Advances in $\mathrm{AdS}_3/\mathrm{CFT}_2$ and $\mathrm{AdS}_2/\mathrm{CFT}_1$

Yolanda Lozano (U. Oviedo)

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ICTEA
Instituto Universitario de Ciencias y Tecnologías del Espacio de Asturias

Universidad de Oviedo
Universidá d'Uviéu
University of Oviedo

1. Introduction

2d CFTs play a prominent role in string theory, and provide the best arena to test the AdS/CFT correspondence and its applications to black hole physics

Canonical example: D1-D5 system

$AdS_3 \times S^3 \times CY_2$ geometry realising (4,4) superconformal symmetry. CFT dual believed to be the free symmetric product orbifold (Gaberdiel, Gopakumar, Eberhardt)

For (0,4) superconformal symmetry:

(0,4) CFTs play a prominent role in the microscopical description of black holes (5d extremal BH have $AdS_3 \times S^2$ near horizon geometries) (Maldacena, Strominger, Vafa'97)

The elliptic genus has been used to compute BPS degeneracies (Minasian, Moore, Tsimpis'99)

Some of these constructions had a known holographic dual:

- Orbifolds of (4,4): $AdS_3 \times S^3/\mathbb{Z}_k \times CY_2$
- Maldacena, Strominger, Witten'97: $AdS_3 \times S^2 \times CY_3$
- Couzens, Lawrie, Martelli, Schafer-Nameki, Wong'17: $AdS_3 \times S^3 \times B_2$

In Y.L., Macpherson, Nunez, Ramirez'19: Classification of AdS_3 spaces with (0,4) supersymmetries in massive IIA

(Recently also in Type IIB (Macpherson, Ramirez'22))

AdS_2 spaces are ubiquitous in black hole physics. Therefore, constructing $\mathrm{AdS}_2/\mathrm{CFT}_1$ pairs is of key importance for the microscopical description program

However, the $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence exhibits unique features compared to its higher dimensional counterparts:

- The AdS_2 space has two disconnected boundaries
- AdS_2 gravity does not support finite energy excitations (circumvent the problem by considering $\mathrm{nAdS}_2/\mathrm{nCFT}_1$)

In certain cases the CFT_1 can be identified as a chiral half of a CFT_2 (Strominger'97; Hartman, Strominger'08; Balasubramanian, de Boer, Sheikh-Jabbari, Simon'09;..)

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Clearly, the $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence is in need of a deeper understanding

In this talk we will review the recent classification of AdS_3 solutions to massive IIA supergravity with (0,4) (small) susy (and SU(2) structure), and the proposal for their 2d dual CFTs

From these solutions:

New AdS_2 solutions with 4 supersymmetries, and a concrete proposal for their dual superconformal quantum mechanics

Interpretation as surface and line CFTs

(Based on papers with C. Couzens, F. Faedo, N. Macpherson, C. Nunez, N. Petri, A. Ramirez, S. Speziali and S. Vandoren 2019-2021)

2. AdS_3 solutions to massive IIA with (0,4) susy

Explicit construction of the Killing spinors that realise the bosonic subalgebra $SO(2,2) \oplus SU(2)$ of $AdS_3 \times S^2$:

$AdS_3 \times S^2 \times M_4 \times I$ solutions to massive IIA, with

- $M_4 = CY_2$: class I
- $M_4 =$ Kähler : class II

$I = S^1$: $M_4 = CY_2$: $AdS_3 \times S^3 \times CY_2$ after T-duality

$M_4 =$ Kähler: $AdS_3 \times S^3 \times B_2$ with $F_3 \neq 0$ after T-duality

Generalisation of the solutions of Couzens, Lawrie, Martelli, Schafer-Nameki, Wong'17 to include D5-branes

Concentrate in class I:

$$ds^2 = \frac{u}{\sqrt{h_4 h_8}}\Big(ds^2_{AdS_3} + \frac{h_8 h_4}{4h_8 h_4 + u'^2} ds^2_{S^2}\Big) + \sqrt{\frac{h_4}{h_8}} ds^2_{CY_2} + \frac{\sqrt{h_4 h_8}}{u} d\rho^2$$

u, h_8 : Linear functions in ρ ; h_4 : Function of ρ and CY_2

Non-vanishing B_2 such that: one NS5 at $\rho = 2n\pi$

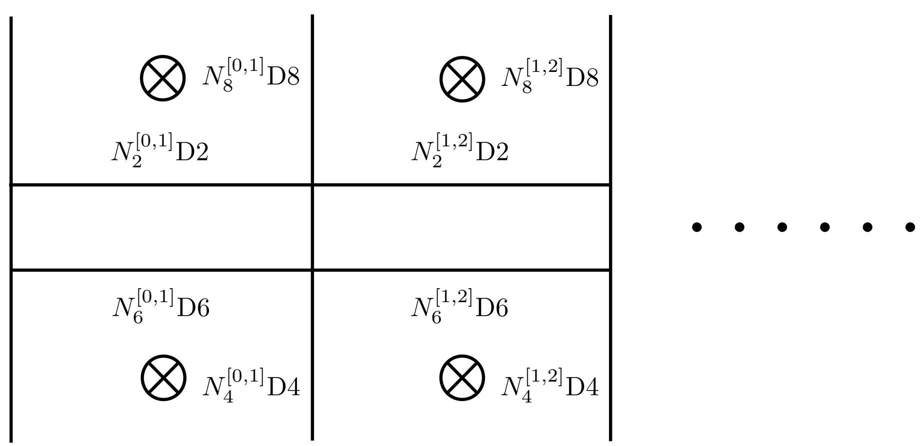
Page fluxes	Branes
$F_0 = h_8'$	D8
$\hat{F}_2 = \Big(h_8 - (\rho - 2n\pi)h_8'\Big)\mathrm{vol}_{S^2}$	D6
$\hat{F}_4 = h_4' \mathrm{vol}_{CY_2}$	D4
$\hat{F}_6 = \Big(h_4 - (\rho - 2n\pi)h_4'\Big)\mathrm{vol}_{CY_2} \wedge \mathrm{vol}_{S^2}$	D2

D2 and D6 branes are stretched between NS5-branes. They play the role of colour branes

D4 and D8 are perpendicular, and play the role of flavour branes

Supported by the analysis of Bianchi identities

Brane set-up:



We will concentrate on solutions with constant u and h_4, h_8 piecewise linear, with the change of slope due to the presence of D4 and D8 source branes, respectively

We will also impose that $h_4(2\pi(P+1)) = h_8(2\pi(P+1)) = 0$ such that the space terminates at $\rho = 2\pi(P+1)$ and we can have a well defined dual CFT

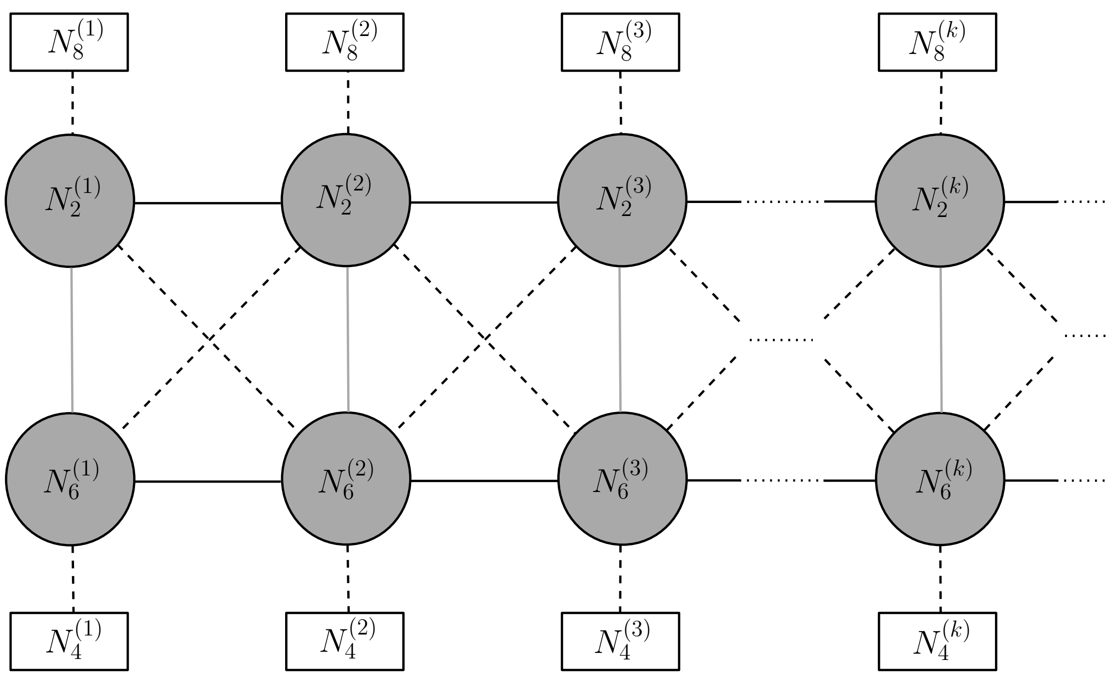
Our proposal: The brane set-up describes a (0,4) QFT that flows to a strongly coupled CFT in the IR, dual to our solutions

3. The 2d CFT

Main features of the brane set-up:

D2 and (wrapped) D6 colour branes

D4 and (wrapped) D8 flavour branes



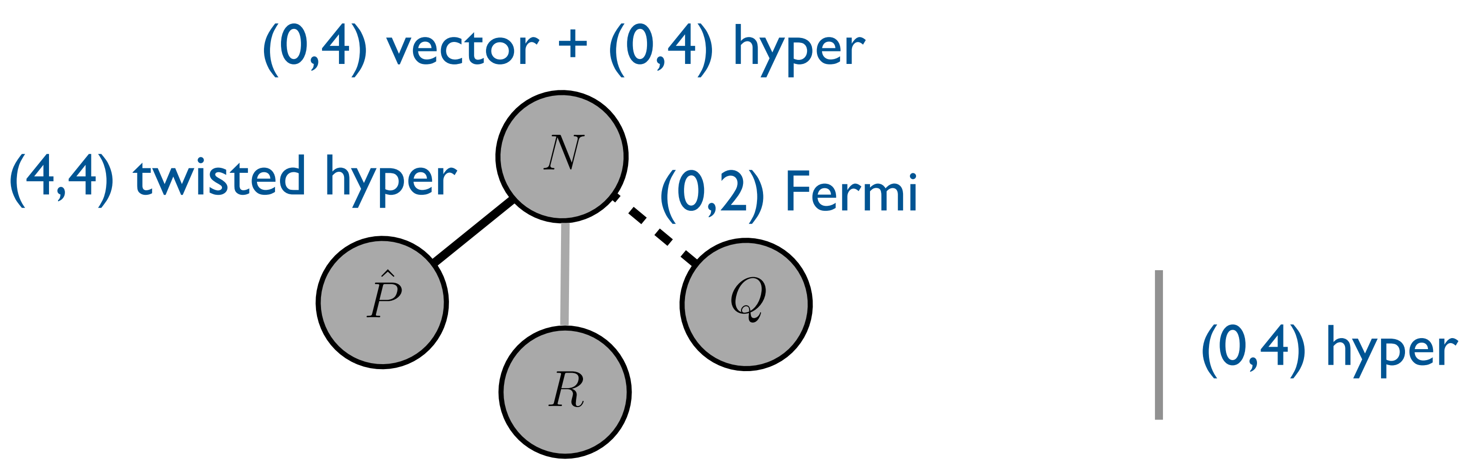
We can build 2d (0,4) quivers with (0,4) vector multiplets, hyper multiplets, twisted hypermultiplets and (0,2) Fermi multiplets

Tong'15

These come from the quantisation of the open strings with ends attached to the different stacks of branes

The resulting gauge theories can be anomalous, since left and right moving fermions need not be paired together

Non-anomalous quivers are obtained by assembling the building block:



This building block is non-anomalous if $2R = Q$

This must be satisfied at each node of the quiver

This is indeed satisfied with the quantised charges associated to the solutions!

We can then compute the central charge of the CFT in the IR, using that the (0,4) superconformal algebra relates the central charge with the R-symmetry anomaly (= level of the superconformal R-symmetry)[1]

$$c = 3k = 6(n_{hyp} - n_{vec})$$

[1] The energy momentum tensor and the R-symmetry current sit in the same multiplet

The holographic central charge, in turn, is computed from

$$c_{hol} = \frac{3R}{2G_3} = \frac{3}{\pi} \int d\rho \, h_4 h_8$$

(Brown, Henneaux'86)

Both expressions should agree in the holographic limit.

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Both expressions should agree in the holographic limit.

Taking $h_4(\rho) = \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k)$ and

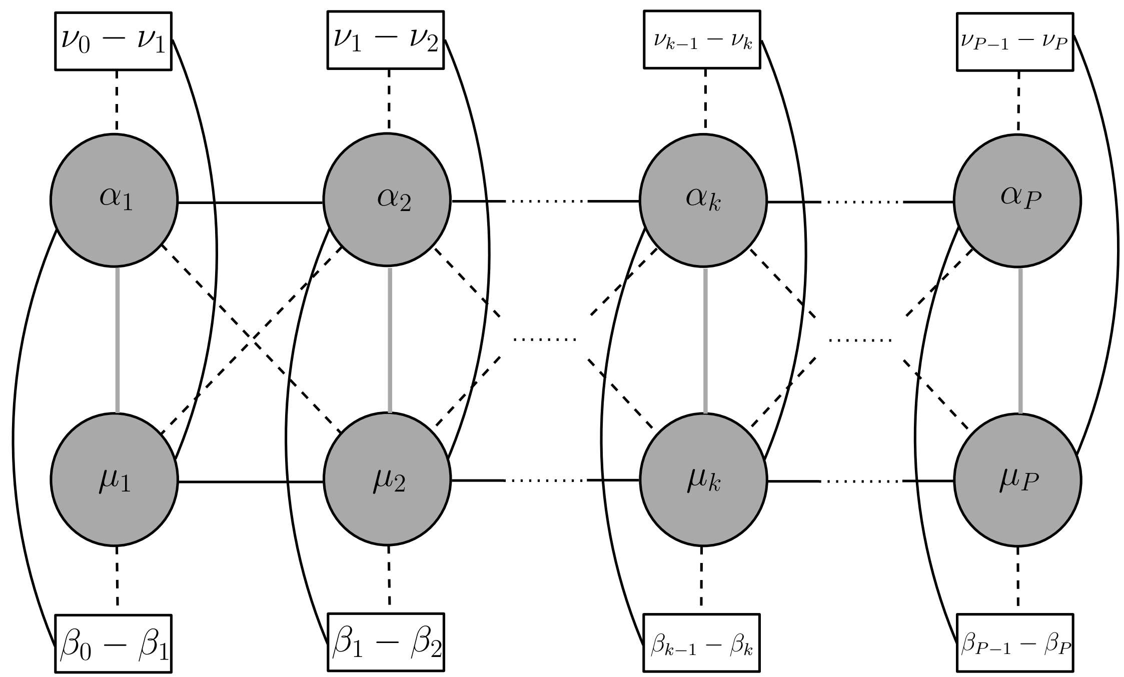
$h_8(\rho) = \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k)$ for $\rho \in [2\pi k, 2\pi(k+1)]$

we get

$c = 6\sum_{k=1}^{P} \alpha_k \mu_k$ to leading order from both expressions!

(Couzens, Y.L., Petri, Vandoren'21)

The non-anomalous quivers are:



4. New AdS_2 solutions in Type II with 4 SUSY

One class in Type IIB is constructed by T-dualising the $AdS_3 \times S^2 \times \mathrm{CY}_2 \times I$ solutions to massive IIA, along the Hopf fibre of AdS_3 :

$$ds^2_{AdS_3} = (d\psi - \sinh r dt)^2 + ds^2_{AdS_2}$$

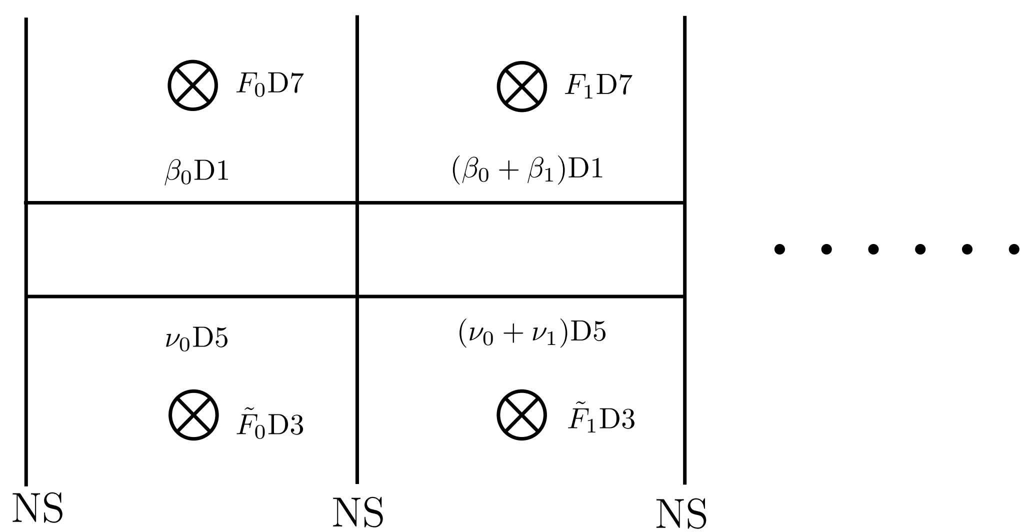
This gives a family of $AdS_2 \times S^2 \times \mathrm{CY}_2 \times S^1 \times I$ solutions with 4 supersymmetries.

These solutions extend the class of $AdS_2 \times S^2 \times \mathrm{CY}_2 \times \Sigma_2$ solutions of Type IIB by Chiodaroli, Gutperle, Krym'09; Chiodaroli, D'Hoker, Gutperle'09, for Σ_2 an annulus, to include D7-branes

In the previous parametrisation the boundaries of AdS_3 are two null cylinders at $r \to \pm\infty$

$\Rightarrow$ The dual SCQM arises as a light-like compactification of the 2d CFT dual to the AdS_3 sols

From the geometry, one constructs the Hanany-Witten brane set-ups:



Taking $h_4(\rho) = \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k)$ and

$h_8(\rho) = \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k)$ for $\rho \in [2\pi k, 2\pi(k+1)]$

we construct the same quiver gauge theories dual to the $AdS_3 \times S^2 \times \mathrm{CY}_2 \times I$ solutions, now built out of 1d N=4 matter multiplets

These quivers inherit the anomaly cancellation conditions of the 2d theory

Our proposal is that these quantum mechanics flow in the IR to the N=4 SCQM dual to our solutions

A check is provided with the computation of the *central charge*:

In gravity:

$$c_{hol} = \frac{3}{4\pi G_2} = \frac{3}{\pi}\int d\rho\, h_4 h_8$$

If we use $c = 6(n_{hyp} - n_{vec})$ in the field theory side, we find perfect agreement with the holographic result.

This formula looks like a generalisation of expressions in the literature for the number of ground states of quiver QM with gauge group $\prod_v U(N_v)$ and bifundamental fields joining the gauge groups with adjacent ones (Denef'02): $\mathcal{M} = \sum_{v,w} N_v N_w - \sum_v N_v^2 + 1$

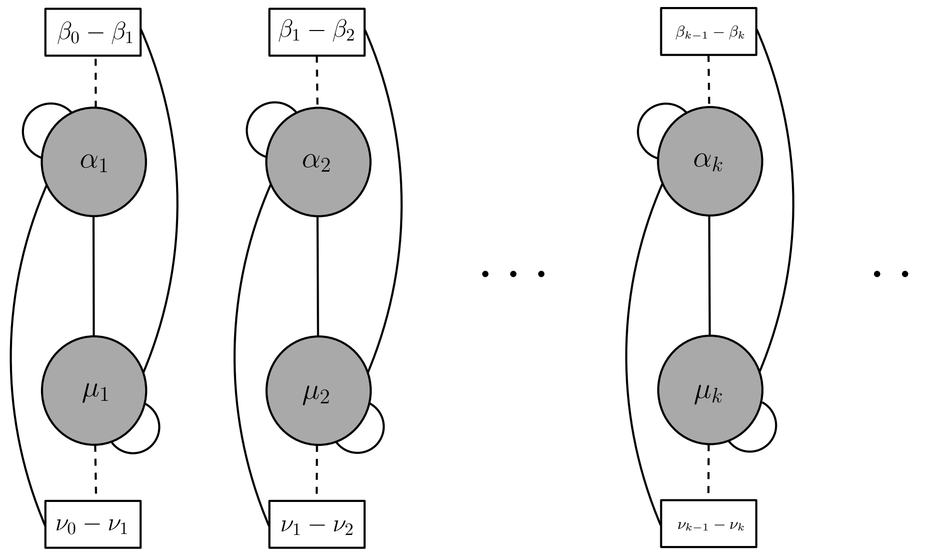
A class of solutions to massive IIA is obtained by analytically continuing the $AdS_3 \times S^2 \times \mathrm{CY}_2 \times I$ solutions.

This gives a family of $AdS_2 \times S^3 \times \mathrm{CY}_2 \times I$ solutions with 4 supersymmetries

The associated brane set-up is

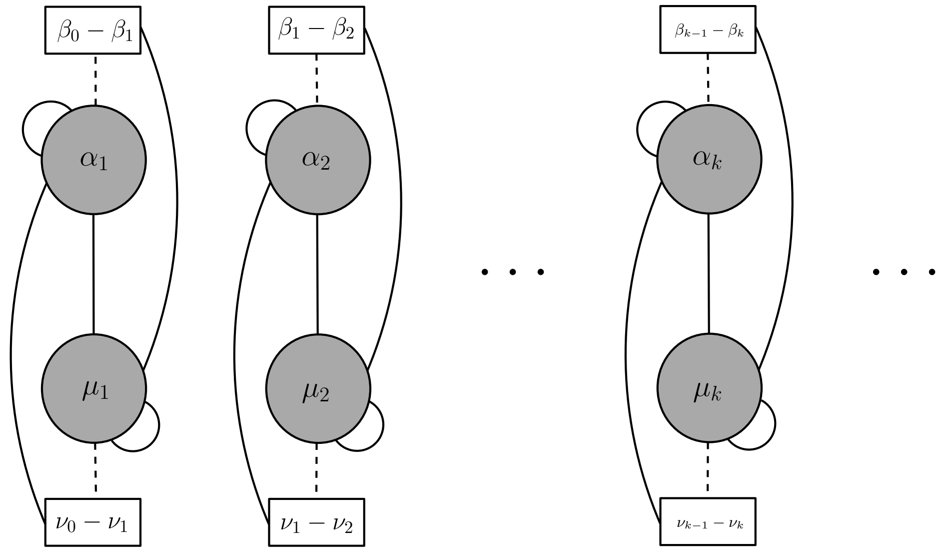
	t	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
$D0$	$\times$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$D4$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$
$D4'$	$\times$	$-$	$-$	$-$	$-$	$-$	$\times$	$\times$	$\times$	$\times$
$D8$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$\times$	$\times$	$\times$	$\times$
$F1$	$\times$	$-$	$-$	$-$	$-$	$\times$	$-$	$-$	$-$	$-$

From it we can construct the quivers



The formula $c = 6(n_{hyp} - n_{vec})$, agrees, once more, with the holographic central charge

From it we can construct the quivers



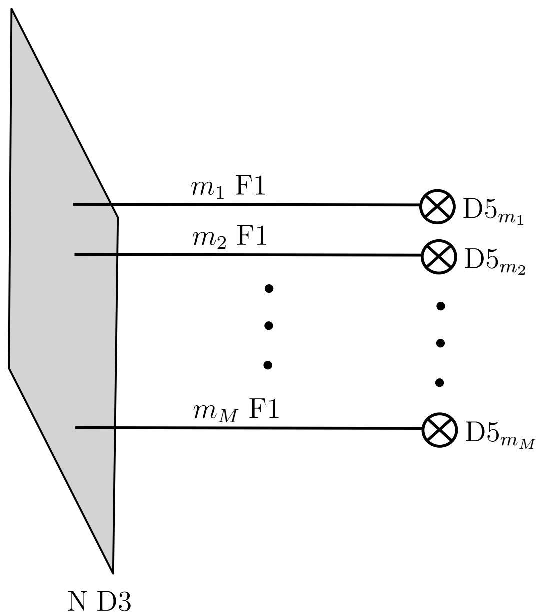
The formula $c = 6(n_{hyp} - n_{vec})$, agrees, once more, with the holographic central charge

In the massless case these solutions arise from AdS_3 spaces in M-theory $\Rightarrow$ They are also null orbifolds

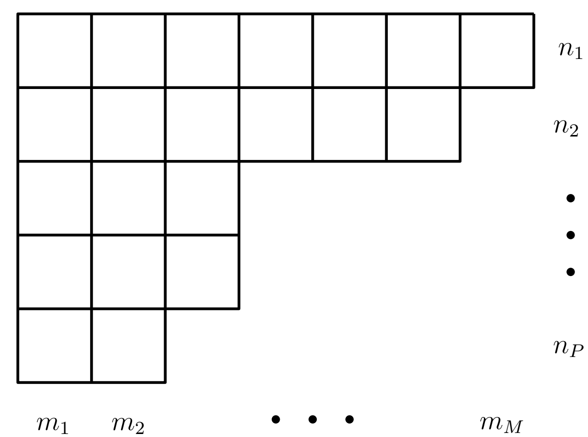
But this is not so in the massive case

The F1-strings describe Wilson loops in antisymmetric reps. of the gauge groups $U(\alpha_k) \times U(\mu_k)$:

Brane realisation of Wilson loops in 4d N=4 SYM
(Yamaguchi'06; Gomis, Passerini'06):



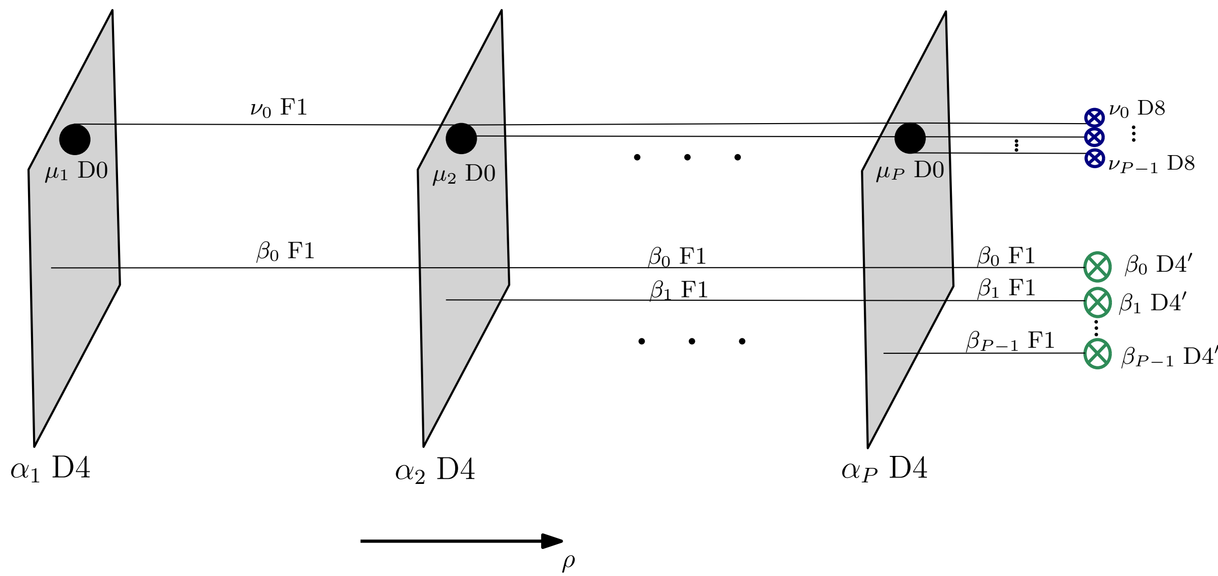
$$
\begin{array}{l|l|lllllllll}
D3: & \times & \times & \times & \times & - & - & - & - & - & - \\
D5: & \times & - & - & - & \times & \times & \times & \times & \times & - \\
F1: & \times & - & - & - & - & - & - & - & - & \times
\end{array}
$$

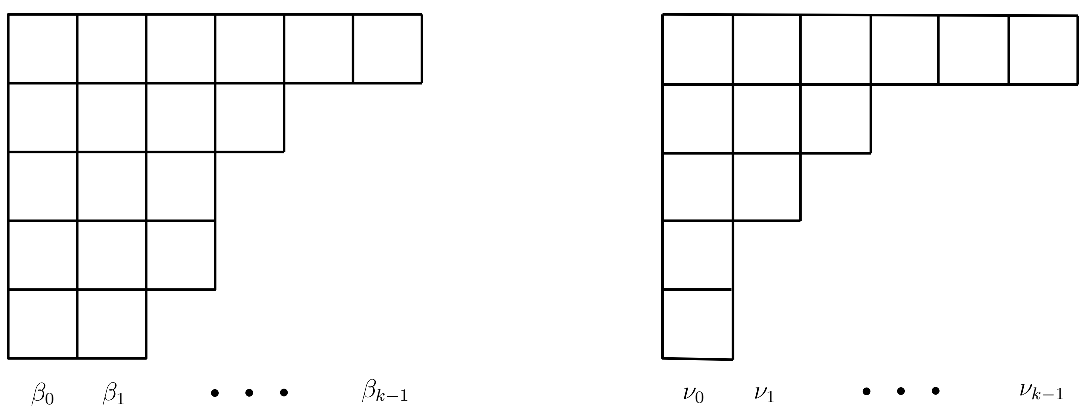


Array of M D5-branes with $(m_1, \ldots, m_M)$ F-strings stretched between them and the N D3 branes

Young tableau labelling the irrep of U(N)

In our case:





Completely antisymmetric representations $(\beta_0, \beta_1, \ldots, \beta_{k-1})$ $(\nu_0, \nu_1, \ldots, \nu_{k-1})$ of $U(\alpha_k)$ and $U(\mu_k)$

5. Defect interpretation

Defect $\leftrightarrow$ Operator insertion realising a deformation of a CFT

When $CY_2 = T^4$ and $h_4(\rho, T^4)$ our previous solutions flow to the $AdS_6 \times S^3 \times I$ solution of Brandhuber, Oz'99, near horizon of D4-D8 branes, asymptotically, with additional fluxes associated to the extra defect branes (Faedo, Y.L., Petri'20)

The defect branes break the supersymmetries by a half, and the conformal symmetry from $SO(5,2) \oplus SU(2)$ to $SO(2,2) \oplus SU(2)$ or $SO(2,1) \oplus SU(2)$

The AdS_3 solutions should be holographically dual to surface operators in the 5d Sp(N) gauge theory

The AdS_2 solutions describe backreacted baryon vertices in the 5d theory

In this interpretation the SCQM arises in the low energy limit of a system of D4'-D8 branes, dual to a 5d N=1 theory, in which these 1d defects are introduced

In the IR the gauge symmetries of the D4'-D8 become global and the branes become flavour branes

The D4-D0 branes become the colour branes of the backreacted geometry

6. Conclusions

- AdS_3 solutions to massive IIA with (0,4) susy classified
- 2d dual CFTs identified as IR fixed points of 2d QFTs built out of (0,4) multiplets
- Duality checked with the computation of the central charge
- New classes of AdS_2 solutions in IIB and massive IIA, extending previous classifications
- Explicit realisations of SCQM as chiral halves of 2d CFTs, dual to AdS_2 solutions with constant electric flux (Hartman, Strominger'08)
- Surface and line defect interpretation within the 5d Sp(N) fixed point theory

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THANKS!