D-instantons in string compactifications

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S.A., A.Sen, B.Stefanski arXiv:2108.04265 N=2,Type IIA
arXiv:2110.06949 N=2,Type IIB

S.A., A.Firat, M.Kim, A.Sen, B.Stefanski arXiv:2204.02981 N=1

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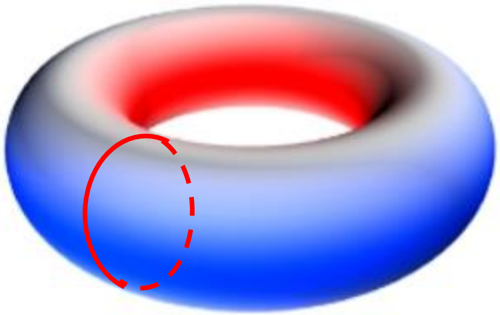
Motivation

Instantons in string theory – *Euclidean branes* wrapped on non-trivial cycles of compactification manifold



Although exponentially suppressed in small g_s limit, they play important role for various reasons:

- crucial for non-perturbative dualities and for going beyond the perturbative formulation
- in N=2, contain information on numerical invariants of compactification manifold $\longrightarrow$ *entropy of BPS black holes*
- in N=1, essential for moduli stabilization

But in contrast to gauge theories, until 2020, *no direct computation* of instanton effects in string theory was possible!!!

Breakthrough: understanding infrared and zero mode divergences through string field theory [A.Sen]

Goals: 1) apply these ideas in the context of Calabi-Yau compactifications of type II string theory to compare with results based on dualities $\rightarrow$ *perfect match*

2) apply them for orientifold compactifications to get new results $\rightarrow$ *instanton induced superpotential*

Instanton corrections in CY compactifications

The effective action of Type II string theory on a CY threefold is determined by the metric on the moduli space

$$\mathcal{M}_{\rm VM} \times \mathcal{M}_{\rm HM}$$

special Kähler
no corrections in string coupling

quaternion-Kähler
corrections in string coupling

D-brane instantons

$d_{\rm cycle}$	**0**	**1**	**2**	**3**	**4**	**5**	**6**
IIA :		×		**D2**		×	
IIB :	**D(-1)**		**D1**		**D3**		**D5**

- D-instanton corrections have been found *exactly* in terms of a *holomorphic contact structure* on the *twistor space* over $\mathcal{M}_{\rm HM}$

[S.A.,Pioline,Saueressig,Vandoren '08]

- The metric has been evaluated explicitly in [S.A., Banerjee '14]

Small string coupling limit

The leading corrections in the small g_s limit

$$\mathrm{d}s^2_{\mathrm{inst}} = \sum_\gamma \frac{\Omega_\gamma\, e^{(5\phi-\mathcal{K})/4}}{64\pi\sqrt{|Z_\gamma|}} \left(\sum_{k=1}^{\infty} k^{-1/2}\, e^{-k\mathcal{T}_\gamma}\right)\left(\mathcal{A}^2 + (\cdots)\mathrm{d}\mathcal{T}_\gamma\right)$$

such terms will be ignored

where

$\mathcal{T}_\gamma = 8\pi\, e^{(\mathcal{K}-\phi)/2}|Z_\gamma| + 2\pi \mathrm{i}\Theta_\gamma$ – instanton action

$\gamma = (p^\Lambda, q_\Lambda)$ – D-brane charge

Ω_γ – Donaldson-Thomas invariant

$Z_\gamma = q_\Lambda z^\Lambda - p^\Lambda F_\Lambda$ – central charge

$\Theta_\gamma = q_\lambda \zeta^\Lambda - p^\Lambda \tilde\zeta_\Lambda$ – axionic coupling

dilaton $(e^\phi \sim g_s^2)$

NS-axion

RR-scalars

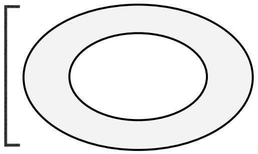
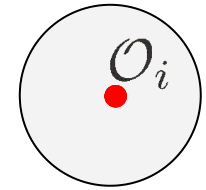
$$\mathcal{A} \;=\; |Z_\gamma| e^{(\mathcal{K}+\phi)/2}\left(\mathrm{d}\sigma + \tilde\zeta_\Lambda \mathrm{d}\zeta^\Lambda - \zeta^\Lambda \mathrm{d}\tilde\zeta_\Lambda + 8e^{-\phi}\mathrm{Im}\partial\log(e^{\mathcal{K}} Z_\gamma)\right)$$
$$+2\mathrm{i}(\mathrm{Im}F)^{\Lambda\Sigma}\left(q_\Lambda - \mathrm{Re}F_{\Lambda\Xi}p^\Xi\right)\left(\mathrm{d}\tilde\zeta_\Sigma - \mathrm{Re}F_{\Sigma\Theta}\mathrm{d}\zeta^\Theta\right) + 2\mathrm{i}\mathrm{Im}F_{\Lambda\Sigma}\, p^\Lambda\, \mathrm{d}\zeta^\Sigma$$

holomorphic prepotential $F(z)$
with Kähler potential $\mathcal{K}$

complex structure moduli (IIA)
complexified Kähler moduli (IIB)

Instanton corrections to string amplitudes

The leading instanton contribution to n-point function:

$$\left\langle \prod_{i=1}^{n} \mathcal{O}_i \right\rangle_{\mathrm{inst}} = e^{-\mathcal{T}} \exp\Big[$$

$$\Big] \prod_{i=1}^{n}$$


But this expression is *not* well-defined because of infrared divergences and the presence of zero modes in the spectrum.

Example: annulus amplitude in type IIB in 10d:

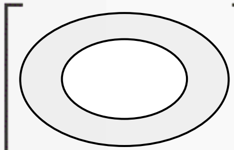
$$\int_0^\infty \frac{dt}{2t}\left[\frac{1}{2}\,\eta(\mathrm{i}t)^{-12}\left(\vartheta_3(0,\mathrm{i}t)^4-\vartheta_4(0,\mathrm{i}t)^4-\vartheta_2(0,\mathrm{i}t)^4+\vartheta_1(0,\mathrm{i}t)^4\right)\right] \overset{?}{=} 0$$

All divergences can be understood from *string field theory* [Sen '20]

- the zero modes related to the collective coordinates of the D-instanton should be left unintegrated till the end of calculation
    - bosonic zero modes produce the momentum conserving delta-function
    - fermionic zero modes require insertion of zero mode vertex operators
- the divergence due to ghost zero modes arises due to the breakdown of the Siegel gauge $b_0|\Psi\rangle = 0$ used to get the worldsheet formulation, which is cured by working with a gauge invariant path integral

Annulus amplitude from string field theory

For a BPS instanton in CY compactification:

$$\exp\Bigg[$$

$$\Bigg] = \frac{\mathrm{i}\,C}{\sqrt{k}}\, g_o\, \Omega_\gamma \underbrace{\int \prod_{\mu} d\xi^\mu}_{\text{bosonic z.m.}} \underbrace{\int \prod_{\delta,\dot\delta=1}^{2} d\chi^\delta\, d\chi^{\dot\delta}}_{\text{fermionic z.m.}}$$

$$C = 2^{-5}\,\pi^{-13/2}$$

counts cycles

open string coupling

$$\mathrm{Re}\mathcal{T}_\gamma = \frac{1}{2\pi^2 g_o^2} \longrightarrow g_o^2 = \frac{2g_s}{V_\gamma} = \frac{e^{(\phi-\mathcal{K})/2}}{16\pi^3|Z_\gamma|}$$

$$\text{bosonic z.m.} \longrightarrow (2\pi)^4\delta^{(4)}\left(\sum_i p_i\right)$$

Metric vs. curvature

We are interested in the effective action for massless scalars. For such fields, 2- and 3-point amplitudes vanish $\longrightarrow$ we need *4-point* function

The simplest 4-point function affected by the metric on $\mathcal{M}_{\rm HM}$ is generated by

$$\int d^4x\, \mathcal{R}_{ijkl}(\chi^i\bar{\chi}^j)(\chi^k\bar{\chi}^l)$$

Symmetric part of (the *Sp(n)* part of) the curvature on $\mathcal{M}_{\rm HM}$

very complicated!

fermions from hypermultiplets

Solution:

do not impose momentum conservation!

Amplitudes and effective action

The effective action:

$$-\frac{1}{2}\int d^4x\, G_{ij}(\vec{\varphi})\,\partial_\mu\varphi^i\partial^\mu\varphi^j$$

$$G_{ij} = g_{ij} + \sum_\gamma e^{-\mathcal{T}_\gamma}\left(h_{ij}^{(\gamma)} + \cdots\right)$$

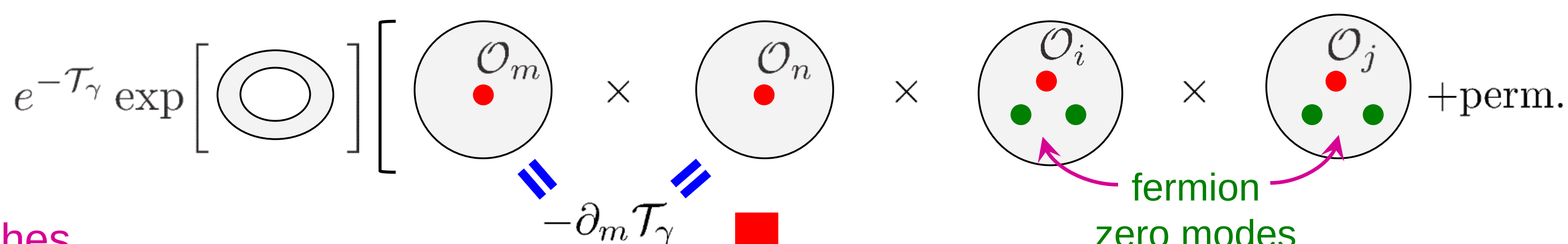
$$\varphi^i = \phi^i + \lambda^i$$

quantum fluctuations

The leading 4-λ term:

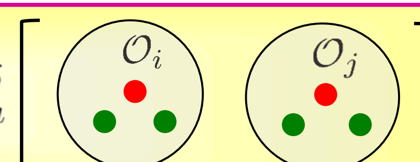
$$-\frac{1}{4}\int d^4x\, e^{-\mathcal{T}_\gamma}\underbrace{\partial_m\mathcal{T}_\gamma\,\partial_n\mathcal{T}_\gamma}_{\text{because } \mathcal{T}_\gamma\sim 1/g_s} h_{ij}^{(\gamma)}(\vec{\phi})\,\lambda^m\,\lambda^n\,\partial_\mu\lambda^i\partial^\mu\lambda^j$$

$$\langle\mathcal{O}_m\mathcal{O}_n\mathcal{O}_i\mathcal{O}_j\rangle_{\rm inst} = (2\pi)^4\delta^{(4)}\left(\sum_\alpha p^{(\alpha)}\right) e^{-\mathcal{T}_\gamma}\left[\partial_m\mathcal{T}_\gamma\,\partial_n\mathcal{T}_\gamma\,h_{ij}^{(\gamma)}(\vec{\phi})\,(p^{(3)}\cdot p^{(4)}) + \text{perm.}\right]$$

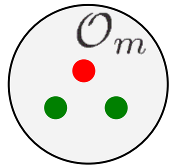


vanishes for $p=p'$

$$(p\cdot p')\,h_{ij}^{(\gamma)} = \sum_{k|\gamma}\frac{C}{\sqrt{k}}\,g_o\,\Omega_{\gamma/k}\int\prod_{\delta,\dot\delta=1}^{2} d\chi_a^\delta d\chi_a^{\dot\delta}\left[\ \right]$$



Final result



$$= \mathrm{i}\, a_m\, p_\mu\, \gamma^\mu_{\dot\alpha\alpha}\, \chi^\alpha \chi^{\dot\alpha}$$

$$\boxed{ds^2_{\text{inst}} = \sum_\gamma 2\pi e^\phi\, g_o\, \Omega_\gamma \left(\sum_{k=1}^\infty k^{-1/2}\, e^{-k\mathcal{T}_\gamma}\right)\left(\sum_m a_m\, d\lambda^m\right)^2 + \mathcal{O}(d\mathcal{T}_\gamma)}$$

Caveat: this procedure is insensitive to the field redefinitions

$$\varphi^m \to \varphi^m + e^{-\mathcal{T}_\gamma}\xi^m(\vec\varphi)$$

leading order

$$d\varphi^m \to d\varphi^m - e^{-\mathcal{T}_\gamma}\xi^m(\vec\varphi)\, d\mathcal{T}_\gamma$$

Terms $\sim d\mathcal{T}_\gamma$ cannot be compared

Evaluation of the coefficients a_m (complicated)

Perfect match with the instanton corrected metric predicted by dualities both in Type IIA and type IIB!

Compactification on an orientifold

Due to breaking of supersymmetry to N=1, fermions become massive and scalars get a potential

$$-\int d^4x \left[\frac{1}{2}\left(e^{\mathcal{K}/2}(\nabla_I\nabla_J W)\,\varepsilon_{\alpha\beta}\psi^{I\alpha}\psi^{J\beta} + \text{h.c.}\right) + e^{\mathcal{K}}\left(\mathcal{K}^{I\bar{J}}\nabla_I W \bar{\nabla}_{\bar{J}}\bar{W} - 3|W|^2\right)\right]$$

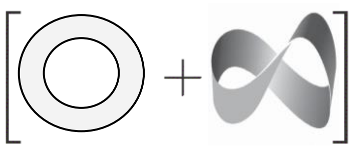
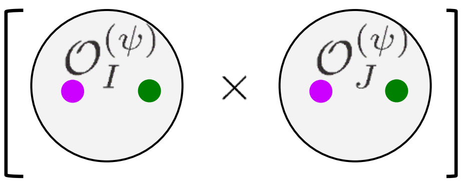
holomorphic superpotential

$$\nabla_I W = \partial_I W + (\partial_I \mathcal{K})W$$

The leading instanton contribution to the fermion mass term:

$$-\frac{1}{2}\int d^4x \left(e^{\mathcal{K}/2}(\partial_I \mathcal{T}_\gamma)\,(\partial_J \mathcal{T}_\gamma)\,W_\gamma\,\varepsilon_{\alpha\beta}\psi^{I\alpha}\psi^{J\beta} + \text{h.c.}\right) \qquad W_\gamma = \mathcal{A}_\gamma\, e^{-\mathcal{T}_\gamma}$$

for *rigid* O(1) instanton

$e^{-\mathcal{T}_\gamma}\exp$  

$$\frac{1}{2}K_0 \int \prod_\mu \frac{d\xi^\mu}{\sqrt{2\pi}} \int \prod_{\delta=1}^{2} d\chi^\delta$$

$$-2\pi \mathrm{i}\partial_I \mathcal{T}_\gamma\,\varepsilon_{\alpha\beta}\chi^\beta$$

contribution of massive modes, complicated (but *finite!*) integral depending on the moduli

Instanton induced superpotential

$$W_\gamma = \frac{\kappa_4^3\, e^{\mathrm{i}\xi}}{32\pi^4 g_o^2}\, K_0\, e^{-\mathcal{K}/2}\, e^{-\mathcal{T}_\gamma}$$

$\xi \in \mathbb{R}$ appears due to a change of coordinates. Its proper choice ensures holomorphicity of W

Conclusions

- String field theory is able to fix all apparent divergences and ambiguities in instanton contributions to string amplitudes
- String amplitudes perfectly reproduce the results predicted by dualities, which provides a highly non-trivial test for both approaches
- The same technique allows to get an instanton induced superpotential in N=1 compactifications where dualities are not powerful enough

Future directions:

- Extend to more general types of instantons in orientifold compactifications
- Include the effect of fluxes
- Get insights about *NS5-brane instantons* in CY compactifications which remain not fully understood
- Go beyond the leading order in the string coupling

Thank you!