

Kaluza-Klein Spectrometry for String Theory Compactifications

Emanuel Malek

Humboldt-Universität zu Berlin



Eurostrings
25th April 2022

with Bobev, Giambrone, Guarino, Nicolai, Robinson, Samtleben, Sterckx,
Trigiante, van Muiden

The importance of Kaluza-Klein spectra

- ▶ Compactification \Rightarrow massive Kaluza-Klein modes arise
- ▶ Kaluza-Klein spectrum important
 - ▶ AdS/CFT: conformal dimensions
 - ▶ Stability of non-SUSY vacua?



Dangers of trusting lower-dimensional supergravity!

Computing Kaluza-Klein spectra is hard

- ▶ Free scalar on S^1 :

$$0 = \partial_x^2 \phi(x, y) + \partial_y^2 \phi(x, y),$$
$$\phi(x, y) = \phi^{(k)}(x) e^{i k y/R}, \quad m^2 = \frac{k^2}{R^2}.$$

Computing Kaluza-Klein spectra is hard

- ▶ Free scalar on S^1 :

$$0 = \partial_x^2 \phi(x, y) + \partial_y^2 \phi(x, y),$$

$$\phi(x, y) = \phi^{(k)}(x) e^{i k y/R}, \quad m^2 = \frac{k^2}{R^2}.$$

- ▶ SUGRA: (linearised) EoMs mix metric & fluxes \Rightarrow eigenmodes?

$$\nabla_Q f^{QMN P} + \frac{1}{2} F^{QMNP} \nabla_Q h_R{}^R - \nabla_Q \left(h^{QR} F_R{}^{MNP} \right) - 3 \nabla^Q \left(h^{S[M} F_{QS}{}^{NP]} \right) = -\frac{1}{288} \epsilon^{MNPQ_1\dots Q_8} F_{Q_1\dots Q_4} f_{Q_5\dots Q_8}.$$

Computing Kaluza-Klein spectra is hard

- ▶ Free scalar on S^1 :

$$0 = \partial_x^2 \phi(x, y) + \partial_y^2 \phi(x, y),$$

$$\phi(x, y) = \phi^{(k)}(x) e^{i k y/R}, \quad m^2 = \frac{k^2}{R^2}.$$

- ▶ SUGRA: (linearised) EoMs mix metric & fluxes \Rightarrow eigenmodes?

$$\nabla_Q f^{QMN P} + \frac{1}{2} F^{QMNP} \nabla_Q h_R{}^R - \nabla_Q \left(h^{QR} F_R{}^{MNP} \right) - 3 \nabla^Q \left(h^{S[M} F_{QS}{}^{NP]} \right) = -\frac{1}{288} \epsilon^{MNPQ_1\dots Q_8} F_{Q_1\dots Q_4} f_{Q_5\dots Q_8}.$$

- ▶ Previously, only two cases understood:

- ▶ Spin-2 fields [Bachas, Estes '11] ✓
- ▶ $M_{int} = \frac{G}{H}$ ✓

Another tool: Consistent truncations

- ▶ Non-linear truncation to subset of KK-modes
- ▶ Solutions are solutions to higher-dim theory
- ▶ Compute subset of masses for any vacuum!

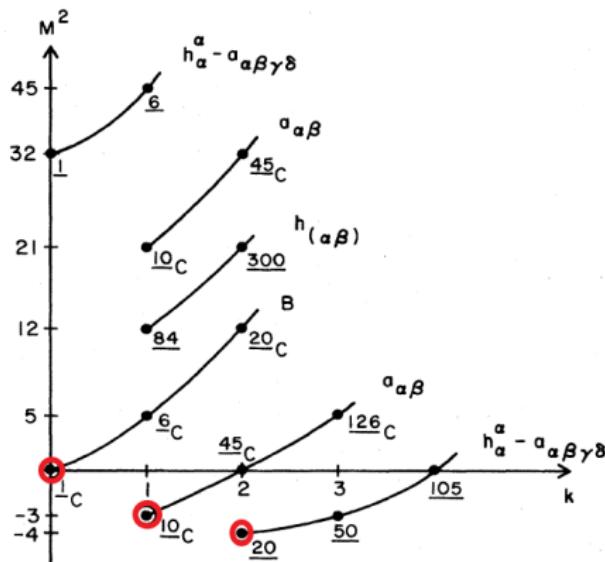
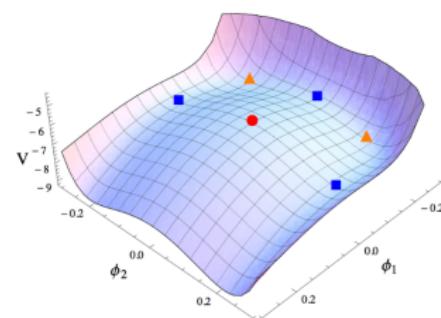


FIG. 2. Mass spectrum of scalars.



Another tool: Consistent truncations

- ▶ Non-linear truncation to subset of KK-modes
- ▶ Solutions are solutions to higher-dim theory
- ▶ Compute subset of masses for any vacuum!

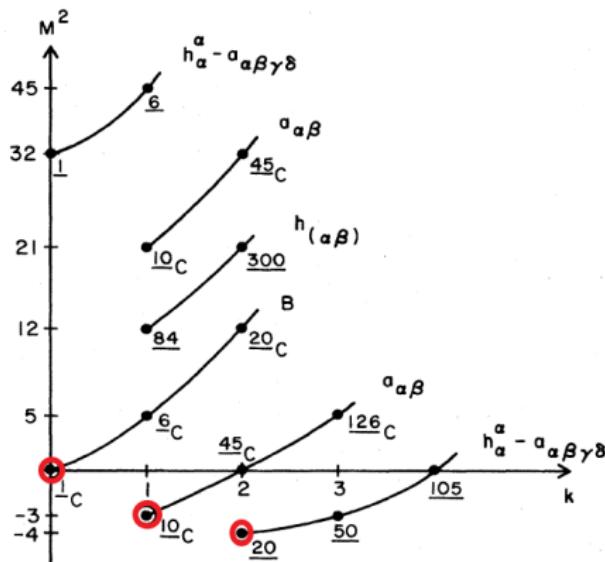
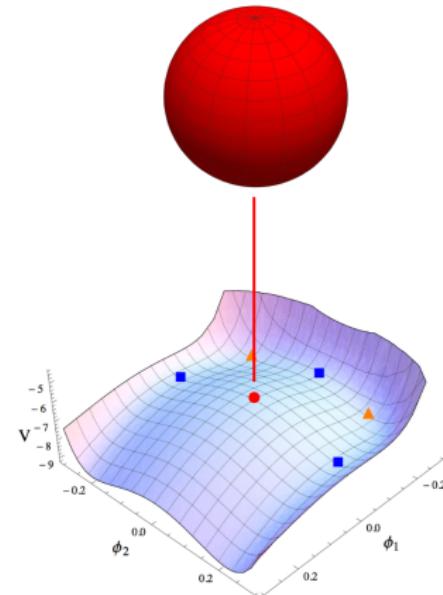


FIG. 2. Mass spectrum of scalars.



Another tool: Consistent truncations

- ▶ Non-linear truncation to subset of KK-modes
- ▶ Solutions are solutions to higher-dim theory
- ▶ Compute subset of masses for any vacuum!

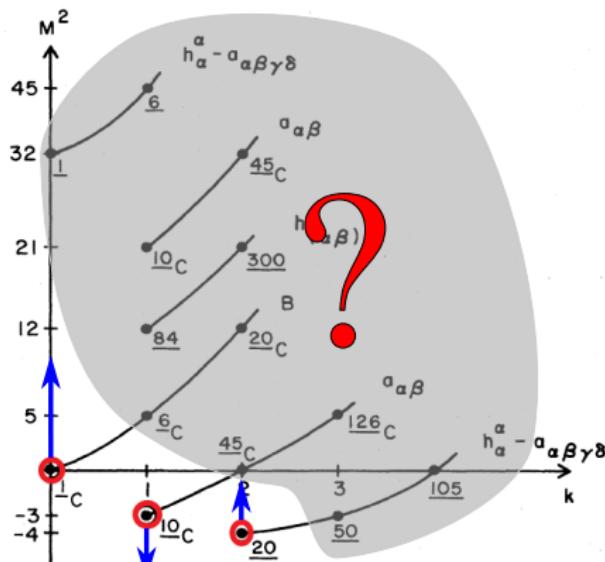
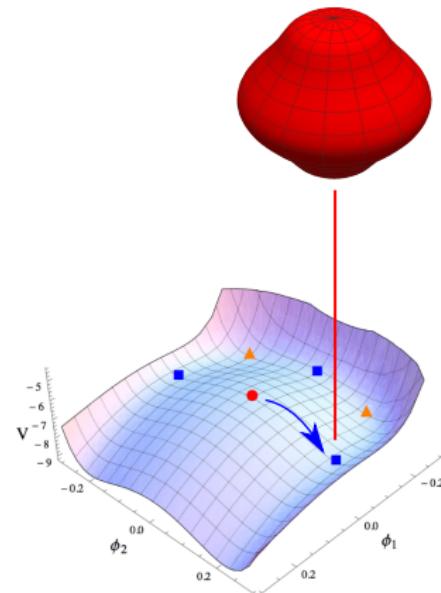


FIG. 2. Mass spectrum of scalars.



Another tool: Consistent truncations

- ▶ No
- ▶ Sol
- ▶ Con

[EM, Samtleben '20]

Extend this to full KK spectrum using ExFT!

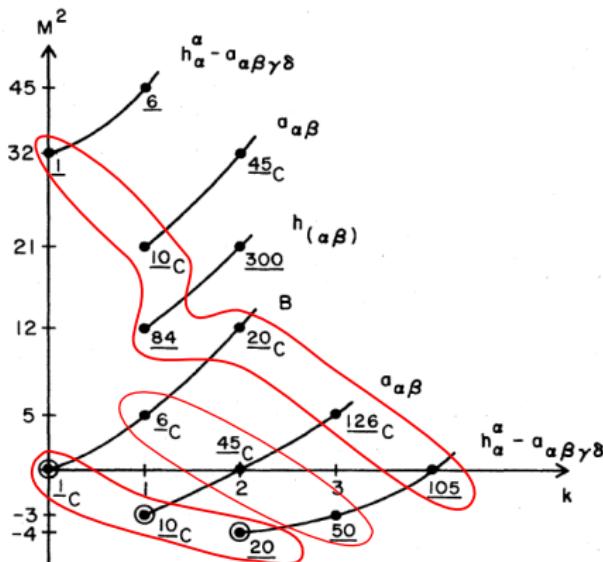
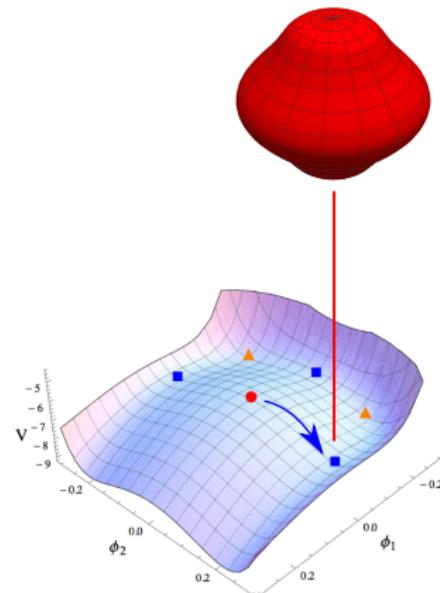
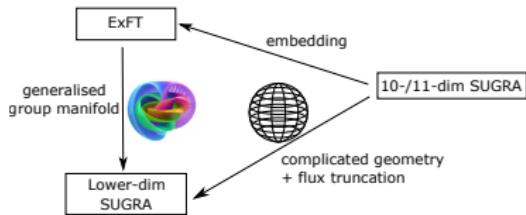


FIG. 2. Mass spectrum of scalars.



Exceptional Field Theory & consistent truncations



Kaluza-Klein spectroscopy

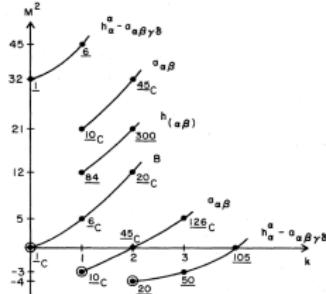
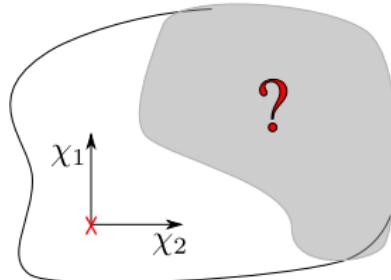


FIG. 2. Mass spectrum of scalars.

Applications



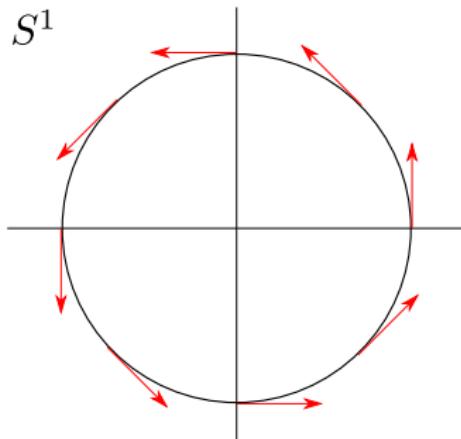
Consistent truncation

Non-linear embedding of lower-dimensional theory
into 10-/11-d supergravity

- ▶ All solutions of lower-d SUGRA → solutions of 10-/11-d SUGRA
- ▶ Non-linearity: highly non-trivial!
- ▶ Symmetry arguments crucial

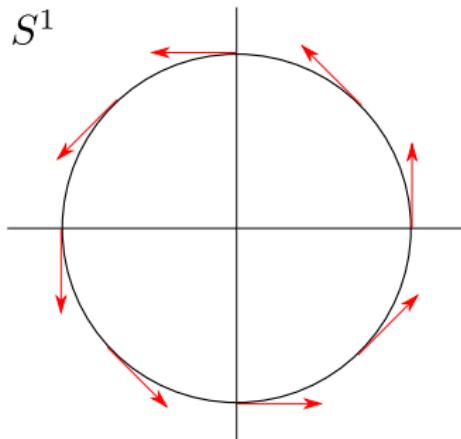
Consistent truncation on group manifold

Symmetry arguments crucial for consistency, e.g.
group manifold



Consistent truncation on group manifold

Symmetry arguments crucial for consistency, e.g.
group manifold



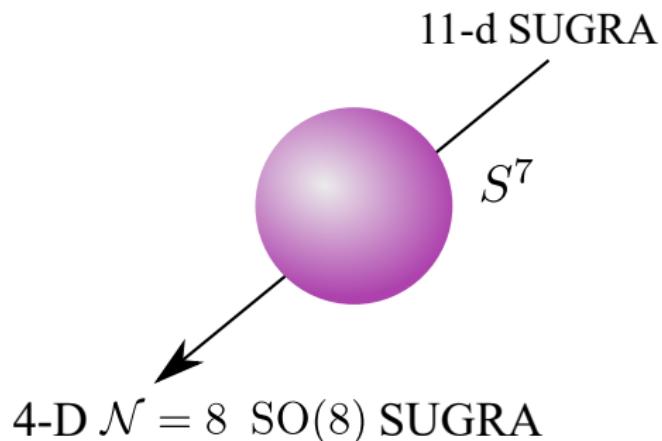
$$U_m{}^\mu \in \mathrm{GL}(D)$$

$$\mathcal{L}_{U_m} U_n = f_{mn}{}^\rho U_\rho$$

$$g_{\mu\nu}(x, y) = g_{mn}(x) (U^{-1})_\mu{}^m(y) (U^{-1})_\nu{}^n(y)$$

Consistent truncations beyond group manifolds

Consistent truncations of 10-d/11-d SUGRA beyond
group manifolds?



[de Wit, Nicolai '82]

Exceptional Field Theory

..., [Berman, Perry '10], [Coimbra, Strickland-Constable, Waldram '11],
[Hohm, Samtleben, '13], ...

Exceptional Field Theory: Unify metric + fluxes of supergravity

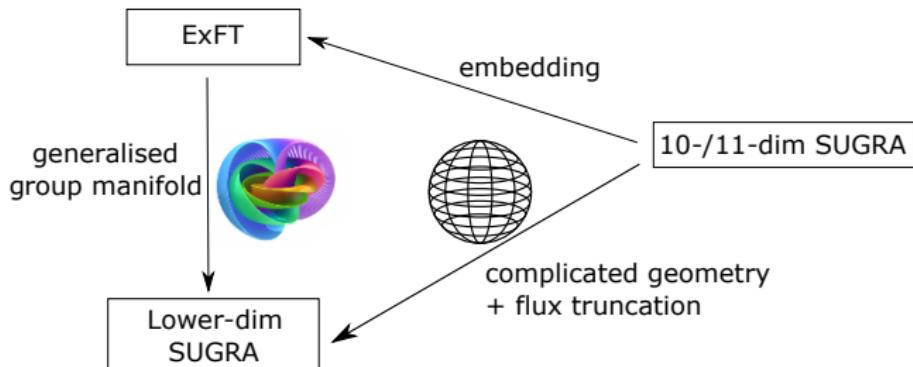
IIB supergravity on $M_4 \times C_6$:

$$\{g, \Phi, B_{(2)}, A_{(0)}, A_{(2)}, A_{(4)}, \dots\} = \mathcal{M}_{MN} \in \frac{E_{7(7)}}{\text{SU}(8)}.$$

Diffeo + gauge transf \rightarrow generalised vector field $V^M \in \mathbf{56}$ of $E_{7(7)}$
Lie derivative \rightarrow generalised Lie derivative

Exceptional Field Theory and consistent truncations

Consistent truncations captured by
“generalised group manifolds” in ExFT



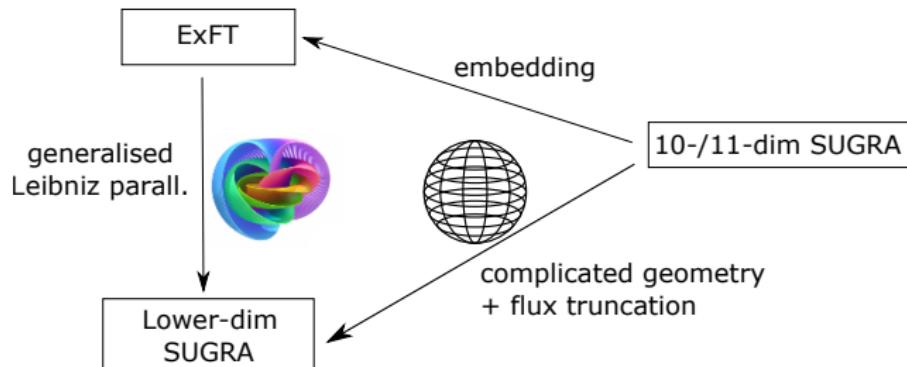
$$U_A{}^M \in E_{7(7)}$$

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C$$

$$\mathcal{M}_{MN}(x, Y) = M_{AB}(x)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

Exceptional Field Theory and consistent truncations

Consistent truncations captured by
“generalised Leibniz parallelisable manifolds” in ExFT



$$U_A{}^M \in E_{7(7)}$$

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C$$

$$\mathcal{M}_{MN}(x, Y) = M_{AB}(x)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

Kaluza-Klein spectroscopy

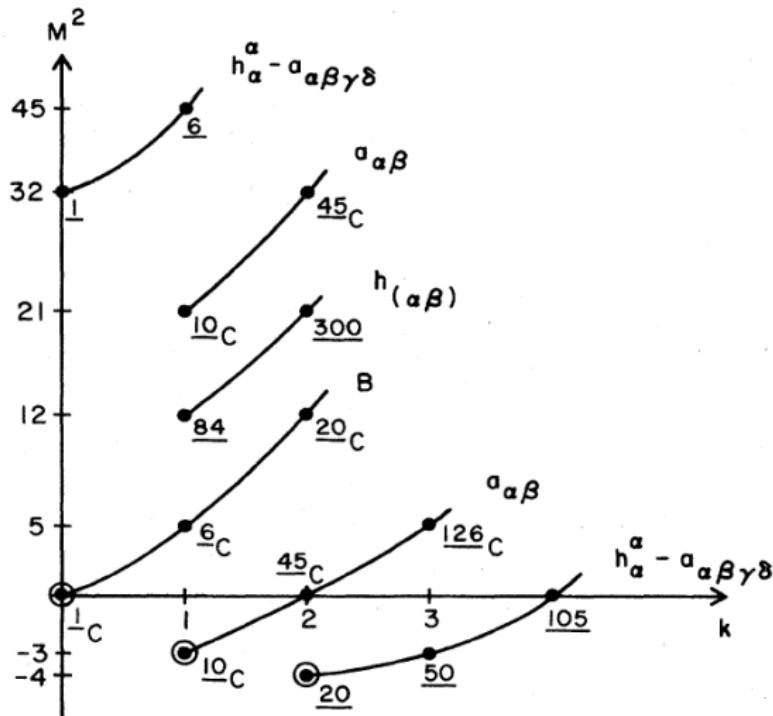


FIG. 2. Mass spectrum of scalars.

KK spectroscopy strategy

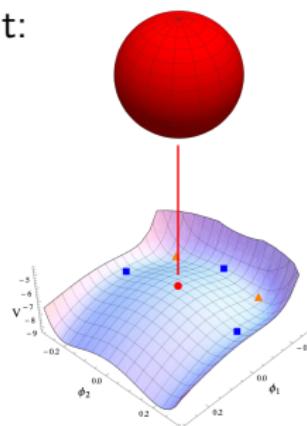
Traditional KK Ansatz: $\phi(x, y) = \phi^\Sigma(x) \underbrace{y_\Sigma(y)}_{\text{harmonics}}$

KK spectroscopy strategy

Traditional KK Ansatz: $\phi(x, y) = \phi^\Sigma(x) \underbrace{\mathcal{Y}_\Sigma(y)}_{\text{harmonics}}$

ExFT KK Ansatz: $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{harmonics}}_{\text{linear}}$

First at max symmetric point:

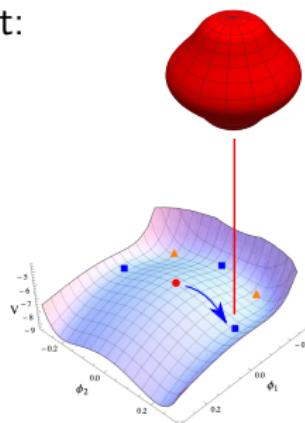


KK spectroscopy strategy

Traditional KK Ansatz: $\phi(x, y) = \phi^\Sigma(x) \underbrace{\mathcal{Y}_\Sigma(y)}_{\text{harmonics}}$

ExFT KK Ansatz: $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{harmonics}}_{\text{linear}}$

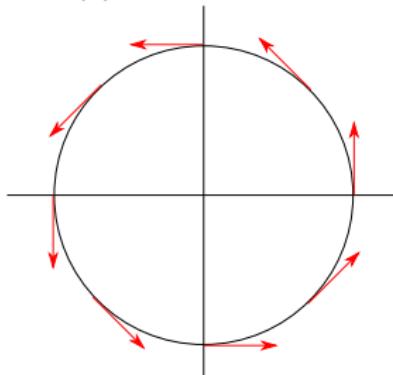
Then at less symmetric point:



Warped compactifications with few / no remaining
(super-)symmetries!

KK spectroscopy at max. symmetric point

$U_A^M \in E_{7(7)}$ give basis for all fields



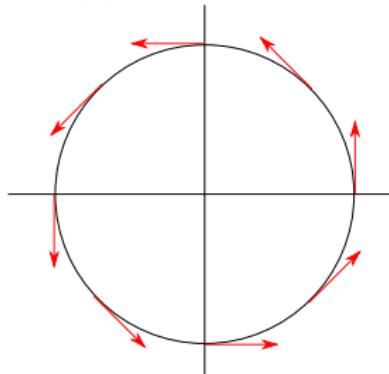
Only need scalar harmonics: \mathcal{Y}_Σ

c.f. $h_{ij}(x, y) = \sum_\ell h^{(\ell)}(x) \mathcal{Y}_{(ij)}^{(\ell)}(y), \quad b_{ij}(x, y) = \sum_\ell b^{(\ell)}(x) \mathcal{Y}_{[ij]}^{(\ell)}(y)$

“ $\mathcal{N} = 8$ supermultiplet contains all SUGRA fields”

KK spectroscopy at max. symmetric point

$U_A^M \in E_{7(7)}$ give basis for all fields

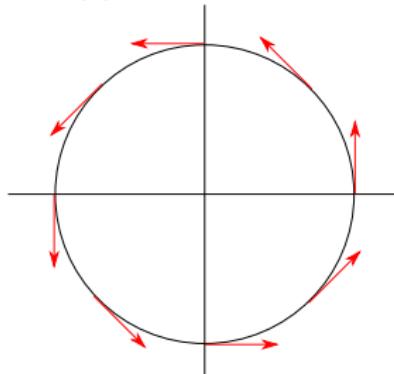


Only need scalar harmonics: \mathcal{Y}_Σ

$\mathcal{M}_{MN}(x, Y) \in E_{7(7)}/\text{SU}(8)$

KK spectroscopy at max. symmetric point

$U_A{}^M \in E_{7(7)}$ give basis for all fields



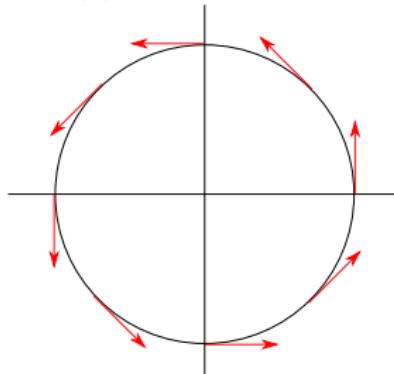
Only need scalar harmonics: \mathcal{Y}_Σ

$$\mathcal{M}_{MN}(x, Y) = (\delta_{AB} + j_{AB}(x)) (U^{-1})_M{}^A(Y) (U^{-1})_N{}^B(Y)$$

$$j_{AB} \in \mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$$

KK spectroscopy at max. symmetric point

$U_A{}^M \in E_{7(7)}$ give basis for all fields

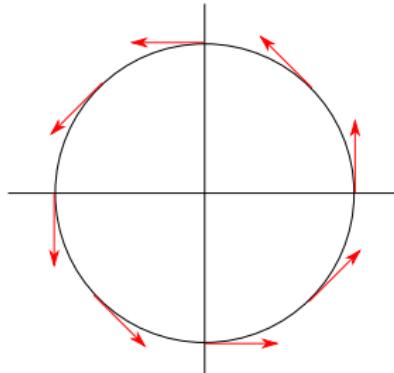


Only need scalar harmonics: \mathcal{Y}_Σ

$$\mathcal{M}_{MN}(x, Y) = (\delta_{AB} + j_{AB}{}^\Sigma(x) \mathcal{Y}_\Sigma)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$
$$j_{AB}{}^\Sigma \in \mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$$

KK spectroscopy at max. symmetric point

$U_A{}^M \in E_{7(7)}$ give basis for all fields



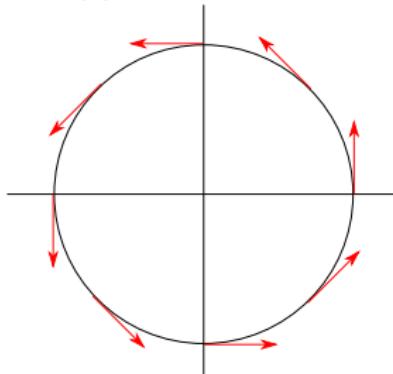
Only need scalar harmonics: \mathcal{Y}_Σ

$$\mathcal{M}_{MN}(x, Y) = (\delta_{AB} + j_{AB}{}^\Sigma(x) \mathcal{Y}_\Sigma)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$
$$j_{AB}{}^\Sigma \in \mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$$

KK Ansatz = consistent truncation \otimes scalar harmonics

KK spectroscopy at max. symmetric point

$U_A{}^M \in E_{7(7)}$ give basis for all fields

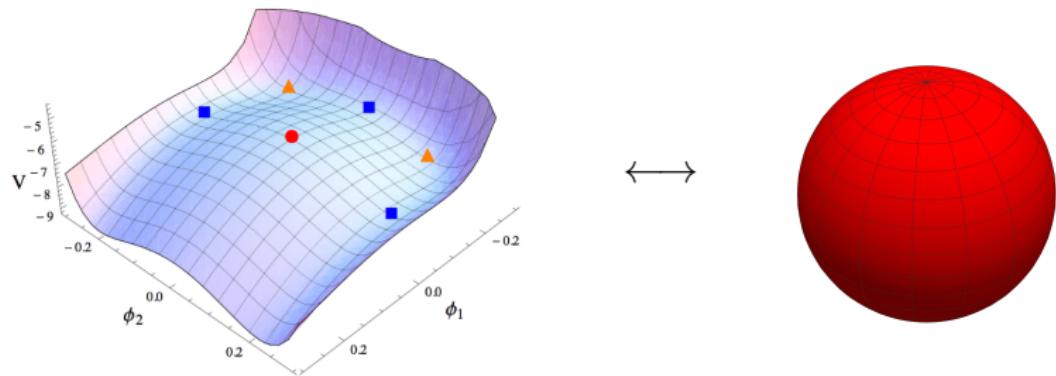


Only need scalar harmonics: \mathcal{Y}_Σ

$$\mathcal{M}_{MN}(x, Y) = (\delta_{AB} + j_{AB}{}^\Sigma(x) \mathcal{Y}_\Sigma)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$
$$j_{AB}{}^\Sigma \in \mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$$

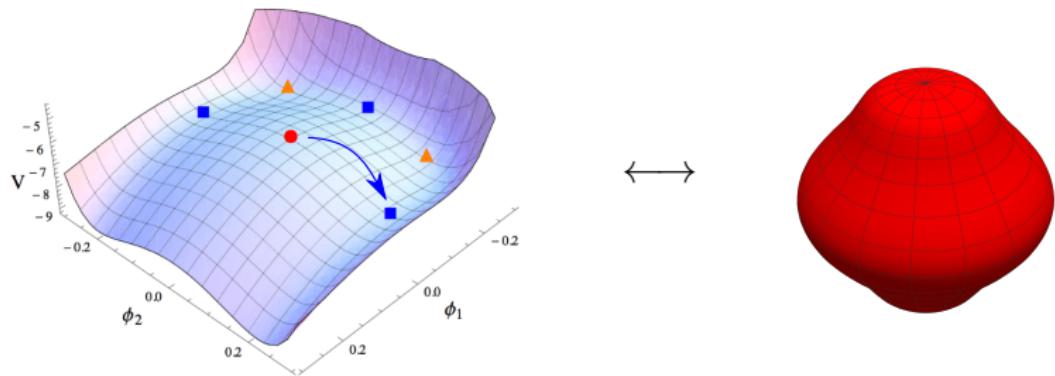
Immediate mass diagonalisation for any vacuum!

KK spectroscopy at less symmetric point



KK Ansatz: $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{scalar harmonics}}_{\text{linear}}$

KK spectroscopy at less symmetric point

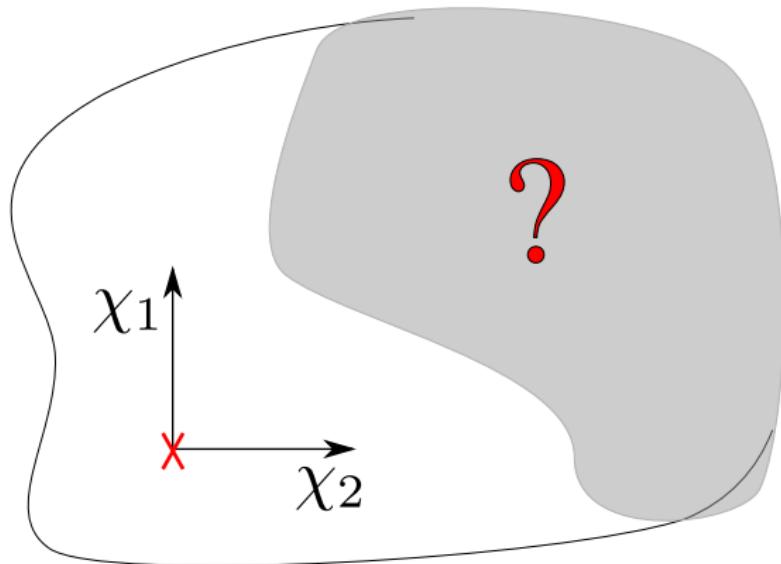


KK Ansatz: $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{scalar harmonics}}_{\text{linear}}$

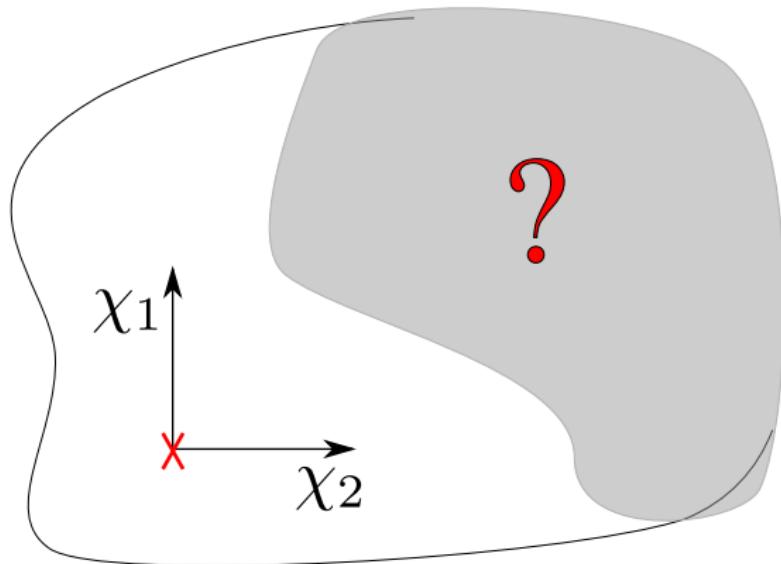
Multiplication by $E_{7(7)}$ matrix, $M_{AB}(x)$!

Use same harmonics as for max. symmetric point

Applications



Applications



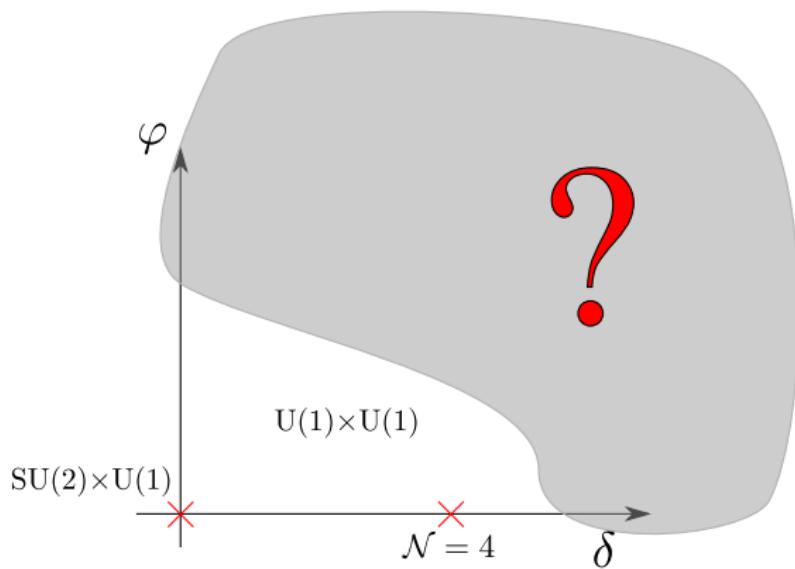
1. Global properties of conformal manifold
2. Stability of non-SUSY AdS

Ex 1. $\mathcal{N} = 2$ AdS₄ family

$[\mathrm{SO}(6) \times \mathrm{SO}(1, 1)] \ltimes \mathbb{R}^{12}$ supergravity

2 moduli $(\varphi, \delta) \in \mathbb{R}_{\geq 0}^2$ in 4-d theory $\Leftrightarrow \mathcal{N} = 2$ conformal manifold

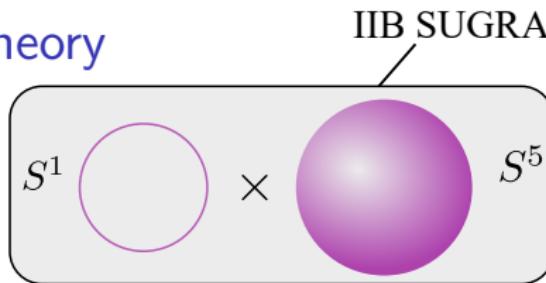
[Guarino, Sterckx, Trigiante '20], [Bobev, Gautason, van Muiden '21]



Expected to be compact e.g. [Perlmutter, Rasteli, Vafa, Valenzuela, '20]

Ex 1. Uplift to IIB string theory

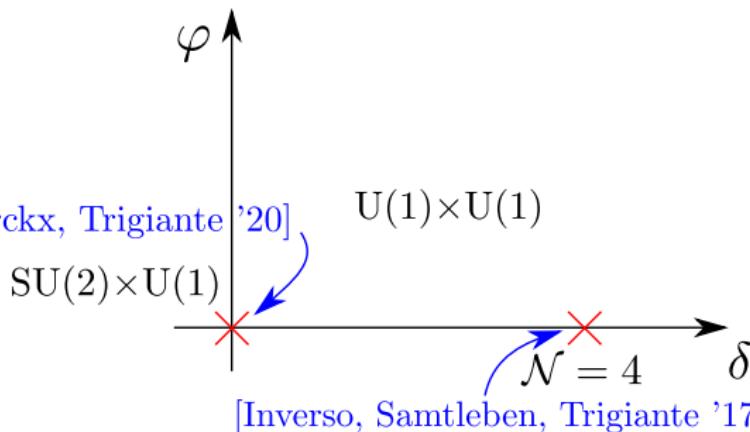
[Inverso, Samtleben, Trigiante '16]



4-D $[\mathrm{SO}(6) \times \mathrm{SO}(1, 1)] \ltimes \mathbb{R}^{12}$ SUGRA

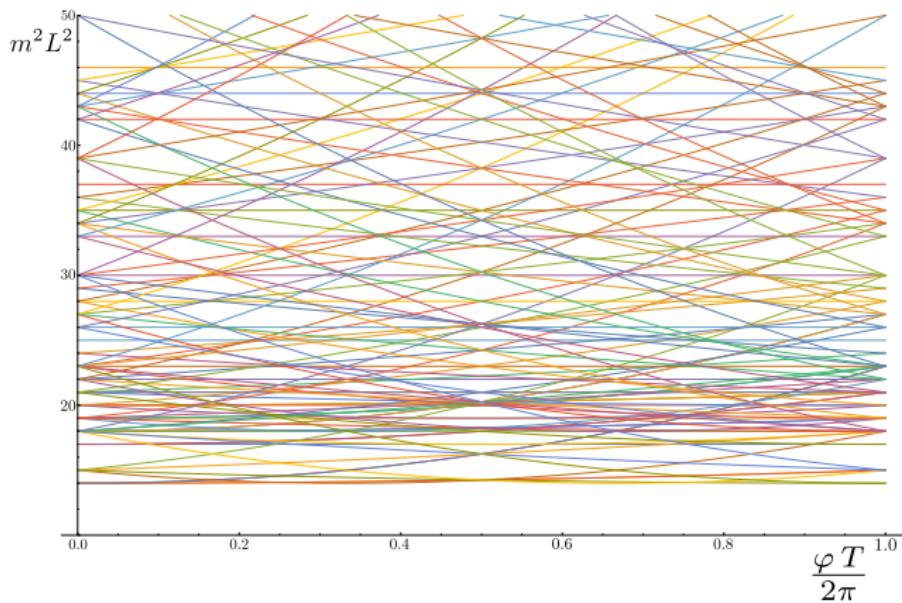
$\mathrm{AdS}_4 \times S^5 \times S^1$ "S-fold" of IIB

[Guarino, Sterckx, Trigiante '20]



Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

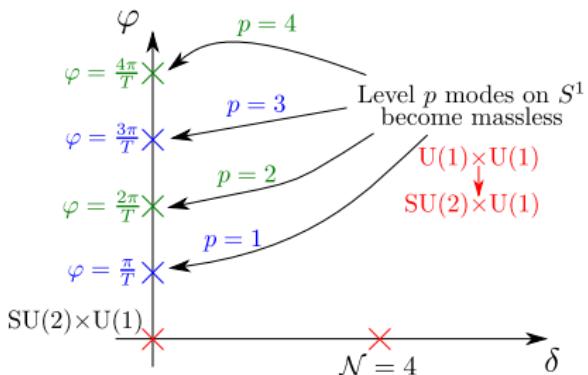
[Giambrone, EM, Samtleben, Trigiante '21]



$$\varphi \sim \varphi + \frac{2\pi}{T}, \quad T \text{ radius of } S^1$$

Ex 1. Space invaders

Higher KK modes become massless when $\varphi = \frac{p\pi}{T}$, $p \in \mathbb{Z}$
[Giambrone, EM, Samtleben, Trigiante '21]



Spectrum identical for $\varphi = \frac{2p\pi}{T}$, $p \in \mathbb{Z}$

Spectrum differs for $\varphi = \frac{(2p+1)\pi}{T}$, $p \in \mathbb{Z}$

Ex 1. Compactness of $\mathcal{N} = 2$ moduli space

[Giambrone, EM, Samtleben, Trigiante '21]

$\varphi \in \mathbb{R}^+$ is a 4-d artefact

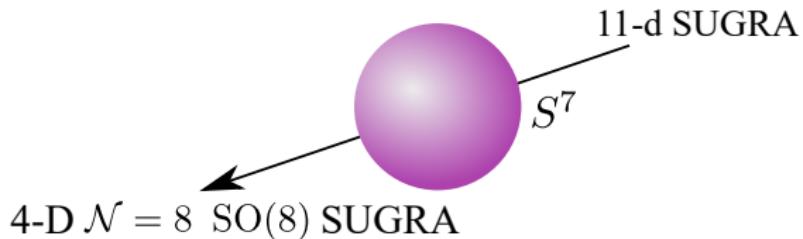
$\varphi \in [0, \frac{2\pi}{T})$ in 10 dimensions

$\varphi \rightarrow \mathbb{C}$ -structure modulus on $S^5 \times S^1$

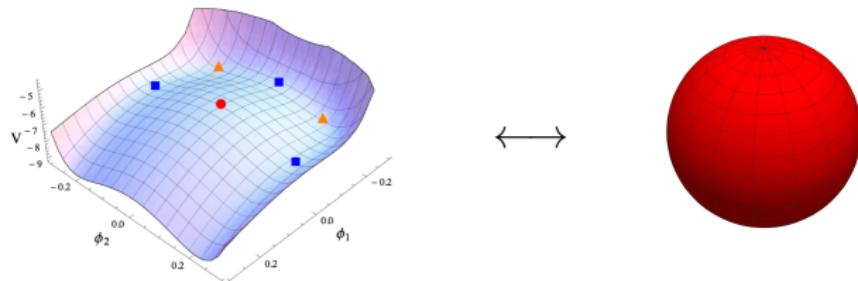
$\varphi \rightarrow$ locally coordinate transformation

δ direction non-compact? [Bobev, Gautason, van Muiden '21], [Cesàro, Larios, Varela '21]

Ex 2. Non-SUSY $\text{SO}(3) \times \text{SO}(3)$ AdS_4 vacua

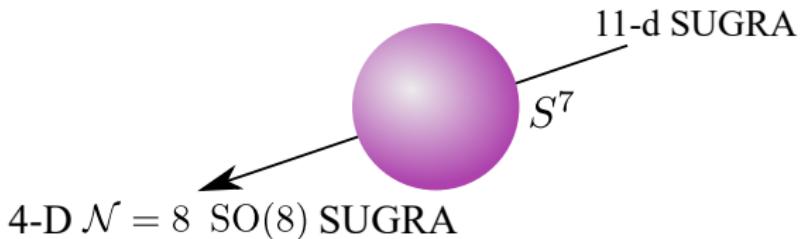


- ▶ Only one non-SUSY vacuum that is stable in 4-d! [Fischbacher, Pilch, Warner '10], [Comsa, Firsching, Fischbacher '19]
- ▶ Non-SUSY $\text{SO}(3) \times \text{SO}(3)$ AdS_4 vacuum [Warner '83]

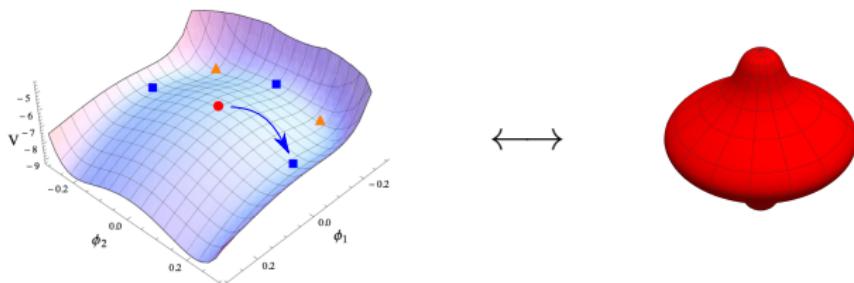


- ▶ Instability?

Ex 2. Non-SUSY $\text{SO}(3) \times \text{SO}(3)$ AdS_4 vacua



- ▶ Only one non-SUSY vacuum that is stable in 4-d! [Fischbacher, Pilch, Warner '10], [Comsa, Firsching, Fischbacher '19]
- ▶ Non-SUSY $\text{SO}(3) \times \text{SO}(3)$ AdS_4 vacuum [Warner '83]



- ▶ Instability?

Ex 2. Perturbative stability?

4-d “zero-mode” stability enough for 11-d perturbative stability?

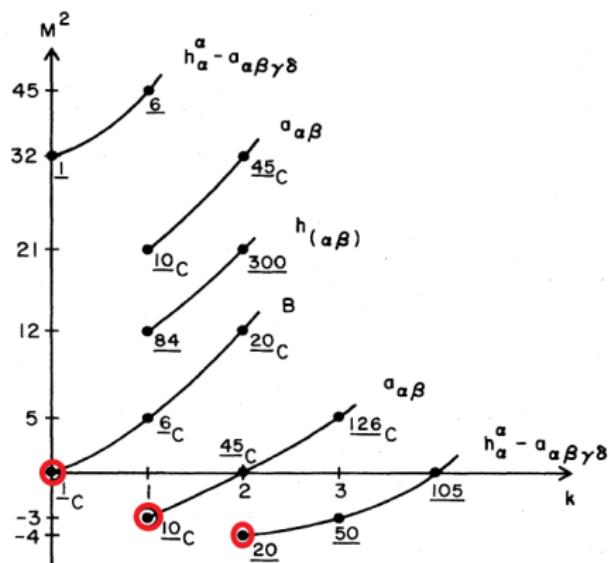
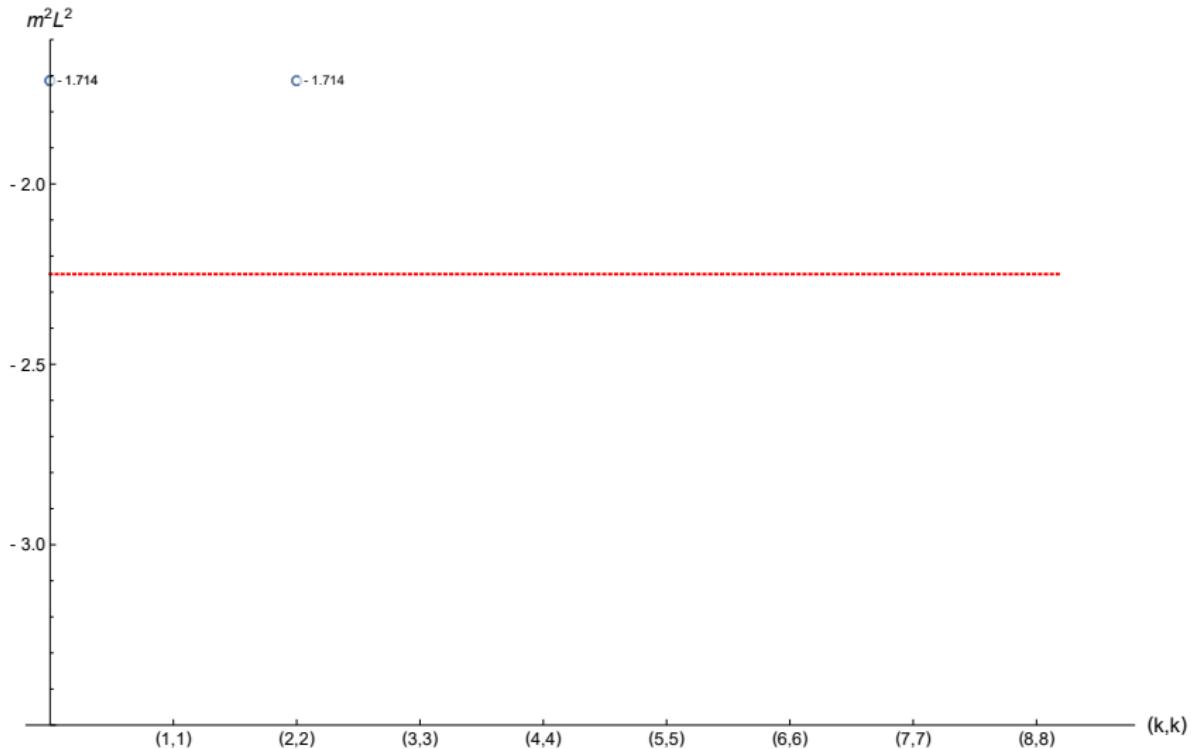


FIG. 2. Mass spectrum of scalars.

Tachyonic KK modes

Modes $\ell = 0$: $\mathcal{N} = 8$ supergravity multiplet

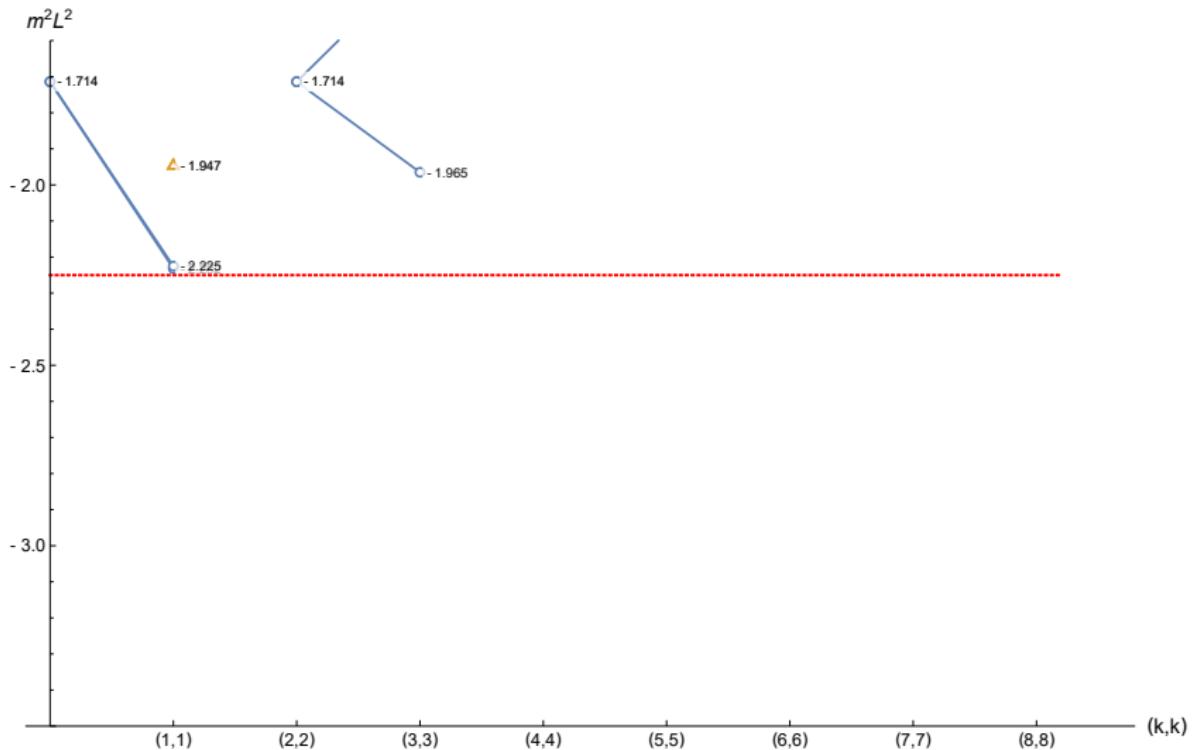
[Fischbacher, Pilch, Warner '10]



Tachyonic KK modes

Modes $\ell \leq 1$: still stable!

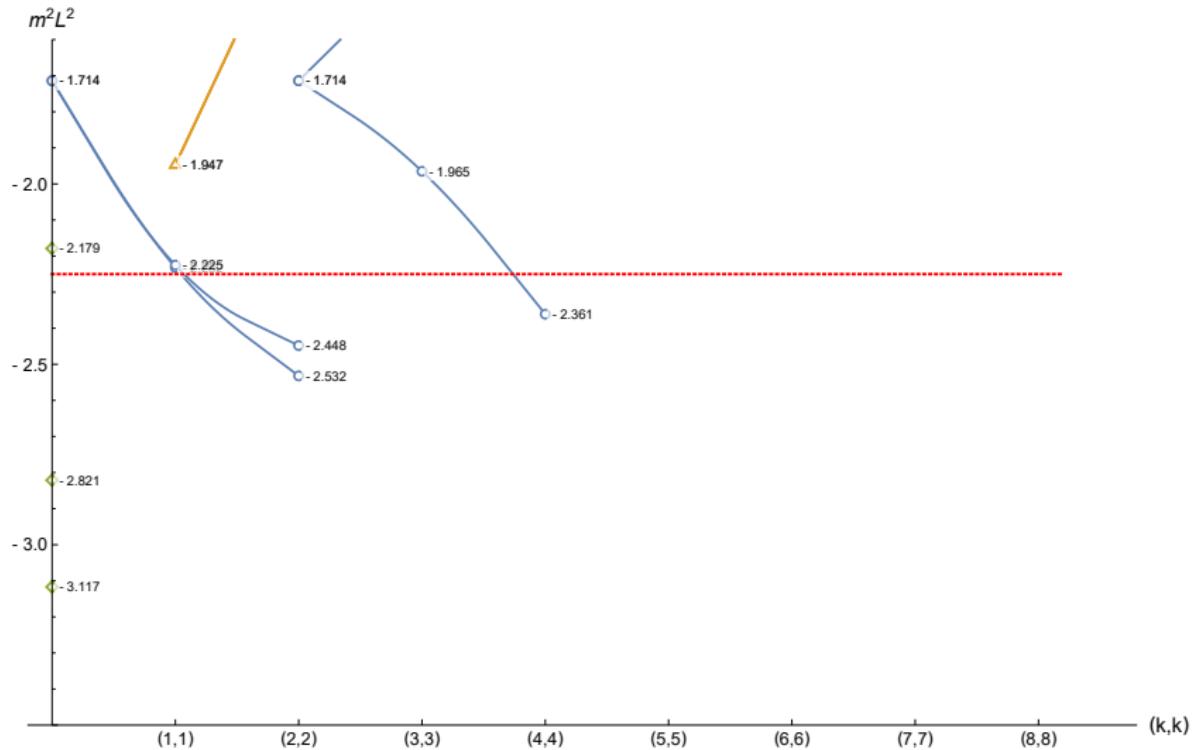
[EM, Nicolai, Samtleben '20]



Tachyonic KK modes

Modes $\ell \leq 2$: **tachyons!**

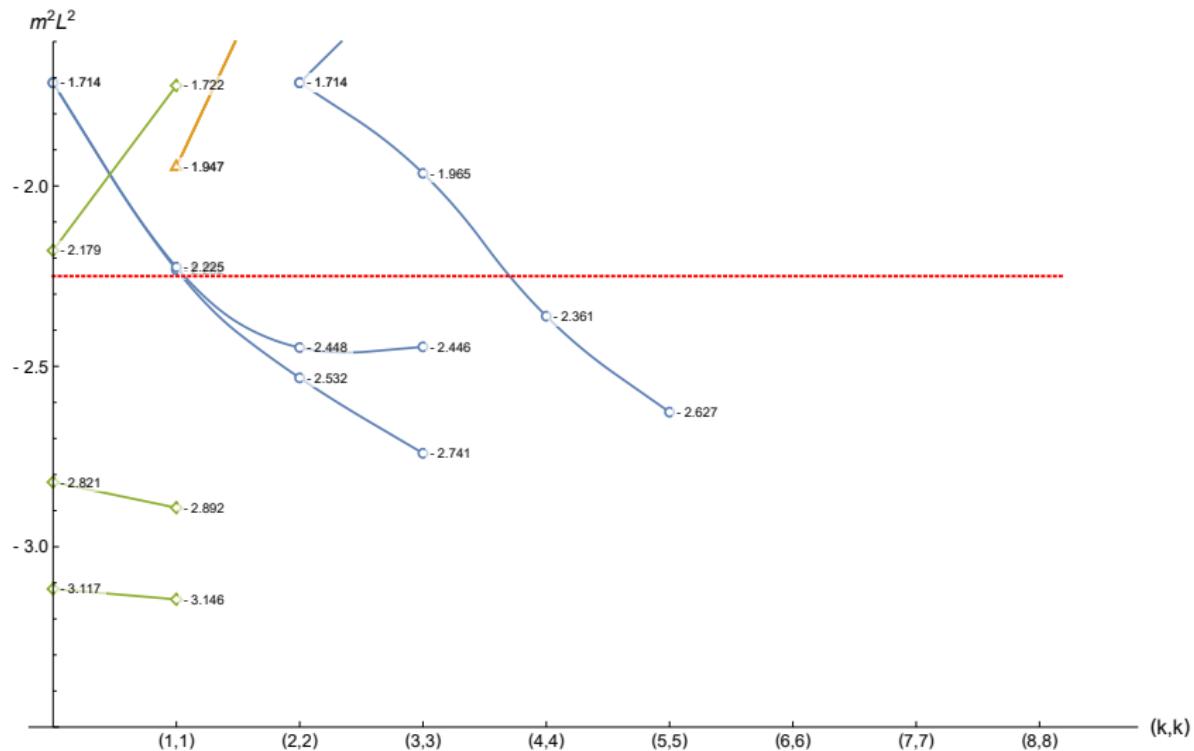
[EM, Nicolai, Samtleben '20]



Tachyonic KK modes

Modes $\ell \leq 3$

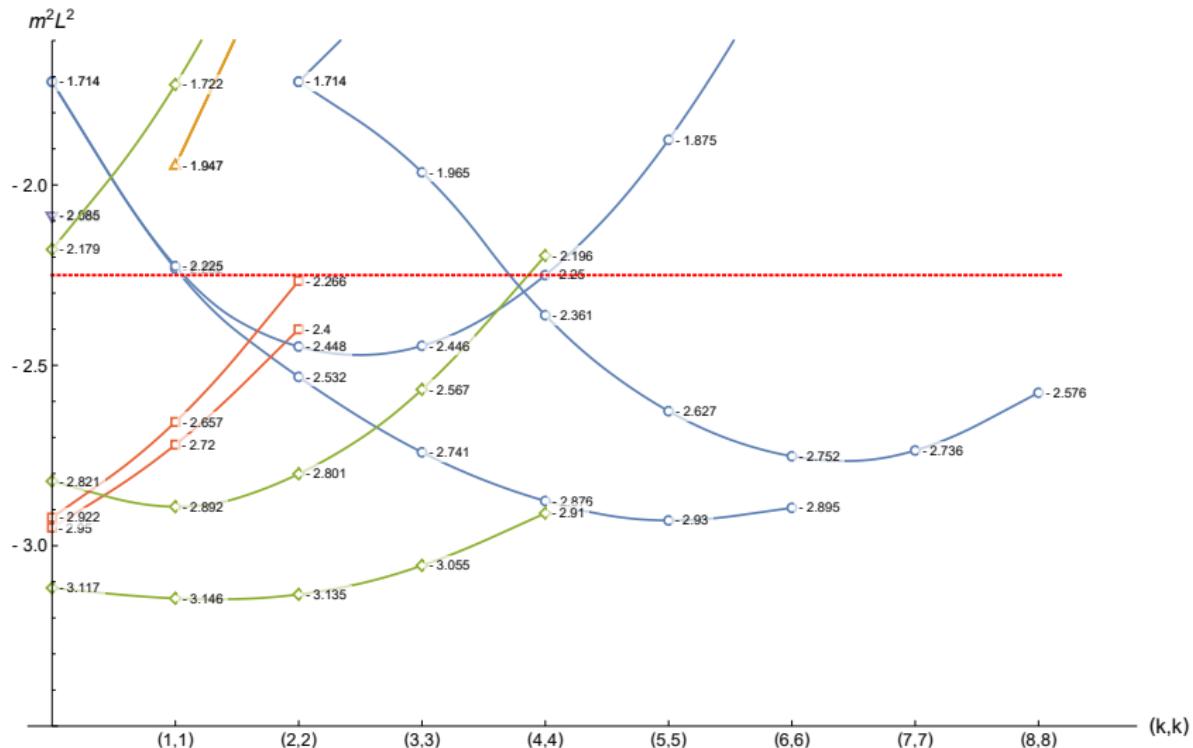
[EM, Nicolai, Samtleben '20]



Tachyonic KK modes

Modes $\ell \leq 6$

[EM, Nicolai, Samtleben '20]



Ex 2. Kaluza-Klein instability

Higher KK modes are tachyonic!

[EM, Nicolai, Samtleben '20]

- ▶ Non-SUSY $\text{SO}(3) \times \text{SO}(3)$ AdS_4 [Warner '83] is perturbatively unstable
- ▶ “Zero-mode” stability $\not\Rightarrow$ perturbative stability in higher dimensions
- ▶ Examples of perturbatively stable non-SUSY AdS_4 vacua in 10-d
[Guarino, EM, Samtleben '20]
Non-SUSY exactly marginal deformation?
[Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

Conclusions

ExFT: Compute full KK spectrum for warped compactifications with few/no remaining (super-)symmetries

- ▶ Danger of trusting lower-dimensional gauged supergravity!
- ▶ Higher KK modes crucial for physics, e.g. compactness, stability
- ▶ AdS/CFT: KK spectrum \Leftrightarrow Anomalous dimensions
Comparison with SUSY index
[Bobev, EM, Robinson, Samtleben, van Muiden '20]

Conclusions

ExFT: Compute full KK spectrum for warped compactifications with few/no remaining (super-)symmetries

- ▶ Danger of trusting lower-dimensional gauged supergravity!
- ▶ Higher KK modes crucial for physics, e.g. compactness, stability
- ▶ AdS/CFT: KK spectrum \Leftrightarrow Anomalous dimensions
Comparison with SUSY index
[Bobev, EM, Robinson, Samtleben, van Muiden '20]

Open questions

- ▶ Correlation functions?
- ▶ Vacua of less SUSY gSUGRA?
- ▶ $\Lambda \geq 0$?

Thank you!