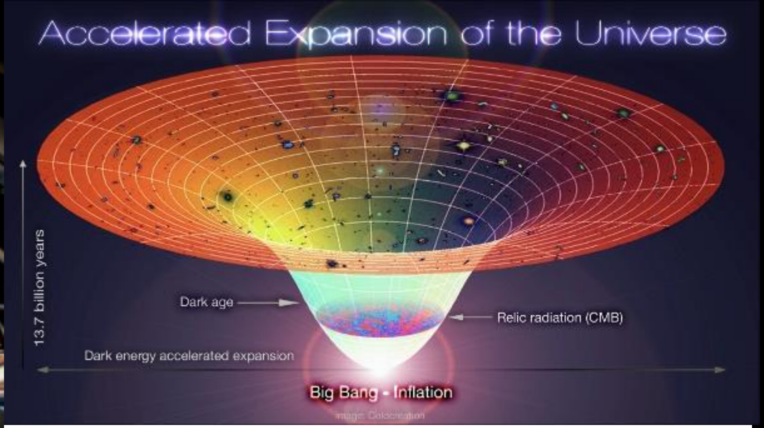
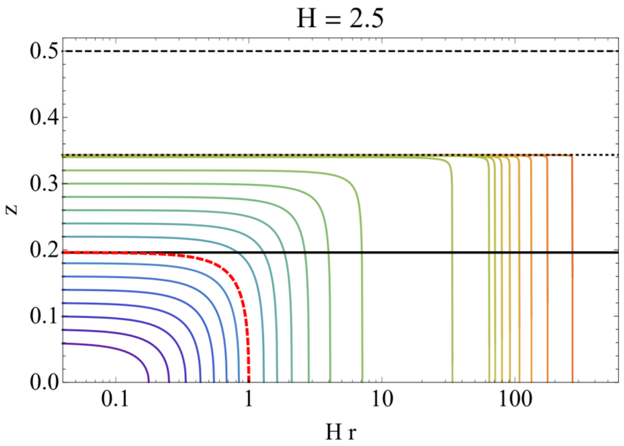


HYDRODYNAMISATION AND ENTANGLEMENT IN HOLOGRAPHY WITH DYNAMICAL BOUNDARY GRAVITY

HOLOGRAPHIC QFTS ON CURVED BACKGROUNDS

With Jorge Casalderrey-Solana, Christian Ecker, Elias Kiritsis and David Mateos

Reference: 2011.08194 (JHEP), 2109.10355 (JHEP) and to appear



Cosmological horizon

Wilke van der Schee

Eurostrings 2022, April 29, Lyon

MOTIVATION

Strongly coupled QFT on de Sitter space

- Maybe relevant for very early universe?
- Many first principle questions arise in de Sitter: thermodynamics, entropy etc
- Can we understand inflation/reheating?

Entanglement in de Sitter space-time

- A playground for (bulk) event and apparent horizons
- From boundary cosmological horizon to entanglement horizon/shadow

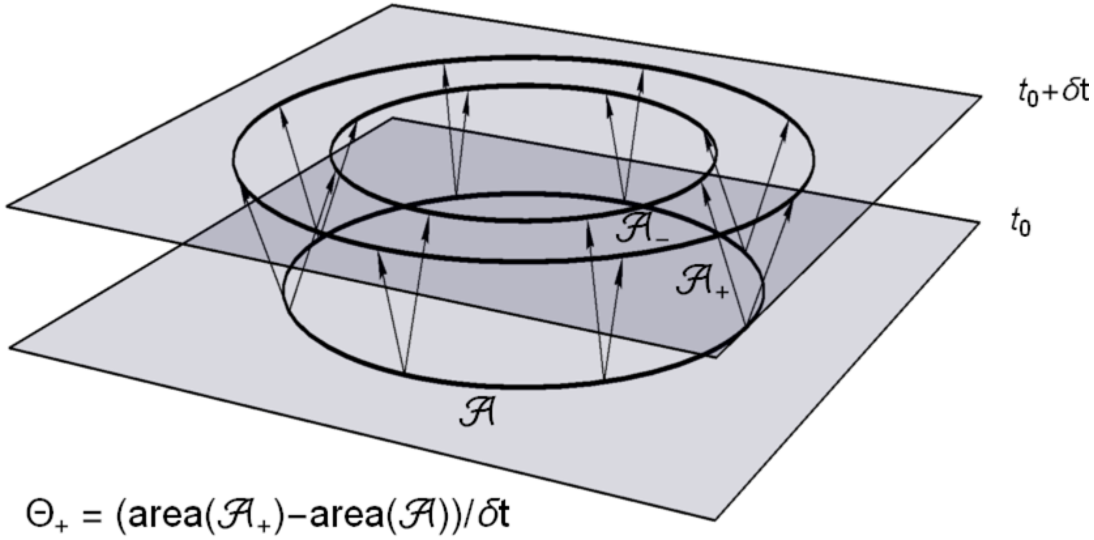
Evolution with dynamical boundary gravity

- Semi-classical Einstein Equations on the boundary
- A technical subtlety: how to extract the log terms?
- Evolutions for de Sitter, asymptotically flat and Big Crunch geometries

Major caveat: this is *not quantum gravity on de Sitter*

EVENT AND APPARENT HORIZONS

Trapped surface: expanding light surfaces contract



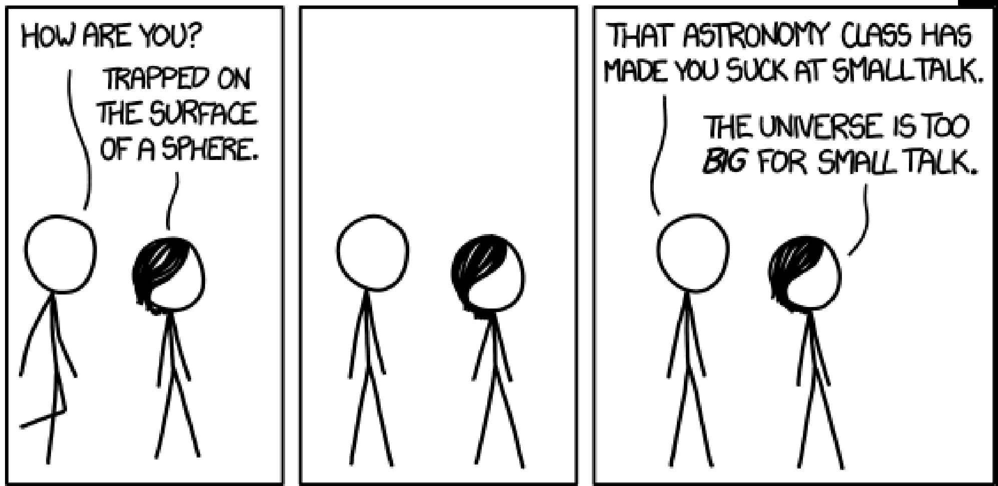
Apparent horizon: outermost trapped surface

- Subtlety: depends on time slicing

Event horizon: outermost surface to causally reach infinity

- Subtlety: depends on entire future

Event and apparent coincide when stationary

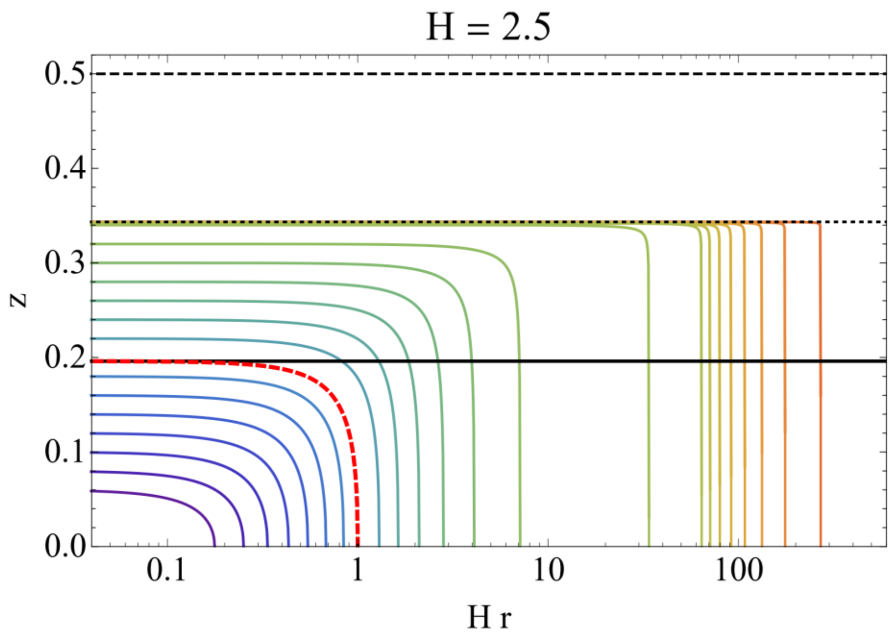


$$ds_{\text{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2\,. \quad S_0(t) = e^{Ht}$$

BULK GEOMETRY WITH dS BOUNDARY

Event, apparent and 'entanglement' horizons for bulk dual to empty de Sitter

- The apparent horizon is much beyond the event horizon
- Also shown: extremal surfaces
- `Temperature' of AH is negative; curiosity?
- Bulk confirms boundary intuition on Hawking temperature and boundary cosmological horizon

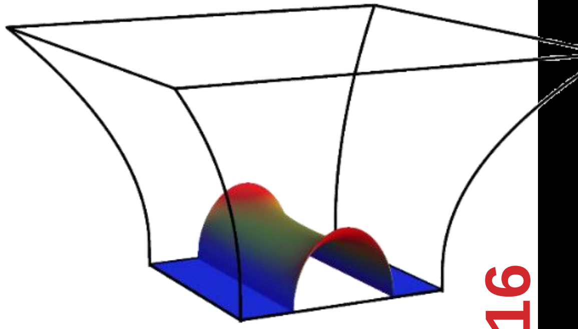


Apparent horizon: $T = -\frac{H}{2\pi}$

Entanglement horizon: $T = 0$

Event horizon: $T = \frac{H}{2\pi}$

Boundary



Cosmological horizon

Jorge Casalderrey, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

NON-CONFORMAL MODEL ON DE SITTER$_4$

De Sitter is conformally flat: almost trivial for CFT

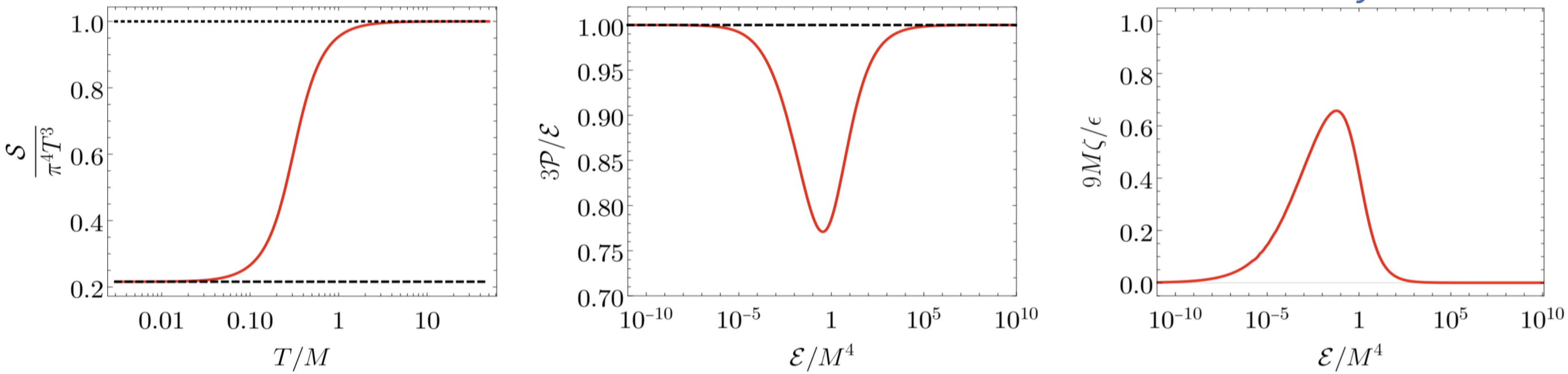
- Break scale invariance by V(Φ) with source M=1:

$$S = \frac{2}{8\pi G}\int_{\mathcal{M}} d^5x\sqrt{-g}\left(\frac{1}{4}R[g] - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

$$L^2V(\phi) = -3 - \frac{3}{2}\phi^2 - \frac{1}{3}\phi^4 + \left(\frac{1}{3\phi_M^2} + \frac{1}{2\phi_M^4}\right)\phi^6 - \frac{1}{12\phi_M^4}\phi^8$$

- Leads to non-trivial EOS and bulk viscosity (no shear considered):

Entropy density *Trace anomaly* *Bulk viscosity*

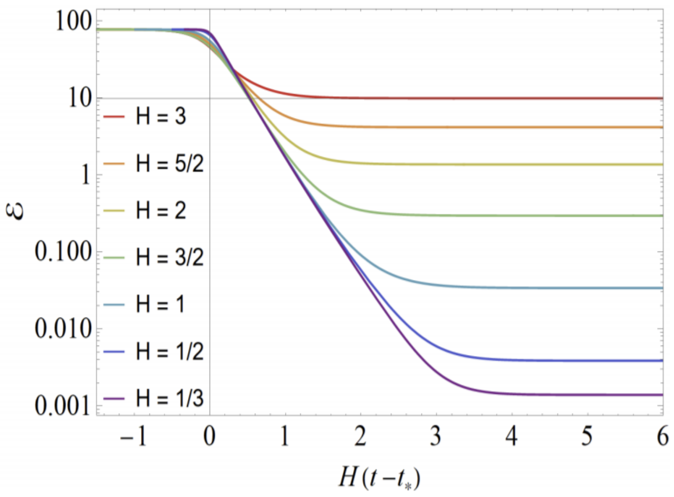


TIME EVOLUTION OF THE PROTOCOL

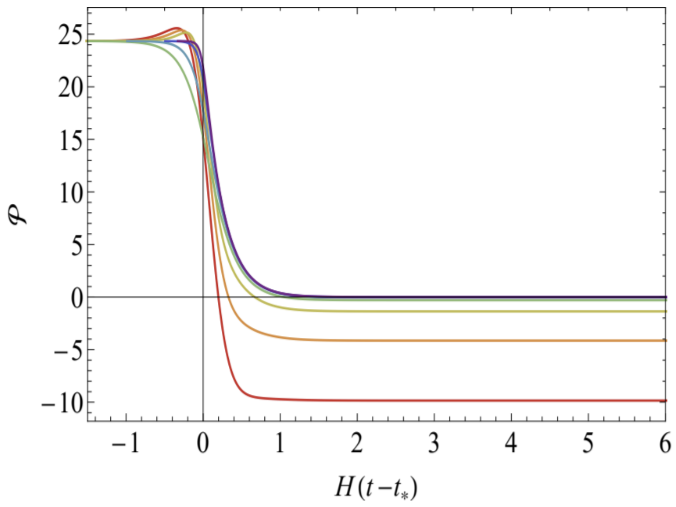
Evolution of stress tensor for different Hubble constants

- Energy density decreases towards vacuum energy (VE) (afterwards renormalised to zero)
- Pressure decreases, changes sign and becomes –VE
- Enthalpy is scheme independent, decays due to expansion

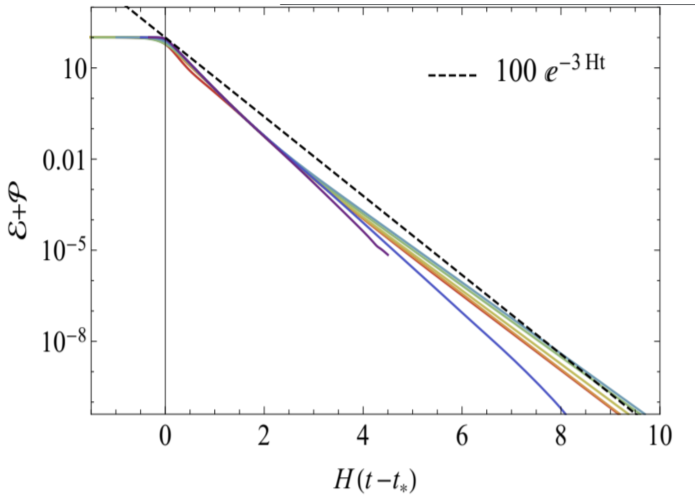
Energy density



Pressure



Enthalpy



THE APPROACH TOWARDS HYDRODYNAMICS

$$\Pi = -\zeta\left(\nabla\cdot u\right) + \zeta\tau_{\Pi} D\left(\nabla\cdot u\right) + \xi_1 \sigma^{\mu\nu}\sigma_{\mu\nu} + \xi_2\left(\nabla\cdot u\right)^2$$
$$+\xi_3 \Omega^{\mu\nu}\Omega_{\mu\nu} + \xi_4 \nabla_\mu^\perp \ln s \nabla_\perp^\mu \ln s + \xi_5 R + \xi_6 u^\alpha u^\beta R_{\alpha\beta}\,.$$

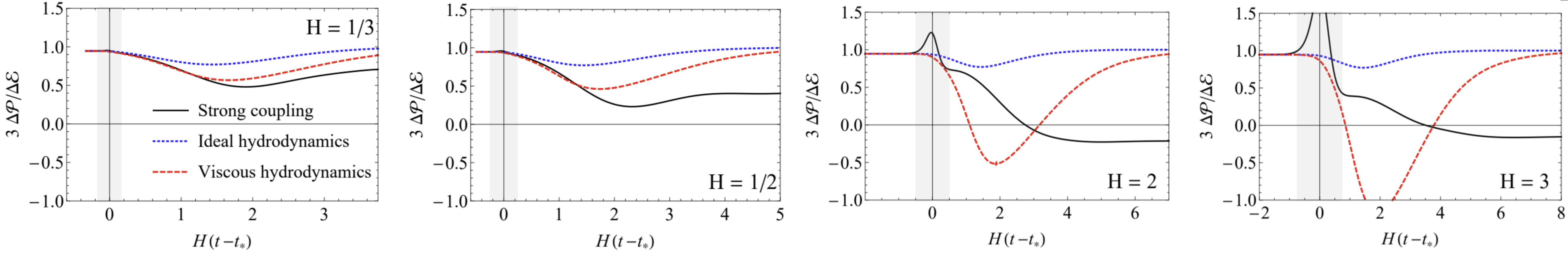
Non-trivial hydrodynamic prediction

$$\Delta\mathcal{P}^{\text{hydro}}(t) \equiv \Delta p_{\text{eq}}(\Delta\mathcal{E}(t)) - 3H\zeta(\Delta\mathcal{E}(t)) + O(H^2)\,,$$

- Δ captures our renormalisation scheme
- Conjecture: scheme ambiguities cancelled by higher order transport coefficient ξ_5

Results

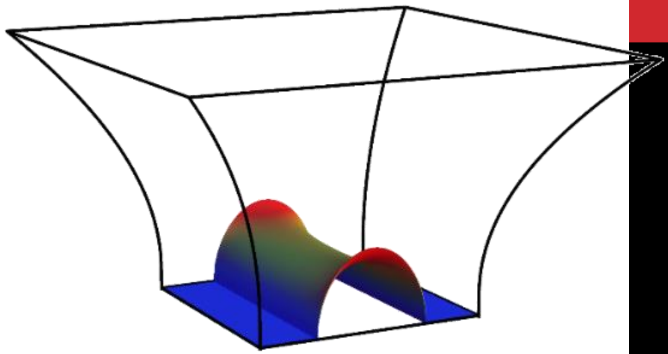
- Viscous hydro works for small H (gradients). Negative `EOS' for large H.



Paul Romatschke, Relativistic Viscous Fluid Dynamics and Non-Equilibrium Entropy (2009)

$$ds^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2\mathrm{d}\vec{x}^2$$

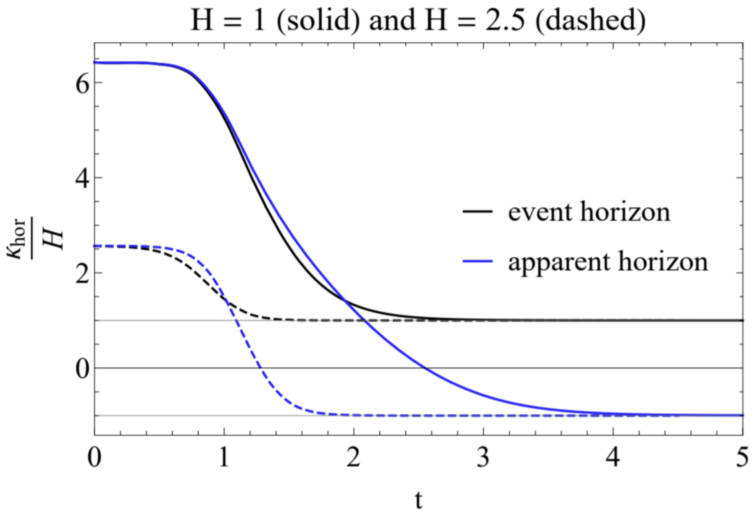
BLACK HOLE THERMODYNAMICS



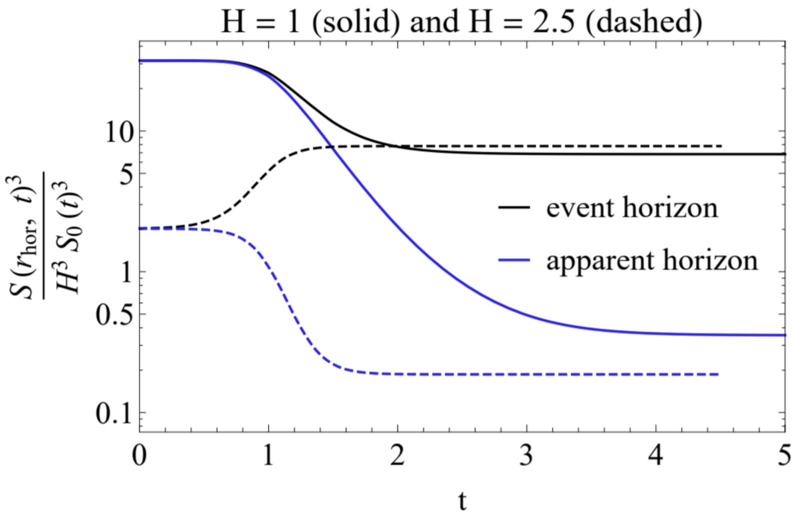
Keep track of bulk event and apparent horizons (EF coordinates)

- Dynamical setting: horizons not coincide at late times:
- Surface gravities can be shown analytically: $\kappa_{\mathrm{EH}} = -\kappa_{\mathrm{AH}} = H$
  EH confirms Hawking's temperature in de Sitter
- Area density apparent horizon vanishes for conformal theory

Surface gravity



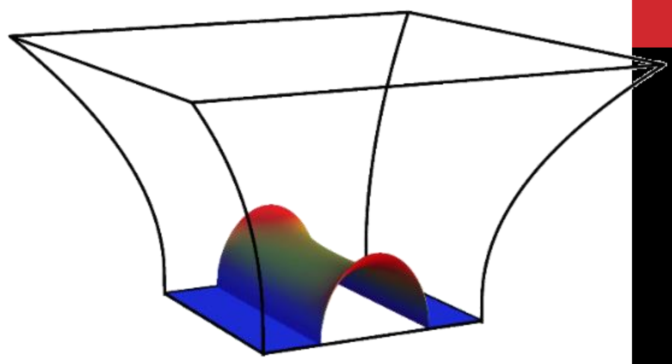
Area densities



Willy Fischler, Sandipan Kundu and Juan Pedraza, Entanglement and out-of-equilibrium dynamics in holographic models of de Sitter QFTs (2013)
Alex Buchel, Entanglement entropy of N = 2* de Sitter vacuum (2019)

$$ds^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2\mathrm{d}\vec{x}^2$$

BLACK HOLE THERMODYNAMICS

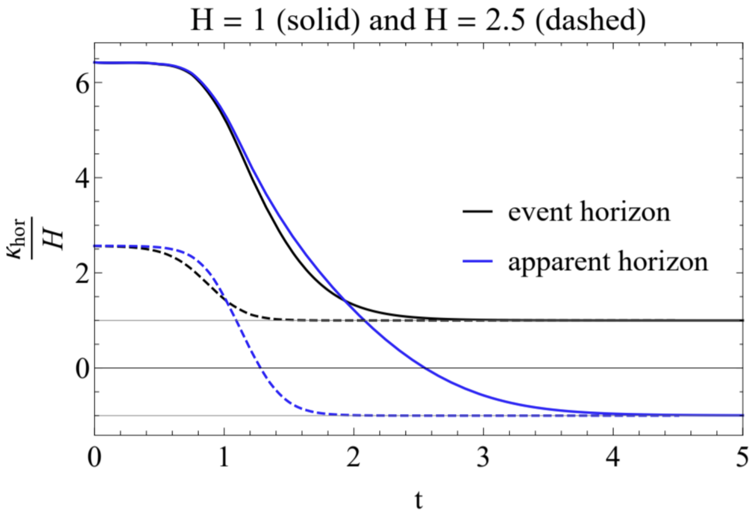


Several interpretational issues

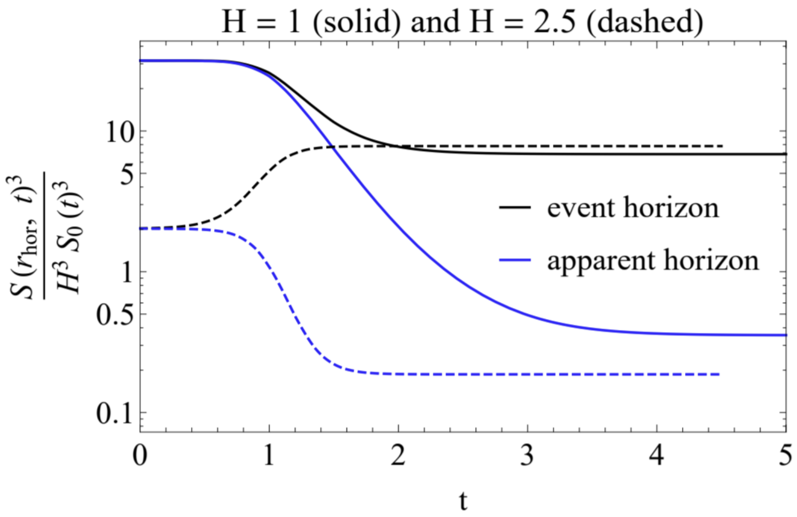
- Expanding space: mapping boundary to bulk horizon not clear
- Apparent horizon: time slicing dependent
- In general: no volume law entropy density expected

Resolution → entanglement entropy is well defined

Surface gravity



Area densities

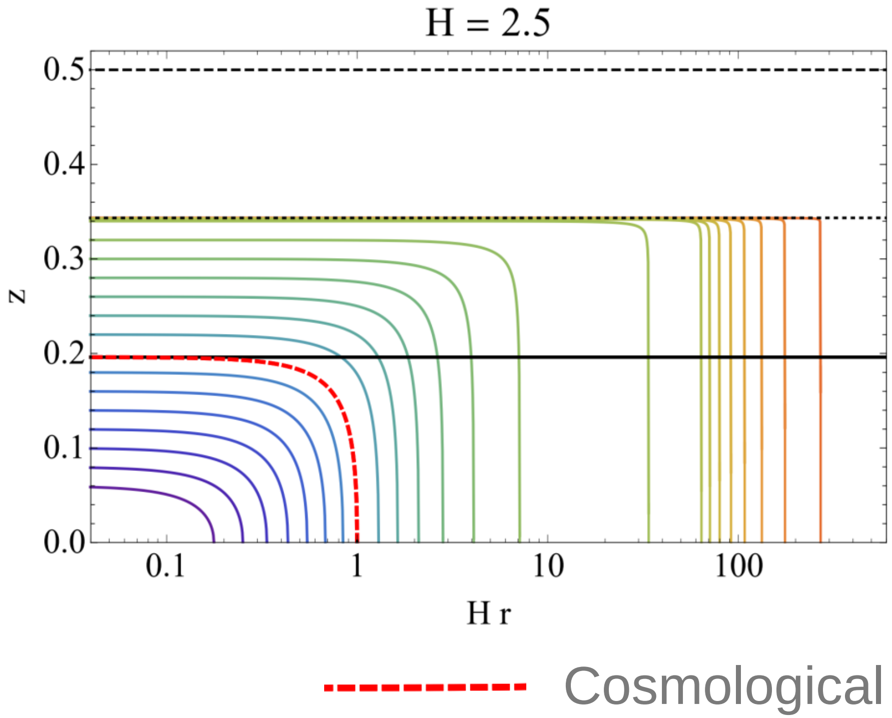


Willy Fischler, Sandipan Kundu and Juan Pedraza, Entanglement and out-of-equilibrium dynamics in holographic models of de Sitter QFTs (2013)
Alex Buchel, Entanglement entropy of N = 2* de Sitter vacuum (2019)

ENTANGLEMENT IN DE SITTER

Extremal surfaces dual to spherical entangling regions:

- Surfaces probe beyond event horizon if and only if entangling region bigger than visible universe
- Only `superobservers' can see behind event horizon

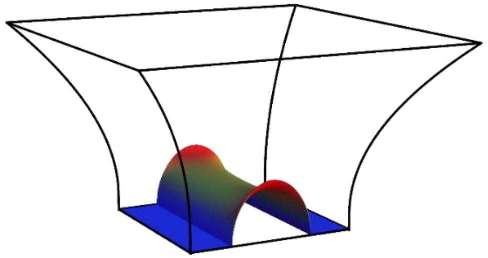


Apparent horizon: $T = -\frac{H}{2\pi}$

Entanglement horizon: $T = 0$

Event horizon: $T = \frac{H}{2\pi}$

Boundary

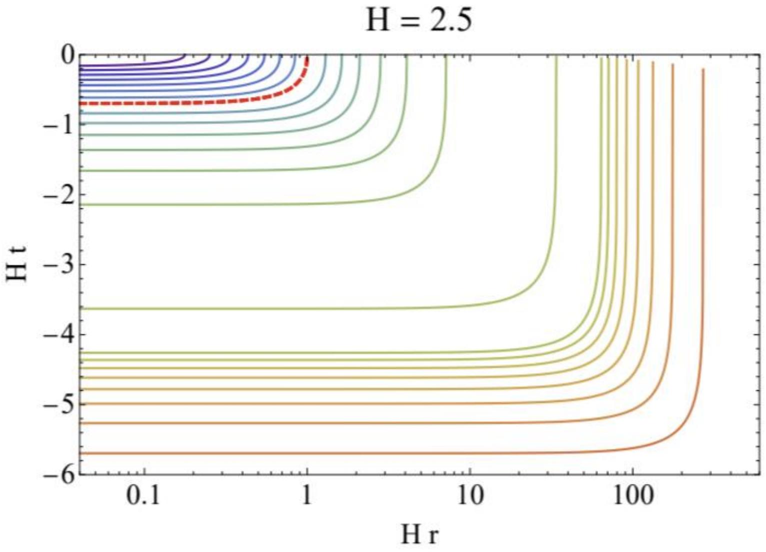


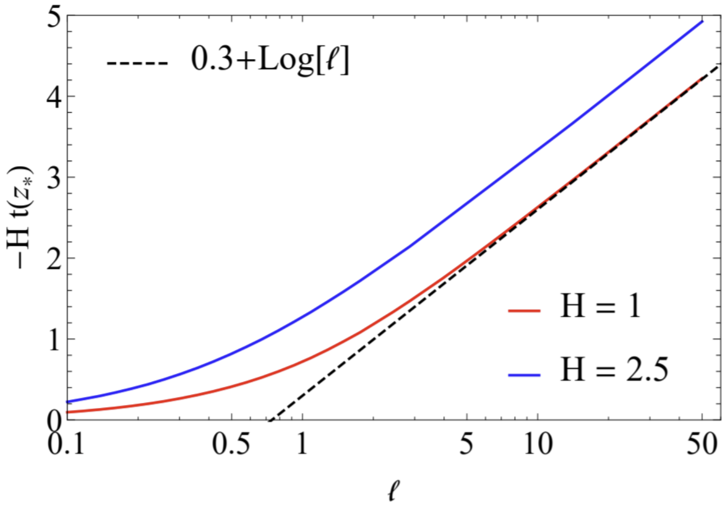
Jorge Casalderrey, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

ENTANGLEMENT IN DE SITTER

Extremal surfaces go backward in time

- Time at the deepest point grows exactly as log(l) for large l
- Implies that `entanglement horizon' contribution has a **constant instead of volume law** contribution
- Standard `area law' divergence still applies





Cosmological horizon

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

BOUNDARY GRAVITY

Study dynamics including semi-classical gravity:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G_{N,4} \langle T_{\mu\nu} \rangle$$

- Stress-tensor includes possible cosmological constant
- NB: renormalisation counterterms are now physical
- We treat the boundary Newton constant as a (small) parameter

Dynamics of scale factor $S_0(t)$ is now consequence of Friedmann equations

How to initialise the dynamics

- Start with thermal Minkowski profile and small boundary $G_{N,4}$

Jewel Ghosh, Elias Kiritsis, Francesco Nitti and Lukas Witkowski, Back-reaction in de Sitter QFTs: holography, gravitational DBI action and f(R) gravity (2020)
Paul Chesler and Abraham Loeb, Holographic duality and mode stability of de Sitter space in semiclassical gravity (2020)

BOUNDARY GRAVITY – CONSISTENT INITIAL CONDITIONS

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

Near-boundary expansion scalar (and metric):

$$\phi = \frac{M}{r} + \frac{\phi_2(t)}{r^3} + \frac{1}{r}\sum_{n\geq 3}\frac{\phi_n(t)}{r^n} + \frac{1}{r}\sum_{n\geq 2}\frac{\psi_n(t)\log r}{r^n} + \cdots$$

- Crucial subtlety: logarithmic terms ψ_n solely determined by source $S_0(t)$
- Consistent IC not much of a problem with known source (ψ_n known analytically)

Dynamics of scale factor $S_0(t)$ is now consequence of Friedmann equations

Sufficiently smooth solution requires knowledge of sufficient # of log's

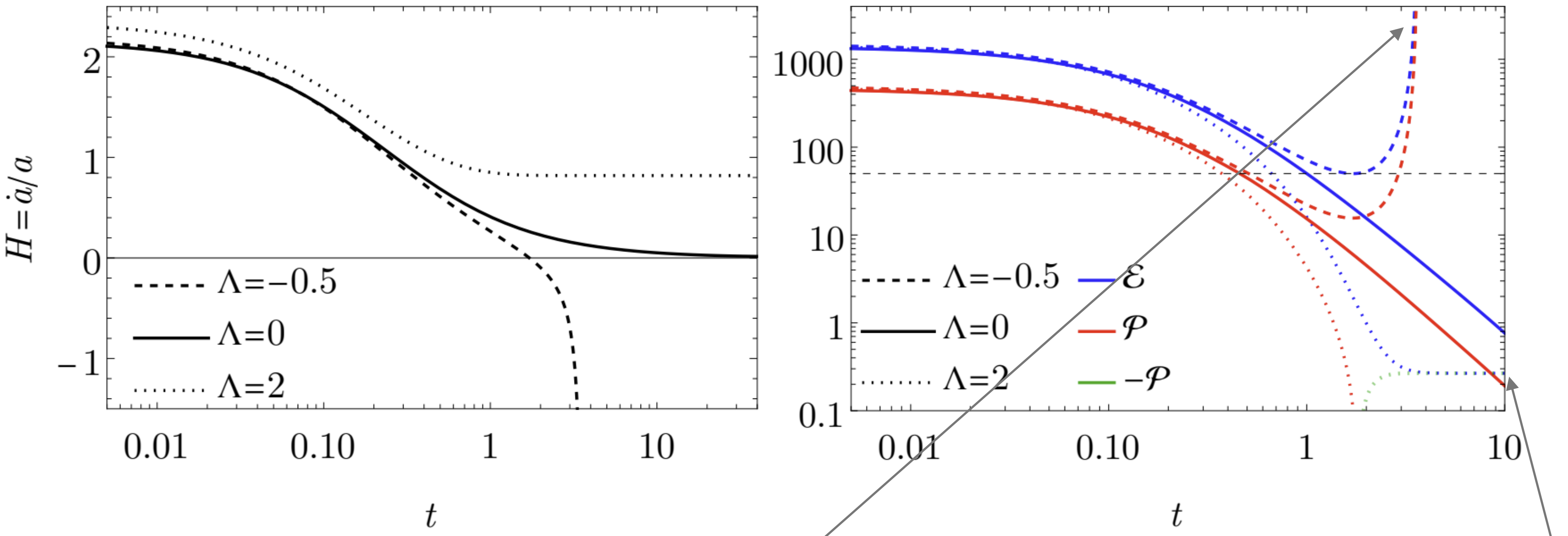
- Log's depend on time derivatives of S0(t)
- Solution: extract $\partial_t^4 S_0(t)$ from near-boundary expansion of scalar (extract ϕ_n)
- Use time derivates to treat first few logs analytically

Sebastian de Haro, Kostas Skenderis and Sergey N. Solodukhin, Holographic Reconstruction of Spacetime and Renormalization in the AdS/CFT Correspondence (2001)

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

BOUNDARY GRAVITY

Stress-energy tensor for zero, positive and negative Λ



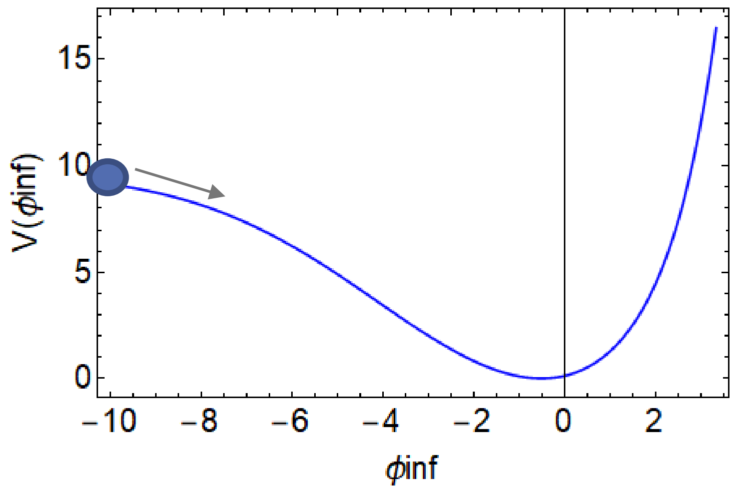
Positive Λ settles down to de Sitter state as studied before, includes (small) Casimir energy

Now also asymptotically flat + Big Crunch geometries

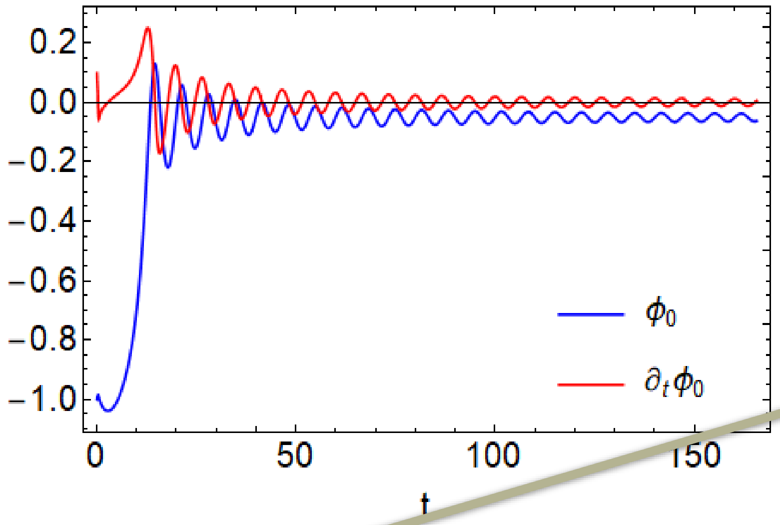
BOUNDARY GRAVITY WITH AN INFLATON

Similar idea; promote constant source M to dynamical inflaton

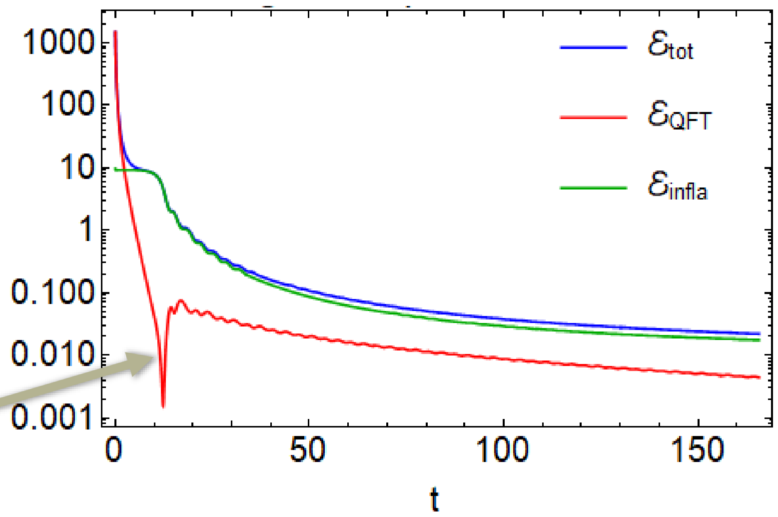
Inflaton potential



Inflaton dynamics



Inflaton and QFT energies



Allows to study reheating?

With Christian Ecker and Elias Kiritsis, in progress

DISCUSSION

Empty de Sitter space

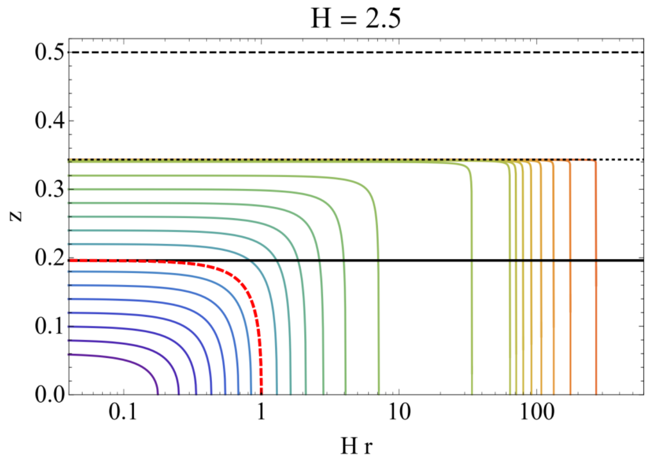
- Interesting event, apparent and entanglement horizons
- Temperature matches Hawking temperature
- Do we understand entropy of de Sitter?

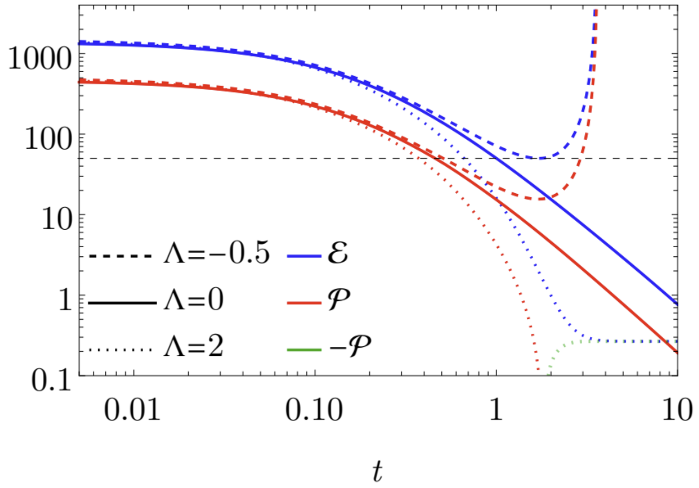
Dynamical gravity on the boundary

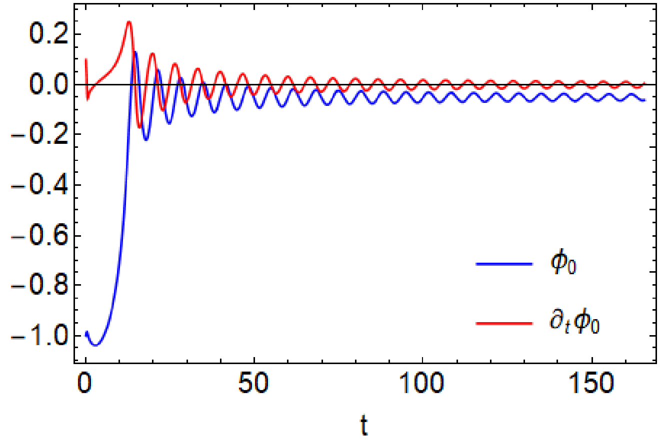
- Technical progress: how to deal with the logs?
- Several interesting evolution: flat space, Big Crunch

Future applications

- Inflaton potential; reheating?
- Relaxing symmetries?







BACK-UP

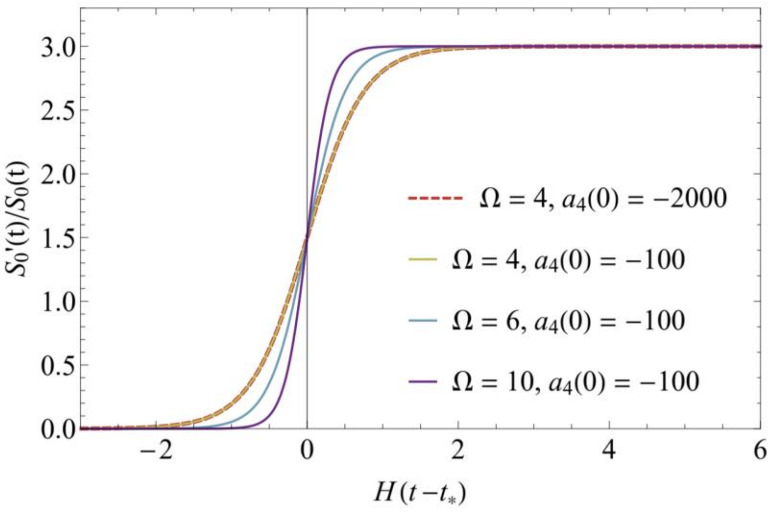
HOW WE SET UP A STATE

Non-trivial boundary metric: $ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2 \,. \quad S_0(t) = e^{Ht}$

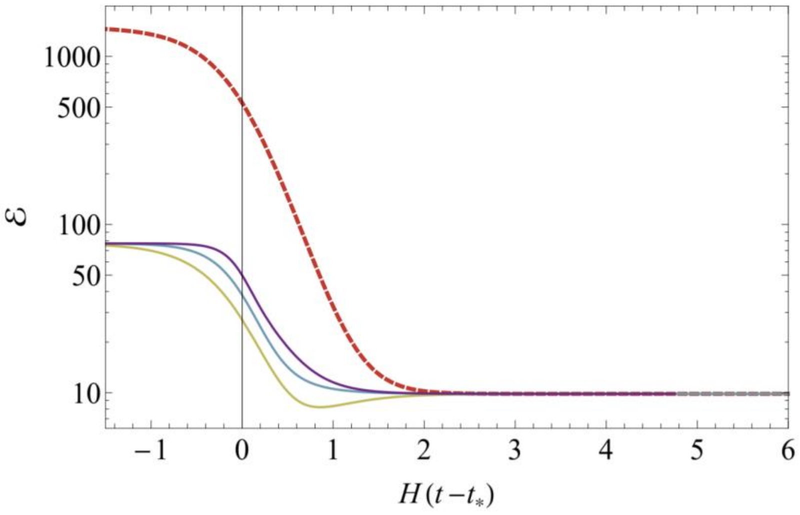
Start with thermal (high-temperature) state in flat space

- Quench system by suitable fast tanh to constant Hubble parameter
- Energy density decreases towards final `vacuum energy' (VE)
- Final (Bunch-Davis)-VE is ambiguous → chose scheme with zero VE

Hubble parameter



Energy density



$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2 . \quad S_0(t) = e^{Ht}$$

THE NUMERICAL SCHEME

Eddington-Finkelstein metric ansatz: $ds^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2 \mathrm{d}\vec{x}^2$

- Advantage: Einstein-scalar equations factorise in nested set of linear equations

Perform near-boundary expansion of A **and** S

- Subtract first few divergent and log terms → treat them analytically
- Expand remainder on finite element mesh (6 domains with 12 spectral points)
- High precision time evolution (90 digit precision, 10000 time steps)
- Evolution only 1+1D, but still takes ~1 day on a laptop

THE APPROACH TOWARDS HYDRODYNAMICS

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2 . \quad S_0(t) = e^{Ht}$$

Comparing with the hydrodynamic constituent relations:

$$T_{\perp}^{\mu\nu} = P(\varepsilon)\Delta^{\mu\nu} - \eta(\varepsilon)\sigma^{\mu\nu} - \zeta(\varepsilon)\Delta^{\mu\nu}(\nabla\cdot u) ,$$
$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

Symmetric set-up: only non-trivial part is bulk viscosity:

$$\Delta\mathcal{P}^{\text{hydro}}(t) \equiv \Delta p_{\text{eq}}(\Delta\mathcal{E}(t)) - 3H\zeta(\Delta\mathcal{E}(t)) + O(H^2) ,$$

A subtlety: EOS and viscosity computed in flat space;
what is the energy density in de Sitter space?

We decided to fix renormalisation freedom so that late time solution has zero energy density

(in any case: ambiguity is order H^2)
(also, note that scheme depends on H for our choice)

HOLOGRAPHIC RENORMALISATION

$$ds_{\rm b}^2 = -{\rm d}t^2 + S_0(t)^2 {\rm d}\vec{x}^2 \,. \quad S_0(t) = e^{Ht}$$

$$ds^2 = -A(r,t){\rm d}t^2 + 2{\rm d}r{\rm d}t + S(r,t)^2{\rm d}\vec{x}^2 \,, \quad \phi = \phi(r,t)\,,$$

Action needs (scheme-dependent) counter-terms:

$$S_{\rm ct} = \frac{1}{8\pi G}\int {\rm d}^4x\sqrt{-\gamma}\left[\left(-\frac{1}{8}R[\gamma] - \frac{3}{2} - \frac{1}{2}\phi^2\right) + \frac{1}{2}(\log\rho)\,\mathcal{A} + \left(\alpha\mathcal{A} + \beta\phi^4\right)\right],$$

$$\mathcal{A} = \mathcal{A}_g + \mathcal{A}_\phi\,, \qquad \mathcal{A}_g = \frac{1}{16}\left(R^{ij}R_{ij} - \frac{1}{3}R^2\right), \qquad \mathcal{A}_\phi = -\frac{\phi^2}{12}R$$

Leads to an ambiguity in the stress-tensor:

$$\begin{aligned}\mathcal{E}(t) = &-\frac{3a_{(4)}(t)}{4} - M\bar\phi_{(2)}(t) + \frac{3S_0'(t)^4}{16S_0(t)^4} + M^2\left(\xi(t)^2 + \frac{S_0'(t)^2}{8S_0(t)^2} + \frac{2S_0''(t)}{3S_0(t)}\right)\\ &- M^2\alpha\frac{S_0'(t)^2}{2S_0(t)^2} - M^4\left(\beta - \frac{7}{36}\right),\\ \mathcal{P}(t) = &-\frac{a_{(4)}(t)}{4} + \frac{1}{3}M\bar\phi_{(2)}(t) + \frac{S_0'(t)^2\left(S_0'(t)^2 - 4S_0(t)S_0''(t)\right)}{16S_0(t)^4}\\ &-\frac{M^2}{3}\left(\xi(t)^2 + \frac{S_0'(t)^2}{8S_0(t)^2} + \frac{13S_0''(t)}{12S_0(t)}\right) + M^2\alpha\left(\frac{S_0'(t)^2}{6S_0(t)^2} + \frac{S_0''(t)}{3S_0(t)}\right) + M^4\left(\beta - \frac{5}{108}\right)\end{aligned}$$

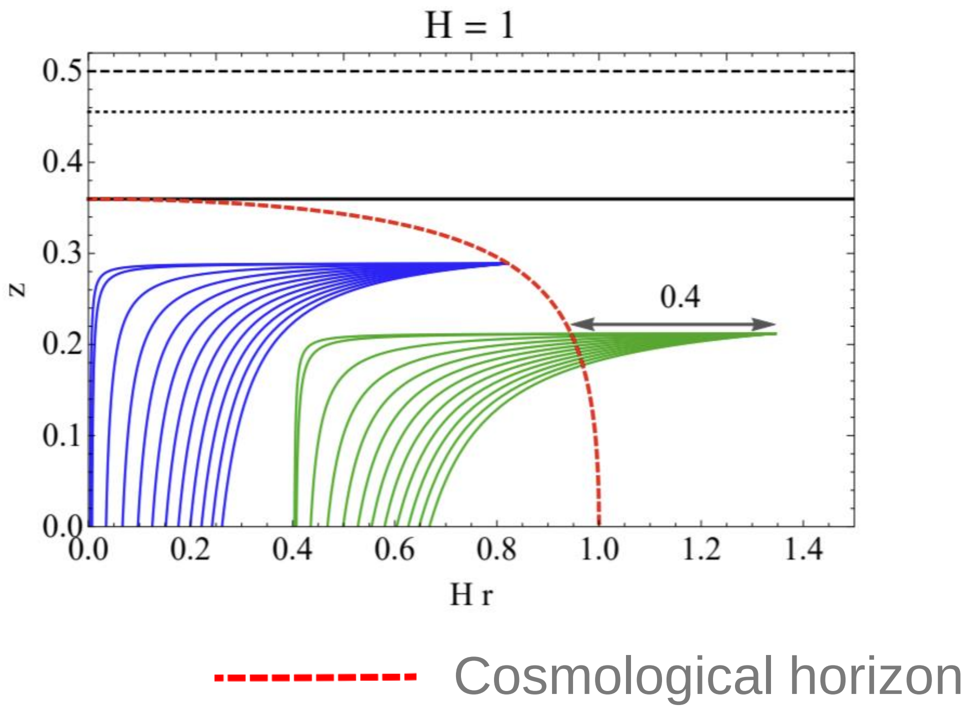
α and β encode scheme dependencies (cosmological constant); fixed such that late time solution has vanishing energy

Ambiguities come in at order H^2

COSMOLOGICAL HORIZON

Extremal surface dual to cosmological horizon:

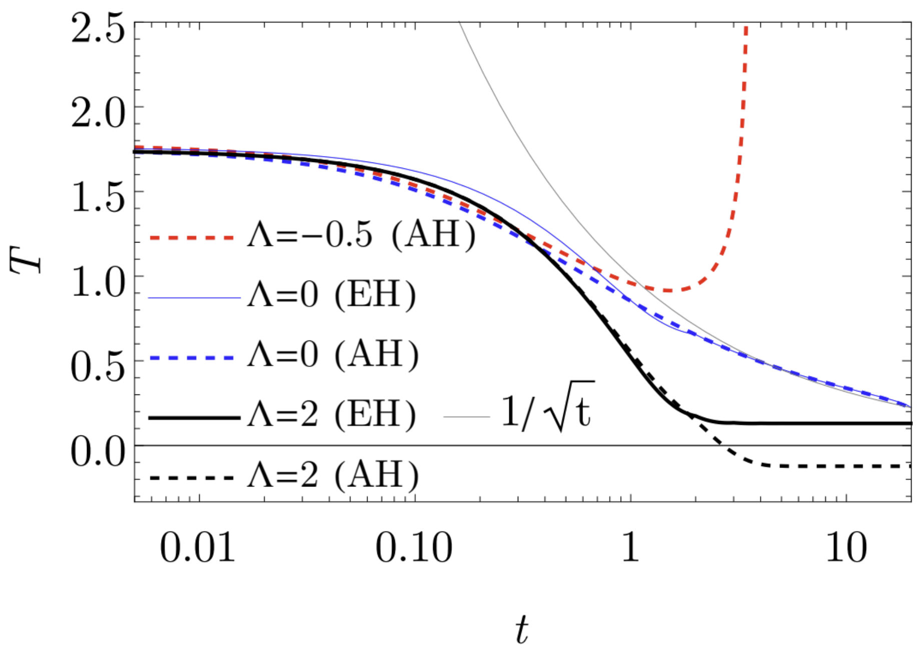
- Separates points in the bulk from which light can reach the (boundary) origin
- *Boundary cosmological horizon → full bulk cosmological horizon*



Jorge Casalderrey, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

BOUNDARY GRAVITY - TEMPERATURES

Temperatures extracted from surface gravity event and apparent horizons



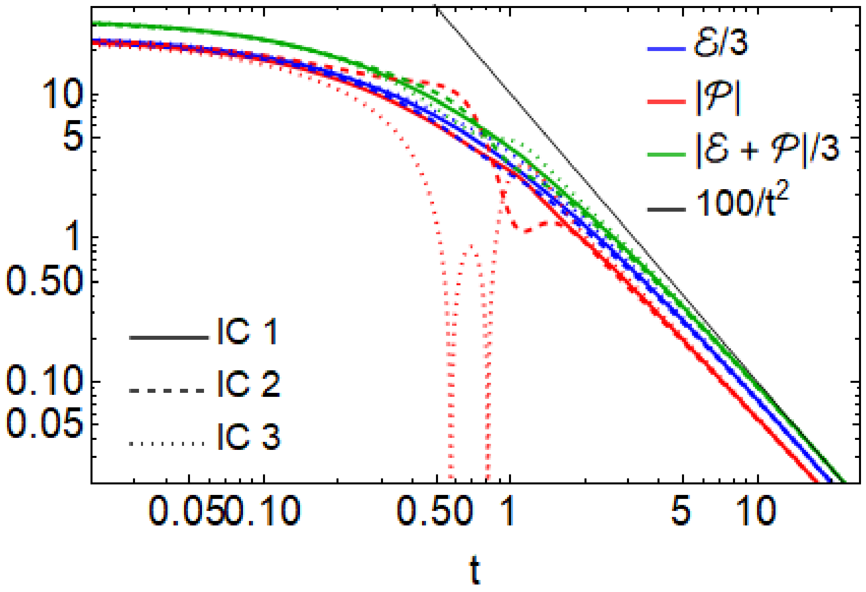
De Sitter case again settles down with negative AH 'temperature'

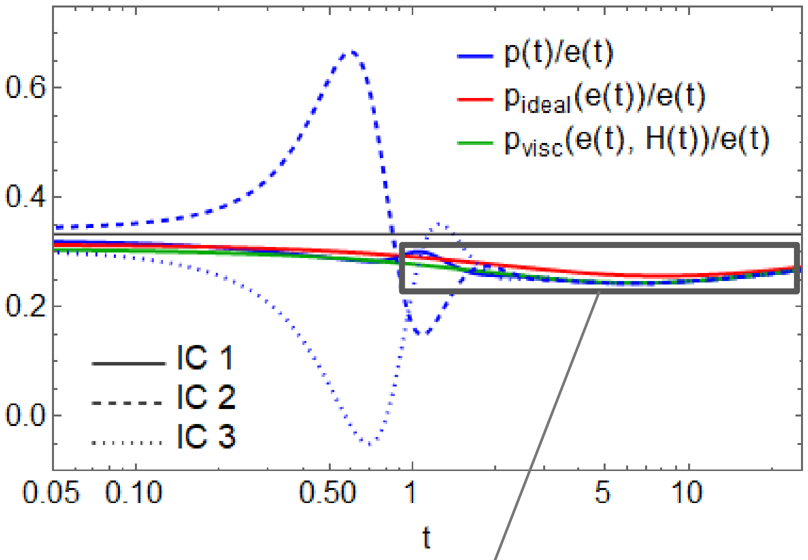
For the big crunch geometry the EH location is not obvious (no future timelike infinity)

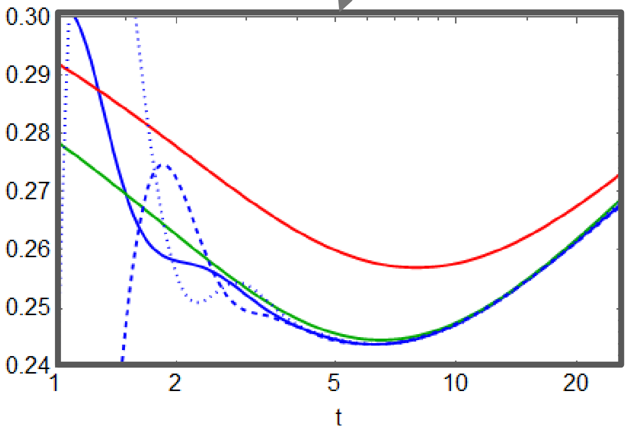
HYDRODYNAMISATION AND BOUNDARY GRAVITY

Three different initial conditions for Λ = 0:

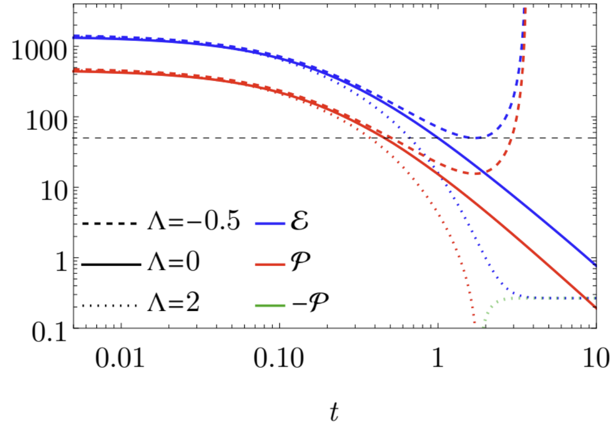
Stress-tensor, and hydrodynamisation:
all hydrodynamise within a time of ~1/T







HYDRODYNAMISATION AND BOUNDARY GRAVITY



AdS case: fast hydrodynamisation, hydro all the way till big crunch

dS case: hydrodynamisation, but falls out of equilibrium.
Casimir energy important.