Exploring SCFTs with Magnetic Quivers

Eurostrings 2022, Lyon

Antoine Bourget

IPhT, CEA, Saclay Ecole Normale Supérieure, Paris

April 27, 2022

Based on long time collaboration with G. Arias-Tamargo, M. van Beest, S. Cabrera, S. Giacomelli, J. Grimminger, A. Hanany, R. Kalveks, M. Martone, A. Pini, S. Schäfer-Nameki, M. Sperling, G. Zafrir, Z. Zhong...

Today: mostly [2006.16994],[2110.11365]

40 + 40 + 43 + 43 +

Introduction

CFTs: Central role among QFTs:

- They are seeds to explore the landscape of QFTs via RG flow
- They encode aspects of quantum gravity via holography.

Supersymmetry: gives exact analytic control over sectors of a QFT.

Introduction

CFTs: Central role among QFTs:

- They are seeds to explore the landscape of QFTs via RG flow
- They encode aspects of quantum gravity via holography.

Supersymmetry: gives exact analytic control over sectors of a QFT.

In this talk, focus on 8 supercharges (+8 superconformal).

 \rightarrow What is the landscape of SCFTs?

Introduction

CFTs: Central role among QFTs:

- They are seeds to explore the landscape of QFTs via RG flow
- They encode aspects of quantum gravity via holography.

Supersymmetry: gives exact analytic control over sectors of a QFT.

In this talk, focus on 8 supercharges (+8 superconformal).

 \rightarrow What is the landscape of SCFTs?

Two approaches:

- "Top-down" explicit construction (Lagrangian, geometric engineering, brane systems, compactifications, ...)
- "Bottom-up" constraints (moduli space geometry, bootstrap, ...)

Superconformal Algebras

Dimension	Susy	Bosonic subalgebra		SCA
d = 6	$\mathcal{N}=(1,0)$	$\mathfrak{so}(6,2)\oplus\mathfrak{su}(2)_H$	\subset	$\mathfrak{osp}(6,2 1)$
d = 5	$\mathcal{N}=1$	$\mathfrak{so}(5,2)\oplus\mathfrak{su}(2)_H$	\subset	$\mathfrak{f}(4)$
d = 4	$\mathcal{N}=2$	$\mathfrak{so}(4,2) \oplus \mathfrak{su}(2)_H \oplus \mathfrak{u}(1)_C$	\subset	$\mathfrak{su}(2,2 2)$
d = 3	$\mathcal{N}=4$	$\mathfrak{so}(3,2) \oplus \mathfrak{su}(2)_H \oplus \mathfrak{su}(2)_C$	\subset	$\mathfrak{osp}(4 4)$

Superconformal Algebras

Dimension	Susy	Bosonic subalgebra		SCA
d = 6	$\mathcal{N}=(1,0)$	$\mathfrak{so}(6,2)\oplus\mathfrak{su}(2)_H$	\subset	$\mathfrak{osp}(6,2 1)$
d = 5	$\mathcal{N}=1$	$\mathfrak{so}(5,2)\oplus\mathfrak{su}(2)_H$	\subset	f(4)
d = 4	$\mathcal{N}=2$	$\mathfrak{so}(4,2) \oplus \mathfrak{su}(2)_H \oplus \mathfrak{u}(1)_C$	\subset	$\mathfrak{su}(2,2 2)$
d = 3	$\mathcal{N}=4$	$\mathfrak{so}(3,2) \oplus \mathfrak{su}(2)_H \oplus \mathfrak{su}(2)_C$	\subset	$\mathfrak{osp}(4 4)$

SCFTs are

- "Rare" in 6d / 5d isolated, rely on exceptional isomorphisms, non Lagrangian.
- More common in 4d (some Lagrangian; existence of conformal manifolds).
 Classification?
- Very large number in 3d.

Superconformal Algebras

Dimension	Susy	Bosonic subalgebra		SCA
d = 6	$\mathcal{N}=(1,0)$	$\mathfrak{so}(6,2)\oplus\mathfrak{su}(2)_H$	\subset	$\mathfrak{osp}(6,2 1)$
d = 5	$\mathcal{N}=1$	$\mathfrak{so}(5,2)\oplus\mathfrak{su}(2)_H$	\subset	f(4)
d = 4	$\mathcal{N}=2$	$\mathfrak{so}(4,2) \oplus \mathfrak{su}(2)_H \oplus \mathfrak{u}(1)_C$	\subset	$\mathfrak{su}(2,2 2)$
d = 3	$\mathcal{N}=4$	$\mathfrak{so}(3,2) \oplus \mathfrak{su}(2)_H \oplus \mathfrak{su}(2)_C$	\subset	$\mathfrak{osp}(4 4)$

SCFTs are

- "Rare" in 6d / 5d isolated, rely on exceptional isomorphisms, non Lagrangian.
- More common in 4d (some Lagrangian; existence of conformal manifolds).
 Classification?
- Very large number in 3d.

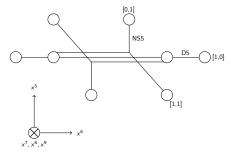
Existence of Moduli space of vacua, always contains Higgs branch.

Top-down construction (example: 5d)

• Geometric engineering : M-theory on canonical threefold singularity.

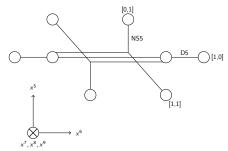
Top-down construction (example: 5d)

- Geometric engineering : M-theory on canonical threefold singularity.
- Brane systems : example of type IIB brane web



Top-down construction (example: 5d)

- Geometric engineering : M-theory on canonical threefold singularity.
- Brane systems : example of type IIB brane web



Mixture of both: IIA with D6 on fibered ALE space.

[Intriligator, Morrison, Seiberg, Aharony, Hanany, Kol, Bergman, Rodrígez-Gómez, Zafrir, Del Zotto, Heckman, Jefferson, Katz, Kim, Vafa, Xie, Yau, Closset, Schäfer-Nameki, Wang, Hayashi, Yagi, ...]

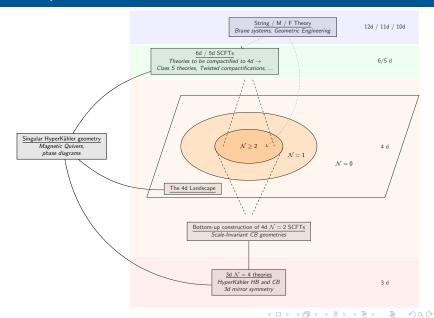
Bottom-up constraints

Classification of 4d rank 1 $\mathcal{N}=2$ SCFT Coulomb branch geometries:

Flavor	CB geometry and deformation	$\Delta(u)$
E ₈	$II^* ightarrow \{I_1^{10}\}$	6
E_7	$III^* ightarrow \{\hat{m{I}}_1^9\}$	4
E_6	$IV^* o \{I_1^8\}$	3
D_4	$I_0^* \to \{I_1^{6}\}$	2
A_2	$IV \rightarrow \{I_1^4\}$	3/2
A_1	$III \rightarrow \{I_3^{\frac{1}{3}}\}$	4/3
Ø	$II \rightarrow \{I_1^{\frac{1}{3}}\}$	6/5
C ₅	$II^* \to \{I_1^6, I_4\}$	6
C_3A_1	$III^* \to \{I_1^5, I_4\}$	4
C_2U_1	$IV^* ightarrow \{I_1^4, I_4\}$	3
C_1	$I_0^* \to \{I_1^2, I_4\}$	2
$A_3 \rtimes \mathbb{Z}_2$	$II^* \to \{I_1^3, I_1^*\}$	6
$A_1U_1 \rtimes \mathbb{Z}_2$	$III^* o \{\overline{I_1^2}, \overline{I_1^*}\}$	4
U_1	$IV^* o \{\hat{I_1^1}, \hat{I_1^*}\}$	3
$A_2 \rtimes \mathbb{Z}_2$	$II^* \to \{I_1^2, IV_{Q=1}^*\}$	6
$U_1 \rtimes \mathbb{Z}_2$	$III^* ightarrow \{I_1, IV_{Q=1}^*\}$	4
Ø	$IV_{Q=1}^*$	3
<i>C</i> ₁	$I_0^* o \{I_2^3\}$	2

[Argyres, Lotito, Lü, Martone 18]

Landscape of SCFTs



The ID card of a 4d $\mathcal{N}=2$ SCFT

These data can be **derived** from realizations of the theory, or can be **constrained** from bottom-up.

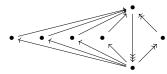
- Flavor symmetry
- Central charges
- Coulomb branch geometry
- Higgs branch geometry
- Seiberg-Witten curve / Integrable system
- Spectrum of BPS states
- Superconformal index [Kinney, Maldacena, Minwalla, Raju 05]
- VOA [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees 13]

• Flavor symmetry : $\mathfrak{sp}(5)$, level k = 7.

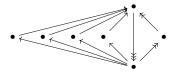
- Flavor symmetry : $\mathfrak{sp}(5)$, level k = 7.
- Central charges : $a = \frac{41}{12}$, $c = \frac{49}{12}$. Effective number of vectors $n_v = 11$, hypers $n_h = 27$.

- Flavor symmetry : $\mathfrak{sp}(5)$, level k = 7.
- Central charges : $a = \frac{41}{12}$, $c = \frac{49}{12}$. Effective number of vectors $n_v = 11$, hypers $n_h = 27$.
- Coulomb branch geometry:
 - Complex Dimension = rank r = 1.
 - Scaling dimension $\Delta = 6$. Characteristic dimension $\kappa = 6$.
 - Singularity and deformation $II^* o \{I_1^6, I_4\}$.

- Flavor symmetry : $\mathfrak{sp}(5)$, level k = 7.
- Central charges : $a = \frac{41}{12}$, $c = \frac{49}{12}$. Effective number of vectors $n_v = 11$, hypers $n_h = 27$.
- Coulomb branch geometry:
 - Complex Dimension = rank r = 1.
 - Scaling dimension $\Delta = 6$. Characteristic dimension $\kappa = 6$.
 - Singularity and deformation $II^* \rightarrow \{I_1^6, I_4\}$.
- Spectrum of BPS states [Cecotti, Del Zotto 14], [Del Zotto, Garcí a Etxebarria 22]

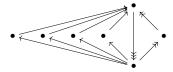


- Flavor symmetry : $\mathfrak{sp}(5)$, level k = 7.
- Central charges : $a = \frac{41}{12}$, $c = \frac{49}{12}$. Effective number of vectors $n_v = 11$, hypers $n_h = 27$.
- Coulomb branch geometry:
 - Complex Dimension = rank r = 1.
 - Scaling dimension $\Delta = 6$. Characteristic dimension $\kappa = 6$.
 - Singularity and deformation $II^* \rightarrow \{I_1^6, I_4\}$.
- Spectrum of BPS states [Cecotti, Del Zotto 14], [Del Zotto, Garcí a Etxebarria 22]



Superconformal index : deduced from class S construction [Chacaltana, Distler
 11]

- Flavor symmetry : $\mathfrak{sp}(5)$, level k = 7.
- Central charges : $a=\frac{41}{12}$, $c=\frac{49}{12}$. Effective number of vectors $n_v=11$, hypers $n_h=27$.
- Coulomb branch geometry:
 - Complex Dimension = rank r = 1.
 - Scaling dimension $\Delta = 6$. Characteristic dimension $\kappa = 6$.
 - Singularity and deformation $II^* \to \{I_1^6, I_4\}$.
- Spectrum of BPS states [Cecotti, Del Zotto 14], [Del Zotto, Garcí a Etxebarria 22]



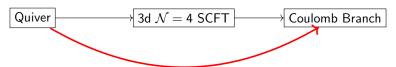
- Superconformal index : deduced from class S construction [Chacaltana, Distler
 11]
- Higgs branch geometry:
 - Quaternionic dimension $d_{HB} = n_h n_v = 24(c a) = 16$
 - Magnetic Quiver (see below)

The Higgs branch of an $SCFT_{8 \text{ susy}}$ is a hyperKähler singular cone due to $\mathfrak{su}(2)_H$. The Coulomb branch of a 3d $\mathcal{N}=4$ SCFT is also a hyperKähler singular cone due to $\mathfrak{su}(2)_C$.



[Cremonesi, Hanany, Zaffaroni 14] [Bullimore, Dimofte, Gaiotto 15] [Braverman, Finkelberg, Nakajima 15]

The Higgs branch of an $SCFT_{8 \text{ susy}}$ is a hyperKähler singular cone due to $\mathfrak{su}(2)_H$. The Coulomb branch of a 3d $\mathcal{N}=4$ SCFT is also a hyperKähler singular cone due to $\mathfrak{su}(2)_C$.



[Cremonesi, Hanany, Zaffaroni 14]

[Bullimore, Dimofte, Gaiotto 15]

[Braverman, Finkelberg, Nakajima 15]

A magnetic quiver is a combinatorial way to encode a hyperKähler singular cone.

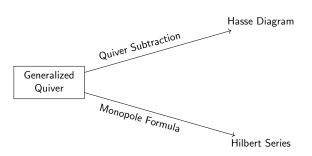
Quiver ----- Generalized Quiver

Let X be a hyperKähler singular cone (technically a *symplectic singularity* [Beauville 00]). We say that the (generalized) quiver Q is a **magnetic quiver** for X if

$$\mathcal{C}^{\mathrm{3d} \mathcal{N}=4}(Q)=X.$$

Let X be a hyperKähler singular cone (technically a *symplectic singularity* [Beauville 00]). We say that the (generalized) quiver Q is a **magnetic quiver** for X if

$$\mathcal{C}^{\mathrm{3d} \mathcal{N}=4}(Q)=X.$$



[Cremonesi, Hanany, Zaffaroni 14]

[AB, Cabrera, Grimminger, Hanany, Sperling, Zajac, Zhong 20]

Higgsing diagrams

Higgsing corresponds to quiver subtraction / Higgs branch singularities.

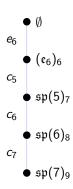
5d 4d

$$\mathfrak{su}(2)_0 + 6F \rightarrow (\mathfrak{e}_6)_6$$

$$\mathfrak{su}(3)_0 + 8 F \quad \to \quad \mathfrak{sp}(5)_7$$

$$\mathfrak{su}(4)_0 + 10F \rightarrow \mathfrak{sp}(6)_8$$

$$\mathfrak{su}(5)_0 + 12F \rightarrow \mathfrak{sp}(7)_9$$



Rank-1 4d $\mathcal{N}=2$ magnetic quivers

Flavor	$dim_{\mathbb{H}}(\mathit{HB})$	Magnetic Quiver
E ₈	29	
E_7	17	Affine
E_6	11	
D_4	5	Dynkin
A_2	2	
A_1	1	Diagrams
Ø	0	

[AB, Grimminger, Hanany, Sperling, Zafrir, Zhong 20]

Flavor	$dim_{\mathbb{H}}(\mathit{HB})$	Magnetic Quiver
C ₅	16	1 2 3 4 5 2
C_3A_1	8	
C_2U_1	4	
C_1	1	
$A_3 \rtimes \mathbb{Z}_2$	9	1 2 3 4
$A_1U_1 \rtimes \mathbb{Z}_2$	3	
U_1	1	
$A_2 \rtimes \mathbb{Z}_2$	5	
$U_1 \rtimes \mathbb{Z}_2$	1	
Ø	0	
<i>C</i> ₁	1	

INPUT:

- A (generalized) quiver
- A list of elementary symplectic singularities with a corresponding magnetic quiver.

INPUT:

- A (generalized) quiver
- A list of elementary symplectic singularities with a corresponding magnetic quiver.

OUTPUT: Structure of nested singularities of the Higgs branch (\leftrightarrow all possible Higgsings).

[Cabrera, Hanany 18]

[AB, Grimminger, Hanany, Sperling, Zhong 21]

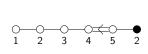
INPUT:

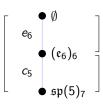
- A (generalized) quiver
- A list of elementary symplectic singularities with a corresponding magnetic quiver.

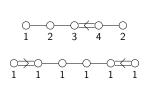
OUTPUT: Structure of nested singularities of the Higgs branch (\leftrightarrow all possible Higgsings).

[Cabrera, Hanany 18]
[AB, Grimminger, Hanany, Sperling, Zhong 21]

Example:







Reductionist approach to Higgs branch geometries :

- What is the list of atoms (elementary symplectic singularities)?
- What are the rules to combine them?

Slice	Framed quiver	Unframed quiver
a_n	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1
b_n		1 0 1 2 2 1
c_n	1 	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
d_n		1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
e_6	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
e_7	2 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	2 1 0 0 1 2 3 4 3 2
e_8	1 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	1 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Slice	Framed quiver	Unframed quiver
f_4		1 0 1 2 3 2
g_2		1 0 0 0 1 2
ac_n	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
ag_2		1 1
cg_2		
$h_{n,k}$		
$\overline{h}_{n,k}$	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
A_n	T ⁿ⁺¹	n+1 1

Higgs Chiral ring (\hat{B} operators)

Hilbert series for the Higgs branch of the $\mathfrak{sp}(5)_7$ theory:

```
 \begin{pmatrix} 1+2t+40t^2+194t^3+1007t^4+4704t^5+18683t^6+67030t^7+220700t^8+657352t^9+1796735t^{10} \\ +4540442t^{11}+10610604t^{12}+23011366t^{13}+46535540t^{14}+87887734t^{15}+155277056t^{16} \\ +257288236t^{17}+400453203t^{18}+859571786t^{19}+807195575t^{20}+1047954388t^{21} \\ +1282842123t^{22}+1481462886t^{23}+1615002952t^{24}+1662191888t^{25}+\cdots palindrome\cdots + t^{50} \\ (-1+t)^{13}(1+t)^{18}(1+t)^{18}(1+t+t^{2})^{16} \end{pmatrix}
```

Higgs Chiral ring (\hat{B} operators)

Hilbert series for the Higgs branch of the $\mathfrak{sp}(5)_7$ theory:

```
 \left(\begin{array}{c} 1+2t+40t^2+194t^3+1007t^4+4704t^5+18683t^6+67030t^7+220700t^8+657352t^9+1796735t^{10}\\ +4540442t^{11}+10610604t^{12}+23011366t^{13}+46535540t^{14}+87887734t^{15}+155277056t^{16}\\ +257288236t^{17}+400453203t^{18}+585971786t^{19}+807195575t^{20}+1047954388t^{21}\\ +1282842123t^{22}+1481462886t^{23}+1615002952t^{24}+1622191888t^{25}+\cdots palindrome\cdots+t^{50} \right)
```

Refined plethystic logarithm:

```
\begin{array}{l} t^2: [20000] \\ t^3: [00001] \\ t^4: -[01000] \\ t^5: -[10010] \\ t^6: -[00200] - [20000] + [01000] \\ \text{etc} \end{array}
```

Higgs Chiral ring (\hat{B} operators)

Hilbert series for the Higgs branch of the $\mathfrak{sp}(5)_7$ theory:

```
 \left( \begin{array}{c} 1 + 2t + 40t^2 + 194t^3 + 1007t^4 + 4704t^5 + 18683t^6 + 67030t^7 + 220700t^8 + 657352t^9 + 1796735t^{10} \\ + 4540442t^{11} + 10610604t^{12} + 23011366t^{13} + 46535540t^{14} + 87887734t^{15} + 155277056t^{16} \\ + 257288236t^{17} + 400453203t^{18} + 585971786t^{19} + 807195575t^{20} + 1047954388t^{21} \\ + 1282842123t^{22} + 1481462886t^{23} + 1615002952t^{24} + 1662191888t^{25} + \cdots \\ palindrome \cdots + t^{50} \right)
```

Refined plethystic logarithm:

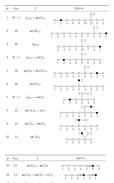
$$t^2$$
: [20000]
 t^3 : [00001]
 t^4 : -[01000]
 t^5 : -[10010]
 t^6 : -[00200] - [20000] + [01000]
etc

Highest weight generating function [Hanany, Kalveks 16]

$$\text{PE}\left[\sum_{i=1}^{4}\mu_{i}^{2}t^{2i}+t^{4}+\mu_{5}(t^{3}+t^{5})\right]$$

4d $\mathcal{N} = 2$ SCFTs at rank 2

[AB, Grimminger, Martone, Zafrir 21]



ş.	d_{HH}	1	Quiver
3	23	$\mathfrak{su}(6)_{10}\times\mathfrak{su}(2)_9$	1 2 3 4 5 6 3
4	13	$\mathfrak{su}(4)_{13}\times\mathfrak{su}(2)_7\times\mathfrak{u}(1)$	0 0 000 000 1 2 3 4 3 1
15	11	$\mathfrak{su}(3)_{10}\times\mathfrak{su}(3)_{10}\times\mathfrak{u}(1)$	0-0 00 00 0-0 1 2 3 3 2 1
36	8	$\mathfrak{su}(3)_{10}\times\mathfrak{ou}(2)_0\times\mathfrak{u}(1)$	2-200 A
17	6	$\mathfrak{su}(2)_8 \times \mathfrak{su}(2)_8 \times \mathfrak{u}(1)^2$	2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
18	2	$u(1)^2$	'
19	29	sp(14) ₉	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
40	17	$\mathfrak{su}(2)_n \times \mathfrak{sp}(10)_7$	0 0 0 000 •000 1 2 3 4 5 2 1 B ₁ 0
41	15	$\mathfrak{sp}(2)_5 \times \mathfrak{sp}(8)_7$	C_1 C_2 C_3 C_4 C_5

φ	d_{MB}	f	Quiver
11	12	$\mathfrak{so}(8)_a \times \mathfrak{sta}(2)_5$	2002
12	10	$u(6)_4$	1 2 1 2
13	6	su(2) ¹	1010 01
14	6	$\mathfrak{gu}(3)_0\times\mathfrak{gu}(2)_4$	1 2 2
15	6	gu(5) ₀	- Å-
16	4	${\rm sta}(2)_{15/3}\times {\rm sta}(2)_{11/3}$	0-0-0 1 2 2
17	2	$\mathfrak{su}(2)_{13/2}\times\mathfrak{u}(1)$	Å
18	2	su(2) _{(7/5}	0-0
19	1	$\mathfrak{su}(2)_{16/5}$	1 1
20	1	u(1)	1 1
21	0		Ŷ

ŕ	d_{MN}	j	Quiver
22	22	ap(12)s	1 2 3 4 5 6
23	20	$\mathfrak{sp}(4)_7 \times \mathfrak{sp}(8)_8$	$ \overset{\bigcirc}{D_1} \overset{\bigcirc}{C_1} \overset{\bigcirc}{D_2} \overset{\bigcirc}{C_2} \overset{\bullet}{D_3} \overset{\bullet}{C_4} \overset{\bullet}{D_5} $
24	24	$\mathfrak{su}(2)_{\mathfrak{f}}^2\times[\mathfrak{f}_4]_{12}$	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
25	12	$\mathfrak{su}(2)_4 \times \mathfrak{sp}(8)_6$	0 0 0 0 0 0 0 0 0 0 0 1 2 3 4 2 1
26	11	$\mathfrak{su}(2)_5 \times \mathfrak{sp}(6)_6 \times \mathfrak{u}(1)$	D_1 D_1 D_1 D_2 D_2 D_3 D_4 D_4 D_5
27	12	$\mathfrak{su}(2)_\S^2 \times \mathfrak{so}(7)_n$	0 000 000 1 2 4 4 2
28	16	$[f_4]_{10}\times u(1)$	1 4 6 4 2
29	7	$\mathfrak{sp}(6)_5 \times \mathfrak{u}(1)$	0-000
30	6	$\mathfrak{su}(3)_4 \times \mathfrak{su}(2)_4^2$	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
31	3	sp(4) ₄	0000000 1 2 1
32	2	$\mathfrak{su}(2)_3\times\mathfrak{su}(2)_3$	00000

ě	d_{HB}	j	Quiver
н	19	$\mathfrak{gm}(5)_{(6)}$	0-0-0-000 1 2 3 4 5
15	6	$\mathfrak{su}(3)_{12}\times\mathfrak{u}(1)$	0-0-0
16	3	$\mathfrak{gu}(2)_{10}\times\mathfrak{u}(1)$	1 2 1
17	32	op(12)11	See Table 7
18	8	$\mathfrak{sp}(4)_5 \times \mathfrak{so}(4)_6$	
t)	14	ap(8) ₇	See Table 7
ю	4	sp(4) _{13/8}	
51	28	$\mathfrak{sp}(8)_{13} \times \mathfrak{su}(2)_{26}$	1 3 5 7 9
72	14	$\mathfrak{sp}(4)_0\times\mathfrak{sw}(2)_{16}\times\mathfrak{sw}(2)_{15}$	1 3 5 4 2
53	7	$\mathfrak{su}(2)_7 \times \mathfrak{su}(2)_{14} \times \mathfrak{u}(1)$	**************************************
54	6	$\mathfrak{su}(2)_{\alpha}\times\mathfrak{su}(2)_{\alpha}$	0
33	2	64(2);	000 ● 1 2
56	2	su(2) ₁₀	088 0 0 1 2

ĕ	$d_{\rm HD}$	f	Quiver
57	12	$ \mathfrak{g}_2 _5\times\mathfrak{su}(2)_{14}$	0 0 0 0 0 0 0 1 2 4 6
58	4	$\mathfrak{su}(2)_{18/3} \times \mathfrak{su}(2)_{18}$	1 2 2
50	6	[Ø4] 10/15	2 4 1
60	2	gra(2) ₆	000 0 0 1 2
61	15	$\mathfrak{gu}(3)_{26}\times\mathfrak{u}(1)$	1 3 5 7
62	5	$\mathfrak{u}(1)\times\mathfrak{u}(1)$	1 3 2
63	2	u(1)	1 2
64	8	$\mathfrak{gu}(2)_{16}\times\mathfrak{u}(1)$	1 3 5
65	2	u(1)	1 2
66	10	$\mathfrak{sp}(4)_{14}\times\mathfrak{su}(2)_8$	7
67	2	\$10(2) ₁₄	000 e 1 2

Various methods can be used (in cooperation):

Derivation from intersection numbers in brane systems [Cabrera, Hanany, Yagi
 18], [AB, Cabrera, Grimminger, Hanany, Zhong 19], [Akhond, Carta, Dwivedi, Hayashi, Kim 20], [van Beest, AB, Eckhard, Schäfer-Nameki 20], [Akhond, Carta 21], [Sperling, Zhong 21] ...

- Derivation from intersection numbers in brane systems [Cabrera, Hanany, Yagi 18], [AB, Cabrera, Grimminger, Hanany, Zhong 19], [Akhond, Carta, Dwivedi, Hayashi, Kim 20], [van Beest, AB, Eckhard, Schäfer-Nameki 20], [Akhond, Carta 21], [Sperling, Zhong 21] ...
- Deduction from known magnetic quivers (e.g. compactifications / twisted compactifications from higher dimension [Zafrir 16], [Martone, Zafrir 21])

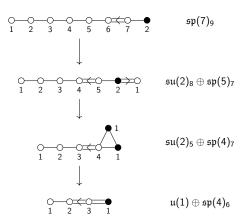
- Derivation from intersection numbers in brane systems [Cabrera, Hanany, Yagi 18], [AB, Cabrera, Grimminger, Hanany, Zhong 19], [Akhond, Carta, Dwivedi, Hayashi, Kim 20], [van Beest, AB, Eckhard, Schäfer-Nameki 20], [Akhond, Carta 21], [Sperling, Zhong 21] ...
- Deduction from known magnetic quivers (e.g. compactifications / twisted compactifications from higher dimension [Zafrir 16], [Martone, Zafrir 21])
- Computation of 3d mirror symmetry (e.g. for Argyres-Douglas theories [Giacomelli, Mekareeya, Sacchi 21], [Carta, Giacomelli, Mekareeya, Mininno 21], [Xie 21], [Dey 21], ...

- Derivation from intersection numbers in brane systems [Cabrera, Hanany, Yagi 18], [AB, Cabrera, Grimminger, Hanany, Zhong 19], [Akhond, Carta, Dwivedi, Hayashi, Kim 20], [van Beest, AB, Eckhard, Schäfer-Nameki 20], [Akhond, Carta 21], [Sperling, Zhong 21] ...
- Deduction from known magnetic quivers (e.g. compactifications / twisted compactifications from higher dimension [Zafrir 16], [Martone, Zafrir 21])
- Computation of 3d mirror symmetry (e.g. for Argyres-Douglas theories [Giacomelli, Mekareeya, Sacchi 21], [Carta, Giacomelli, Mekareeya, Mininno 21], [Xie 21], [Dey 21], ...
- Derivation from geometry of string backgrounds [Collinucci, Valandro 20],
 [Closset, Schäfer-Nameki, Wang 21], ...

- Derivation from intersection numbers in brane systems [Cabrera, Hanany, Yagi 18], [AB, Cabrera, Grimminger, Hanany, Zhong 19], [Akhond, Carta, Dwivedi, Hayashi, Kim 20], [van Beest, AB, Eckhard, Schäfer-Nameki 20], [Akhond, Carta 21], [Sperling, Zhong 21] ...
- Deduction from known magnetic quivers (e.g. compactifications / twisted compactifications from higher dimension [Zafrir 16], [Martone, Zafrir 21])
- Computation of 3d mirror symmetry (e.g. for Argyres-Douglas theories [Giacomelli, Mekareeya, Sacchi 21], [Carta, Giacomelli, Mekareeya, Mininno 21], [Xie 21], [Dey 21], ...
- Derivation from geometry of string backgrounds [Collinucci, Valandro 20],
 [Closset, Schäfer-Nameki, Wang 21], ...
- Guess based on knowledge of the chiral ring [Cabrera, Hanany, Zajac 18], [Arias-Tamargo, AB, Pini 21]
- etc...

RG flows

How is the RG flow read on magnetic quivers? [van Beest, Giacomelli 21]



General patterns have been identified but the detailed rules are under investigation.

We have characterized the Higgs branches of some families of 4d $\mathcal{N}=2$ SCFTs, understood their **phase structure** and how theories are connected via **generalized Higgsing** and **RG flow**.

We have characterized the Higgs branches of some families of 4d $\mathcal{N}=2$ SCFTs, understood their **phase structure** and how theories are connected via **generalized Higgsing** and **RG flow**.

The same methods apply to other dimensions.

We have characterized the Higgs branches of some families of 4d $\mathcal{N}=2$ SCFTs, understood their **phase structure** and how theories are connected via **generalized Higgsing** and **RG flow**.

The same methods apply to other dimensions.

Future directions and open problems:

- What is the scope of magnetic quivers? Various extensions of the notions have already been proposed. What is the generic magnetic "object"?
- What other information can be extracted from magnetic quivers? E.g. [Nawata, Sperling, Wang, Zhong 21]
- Is there a possible bottom-up approach?
 - Classification of possible elementary slices (see recent progress in [Bellamy, Bonnafé, Fu, Juteau, Levy, Sommers 22]
 - How these slices combine.

We have characterized the Higgs branches of some families of 4d $\mathcal{N}=2$ SCFTs, understood their **phase structure** and how theories are connected via **generalized Higgsing** and **RG flow**.

The same methods apply to other dimensions.

Future directions and open problems:

- What is the scope of magnetic quivers? Various extensions of the notions have already been proposed. What is the generic magnetic "object"?
- What other information can be extracted from magnetic quivers? E.g. [Nawata, Sperling, Wang, Zhong 21]
- Is there a possible bottom-up approach?
 - Classification of possible elementary slices (see recent progress in [Bellamy, Bonnafé, Fu, Juteau, Levy, Sommers 22]
 - How these slices combine.

Thank you for your attention!