,\Lambda\rangle\!\rangle}_{\mathfrak{psu}(1,1|2)_1} \underbrace{\|u\rangle\!\rangle}_{\mathbb{T}^4} \underbrace{\|\mathsf{gh}\rangle\!\rangle}_{\rho\sigma\text{ ghosts}}$$

Worldsheet cylinder amplitude (J_0^3 generates spacetime cylinder modulus t)

$$\mathcal{A}_{u|v}(t) = \int_0^\infty d\tau\, \langle\!\langle W,\Lambda,u\| e^{2\pi i\tau(L_0-\frac{c}{24})} e^{2\pi i t J_0^3} \|W,\Lambda,v\rangle\!\rangle$$



Localisation

Can manipulate $\mathcal{A}_{u|v}$ into (again, up to spin structures)

$$\mathcal{A}_{u|v} = \int_0^\infty d\tau \sum_{w=1}^{\infty} \frac{x^{\frac{w}{4}}}{w}\ \delta(\tfrac{t}{w} - \tau) \quad \underbrace{\langle\langle u \| e^{2\pi i \frac{t}{w} J_0^3} \| v \rangle\rangle}_{\text{overlap of } \mathbb{T}^4 \text{ boundary states}}$$

Localisation

Can manipulate $\mathcal{A}_{u|v}$ into (again, up to spin structures)

$$\mathcal{A}_{u|v} = \int_0^\infty d\tau \sum_{w=1}^{\infty} \frac{x^{\frac{w}{4}}}{w} \; \delta(\tfrac{t}{w} - \tau) \quad \underbrace{\langle\!\langle u \| e^{2\pi i \frac{t}{w} J_0^3} \| v \rangle\!\rangle}_{\text{overlap of } \mathbb{T}^4 \text{ boundary states}}$$

$\rightarrow$ localizes at $\tau = \frac{t}{w}$ for $w \in \mathbb{Z} \implies$ unramified covering maps Γ : cyl $\rightarrow$ cyl

Localisation

Can manipulate $\mathcal{A}_{u|v}$ into (again, up to spin structures)

$$\mathcal{A}_{u|v} = \int_0^\infty d\tau \sum_{w=1}^{\infty} \frac{x^{\frac{w}{4}}}{w} \; \delta(\tfrac{t}{w} - \tau) \quad \underbrace{\langle\langle u \| e^{2\pi i \frac{t}{w} J_0^3} \| v \rangle\rangle}_{\text{overlap of } \mathbb{T}^4 \text{ boundary states}}$$

$\rightarrow$ localizes at $\tau = \frac{t}{w}$ for $w \in \mathbb{Z} \implies$ unramified covering maps Γ : cyl $\rightarrow$ cyl

To compare with the dual CFT, go to the grandcanonical ensemble by fixing fugacity p for N [Eberhardt '20]

$$\mathfrak{Z}_{u|v}(p;t) = \exp\left(\sum_{w=1}^{\infty} \frac{p^w}{w} \; {}_{\mathbb{T}^4}\langle\langle u \| e^{2\pi i \frac{t}{w} J_0^3} \| v \rangle\rangle_{\mathbb{T}^4} \right)$$

Maximally-fractional boundary states in $\mathrm{Sym}_N(\mathbb{T}^4)$

Start with the $(\mathbb{T}^4)^{\otimes N}$ boundary state (works for general seed CFT)

$$\underbrace{\|u\rangle\!\rangle_{\mathbb{T}^4} \otimes \ldots \otimes \|u\rangle\!\rangle_{\mathbb{T}^4}}_{N \text{ times}}$$

Maximally-fractional boundary states in $\mathrm{Sym}_N(\mathbb{T}^4)$

Start with the $(\mathbb{T}^4)^{\otimes N}$ boundary state (works for general seed CFT)

$$\underbrace{\|u\rangle\!\rangle_{\mathbb{T}^4} \otimes \ldots \otimes \|u\rangle\!\rangle_{\mathbb{T}^4}}_{N \text{ times}}$$

Go to a $\mathrm{Sym}_N(\mathbb{T}^4)$ boundary state by adding all possible twisted sectors
[independently derived by Belin, Biswas, Sully '21]

$$\|u,\rho\rangle\!\rangle_{\mathrm{Sym}} = \frac{1}{\sqrt{N!}} \sum_{\sigma=\gamma_1\gamma_2\ldots\in S_N} \chi_\rho(\sigma) \bigotimes_r \|u\rangle\!\rangle_{\mathbb{T}^4}^{\gamma_r}$$

Maximally-fractional boundary states in $\mathrm{Sym}_N(\mathbb{T}^4)$

Start with the $(\mathbb{T}^4)^{\otimes N}$ boundary state (works for general seed CFT)

$$\underbrace{\|u\rangle\rangle_{\mathbb{T}^4} \otimes \ldots \otimes \|u\rangle\rangle_{\mathbb{T}^4}}_{N \text{ times}}$$

Go to a $\mathrm{Sym}_N(\mathbb{T}^4)$ boundary state by adding all possible twisted sectors
[independently derived by Belin, Biswas, Sully '21]

$$\|u, \rho\rangle\rangle_{\mathrm{Sym}} = \frac{1}{\sqrt{N!}} \sum_{\sigma=\gamma_1\gamma_2\ldots\in S_N} \chi_\rho(\sigma) \bigotimes_r \|u\rangle\rangle_{\mathbb{T}^4}^{\gamma_r}$$

$\rightarrow$ $\gamma_1, \gamma_2, \ldots$ are disjoint cycles, $\|u\rangle\rangle_{\mathbb{T}^4}^{\gamma_r}$ built on top of the γ_r-twisted vacuum

Maximally-fractional boundary states in $\mathrm{Sym}_N(\mathbb{T}^4)$

Start with the $(\mathbb{T}^4)^{\otimes N}$ boundary state (works for general seed CFT)

$$\underbrace{\|u\rangle\!\rangle_{\mathbb{T}^4} \otimes \ldots \otimes \|u\rangle\!\rangle_{\mathbb{T}^4}}_{N \text{ times}}$$

Go to a $\mathrm{Sym}_N(\mathbb{T}^4)$ boundary state by adding all possible twisted sectors
[independently derived by Belin, Biswas, Sully '21]

$$\|u,\rho\rangle\!\rangle_{\mathrm{Sym}} = \frac{1}{\sqrt{N!}} \sum_{\sigma=\gamma_1\gamma_2\ldots\in S_N} \chi_\rho(\sigma) \bigotimes_r \|u\rangle\!\rangle_{\mathbb{T}^4}^{\gamma_r}$$

→ $\gamma_1, \gamma_2, \ldots$ are disjoint cycles, $\|u\rangle\!\rangle_{\mathbb{T}^4}^{\gamma_r}$ built on top of the γ_r-twisted vacuum

→ for $\rho = \mathrm{id}$, the cylinder correlator in grandcan. ensemble gives

$$\sum_{N=0}^{\infty} p^N \,{}_{\mathrm{Sym}}\langle\!\langle u, \mathrm{id}\| e^{2\pi i t (L_0 - \frac{c}{24})} \|v, \mathrm{id}\rangle\!\rangle_{\mathrm{Sym}} = \ldots = \mathfrak{Z}_{u|v}(p;t)\,!$$

Spherical branes: holographic correspondence

spherical D-branes in $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ at $k = 1$

$$\Longleftrightarrow$$

maximally-fractional boundary states in $\mathrm{Sym}_N(\mathbb{T}^4)$

Spherical branes: holographic correspondence

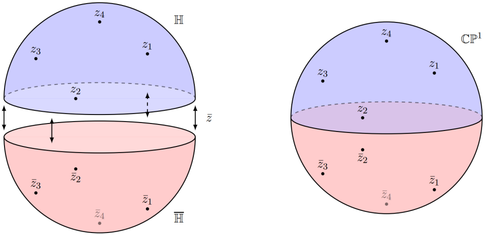
spherical D-branes in $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ at $k = 1$

$\Longleftrightarrow$

maximally-fractional boundary states in $\mathrm{Sym}_N(\mathbb{T}^4)$

Also supported by computing the disk amplitudes (see Bob Knighton's poster)

$$\int \left\langle \prod_{a=1}^{n-1} W(u_a) \prod_{i=1}^{n} V^{w_i}_{m_i,j_i}(x_i, z_i) \right\rangle_{\mathbb{H}}$$



Spherical branes: holographic correspondence

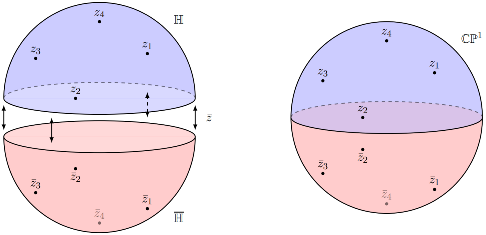
spherical D-branes in $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ at $k = 1$

$\Longleftrightarrow$

maximally-fractional boundary states in $\mathrm{Sym}_N(\mathbb{T}^4)$

Also supported by computing the disk amplitudes (see Bob Knighton's poster)

$$\int \Big\langle \prod_{a=1}^{n-1} W(u_a) \prod_{i=1}^{n} V^{w_i}_{m_i,j_i}(x_i, z_i) \Big\rangle_{\mathbb{H}}$$



$\rightarrow$ gives leading contribution to the disk correlators with max.-fractional BCs

Summary and outlook

Take-home messages:

Summary and outlook

Take-home messages:

$\rightarrow \exists$ setup in 3d where both sides of AdS/CFT can be exactly computed

Summary and outlook

Take-home messages:

→ $\exists$ setup in 3d where both sides of AdS/CFT can be exactly computed

→ some non-perturbative vacua matched on both sides: spherical D-branes in $\mathrm{AdS}_3 \times \mathrm{S}_3 \times \mathbb{T}^4$ identified with maximally fractional boundary states in $\mathrm{Sym}(\mathbb{T}^4)$

Summary and outlook

Take-home messages:

$\rightarrow$ $\exists$ setup in 3d where both sides of AdS/CFT can be exactly computed

$\rightarrow$ some non-perturbative vacua matched on both sides: spherical D-branes in $\mathrm{AdS}_3 \times \mathrm{S}_3 \times \mathbb{T}^4$ identified with maximally fractional boundary states in $\mathrm{Sym}(\mathbb{T}^4)$

Future directions:

Summary and outlook

Take-home messages:

$\rightarrow \exists$ setup in 3d where both sides of AdS/CFT can be exactly computed

$\rightarrow$ some non-perturbative vacua matched on both sides: spherical D-branes in $\mathrm{AdS}_3 \times \mathrm{S}_3 \times \mathbb{T}^4$ identified with maximally fractional boundary states in $\mathrm{Sym}(\mathbb{T}^4)$

Future directions:

Use the spherical branes to compute worldsheet instanton corrections to AdS_3 amplitudes [Sen, Alexandrov, Stefanski '20, '21]

Summary and outlook

Take-home messages:

$\rightarrow$ $\exists$ setup in 3d where both sides of AdS/CFT can be exactly computed

$\rightarrow$ some non-perturbative vacua matched on both sides: spherical D-branes in $\mathrm{AdS}_3 \times \mathrm{S}_3 \times \mathbb{T}^4$ identified with maximally fractional boundary states in $\mathrm{Sym}(\mathbb{T}^4)$

Future directions:

Use the spherical branes to compute worldsheet instanton corrections to AdS_3 amplitudes [Sen, Alexandrov, Stefanski '20, '21]

Explore the duality for other non-perturbative vacua (AdS_2 branes, symmetry-breaking branes)

Summary and outlook

Take-home messages:

$\rightarrow$ $\exists$ setup in 3d where both sides of AdS/CFT can be exactly computed

$\rightarrow$ some non-perturbative vacua matched on both sides: spherical D-branes in $\mathrm{AdS}_3 \times \mathrm{S}_3 \times \mathbb{T}^4$ identified with maximally fractional boundary states in $\mathrm{Sym}(\mathbb{T}^4)$

Future directions:

Use the spherical branes to compute worldsheet instanton corrections to AdS_3 amplitudes [Sen, Alexandrov, Stefanski '20, '21]

Explore the duality for other non-perturbative vacua (AdS_2 branes, symmetry-breaking branes)

Consider orientifolds and crosscap states [Gaberdiel, Knighton, JV: work in progress]

Summary and outlook

Take-home messages:

$\rightarrow$ $\exists$ setup in 3d where both sides of AdS/CFT can be exactly computed

$\rightarrow$ some non-perturbative vacua matched on both sides: spherical D-branes in $\mathrm{AdS}_3 \times \mathrm{S}_3 \times \mathbb{T}^4$ identified with maximally fractional boundary states in $\mathrm{Sym}(\mathbb{T}^4)$

Future directions:

Use the spherical branes to compute worldsheet instanton corrections to AdS_3 amplitudes [Sen, Alexandrov, Stefanski '20, '21]

Explore the duality for other non-perturbative vacua (AdS_2 branes, symmetry-breaking branes)

Consider orientifolds and crosscap states [Gaberdiel, Knighton, JV: work in progress]

Consider D-branes in the analogous $\mathrm{AdS}_5 \times \mathrm{S}^5$ setup [Gaberdiel, Gopakumar '21]

Thank you!