The BMS algebra at spatial infinity ($D = 4$ and $D > 4$)

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Introduction

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Will give here three reasons.

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it is important to unveil its action on spacelike (Cauchy) hypersurfaces, and thus, at spatial infinity.

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New, consistent boundary conditions have been proposed, which are invariant under the full infinite-dimensional BMS group,

providing a standard, non-trivial, canonical realization of the BMS symmetry.

This establishes also the important fact that the BMS symmetry is a symmetry of the theory and not just a symmetry at null infinity.

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This requires understanding the action of all the BMS supertranslations at spatial infinity.

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the realization of which exhibits (somewhat unexpectedly) a very interesting nonlinear structure.

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(Work done in collaboration with Cédric Troessaert, with also Oscar Fuentealba, Sucheta Majumdar, Javier Matulich and Turmoli Neogi who joined more recently)

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The definition of the theory is completed by providing boundary conditions on the dynamical variables (definition of phase space), which are assumed to make the off-shell action finite.

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For a canonical action of the Lorentz boosts, one imposes moreover that U_{ij} be parametrized by an $O(1)$ odd function of the angles $\overline{U}(\mathbf{n}^k) = O(1) = -U(-\mathbf{n}^k)$. Furthermore, V^{ij} is parametrized by an $O(1)$ even function of the angles $V(\mathbf{n}^k) = V(-\mathbf{n}^k)$.

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$$\xi = b_i x^i + T(\mathbf{n}) + O\left(r^{-1}\right)$$

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The zero mode of T and the first spherical harmonic component of W describe translations.

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The algebra of the generators can be easily verified to be the BMS algebra.

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(and these are necessary to avoid log singularities at null infinity).

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As in 4 dimensions, one must include explicitly the improper gauge symmetries in the asymptotic form of the fields.

A new feature is that improper gauge terms are not at the same order in $\frac{1}{r}$ as the Coulomb part,

while in 4 dimensions, they are at the same order but distinguished by parity conditions.

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See O. Fuentealba, M. Henneaux, J. Matulich and C. Troessaert, e-Print : 2111.09664 [hep-th]

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where

$$g_{rr} = 1 + \frac{2\overline{\lambda}}{r^2} + \frac{h^{(2)}_{rr}}{r^3} + \mathcal{O}\left(r^{-4}\right), \tag{4.1}$$

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as it follows by applying standard canonical methods.

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(first term : ADM contribution ; second term : nonlinear contribution)

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Preliminary analysis indicates that the size of the BMS group does not increase because the new terms in the diffeomorphism generators define proper gauge transformations.

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THANK YOU !