Islands and Page curves in 4d from Type IIB

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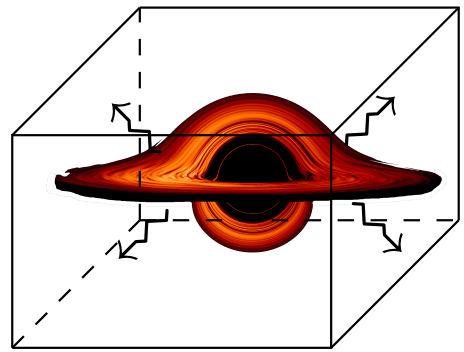
arXiv:2105.00008: "Islands and Page curves in 4d from Type IIB"
arXiv:2011.10050, arXiv:2112.14648 with Lorenzo Coccia

Information paradox & entropy of Hawking radiation

black hole in 'box' (AdS) coupled to bath (non-gravitational CFT)



Entropy of Hawking radiation: Page curves from semi-classical gravity. Based on 2d gravity and bottom-up braneworld models.

Today: String theory realization of 4d black holes coupled to bath
UV complete, 'microscopic' AdS/CFT duals, Page curves

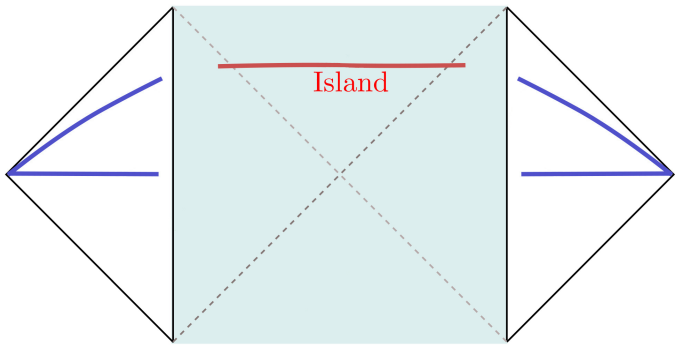
Outline:

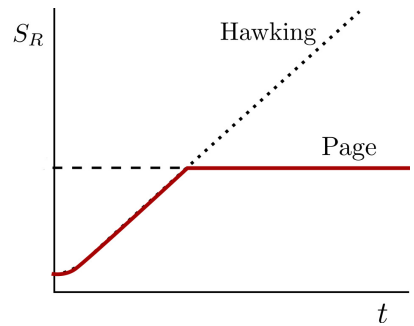
- Islands, braneworlds, double holography
- String theory braneworlds: D3/D5/NS5
- Black holes & Page curves in Type IIB

Islands, braneworlds, double holography

Islands, braneworlds & double holography

Radiation entropy in non-gravitating bath with island contributions:
[Penington,Almheiri,Engelhardt,Marolf,Maxfield,Mahajan,Maldacena,Zhao,. . .]



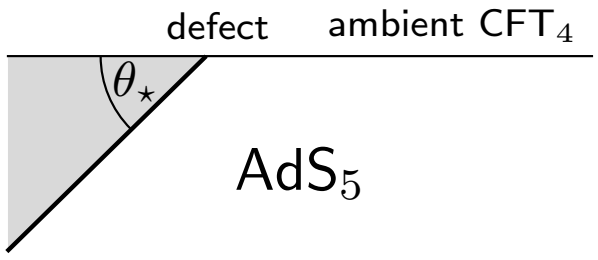


$$S_{\text{rad}} = \min_I \left\{ \text{ext}_I \left[\frac{\text{Area}(\partial I)}{4G_N} + S_{\text{semi-cl}} \left[\Sigma_{\text{rad}} \cup I \right] \right] \right\}$$

Replica wormholes in 2d, quantum extremal surfaces in AdS/CFT, standard R/T surfaces in braneworlds & double holography . . .

Islands, braneworlds & double holography

Braneworld model for 4d gravity coupled to non-gravitational bath:



$$ds^2 = \frac{d\theta^2 + ds^2_{\mathrm{AdS}_4}}{\sin^2\theta}$$

Islands, braneworlds & double holography

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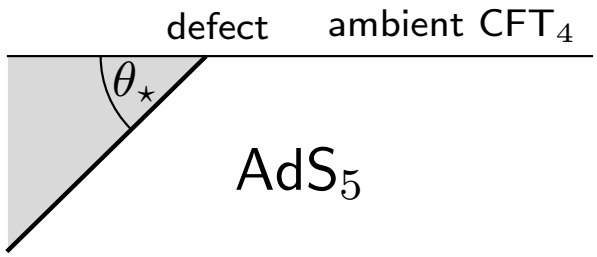

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Double holography & 'intermediate' holographic description:

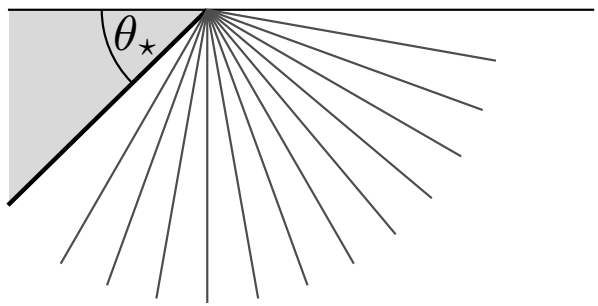
(a) CFT$_4$ on half space coupled to CFT$_3$ on boundary

→ (b) AdS$_4$ gravity coupled to 'ambient' CFT$_4$ on half space
(geometrize only 3d boundary d.o.f.)

(c) geometrize full BCFT: AdS$_5$ gravity + ETW brane

Islands, braneworlds & double holography

4d brane black hole coupled to CFT_4 bath on fixed background:



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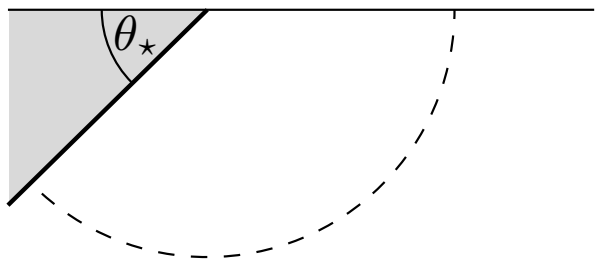

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eternal AdS_4 black hole slices

Islands, braneworlds & double holography

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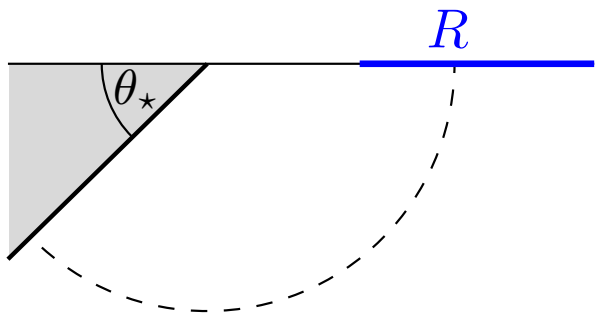

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Collect Hawking radiation in CFT region R, compute entropy:

Islands, braneworlds & double holography

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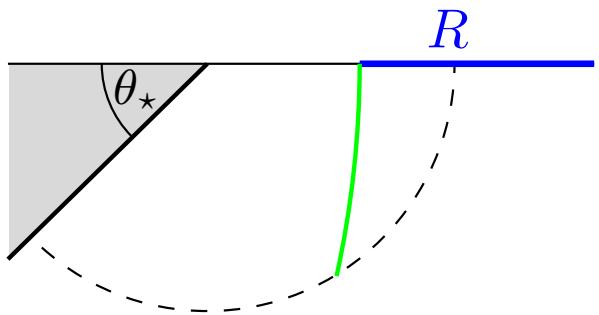

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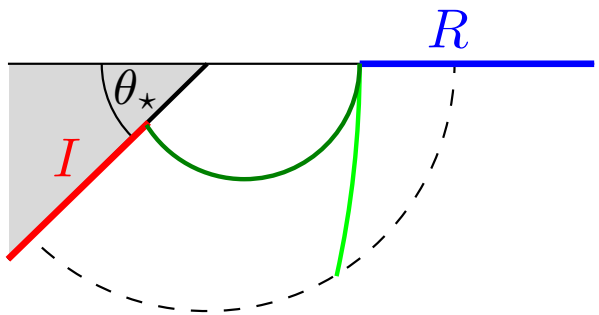

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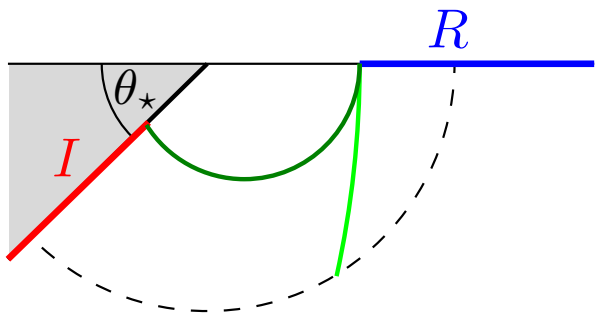

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Island surface in 4d intermediate description from R/T in 5d
Competition between island and HM surfaces $\rightarrow$ Page curves

String theory braneworlds

D3/D5/NS5 BCFTs

BPS boundary conditions for $\mathcal{N} = 4$ SYM: D3 ending on D5/NS5

[Gaiotto,Witten]

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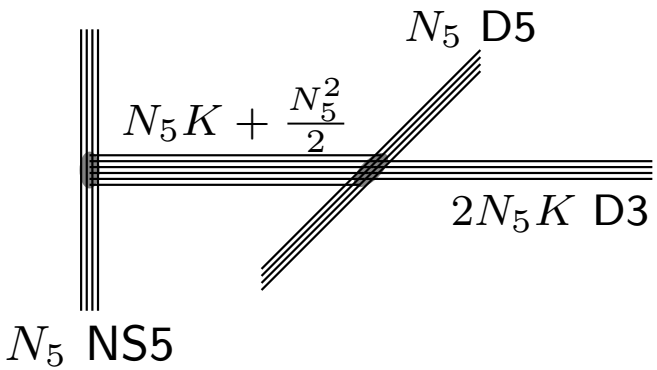

$2N_5K$ semi-infinite D3-branes ending on N_5 D5 and N_5 NS5.
$R = N_5/2 + K$ D3 end on each NS5, $S = N_5/2 - K$ on each D5.

D3/D5/NS5 BCFTs

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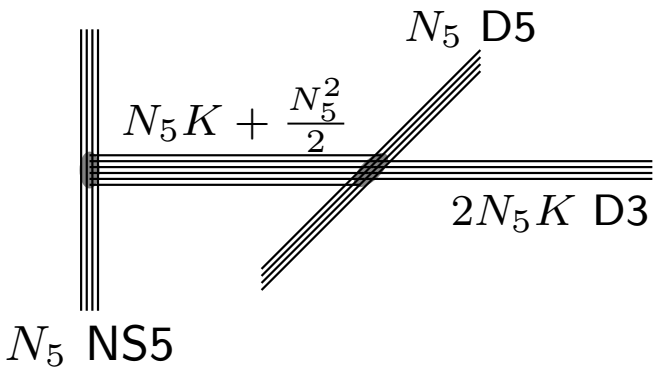

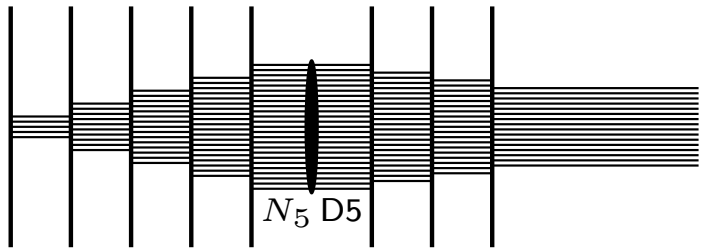

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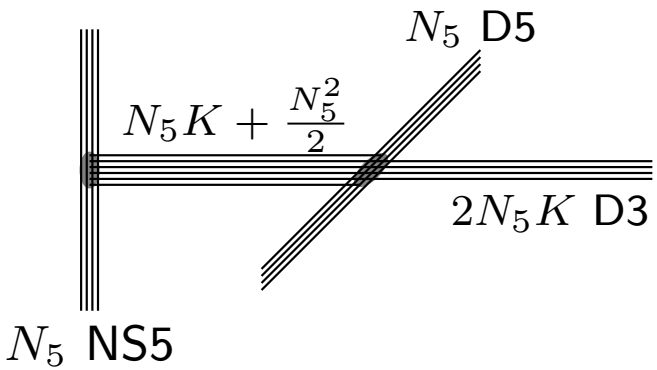

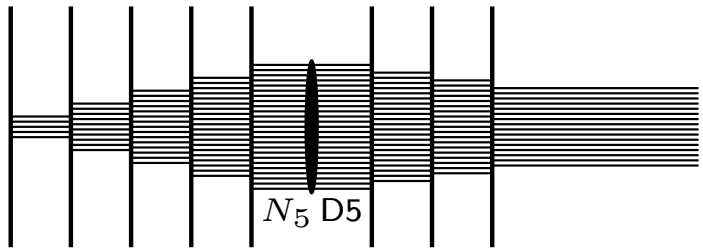

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$$U(R)-U(2R)-\ldots-U(R^2)-\ldots-U(2N_5K+S)-\widehat{U(2N_5K)}$$
$$|$$
$$[N_5]$$

4d $U(2N_5K)$ $\mathcal{N}=4$ SYM on half space, coupled to 3d quiver SCFT with N_5-1 nodes, for $N_5>2K$ with N_5 flavors.

Holographic duals for D3/D5/NS5 BCFTs

AdS$_4$, S^2_1, S^2_2 warped over Riemann surface Σ [D'Hoker,Estes,Gutperle]

$$ds^2 = f_4^2 ds^2_{\mathrm{AdS}_4} + f_1^2 ds^2_{S^2_1} + f_2^2 ds^2_{S^2_2} + 4\rho^2 ds^2_\Sigma$$

Specified by Σ, harmonic h_1, h_2: D3/D5/NS5 (multi) Janus,
3d SCFTs [Assel,Bachas], 4d BCFTs [Aharony,Berdichevsky,Berkooz].

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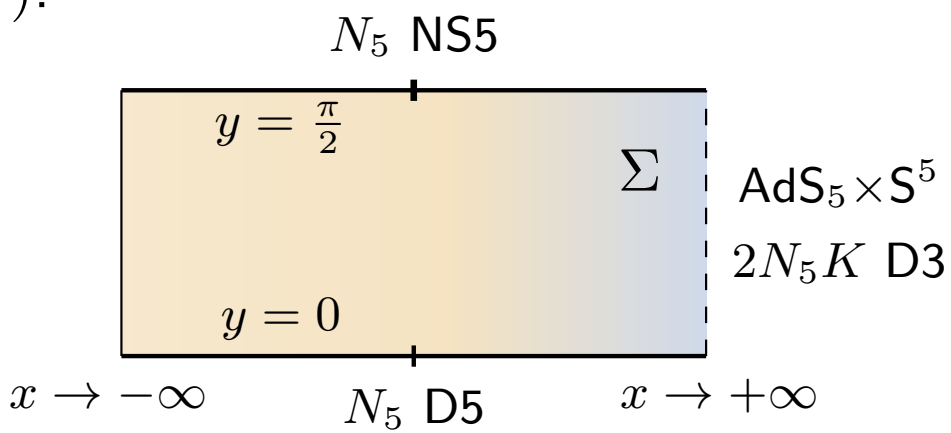
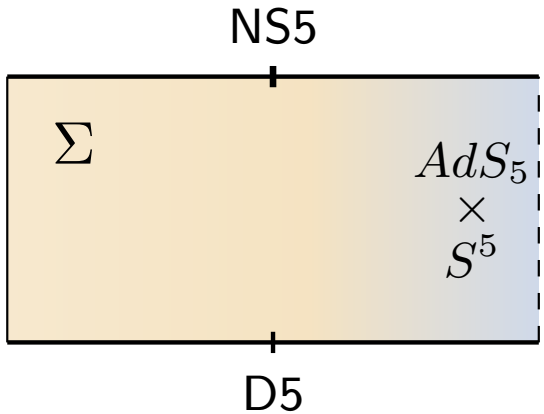
BCFT(N_5, K):

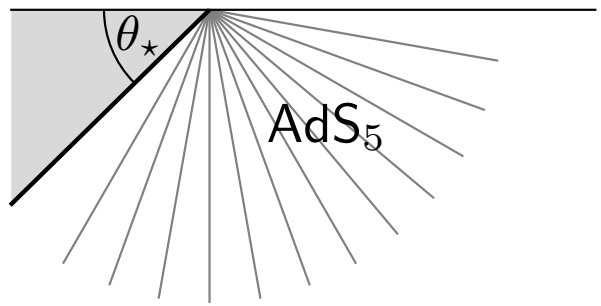


Σ=strip with D5, NS5 sources on boundary, AdS$_5\times$S^5 at $x \to \infty$, geometry closes off smoothly on other boundaries.

Connection to braneworld models

ETW brane 'resolved' into geometry + fluxes around 5-branes, $AdS_5 \times S^5$ region ends smoothly





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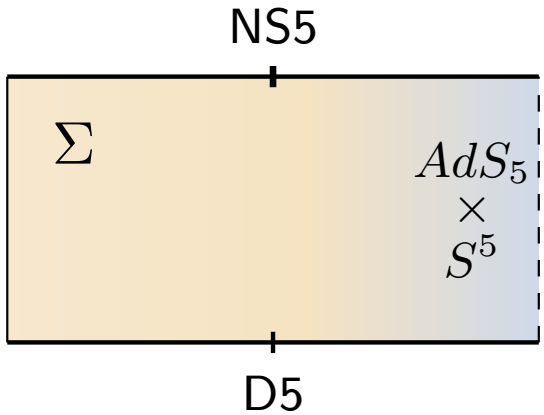

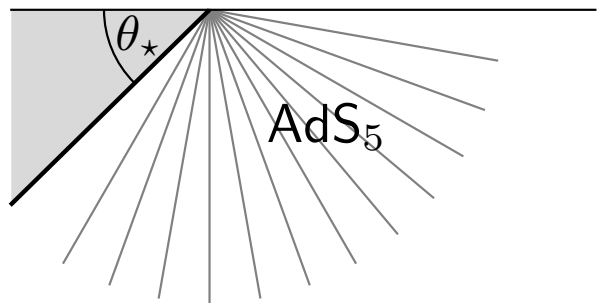

AdS_4 at each point on Σ vs. AdS_4 warped over angular coordinate, fibers joined at 3d boundary.

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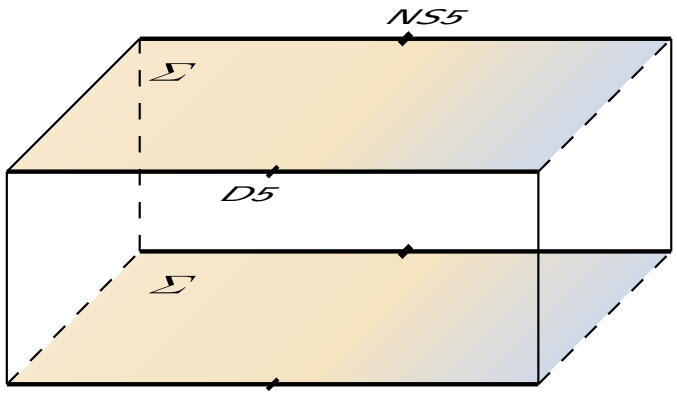

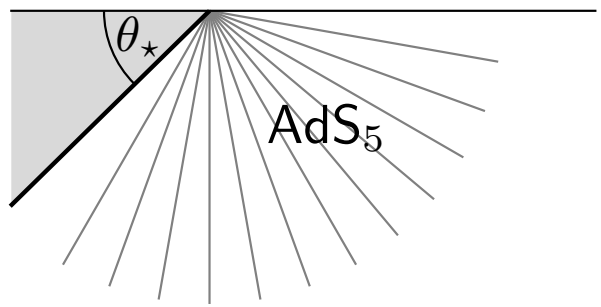

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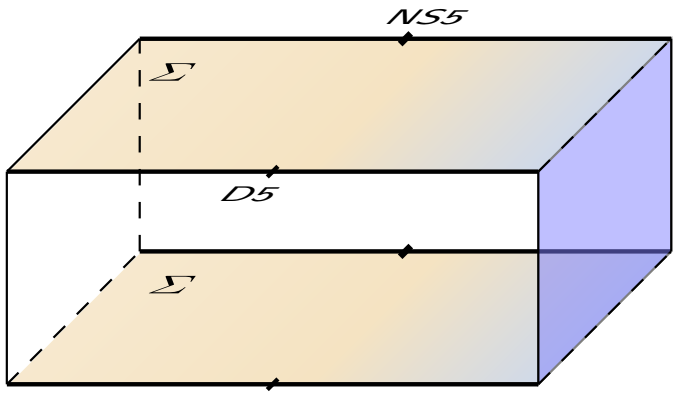

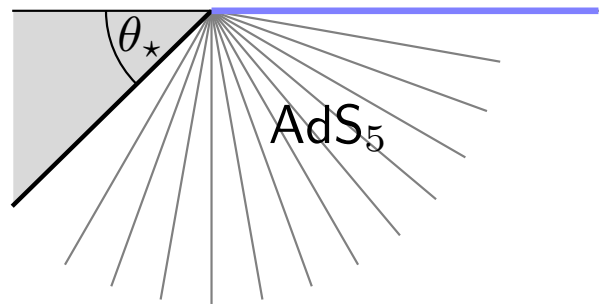

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4d ambient CFT at $x \to \infty$. Intermediate holographic description in 10d: dualize 3d quiver SCFT part [details to appear w/ Karch,Sun].

3d, 4d central charges in D3/D5/NS5

In braneworld models, ETW brane angle $\theta_\star \sim c_{3\mathrm{d}}/c_{4\mathrm{d}}$. Here:

- 4d $\mathcal{N} = 4$ $U(2N_5K)$ SYM: $c_{4\mathrm{d}} \sim N_5^2 K^2$
- 3d *long quivers* $L \gg 1$ nodes, ranks $\mathcal{O}(L^2)$: $F_{S^3} \sim L^4 = N_5^4$
  [2011.10050 Coccia,CU], [Van Raamsdonk,Waddell]

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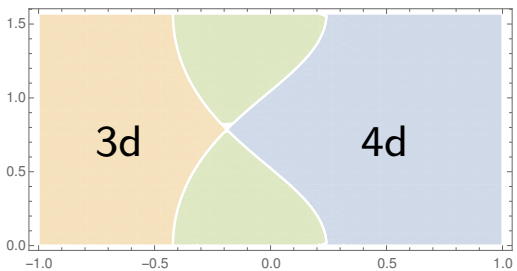
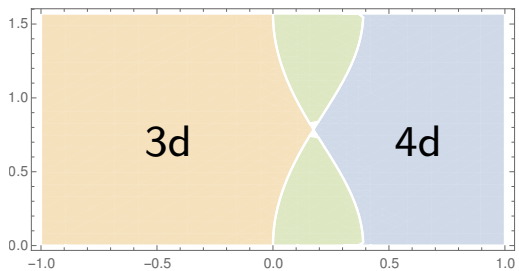
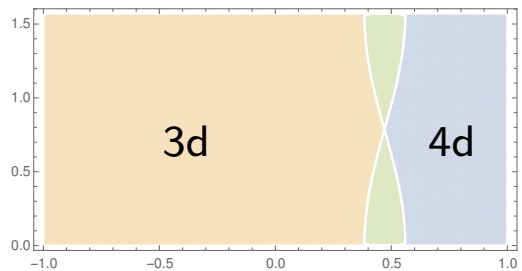


$N_5/K=1$



$N_5/K=2$



$N_5/K=4$

Large rep. Wilson loops in brane setups/supergravity/localization: 'brane coordinates' on $\Sigma \to$ '3d', '4d' regions [2112.14648 Coccia,CU]

Black holes and Page curves

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Black holes in stringy braneworlds:

$$ds^2 = f_4^2 ds^2_{\mathrm{AdS}_4} + f_1^2 ds^2_{S_1^2} + f_2^2 ds^2_{S_2^2} + 4\rho^2 ds^2_{\Sigma}$$

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$\mathrm{AdS}_4 \to \mathrm{AdS}_4$ black hole throughout Σ, solves Type IIB EOM

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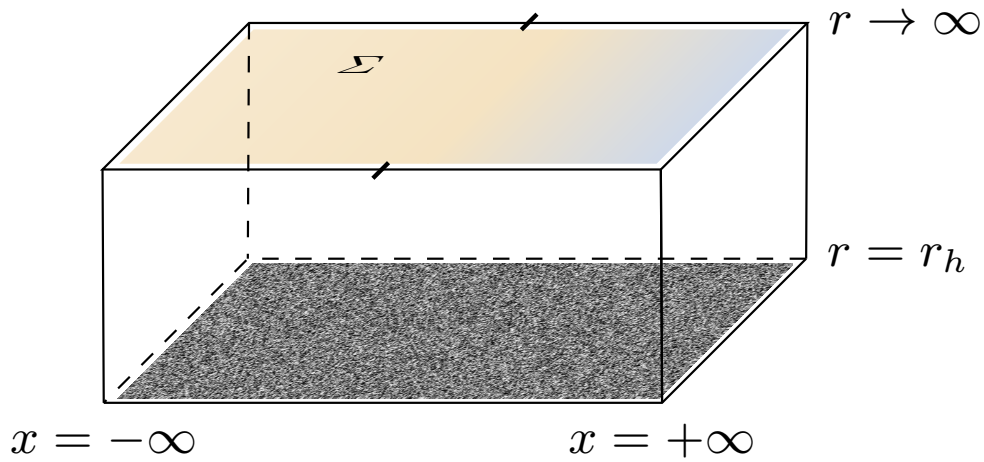

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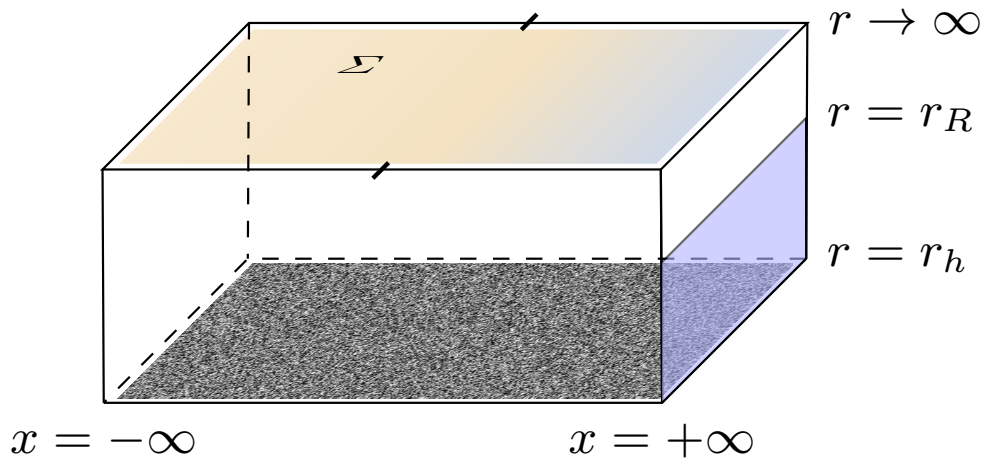
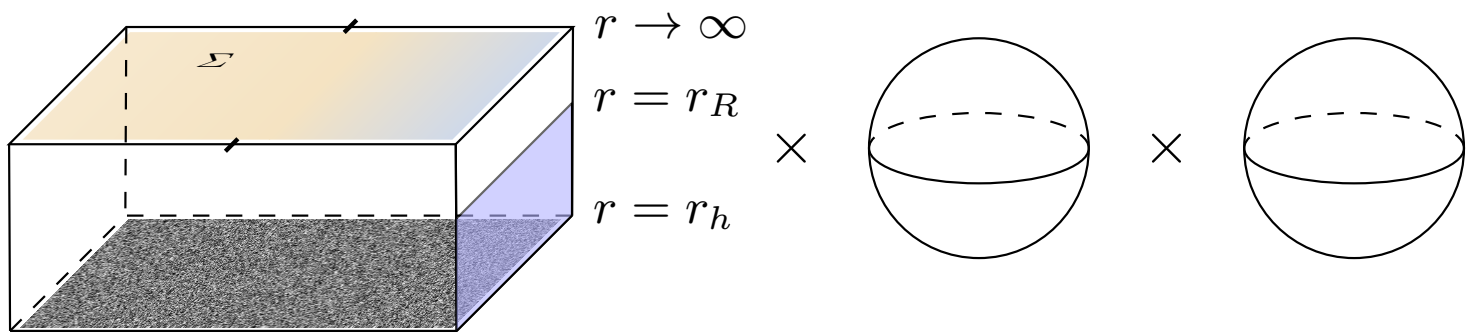


AdS_4 black hole coupled to 4d CFT in intermediate description.
Radiation region in ambient 4d CFT geometry at $x = \infty$.

Black holes and Page curves

Radiation entropy: 8d Ryu/Takayanagi surfaces in 10d geometry, wrap $S^2_{1/2}$, split AdS$_4$ at $x=\infty$, $r=r_R$, fixed t, extend along Σ



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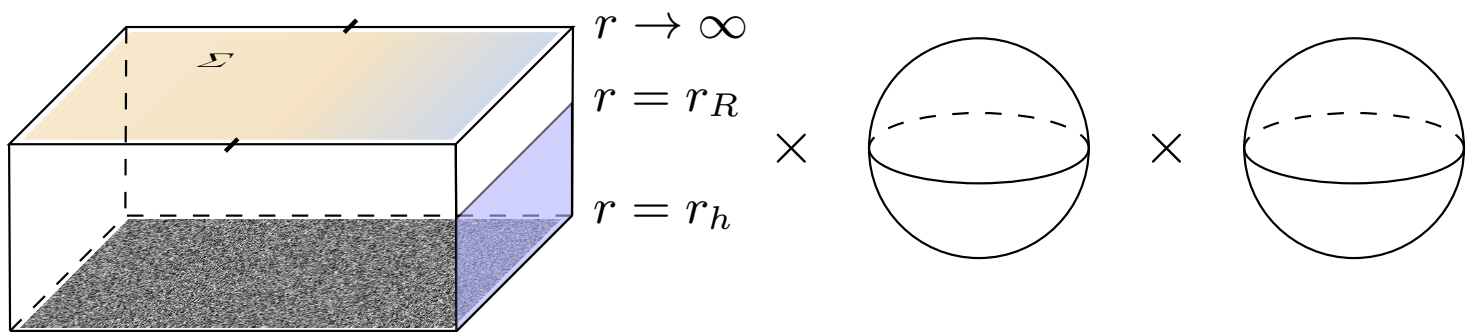

Embedding specified by $r(x,y)$: PDE in background with 5-brane singularities, no help from susy $\rightarrow$ numerics.

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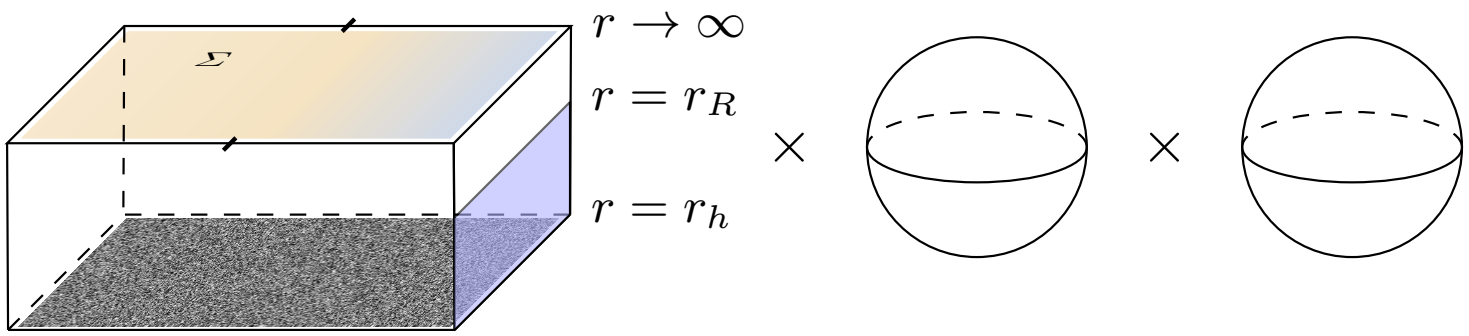
Radiation entropy: 8d Ryu/Takayanagi surfaces in 10d geometry, wrap $S^2_{1/2}$, split AdS_4 at $x=\infty$, $r=r_R$, fixed t, extend along Σ



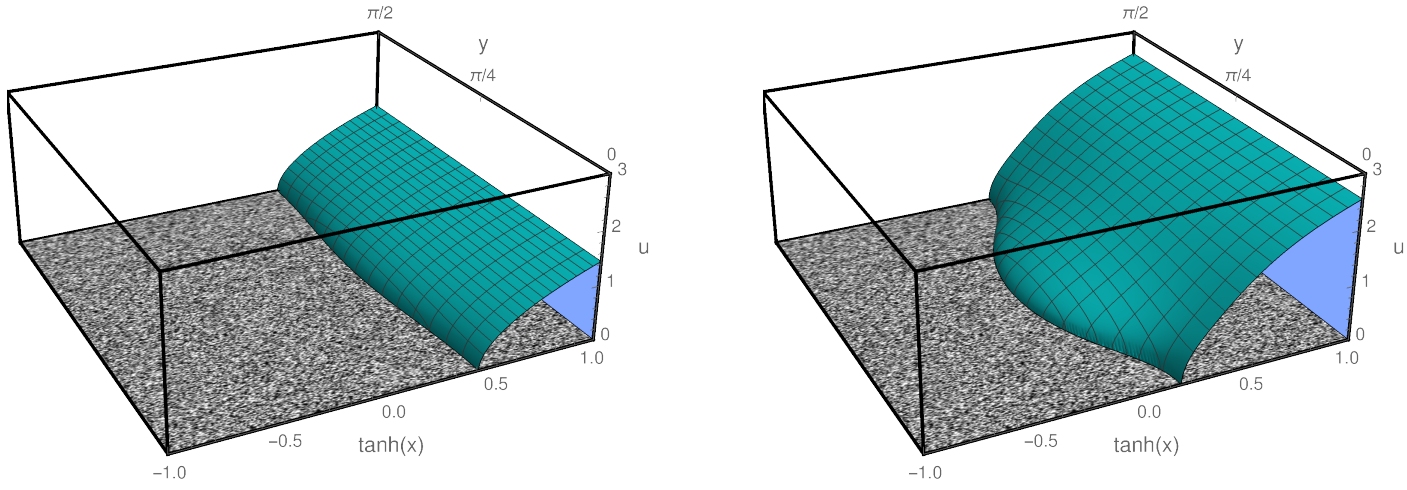
Embedding specified by $r(x,y)$: PDE in background with 5-brane singularities, no help from susy $\rightarrow$ numerics.

Boundary conditions on $\partial\Sigma$ from closing off spheres smoothly
$\Rightarrow$ analog of "Neumann at ETW brane", derived from regularity

Black holes and Page curves

Radiation entropy: 8d Ryu/Takayanagi surfaces in 10d geometry, wrap $S^2_{1/2}$, split AdS_4 at $x = \infty$, $r = r_R$, fixed t, extend along Σ

HM surfaces @$t = 0$:

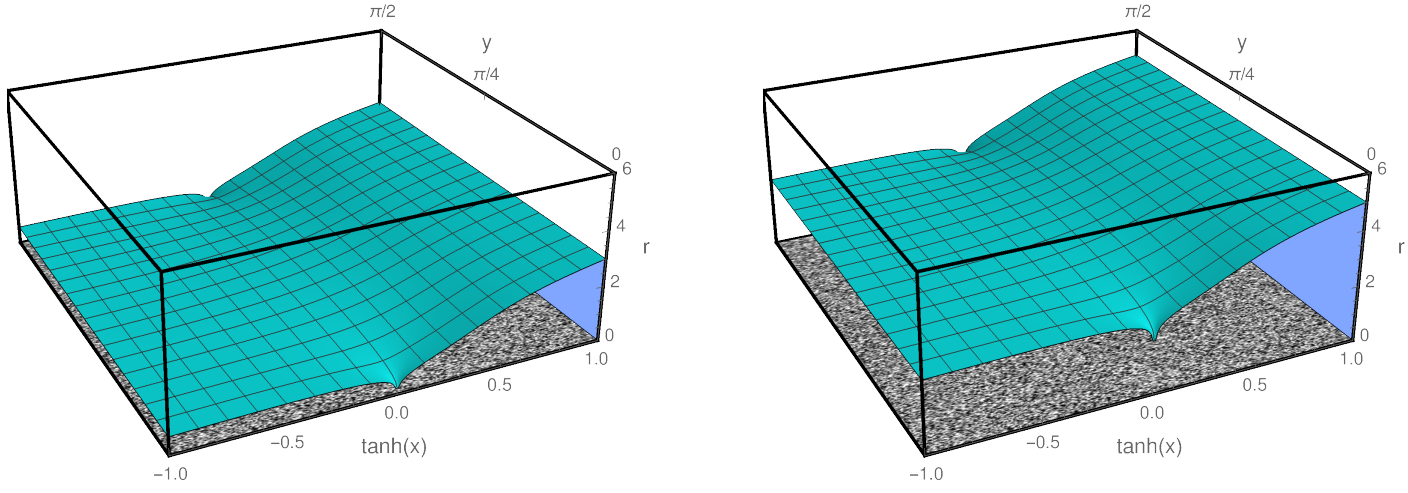


– cross horizon before reaching 'resolved ETW brane region', end in second exterior region $\Rightarrow$ area grows in time

Black holes and Page curves

Radiation entropy: 8d Ryu/Takayanagi surfaces in 10d geometry, wrap $S^2_{1/2}$, split AdS_4 at $x = \infty$, $r = r_R$, fixed t, extend along Σ

Island surfaces:



- stretch all through Σ to $x = -\infty$, detect D5/NS5
- do not cross horizon $\Rightarrow$ constant area, limit entropy growth

Black holes and Page curves

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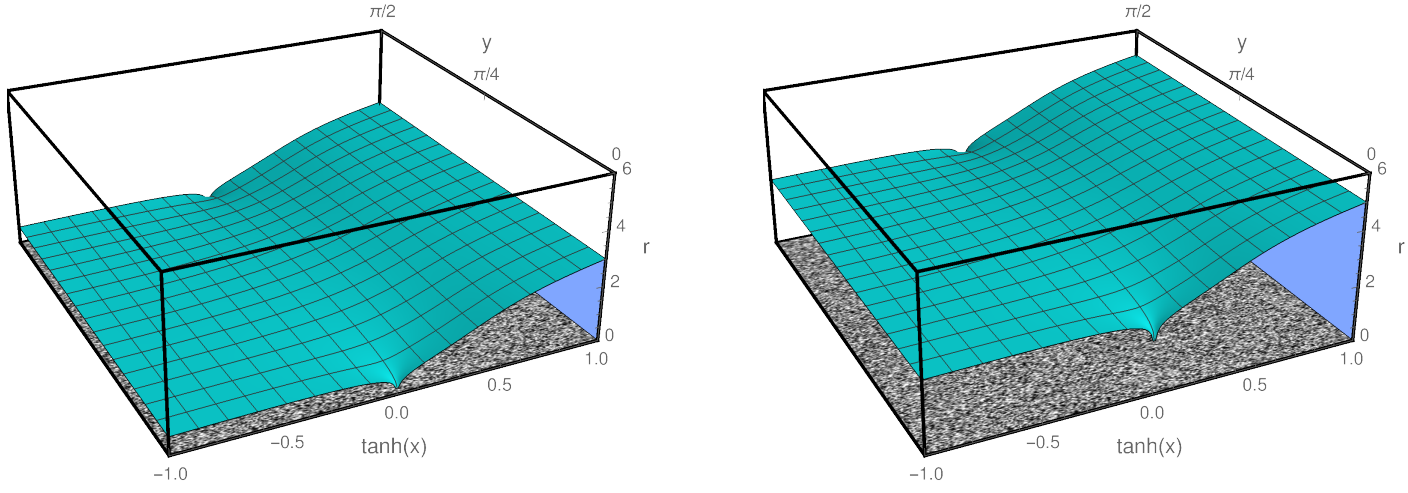


- stretch all through Σ to $x = -\infty$, detect D5/NS5
- do not cross horizon $\Rightarrow$ constant area, limit entropy growth

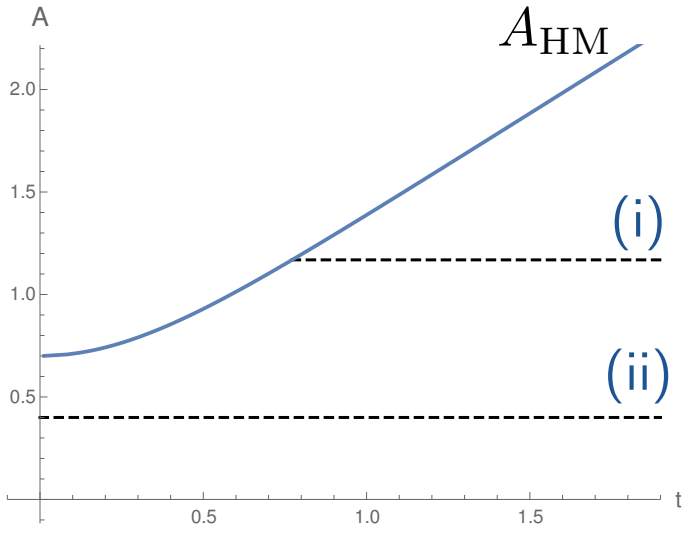
Island surfaces preventing unitarity paradoxes for 4d black holes in 10d Type IIB setups engineered to uplift braneworld models ✓✓

Black holes and Page curves

Entropy curve from competition between island and HM surfaces:

(i) HM dominates initially, island later $\rightarrow$ Page curve

(ii) island dominates right away $\rightarrow$ constant entropy



(i) $A_{\mathrm{island}} - A_{\mathrm{HM}}^{t=0} > 0$

(ii) $A_{\mathrm{island}} - A_{\mathrm{HM}}^{t=0} < 0$

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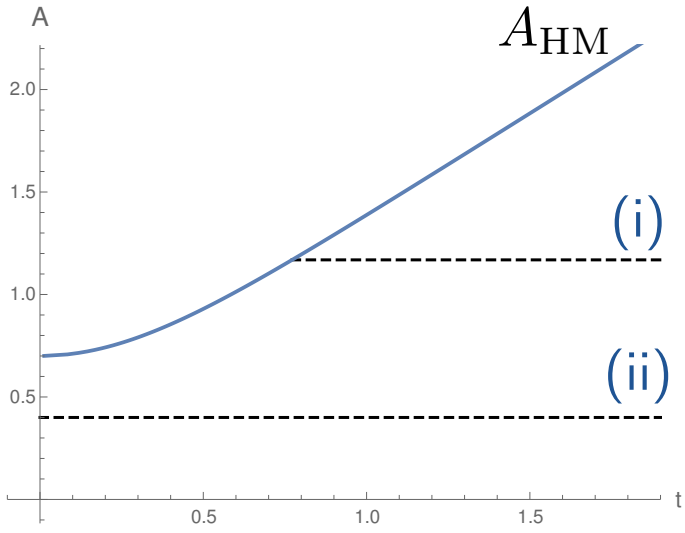

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Both compatible with unitarity. Phase structure from $\Delta A_{t=0}$...

Black holes and Page curves

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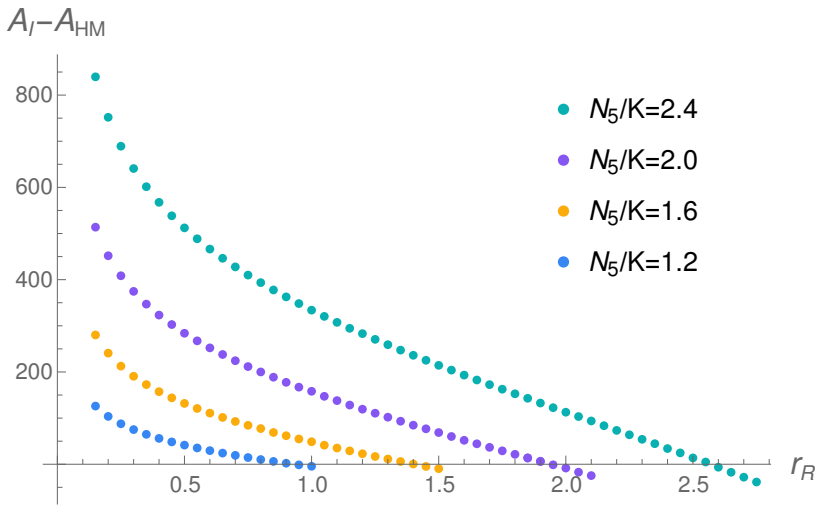

Radiation collected far enough in bath $\rightarrow$ non-trivial entropy curve.

Black holes and Page curves

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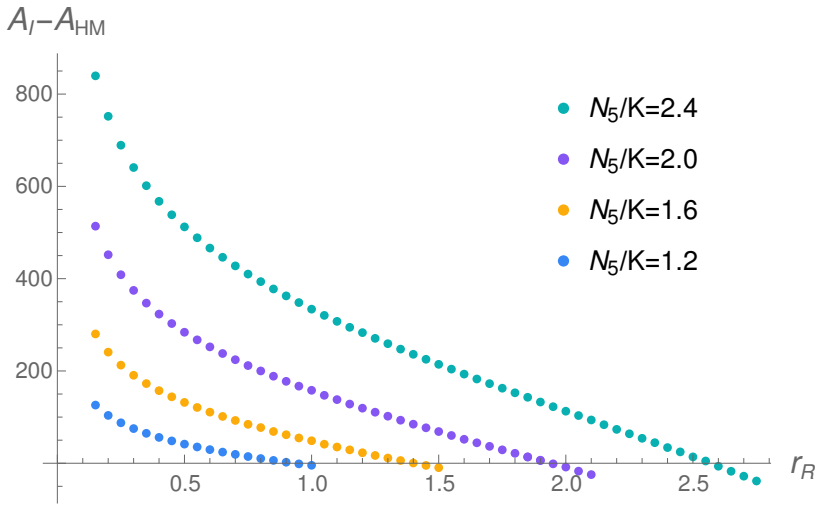

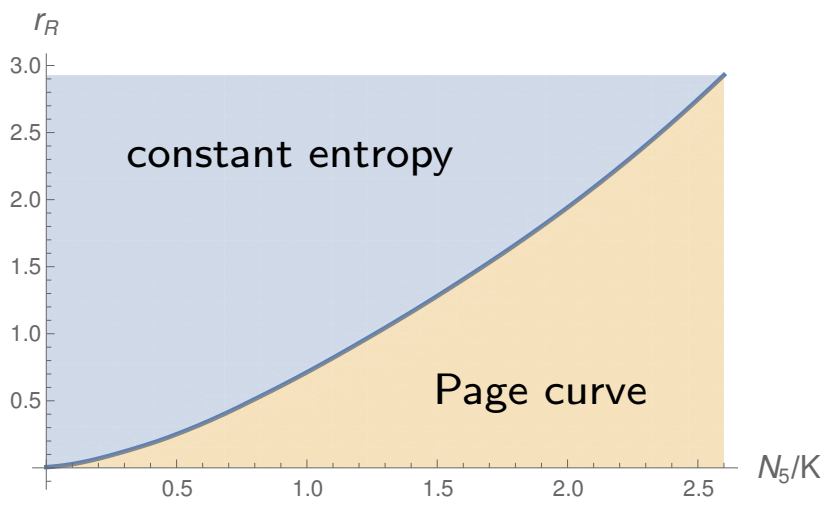

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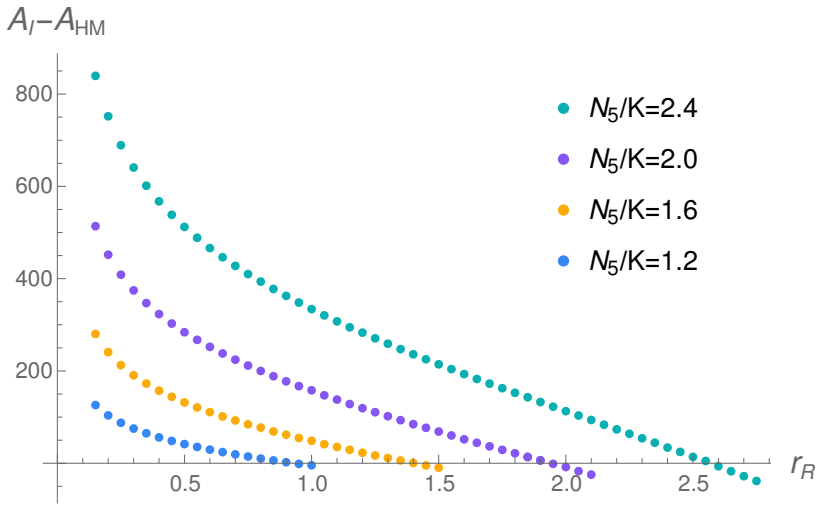

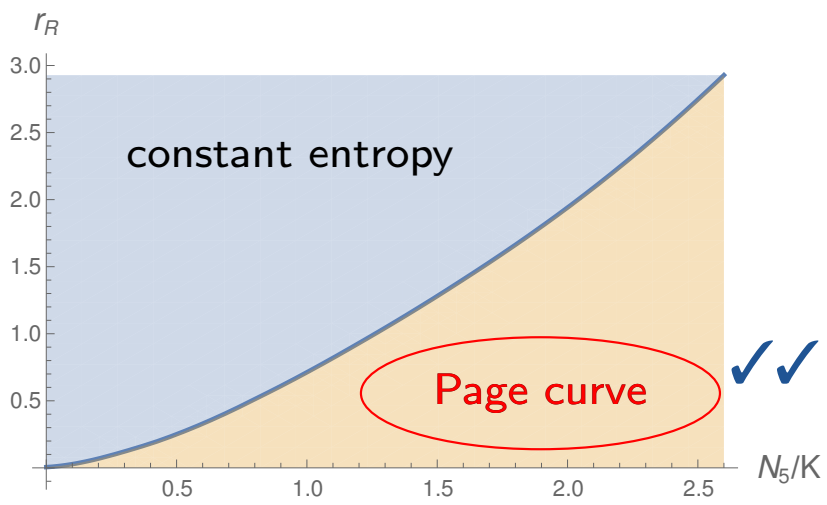

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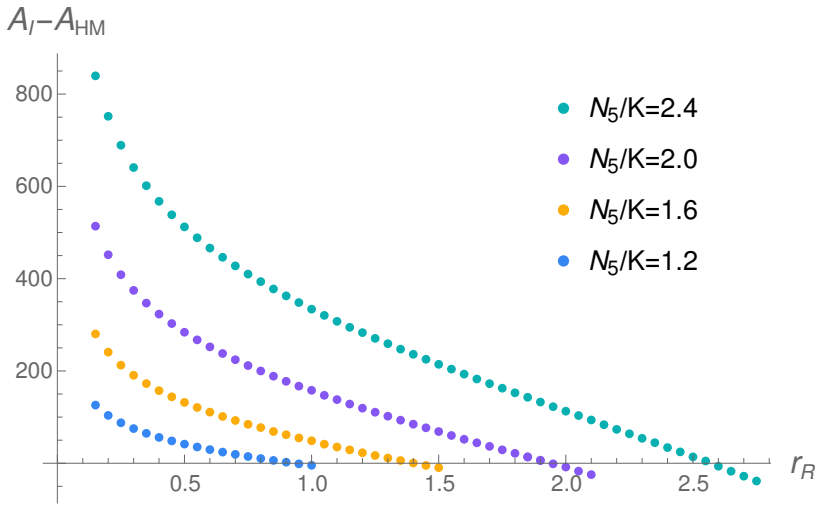
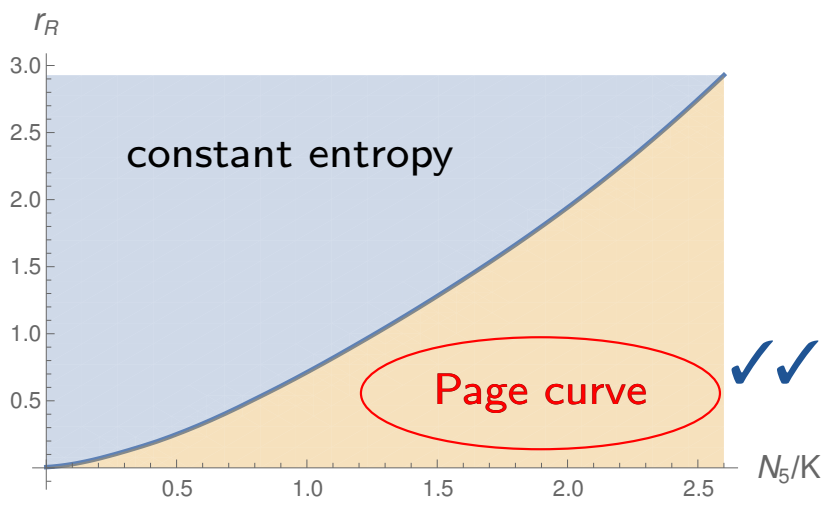
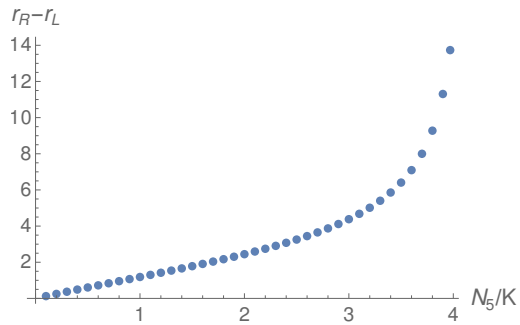


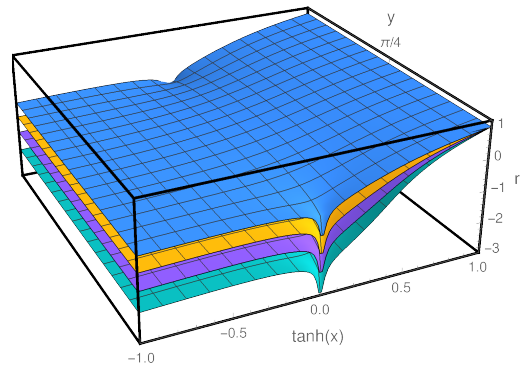


Radiation collected far enough in bath $\rightarrow$ non-trivial entropy curve.
Consistent results in braneworlds, $N_5/K \sim \theta_\star$ [Geng et al 2112.09132]

Black holes and Page curves

Critical parameter at $T = 0$: islands disappear at $(N_5/K)_{\text{crit}} \approx 4$:





Not an information paradox. Braneworld analog [Chen,Myers et al; Geng,Karch et al], recent 10d study [Demulder,Gnecchi,Lavdas,Lüst].

Gravity in intermediate description massive with non-gravitating bath. D3/D5/NS5 setups for gravitating bath: arXiv:2105.00008.

Conclusion

Conclusion

4d black holes coupled to bath from string theory braneworlds, based on D3/D5/NS5 setups & $\text{AdS}_4 \times S^2 \times S^2 \times \Sigma$ solutions.

Island surfaces prevent unitarity puzzles, lead to Page curves. Phases, critical parameters consistent with braneworld analyses.

Concrete field theory duals: $\mathcal{N} = 4$ SYM BCFTs coupled to 3d long quiver SCFTs. Microscopic, UV complete models.

Thank you!