A NEW QUANTUM SPECTRAL CURVE FOR ADS3/CFT2

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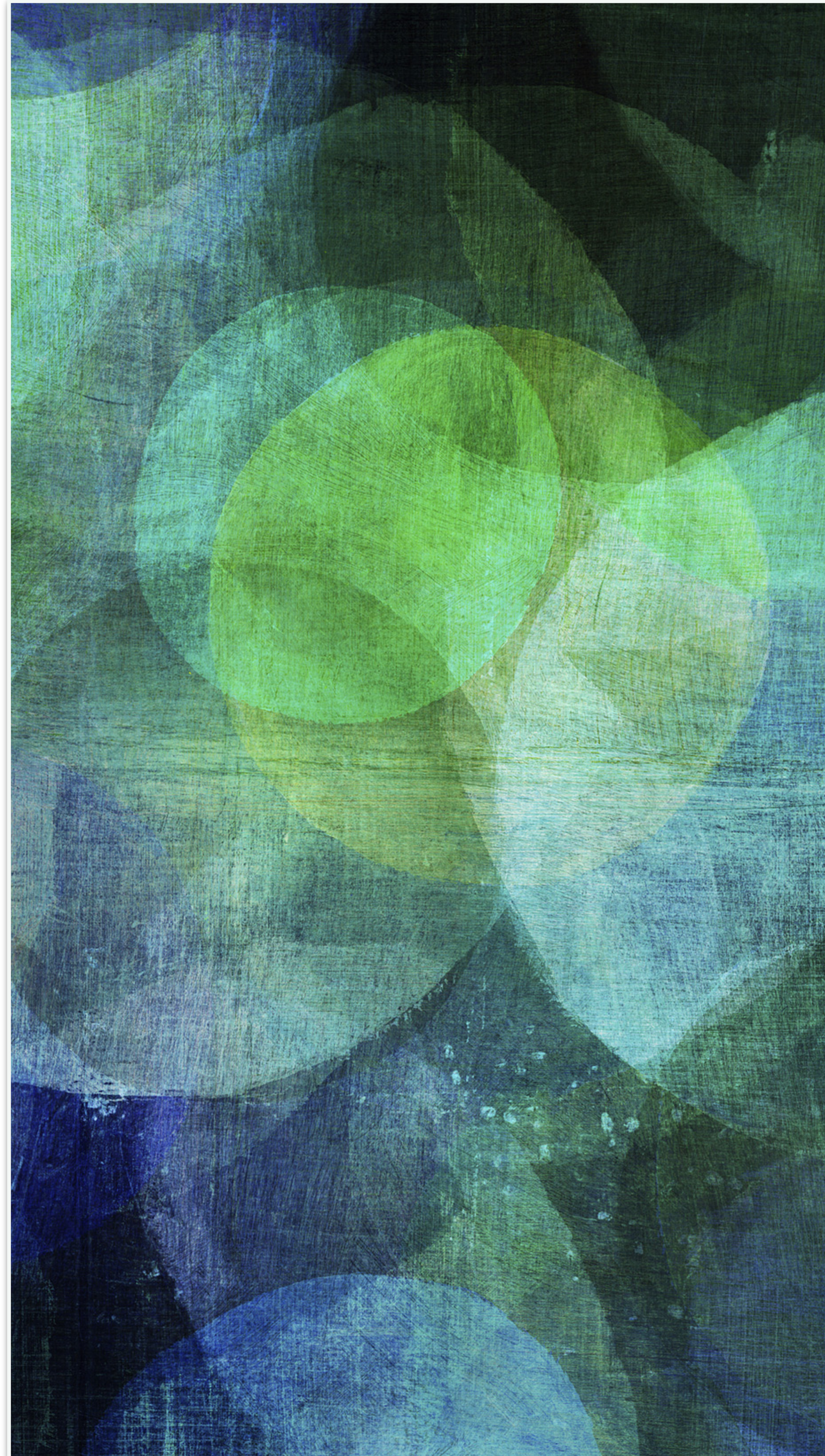
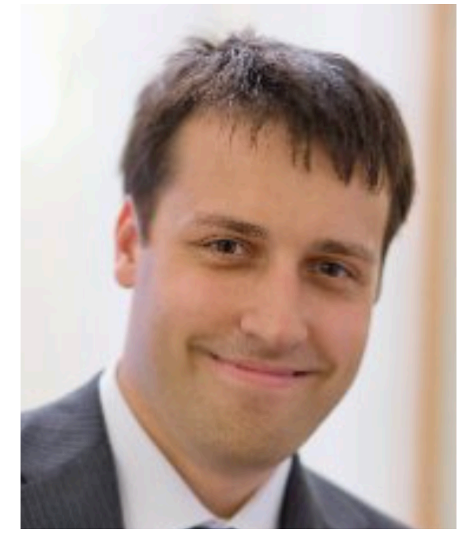
Based on joint work with N. Gromov, B. Stefański, jr. and Alessandro Torrielli

King's College London

European Research Council



Work with three great collaborators:



Alessandro Torrielli, Univ. Surrey

Bogdan Stefański, jr., City Univ. London

Nikolay Gromov, King’s Coll. London

“Quantum spectral curve for AdS/CFT: a proposal”
(hep-th/2109.05500)

Same results obtained independently by
Dmytro Volin and Simon Ekhammar (Uppsala Univ.). Check out their paper too!

“Monodromy bootstrap for SU(2|2) quantum spectral curves:
from Hubbard model to AdS_3/CFT_2”
(hep-th/2109.06164) (same day)

What **AdS$_3$/CFT$_2$** are we studying?

[Maldacena '97]

String theory on $AdS_3 \times S^3 \times T^4$
with **purely R-R flux**
16 super symmetries

Unknown dual CFT:
IR limit of world volume theory
of D1-D5 system

Parameters: g_s in my talk will be 0 ("planar limit")

$\frac{\alpha'}{r^2_{AdS}}$ ("coupling constant") finite

This is the "polar opposite" to the NSNS case on which much progress was made
[Eberhardt, Gaberdiel, Gopakumar '18]

For RR, worldsheet CFT approach is much more complicated if not impossible

Can we solve the theory with **integrability**?

We propose a tool to solve the full **non-protected planar spectrum**
(in the sector with no winding/momentum on the torus)

“Worldsheet” integrability

[Minahan, Zarembo ’02] [Metsaev, Tseytlin ’02]

+ important work of many people for last 20 years

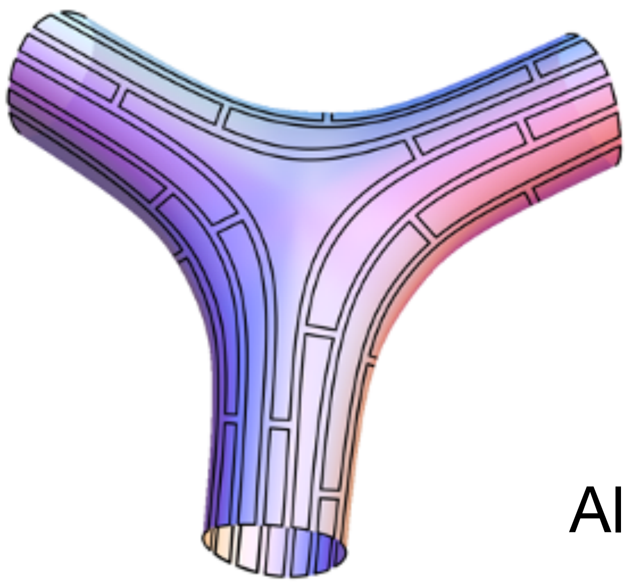
Conjectured for important AdS/CFT dualities

$AdS_5 \times S^5 \leftrightarrow \mathcal{N}$=4 SYM
AdS_5/CFT_4

$AdS_4 \times CP^3 \leftrightarrow$ ABJM
AdS_4/CFT_3

[Babichenko, Stefanski, Zarembo ’09]
[Borsato, Ohlsson-Sax, Sfondrini, Stefanski ’14] + Torrielli ’13,’16]

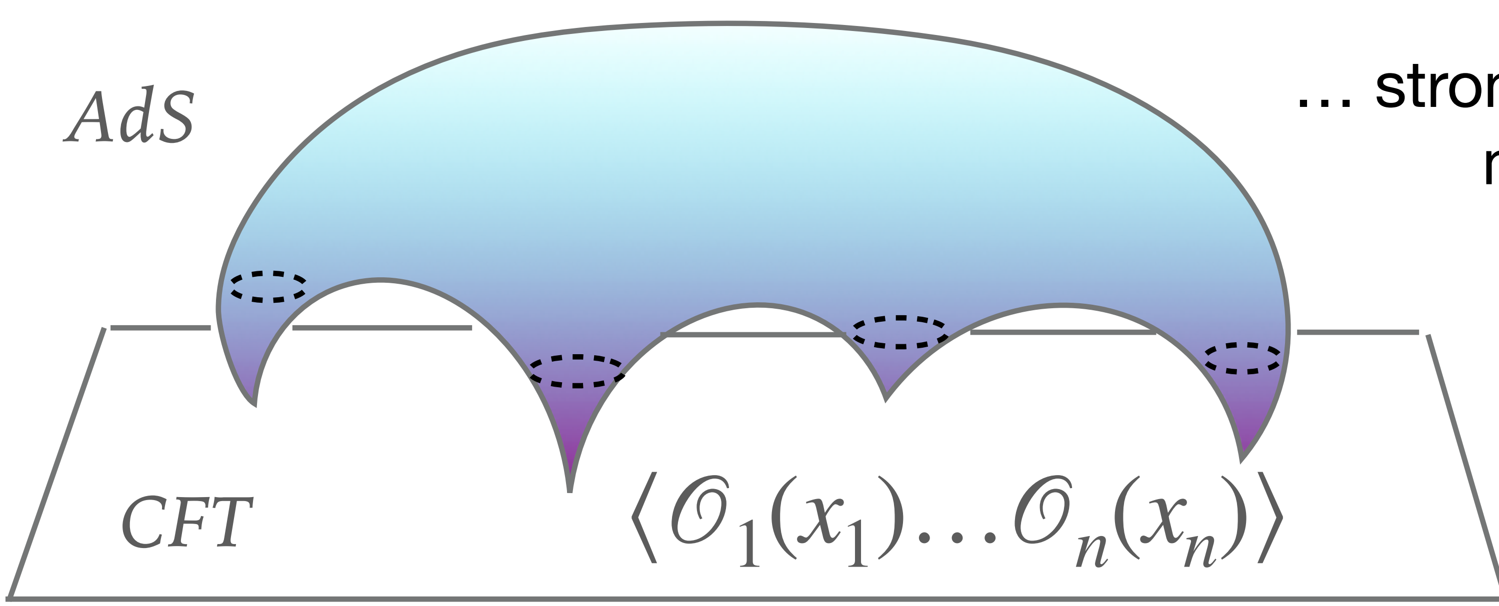
AdS_3/CFT_2
for backgrounds $AdS_3 \times S^3 \times T^4$
$AdS_3 \times S^3 \times S^3 \times S^1$



Integrability lives on the worldsheet / planar Feynman diagrams
Compatible with **higher dimensions** and **chaos** in spacetime!

All string amplitudes / local correlation functions should be solvable at finite coupling α', even order by order in g_s

e.g. [Basso, Komatsu, Vieira ’15],[Bargheer, Caetano, Fleury, Komatsu, Vieira ’17] ….



… strong evidence but still a long way to go,
many ongoing developments…

Focus of today:
solving the planar spectrum

$$\langle \mathcal{O}(x)\bar{\mathcal{O}}(y)\rangle \propto \frac{1}{|x-y|^{2\Delta_{\mathcal{O}}(g)}}$$

Exact spectrum described by the **Quantum Spectral Curve**

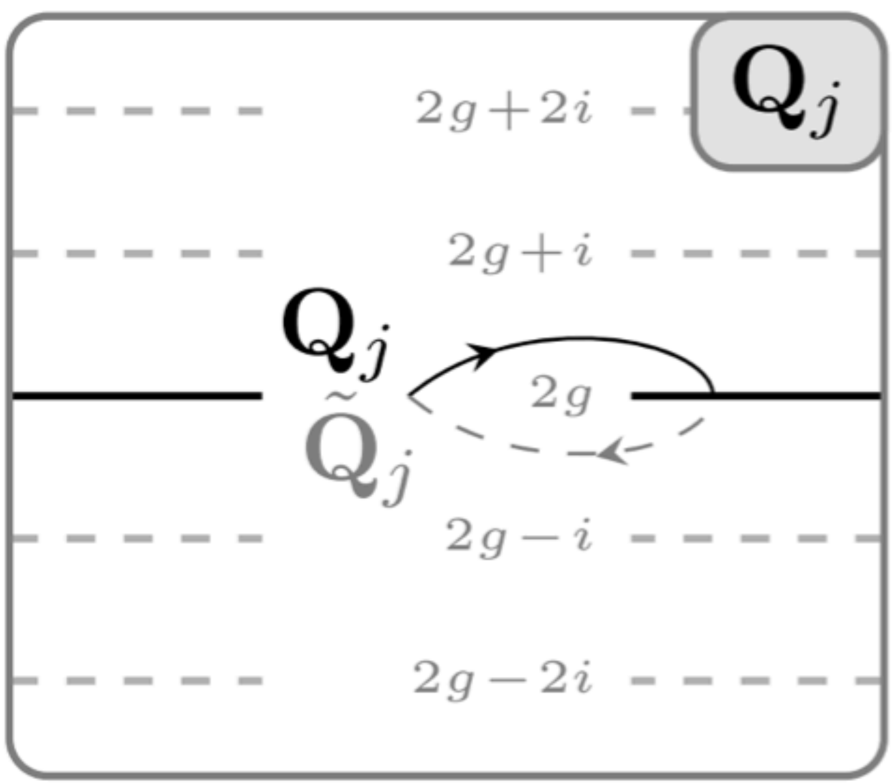
AdS_5/CFT_4 [Gromov, Kazakov, Leurent, Volin '13]

AdS_4/CFT_3 [AC, Fioravanti, Gromov, Tateo '14]

What is it?

A complex analysis problem for "Q-functions" $Q_i(u)$

spectral parameter



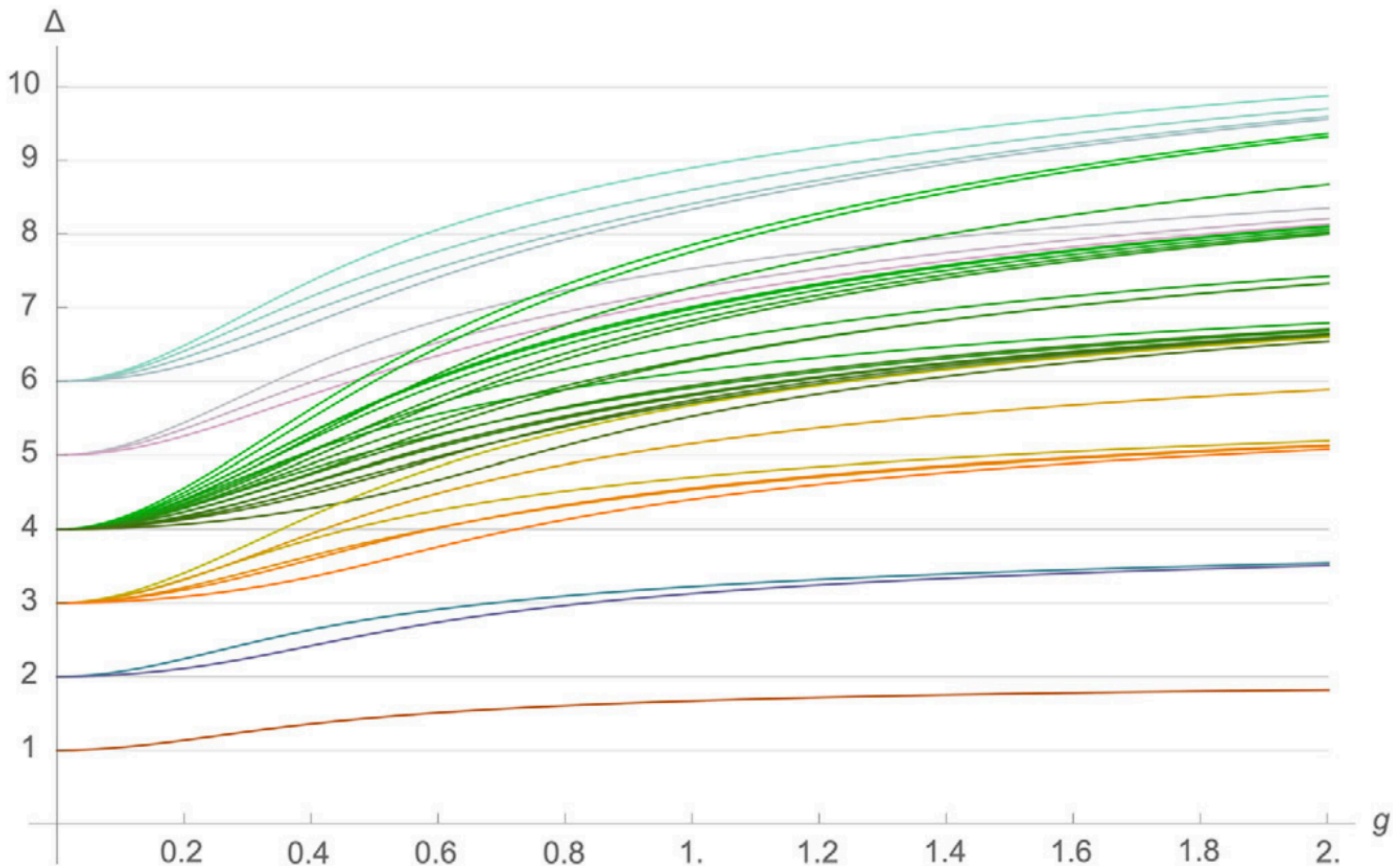
Global charges (incl. Δ)

$$\mathbf{Q}_i(u) \sim u^{\hat{M}_i}, \quad u \sim \infty$$

$\frac{R_{AdS}^2}{4\pi\alpha'} \equiv g \simeq$ position of branch points

Precision spectroscopy with the QSC

We can answer almost any question on the spectrum, many applications! Some examples…

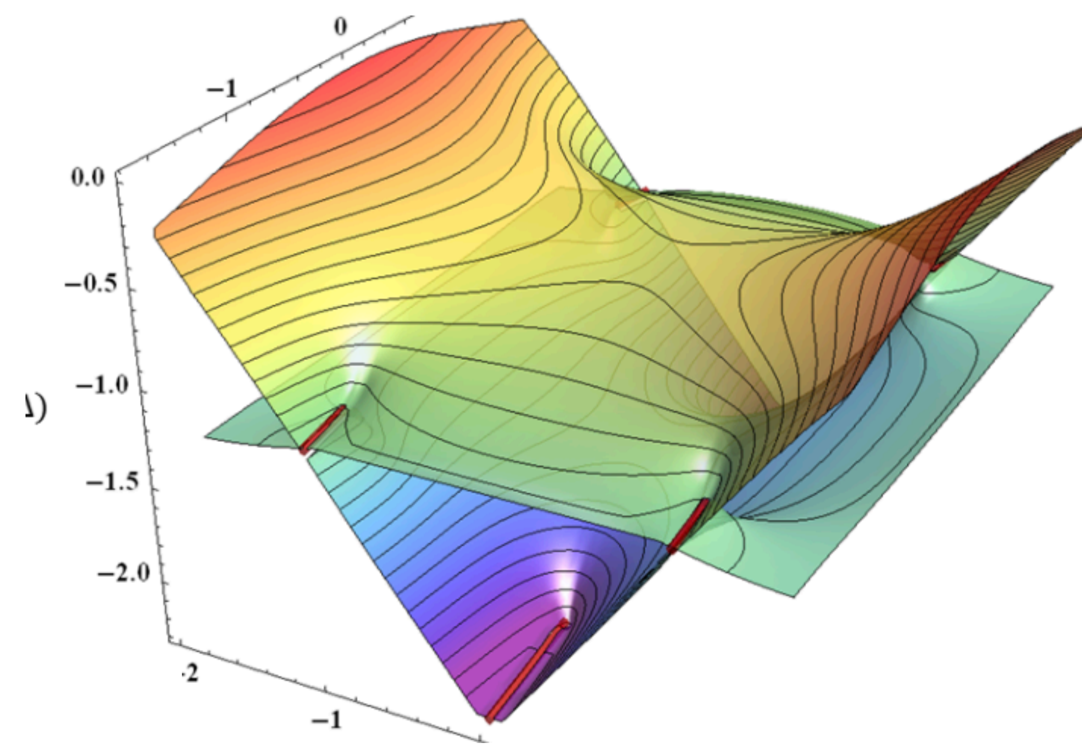


Above: *spectrum of defect CFT on a Wilson line*

[AC, Julius, Gromov, Preti '21]

Analytic continuation in Spin and Regge trajectories

[Gromov, Levkovich-Maslyuk, Sizov '15]



$$
\begin{aligned}
\Delta \;=\;& 4+12g^2-48g^4+336g^6+g^8\big(-2496+576\,\zeta_3-1440\,\zeta_5\big)\\
&+g^{10}\big(15168+6912\,\zeta_3-5184\,\zeta_3^2-8640\,\zeta_5+30240\,\zeta_7\big)\\
&+g^{12}\big(-7680-262656\,\zeta_3-20736\,\zeta_3^2+112320\,\zeta_5+155520\,\zeta_3\,\zeta_5+75600\,\zeta_7-489888\,\zeta_9\big)\\
&+g^{14}\big(-2135040+5230080\,\zeta_3-421632\,\zeta_3^2+124416\,\zeta_3^3-229248\,\zeta_5+411264\,\zeta_3\,\zeta_5\\
&\quad-993600\,\zeta_5^2-1254960\,\zeta_7-1935360\,\zeta_3\,\zeta_7-835488\,\zeta_9+7318080\,\zeta_{11}\big)\\
&+g^{16}\Big(54408192-83496960\,\zeta_3+7934976\,\zeta_3^2+1990656\,\zeta_3^3-19678464\,\zeta_5-4354560\,\zeta_3\,\zeta_5\\
&\quad-3255552\,\zeta_3^2\,\zeta_5+2384640\,\zeta_5^2+21868704\,\zeta_7-6229440\,\zeta_3\,\zeta_7+22256640\,\zeta_5\,\zeta_7\\
&\quad+9327744\,\zeta_9+23224320\,\zeta_3\,\zeta_9+\frac{65929248}{\cdots}\,\zeta_{11}-106007616\,\zeta_{13}-\frac{684288}{\cdots}\,Z_{11}^{(2)}\Big)
\end{aligned}
$$

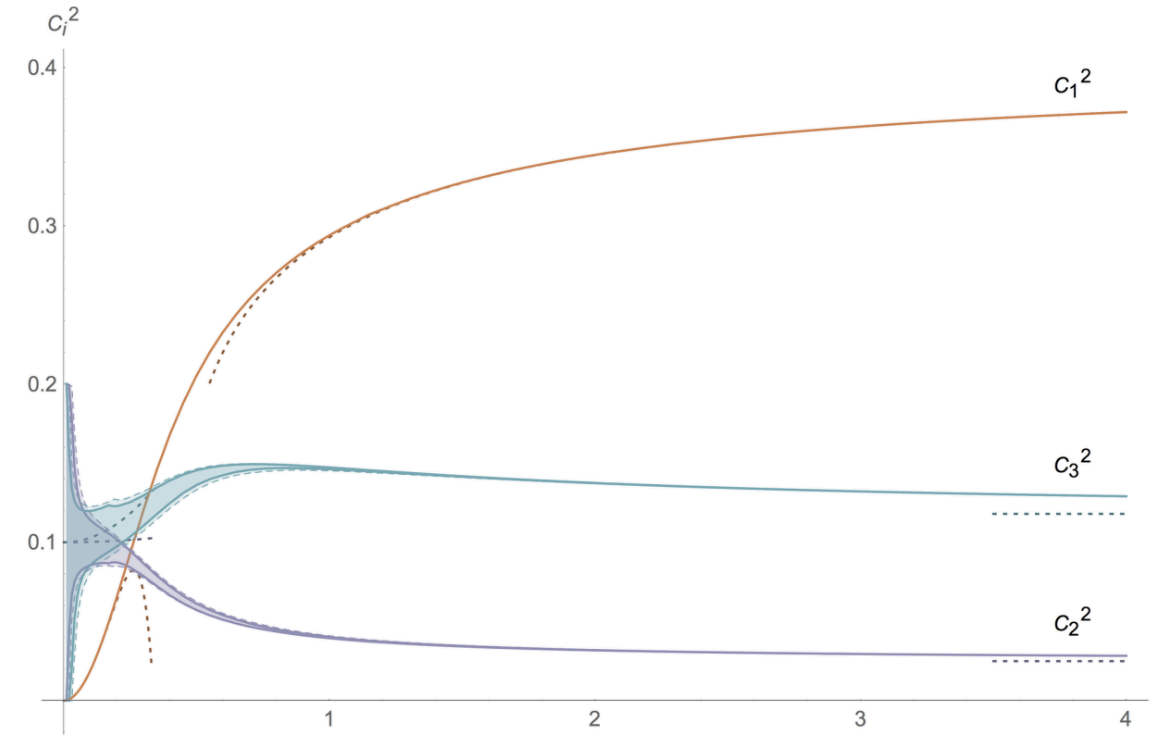
Solve analytically at weak coupling (and other limits)

[Marboe, Volin '14]

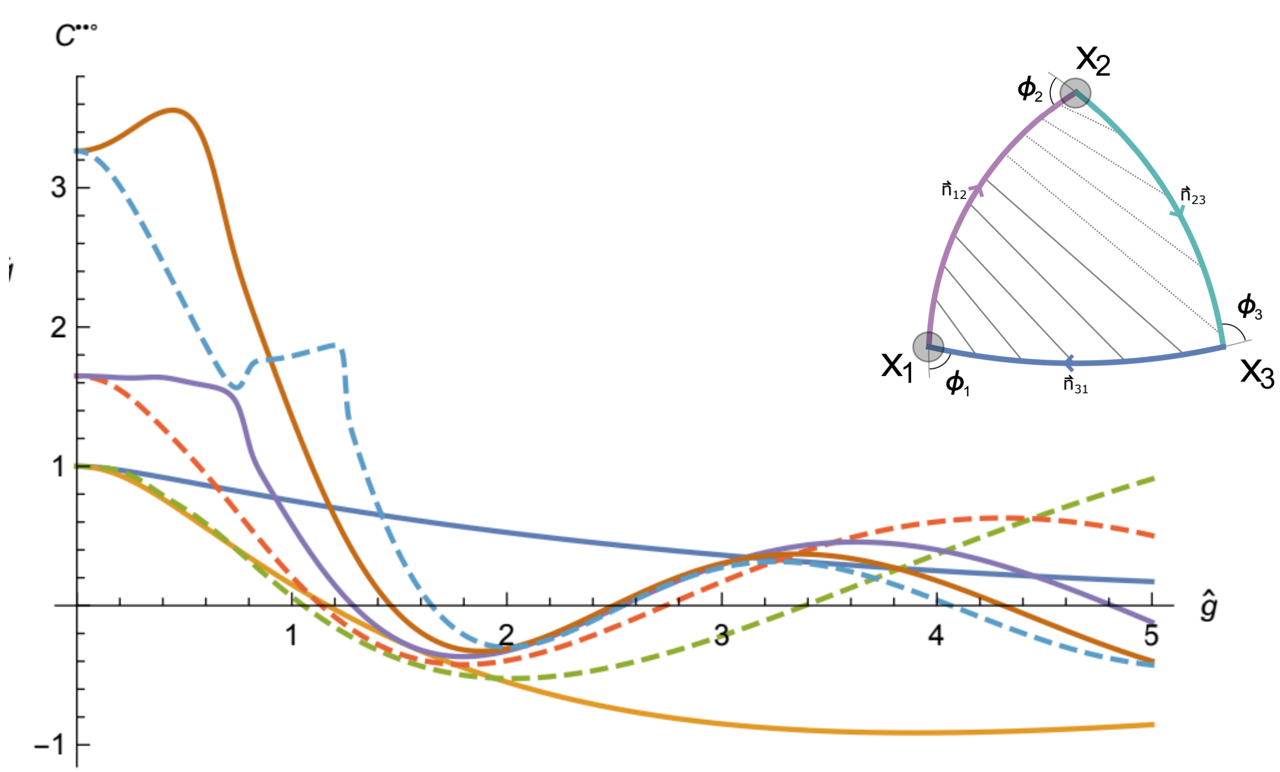
The QSC is also at the center of some approaches to compute correlators

e.g. one can use a synergy with conformal bootstrap methods

[AC, Gromov, Julius, Preti '21 , '22]



Moreover there is evidence the Q-functions can be used to build correlators through the Separation of Variables



$$\simeq C_{123}^{\bullet\bullet\circ} \propto \frac{\int_{|} Q_1 Q_2 e^{-\phi_3 u} \frac{du}{2\pi i u}}{\sqrt{\int_{|} Q_1 Q_1 \frac{du}{2\pi i u}}\sqrt{\int_{|} Q_2 Q_2 \frac{du}{2\pi i u}}}$$

[AC, Gromov, Levkovich-Maslyuk '18]

[Komatsu, Giombi '18] [Jiang, Komatsu, Kostov, Serban '15]+…

What we know on the AdS3/CFT2 integrable system

[Borsato, Ohlsson-Sax, Sfondrini, Stefanski ’14]

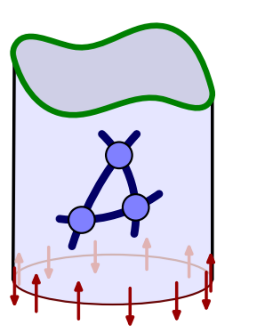
The string in uniform lightcone gauge is a non-relativistic, integrable theory.
Understood in detail in large volume

Dispersion relation of elementary excitations:

$$E(p) = \sqrt{m^2 + 4\, h^2(\alpha')\ \sin^2 \frac{p}{2}}$$

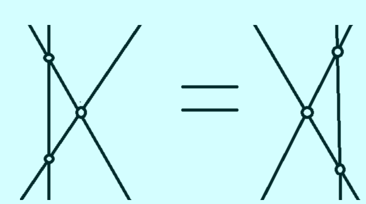
Expect a redefined *integrability* coupling,
(depending also on moduli)

$m = \pm 1$ (massive), or $m = 0$ (**massless**, *new feature!*)



Worldsheet S-matrix bootstrapped at finite coupling!

assuming integrability



[Borsato, Ohlsson-Sax, Sfondrini, Stefanski ’14]

[Borsato, Ohlsson-Sax, Sfondrini, Stefanski, Torrielli ’13,’16]

[Frolov, Sfondrini ’21]

Asymptotic Bethe Ansatz (ABA) - interacting gas of particles in large volume

$$\prod_{i\neq j}\hat{S}(p_i,p_j)=e^{ip_iL}$$

constraint on "Bethe roots" (≃quantised momenta)

[Borsato, Ohlsson-Sax, Sfondrini, Stefanski, Torrielli '13,'16]

anomalous dimension:

$$\delta\Delta=\sum_{i=1}E(p_i)+O(e^{-M_{gap}\,L})$$

finite size correction

In a general sector with massless modes, corrections are $O\left(\frac{1}{L}\right)$

[Abbott, Aniceto '15]

Finite L is encoded in Thermodynamic Bethe Ansatz, for AdS3 written recently [Frolov, Sfondrini '21]

Expected to be related to QSC, but much more complicated

[Beisert, Staudacher '05]
[Bombardelli, Fioravanti, Tateo '09]
[Arutyunov, Frolov '09]
[Gromov, Kazakov, Vieira '09]
[AC, Fioravanti, Tateo '10]
[Gromov, Kazakov, Leurent, Volin '11,'13]

Previous route in AdS_5 and AdS_4:

worldsheet S-matrix → **QSC** (through TBA)

Very involved!

Now:

QSC from general principles

Could lead to discovery of many new cases!

QSC = <u>Symmetry</u> + <u>Analyticity</u>

Symmetry

AdS3: $psu(1,1|2)_L \oplus psu(1,1|2)_R$ **, two copies of a well-understood case**

16 + 16 Q-functions

They satisfy functional “QQ-relations” reflecting the symmetry algebra

Example (details not important)

$$Q_{1|1}(u+\tfrac{i}{2})Q_{2|1}(u-\tfrac{i}{2}) - Q_{1|1}(u-\tfrac{i}{2})Q_{2|1}(u+\tfrac{i}{2})$$

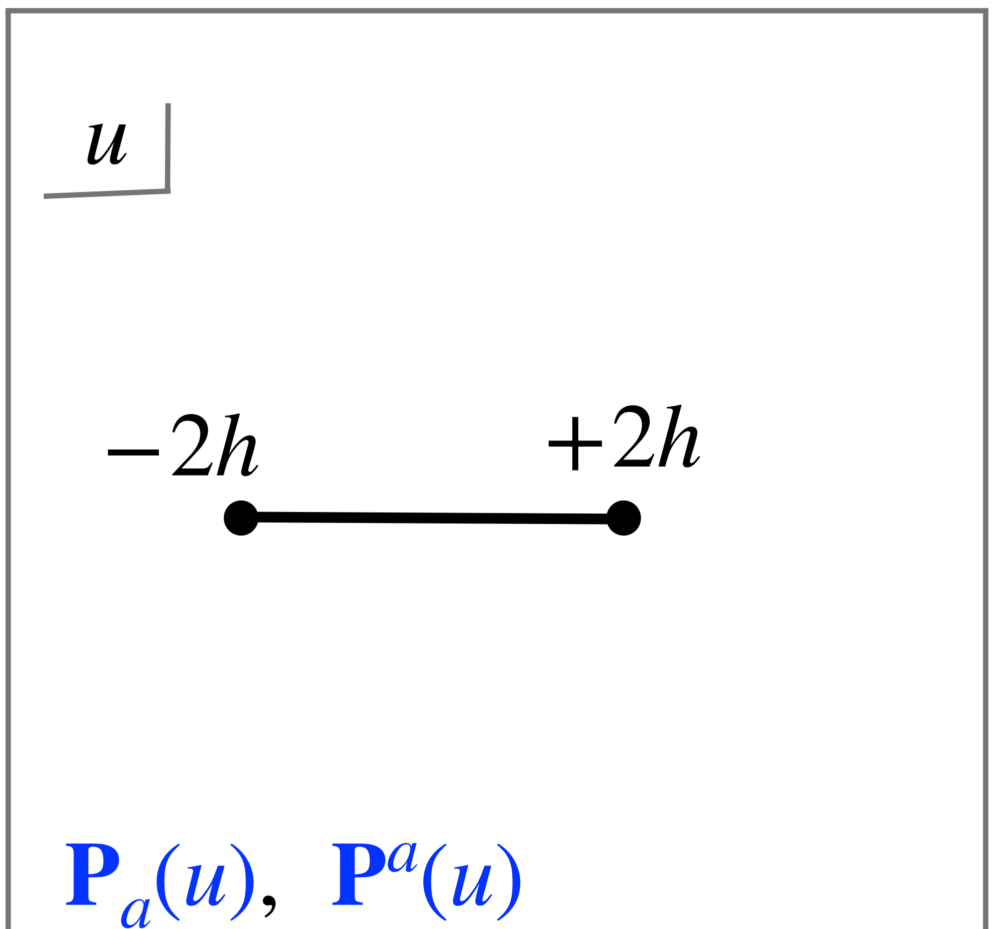
$$= \mathbf{Q}_1(u)\mathbf{Q}^2(u)$$

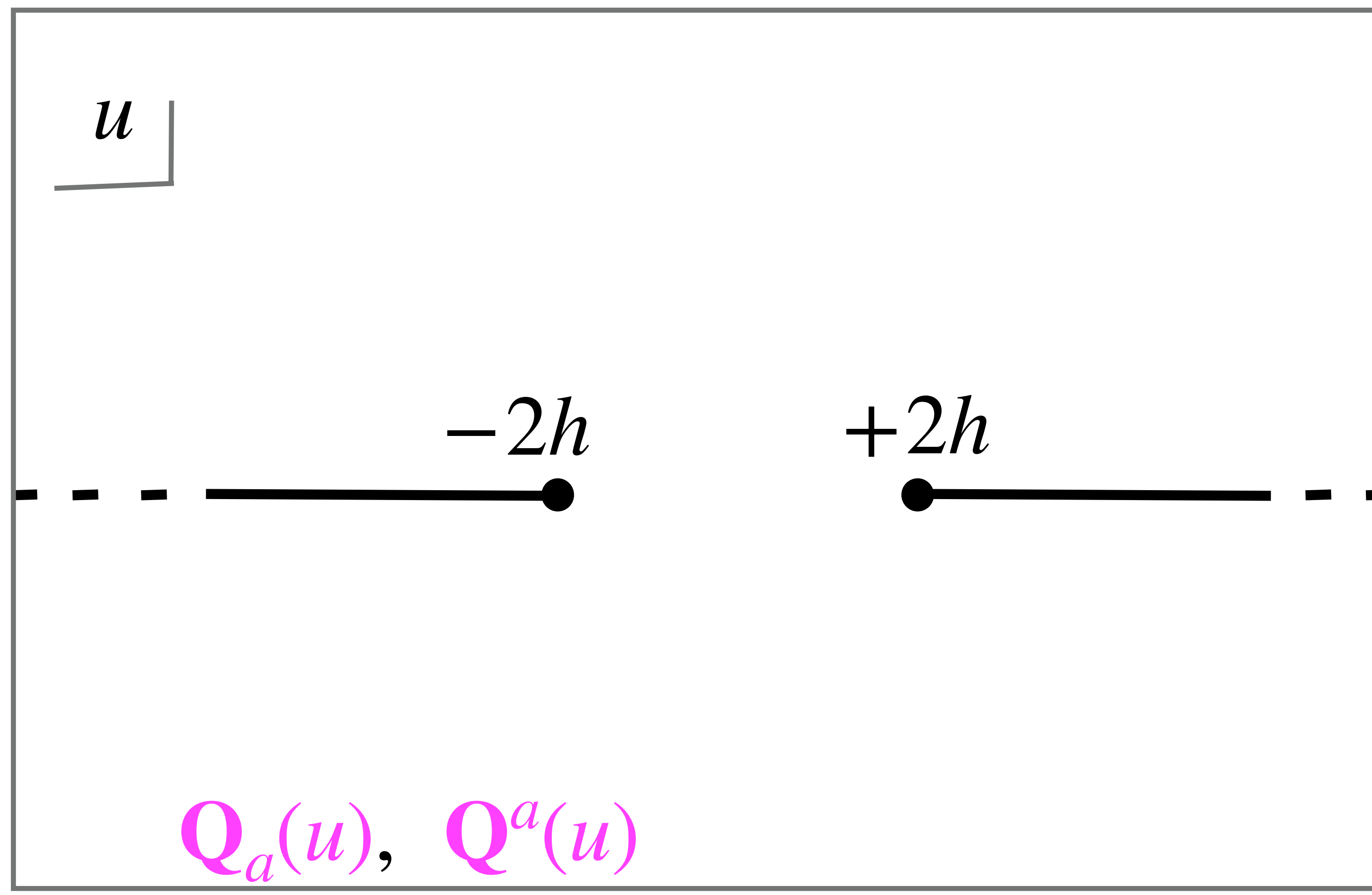
Some Q-functions play a special role. **Analyticity**

In the classical limit,

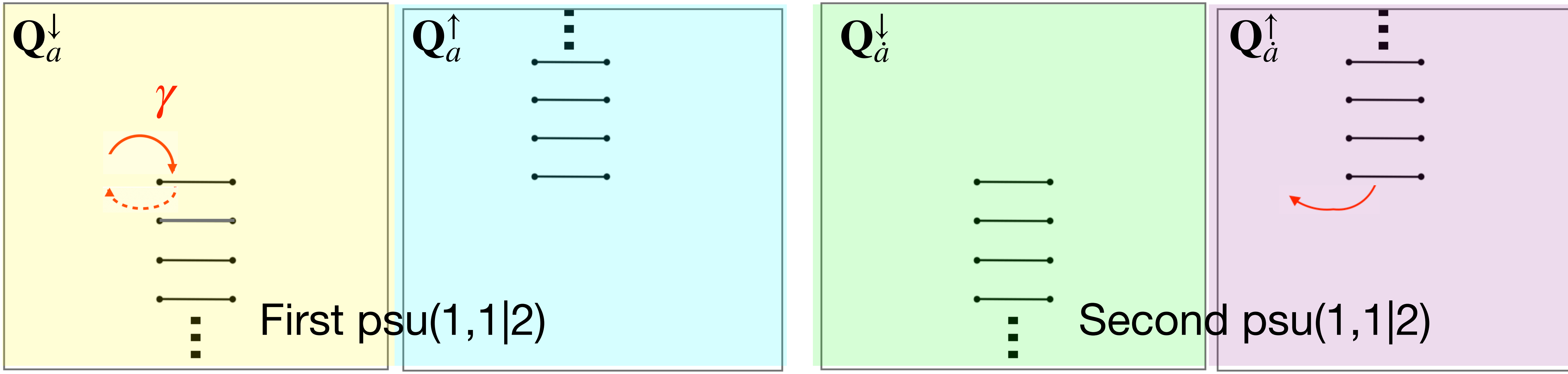
$(\mathbf{Q}_1(u), \mathbf{Q}_2(u) \,|\, \mathbf{P}_1(u), \mathbf{P}_2(u))$ parametrise motion in $\mathbf{AdS_3}$ or $\mathbf{S^3}$

Inspired by the other QSC's we postulate the cut structure:





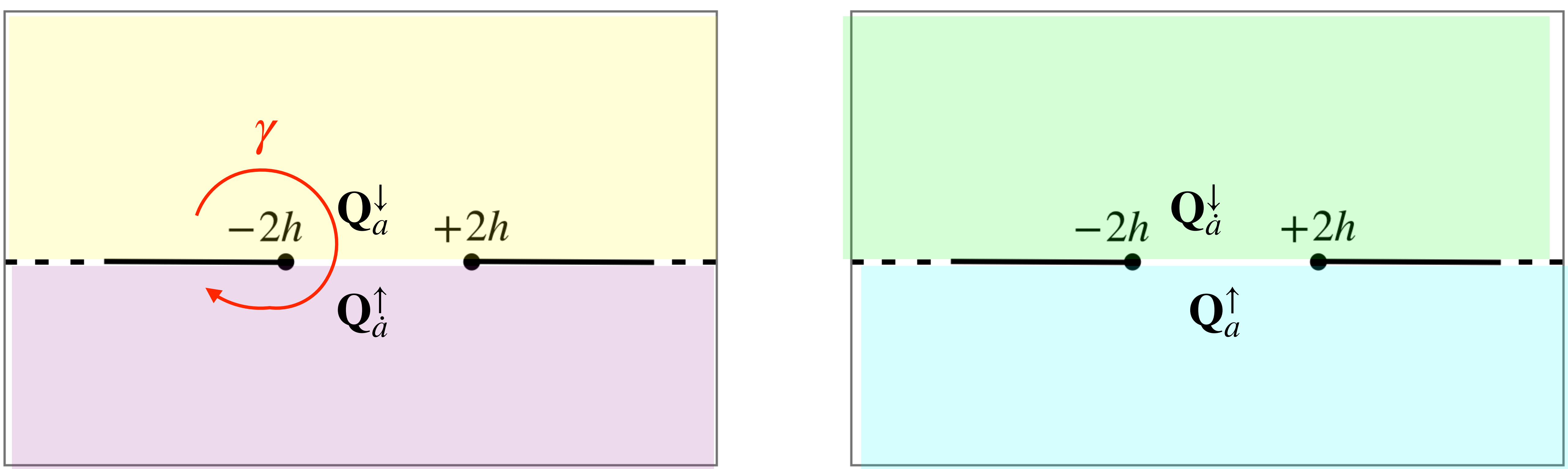
No other singularities on these sheets



QQ-relations + analyticity have some tension…

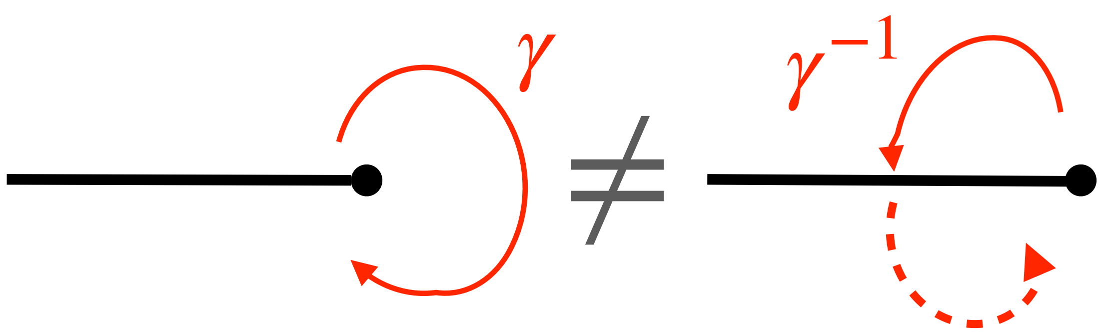
To resolve it we need to impose some gluing of Riemann sheets

In pictures, that's it!

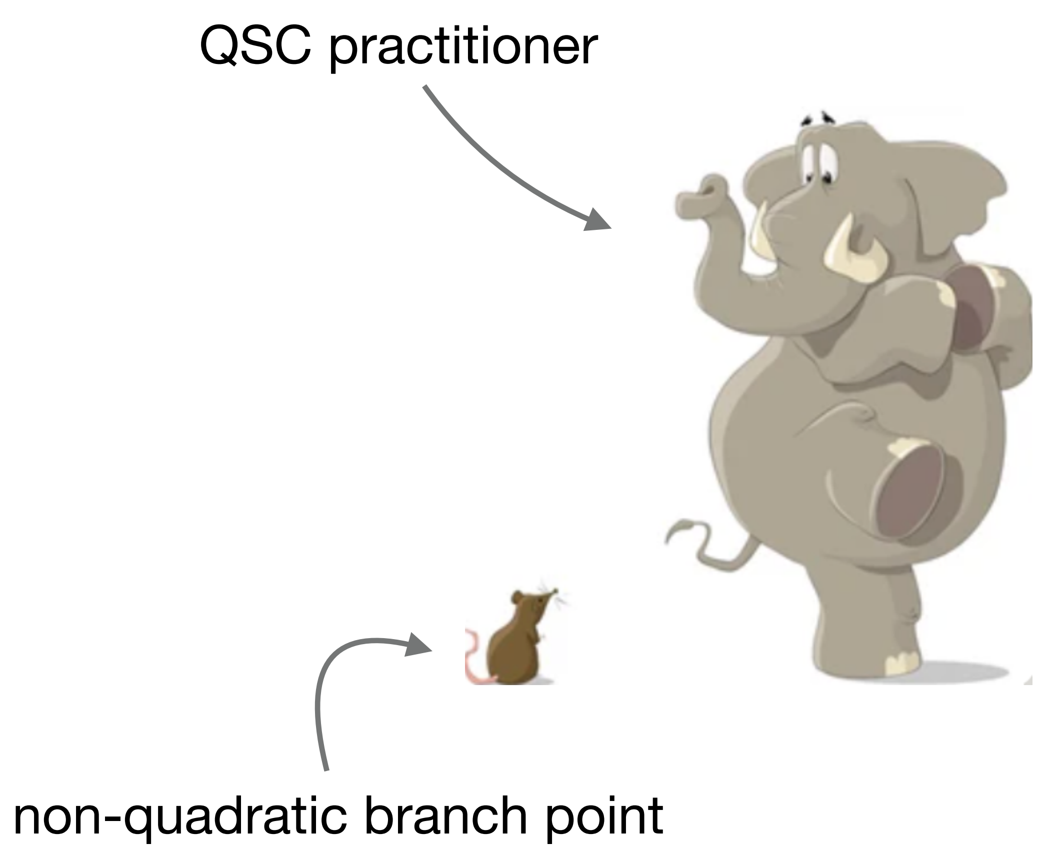


In AdS4 and AdS5: infinitely many branch points, infinitely many sheets,
but branch points were quadratic

In AdS3 they each have infinite order



$$\gamma \neq \gamma^{-1}, \quad \gamma^n \neq 1$$



We think this is a
signature of massless modes.
We have to make friends with it!

What evidence do we have?

We can solve the QSC equations at **large volume** (in the massive sector)

[AC, Gromov, Stefanski, Torrielli '21]
[Ekhammar, Volin '21]

$$J \to \infty \qquad \Delta \sim J + O(1)$$

Explicit solution in the limit

Blocks of the worldsheet S-matrix appear in this solution, and constraints which **reproduce the Asymptotic Bethe Ansatz.**

i.e., the QSC "knows" the worldsheet S-matrix, including the dressing phases

(should resolve ambiguities in the S-matrix bootstrap)

Good indications this extends to include massless modes (but limit is subtle)

[AC, Ekhammar, Gromov, Stefanski, Torrielli, Volin in progress]

Encouraging preliminary numerics at **finite quantum numbers, finite coupling: indicating the QSC has discrete solutions**

[Ekhammar, Gromov, Ryan]

Need to develop new techniques due to non-quadratic branch points.

[in progress]

Conclusions and Outlook

[AC, Gromov, Stefanski, Torrielli '21]
[Ekhammar, Volin '21]

New example of QSC proposed for string theory on $AdS_3 \times S^3 \times T^4$

New tool for quantitative, non-perturbative studies of non-protected states.

For the first time, this comes from a classification-type approach.
Can we use the to deduce further QSC's?
(e.g. other AdS_3/CFT_2 dualities, defect setups…)

Hopefully soon there will be plots of the spectrum…

[Ekhammar, Gromov, Ryan et al] [in progress]

… and new explorations of AdS_3/CFT_2 will become possible!
Stay tuned!

Thank you!

Asymptotic Bethe Ansatz: what it looks like

[Borsato, Ohlsson-Sax,
Sfondrini, Stefanski, Torrielli ’13,’16]

$$
\begin{aligned}
1 &= \prod_{j=1}^{K_2} \frac{y_{1,k} - x_j^+}{y_{1,k} - x_j^-} \prod_{j=1}^{K_{\bar{2}}} \frac{1 - \frac{1}{y_{1,k}\bar{x}_j^-}}{1 - \frac{1}{y_{1,k}\bar{x}_j^+}}, \\
\left(\frac{x_k^+}{x_k^-}\right)^L &= \prod_{j\neq k}^{K_2} \frac{x_k^+ - x_j^-}{x_k^- - x_j^+} \frac{1 - \frac{1}{x_k^+ x_j^-}}{1 - \frac{1}{x_k^- x_j^+}} \sigma^2(x_k, x_j) \prod_{j=1}^{K_1} \frac{x_k^- - y_{1,j}}{x_k^+ - y_{1,j}} \prod_{j=1}^{K_3} \frac{x_k^- - y_{3,j}}{x_k^+ - y_{3,j}} \\
&\times \prod_{j=1}^{K_{\bar{2}}} \frac{1 - \frac{1}{x_k^+ \bar{x}_j^+}}{1 - \frac{1}{x_k^- \bar{x}_j^-}} \frac{1 - \frac{1}{x_k^+ \bar{x}_j^-}}{1 - \frac{1}{x_k^- \bar{x}_j^+}} \tilde{\sigma}^2(x_k, \bar{x}_j) \prod_{j=1}^{K_{\bar{1}}} \frac{1 - \frac{1}{x_k^- y_{\bar{1},j}}}{1 - \frac{1}{x_k^+ y_{\bar{1},j}}} \prod_{j=1}^{K_{\bar{3}}} \frac{1 - \frac{1}{x_k^- y_{\bar{3},j}}}{1 - \frac{1}{x_k^+ y_{\bar{3},j}}}, \\
1 &= \prod_{j=1}^{K_2} \frac{y_{3,k} - x_j^+}{y_{3,k} - x_j^-} \prod_{j=1}^{K_{\bar{2}}} \frac{1 - \frac{1}{y_{3,k}\bar{x}_j^-}}{1 - \frac{1}{y_{3,k}\bar{x}_j^+}},
\end{aligned}
$$

$$
\begin{aligned}
1 &= \prod_{j=1}^{K_{\bar{2}}} \frac{y_{\bar{1},k} - \bar{x}_j^-}{y_{\bar{1},k} - \bar{x}_j^+} \prod_{j=1}^{K_2} \frac{1 - \frac{1}{y_{\bar{1},k} x_j^+}}{1 - \frac{1}{y_{\bar{1},k} x_j^-}}, \\
\left(\frac{\bar{x}_k^+}{\bar{x}_k^-}\right)^L &= \prod_{j\neq k}^{K_{\bar{2}}} \frac{\bar{x}_k^- - \bar{x}_j^+}{\bar{x}_k^+ - \bar{x}_j^-} \frac{1 - \frac{1}{\bar{x}_k^+ \bar{x}_j^-}}{1 - \frac{1}{\bar{x}_k^- \bar{x}_j^+}} \sigma^2(\bar{x}_k, \bar{x}_j) \prod_{j=1}^{K_{\bar{1}}} \frac{\bar{x}_k^+ - y_{\bar{1},j}}{\bar{x}_k^- - y_{\bar{1},j}} \prod_{j=1}^{K_{\bar{3}}} \frac{\bar{x}_k^+ - y_{\bar{3},j}}{\bar{x}_k^- - y_{\bar{3},j}} \\
&\times \prod_{j=1}^{K_2} \frac{1 - \frac{1}{\bar{x}_k^- x_j^-}}{1 - \frac{1}{\bar{x}_k^+ x_j^+}} \frac{1 - \frac{1}{\bar{x}_k^+ x_j^-}}{1 - \frac{1}{\bar{x}_k^- x_j^+}} \tilde{\sigma}^2(\bar{x}_k, x_j) \prod_{j=1}^{K_1} \frac{1 - \frac{1}{\bar{x}_k^+ y_{1,j}}}{1 - \frac{1}{\bar{x}_k^- y_{1,j}}} \prod_{j=1}^{K_3} \frac{1 - \frac{1}{\bar{x}_k^+ y_{3,j}}}{1 - \frac{1}{\bar{x}_k^- y_{3,j}}}, \\
1 &= \prod_{j=1}^{K_{\bar{2}}} \frac{y_{\bar{3},k} - \bar{x}_j^-}{y_{\bar{3},k} - \bar{x}_j^+} \prod_{j=1}^{K_2} \frac{1 - \frac{1}{y_{\bar{3},k} x_j^+}}{1 - \frac{1}{y_{\bar{3},k} x_j^-}}.
\end{aligned}
$$

Dressing phases
(complicated functions)

Reported here: massive sector
equations.