

Dark Gravity confronted with SN, BAO and the CMB

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Dark Gravity : extension of General Relativity aiming at a stable anti-gravitational sector

- Theoretical motivations :
 - Understand negative energy solutions
 - Avoid infinities (Black Hole and Big Bang singularities, vacuum energy, ...)
- Phenomenological motivations :
 - Repellent gravity to accelerate the universe
 - Repellent gravity to mimic Dark Matter phenomenology
- Challenge : instability issues !

From background dependence to Dark Gravity (DG)

How far can we go ?

GR : $g_{\mu\nu}$

DG : $g_{\mu\nu}$ and $\eta_{\mu\nu}$

$$\text{Riem}(\eta_{\mu\nu})=0$$

\Rightarrow $g_{\mu\nu}$ has a twin, « the inverse metric » $\tilde{g}_{\mu\nu}$

$$\tilde{g}_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma} [g^{-1}]^{\rho\sigma}$$

\Rightarrow $(g_{\mu\nu}, \tilde{g}_{\mu\nu})$ is a Janus field



From the Action to DG field equations

The Action must respect the **permutation symmetry** between $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$:

$$\int d^4x (\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) + \int d^4x (\sqrt{g}L + \sqrt{\tilde{g}}\tilde{L})$$

$$\delta g_{\mu\nu} \Rightarrow \delta S = 0$$



$$\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}G^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{G}^{\rho\mu} = -8\pi G(\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}T^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{T}^{\rho\mu})$$

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T})$$

Implications of DG equations

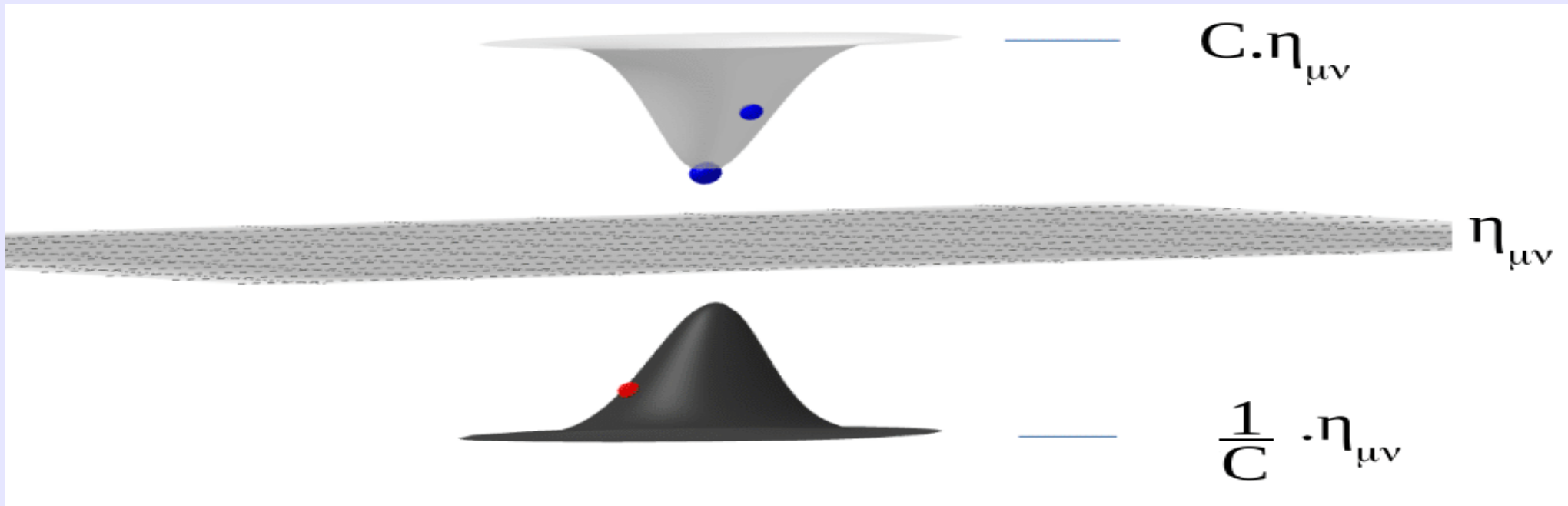
- DG is background dependent yet deviations from GR can remain arbitrarily small provided one side of the Janus Field dominates the other.
- Ghost interaction between Janus and source fields but Janus field not understood to be a quantum field !
 - DG more natural than GR as a semiclassical* theory of gravity
 - Semiclassical DG stability : OK**
- New discrete (permutation) symmetry is very fundamental : will be interpreted as a global time reversal symmetry.

* [arxiv:0802.1978](https://arxiv.org/abs/0802.1978) Mark Albers, Claus Kiefer, Marcel Reginatto, Measurement Analysis and Quantum Gravity : « Despite the many physical arguments which speak in favor of a quantum theory of gravity, it appears that the justification for such a theory must be based on empirical tests and does not follow from logical arguments alone »

[arxiv:1903.01823](https://arxiv.org/abs/1903.01823) A.Tilloy, 2019. Does gravity have to be quantized? Lessons from non-relativistic toy models, proceedings of the DICE 2018 workshop.

** [arxiv:1401.4024](https://arxiv.org/abs/1401.4024) V. A. Rubakov, page 8 : Gradient, tachyonic and ghost instabilities in scalar-tensor theories : « for ghosts, background is QM unstable but classically stable »

The static isotropic solution



- Antigravity without run away !
- Asymptotic C matters : GR corresponds to C infinite

Time reversal and negative energies

- Problem : all fundamental relativistic equations have negative energy solutions banished without really convincing reason (anti-particle, unobserved, fatally unstable)
- Unitary time reversal T is the natural symmetry between $E > 0$ and $E < 0$ fields (phase factor $e^{i(Et/\hbar - \vec{p}\vec{r}/\hbar)}$ must be invariant)
- But to work properly, Unitary T needs an extension of GR in which :
 - T (unlike a mere reparametrization) must transform $g_{\mu\nu}$ into another $\tilde{g}_{\mu\nu}$
 - T needs a privileged fixed origin of time (t, 0, -t) and fixed pivot metric $(g_{\mu\nu}, \eta_{\mu\nu}, \tilde{g}_{\mu\nu})$
- The extension of GR with a non dynamical background $\eta_{\mu\nu}$ is DG :
 - ⇒ Janus field $(g_{\mu\nu}, \tilde{g}_{\mu\nu})$ with permutation symmetry
 - ⇒ new understanding of time reversal

The static isotropic solution

C=1

DG:

$$g_{ii}(r) = A = e^{2MG/r} \approx 1 + 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2}$$

$$-g_{00}(r) = \frac{1}{A} = e^{-2MG/r} \approx 1 - 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2} - \frac{4}{3}\frac{M^3G^3}{r^3}$$

C=∞

RG (Schwarzschild) :

$$g_{ii}(r) = \left(1 + \frac{MG}{2r}\right)^4 \approx 1 + 2\frac{MG}{r} + \frac{3}{2}\frac{M^2G^2}{r^2}$$

$$-g_{00}(r) = \frac{\left(1 - \frac{MG}{2r}\right)^2}{\left(1 + \frac{MG}{2r}\right)^2} \approx 1 - 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2} - \frac{3}{2}\frac{M^3G^3}{r^3}$$

- No Horizon
- Zero Gravitational Waves

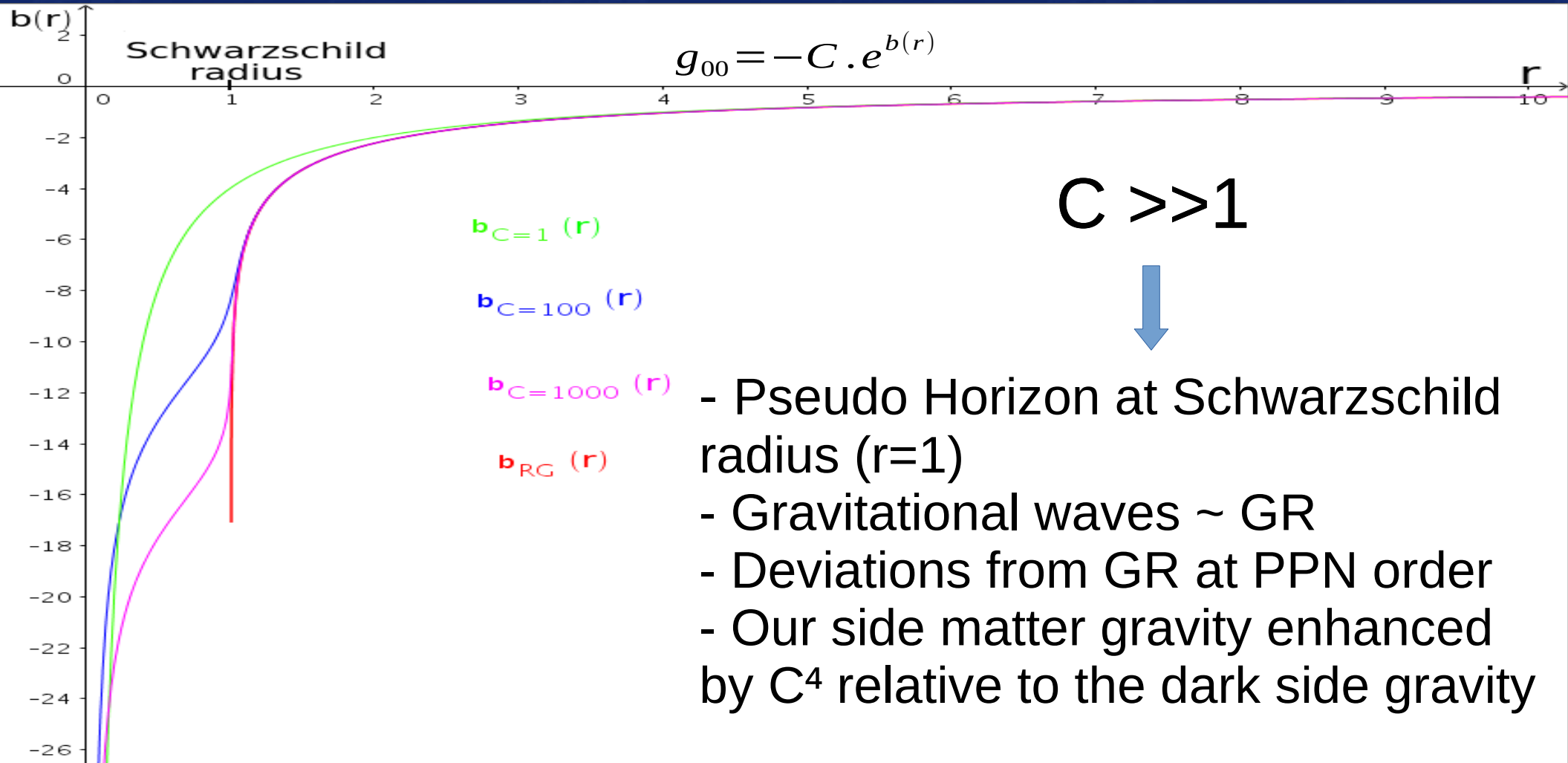
$$\tilde{h}_{\mu\nu} = -h_{\mu\nu} + O(\hbar^2)$$

$$2(R_{\mu\nu}^{(1)} - \frac{1}{2}\eta_{\mu\nu}R_{\lambda}^{(1)\lambda}) = -8\pi G(T_{\mu\nu} - \tilde{T}_{\mu\nu} + t_{\mu\nu} - \tilde{t}_{\mu\nu})$$

→ 0

- Deviations from GR at PPN order only

The static isotropic solution



Homogeneous flat metrics in privileged coordinate system

2 (out of many) possible choices



?



$$-d\tau^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

$$-d\tau_{\text{Minkowski}}^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$-d\tilde{\tau}^2 = -dt^2 + \frac{1}{a^2(t)}(dx^2 + dy^2 + dz^2)$$

Standard metrics

$$-d\tau^2 = a^2(t)(-dt^2 + dx^2 + dy^2 + dz^2)$$

$$-d\tau_{\text{Minkowski}}^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$-d\tilde{\tau}^2 = \frac{1}{a^2(t)}(-dt^2 + dx^2 + dy^2 + dz^2)$$

Conformal metrics



Cosmological equations

$$g_{\mu\nu} = a^2(t) \eta_{\mu\nu}$$

$$\tilde{a}(t) = \frac{1}{a(t)}$$

$$\tilde{g}_{\mu\nu} = a^{-2}(t) \eta_{\mu\nu}$$

- Problem : Homogeneous & isotropic Janus solution is flat but static !

$$a^2 H^2 - \tilde{a}^2 \tilde{H}^2 = \frac{8\pi G}{3} (a^4 \rho - \tilde{a}^4 \tilde{\rho})$$

$$a^2 (2\dot{H} + H^2) - \tilde{a}^2 (2\dot{\tilde{H}} + \tilde{H}^2) = -8\pi G (a^4 p - \tilde{a}^4 \tilde{p})$$

- Solution : Introduce offshell mechanism $\Gamma(t)$: matter-radiation exchange

or variable $\tilde{G}(t) = \frac{1}{G(t)}$

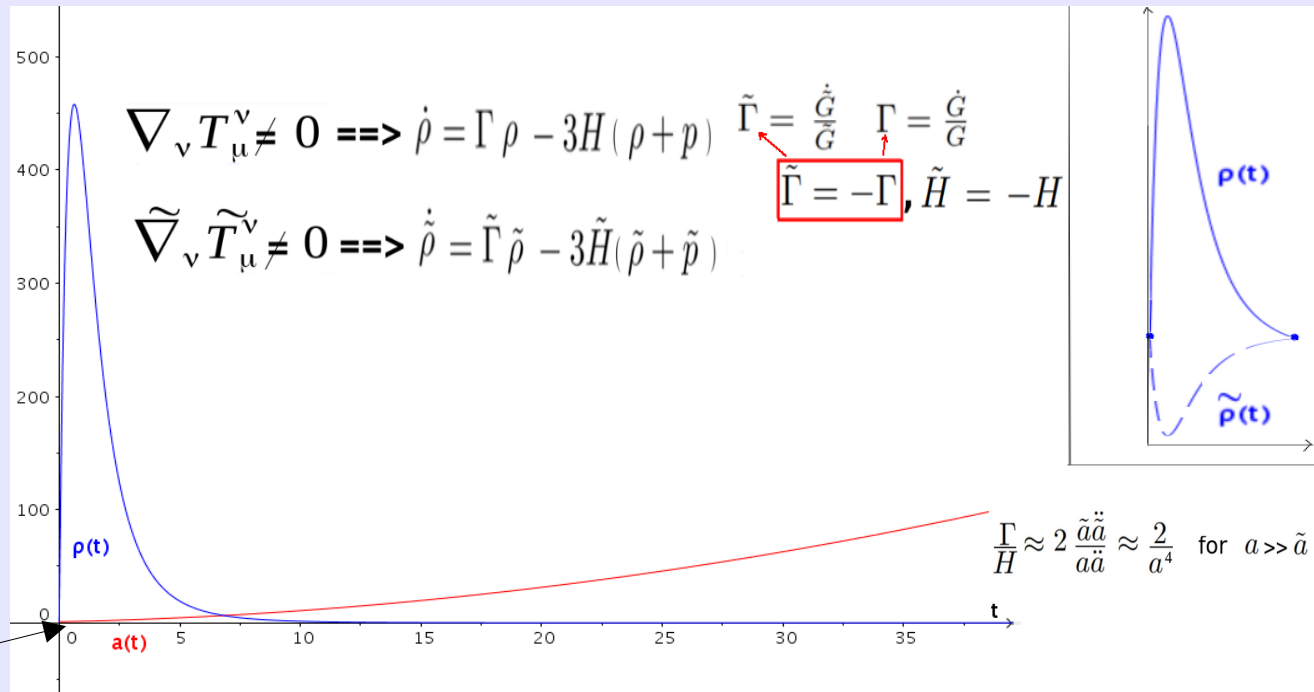
$$\nabla_{\nu} T_{\mu}^{\nu} \neq 0 \implies \dot{\rho} = \Gamma \rho - 3H(\rho + p) \quad \tilde{\Gamma} = \frac{\dot{\tilde{G}}}{\tilde{G}} \quad \Gamma = \frac{\dot{G}}{G}$$

$$\tilde{\nabla}_{\nu} \tilde{T}_{\mu}^{\nu} \neq 0 \implies \dot{\tilde{\rho}} = \tilde{\Gamma} \tilde{\rho} - 3\tilde{H}(\tilde{\rho} + \tilde{p}) \quad \tilde{\Gamma} = -\Gamma, \tilde{H} = -H$$

- Then cosmological equations have realistic solutions

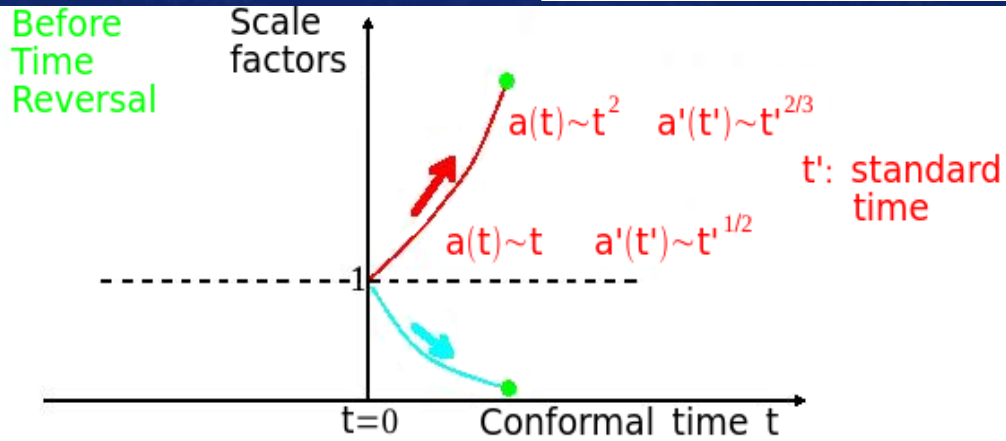
DG solutions with offshell $\Gamma(t)$

- Differential equations can be solved numerically :



DG time reversal

$$a^2 H^2 - \tilde{a}^2 \tilde{H}^2 = \frac{8\pi G}{3} (\rho a^4 - \tilde{\rho} \tilde{a}^4)$$



- Janus scale factors are related by a **global conformal time reversal symmetry T** :

$$\tilde{a}(t) = \frac{1}{a(t)} = a(-t)$$

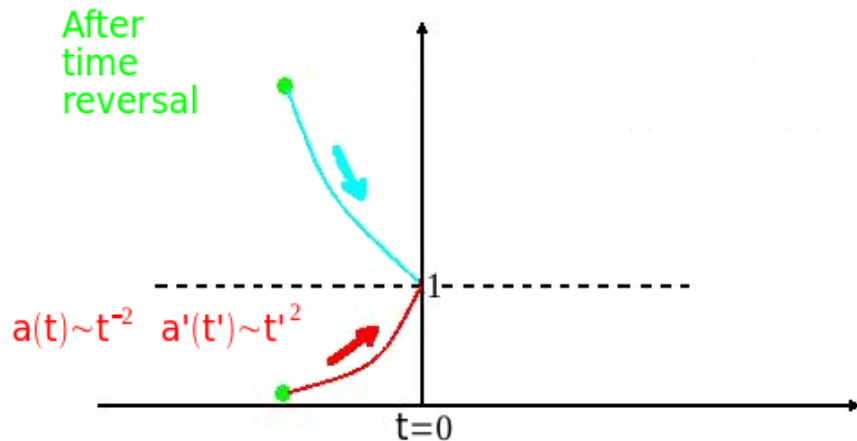
- Discontinuous permutation T** allowed when

$$\rho = \tilde{\rho}$$



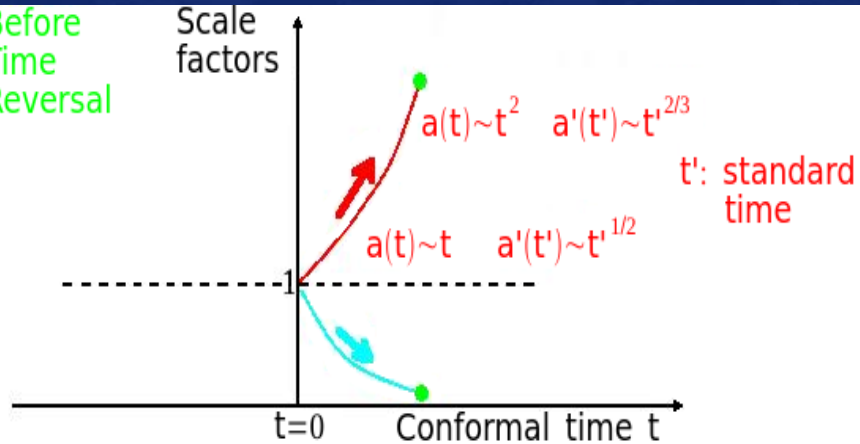
Global time reversal : not going backward in time, but jumping to the opposite time !

⇒ A cyclic Universe ?

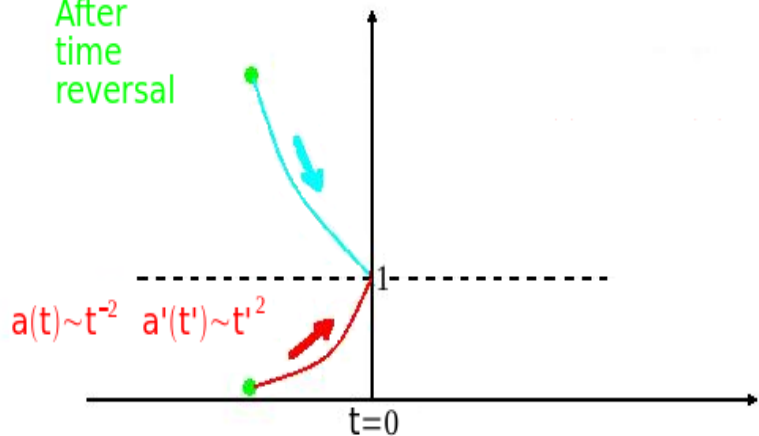


DG Cosmology

Before
Time
Reversal

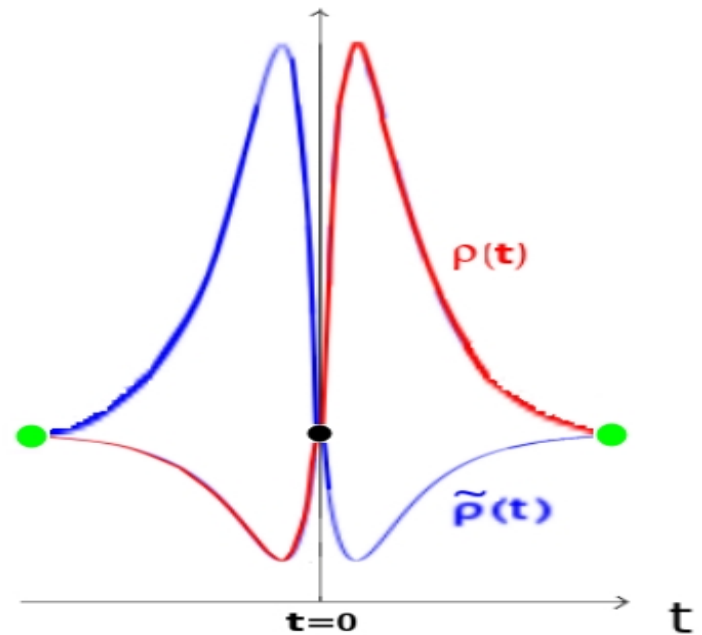


After
time
reversal



$\Gamma(t)$ allows to:

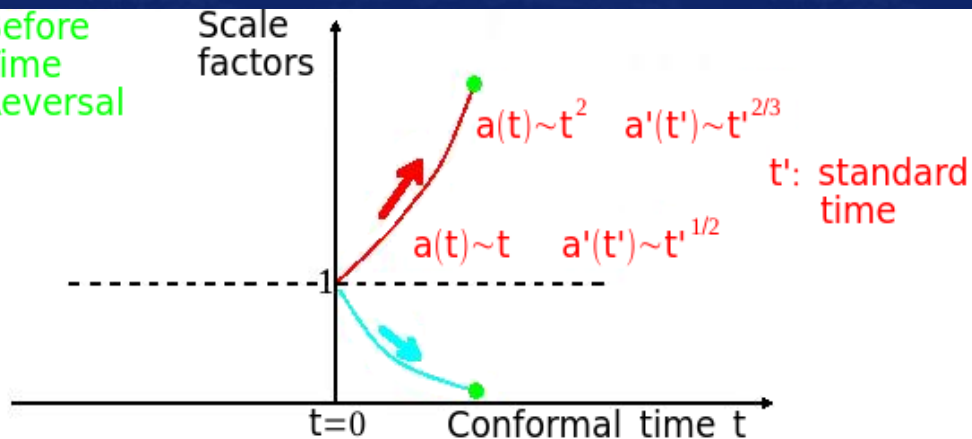
- interpret permutation symmetry as a time reversal symmetry
- get a cyclic universe



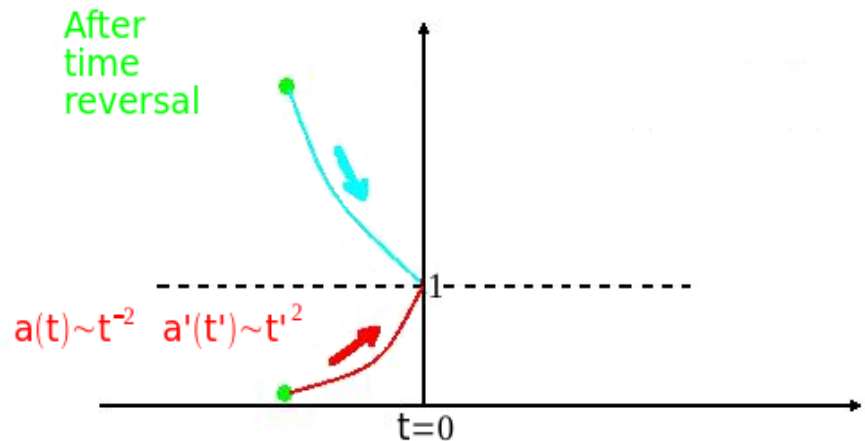
$$\Rightarrow \rho(t=0) = \tilde{\rho}(t=0)$$

DG Cosmology

Before
Time
Reversal



After
time
reversal



> Hyp: $\rho = \tilde{\rho}$ occurred at transition redshift
triggering T $\Rightarrow a'(t) \sim t^2$

> With $H'(t)$ continuous at the transition from $t^{2/3}$ to t^α and assuming same rough universe age as within LCDM ($\approx 1 / H'_0$)

$$z_{\text{tr}} = \left(\frac{2/3 - \alpha}{1 - \alpha} \right)^\alpha - 1$$

$\Rightarrow z_{\text{tr}} = 0.78$ vs observed LCDM $z_{\text{tr}} = 0.67 \pm 0.1$



- Close to LCDM scale factor evolution
- Without DE
- Inflation not needed to get $k=0$
- Without Big Bang singularity
- Cosmological DM still needed
- Dark side effects only since t_{tr} or near $t=0$

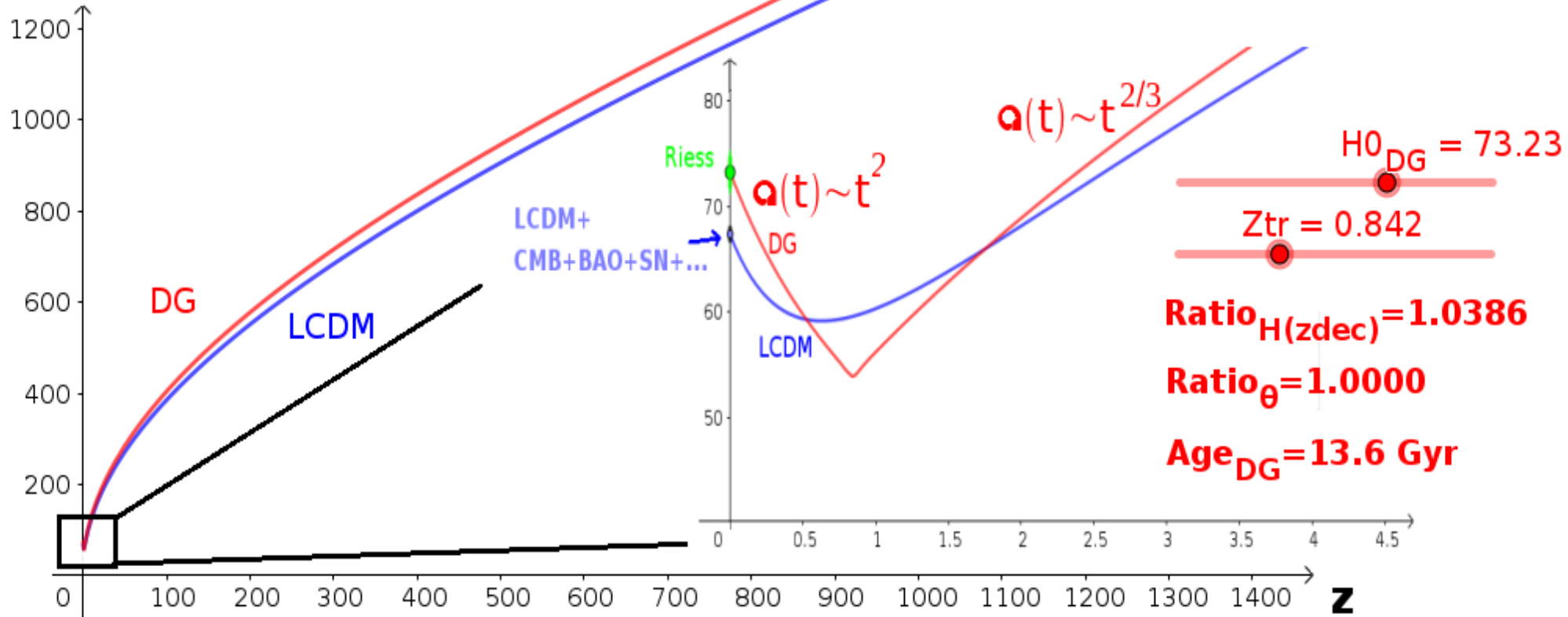
Testing Dark Gravity

Assume a flat cosmological model with:

- Radiation ($\sim t^{1/2}$) then matter ($\sim t^{2/3}$) dominated era (nothing else !)
- Instantaneous transition @ z_{tr}
- Constantly accelerated era ($\sim t^2$)

$H(z)/(1+z)$: Dark Gravity vs LCDM

$H(z)/(1+z)$



A single free parameter : z_{tr}

A single parameter z_{tr} replaces 2 LCDM parameters : $(\Omega_M, \Omega_\Lambda)$

$$\Omega_M(z_{tr}) = \frac{8\pi G\rho_M(z_{tr})}{3H_{tr}^2} = 1 - \Omega_{rad}(z_{tr}) \approx 1$$

$$\rho_M(z_{tr}) = \rho_M(0) \cdot (1+z_{tr})^3; H_{tr} = H_0(1+z_{tr})^{0.5}$$



$$\Omega_M = \frac{8\pi G\rho_M(0)}{3H_0^2} \approx 1/(1+z_{tr})^2$$

SN1A test of a DG transition

(JLA : 740 SN)

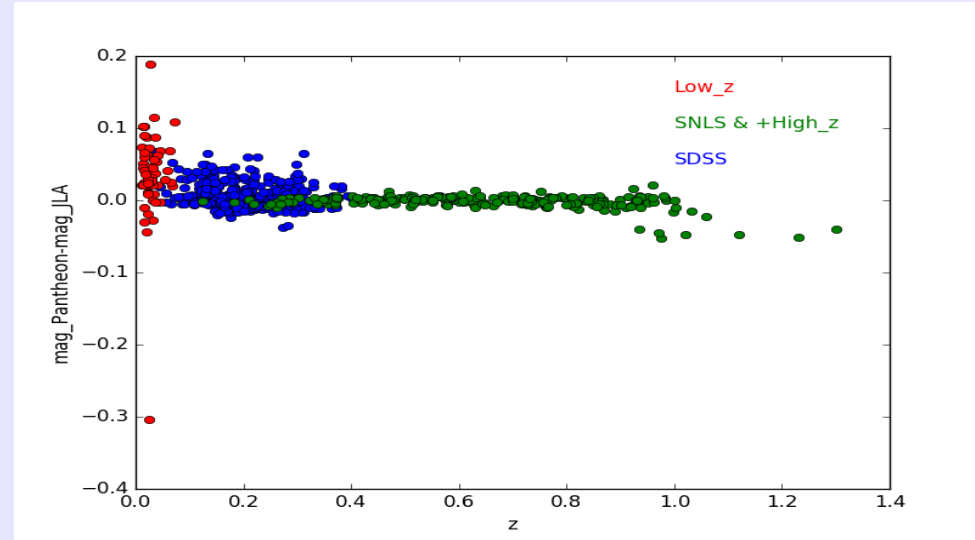
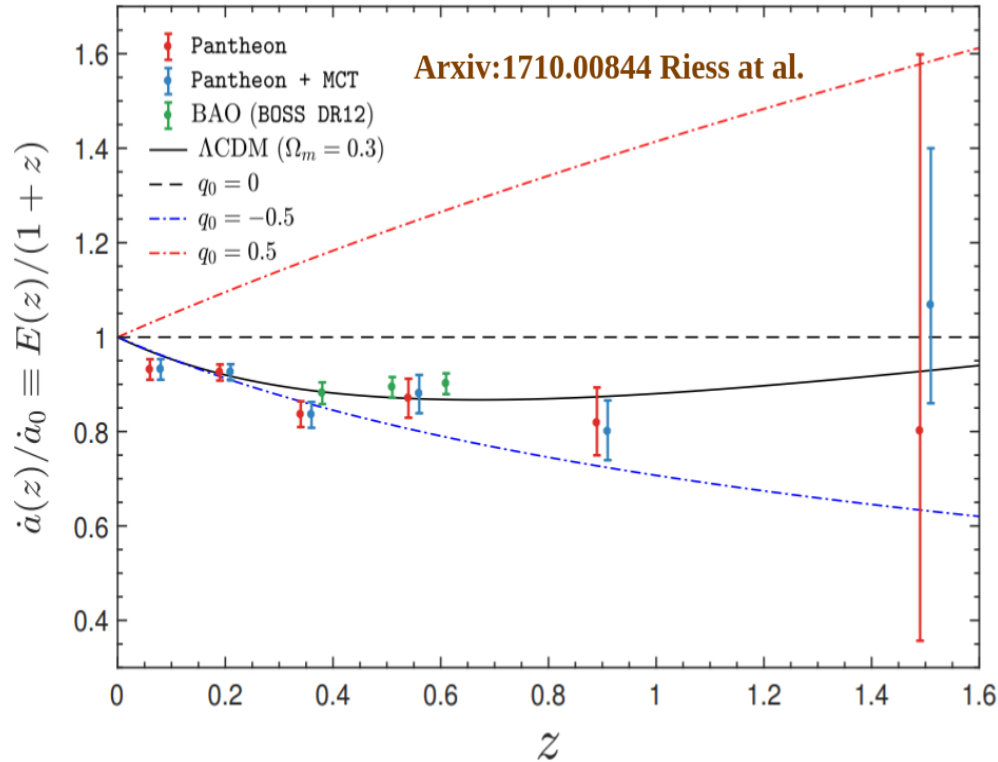
- $a(t) \sim t^2$ ($q_0 \sim -0.5$) : meaningless within LCDM but expected in DG
- Fit between 0 and z_{\max} with free power law t^α
 - $z_{\max}=0.6 \Rightarrow \alpha = 1.85 \pm 0.15$ (1σ from $\alpha = 2.$)
 - $z_{\max}=0.8 \Rightarrow \alpha = 1.78 \pm 0.11$ (2σ from $\alpha = 2.$)

- DG transition from $t^{2/3}$ to t^2 at z_{tr} :

$$z_{\text{tr}} = 0.67 + 0.24 - 0.12 \text{ (Minos errors)}$$

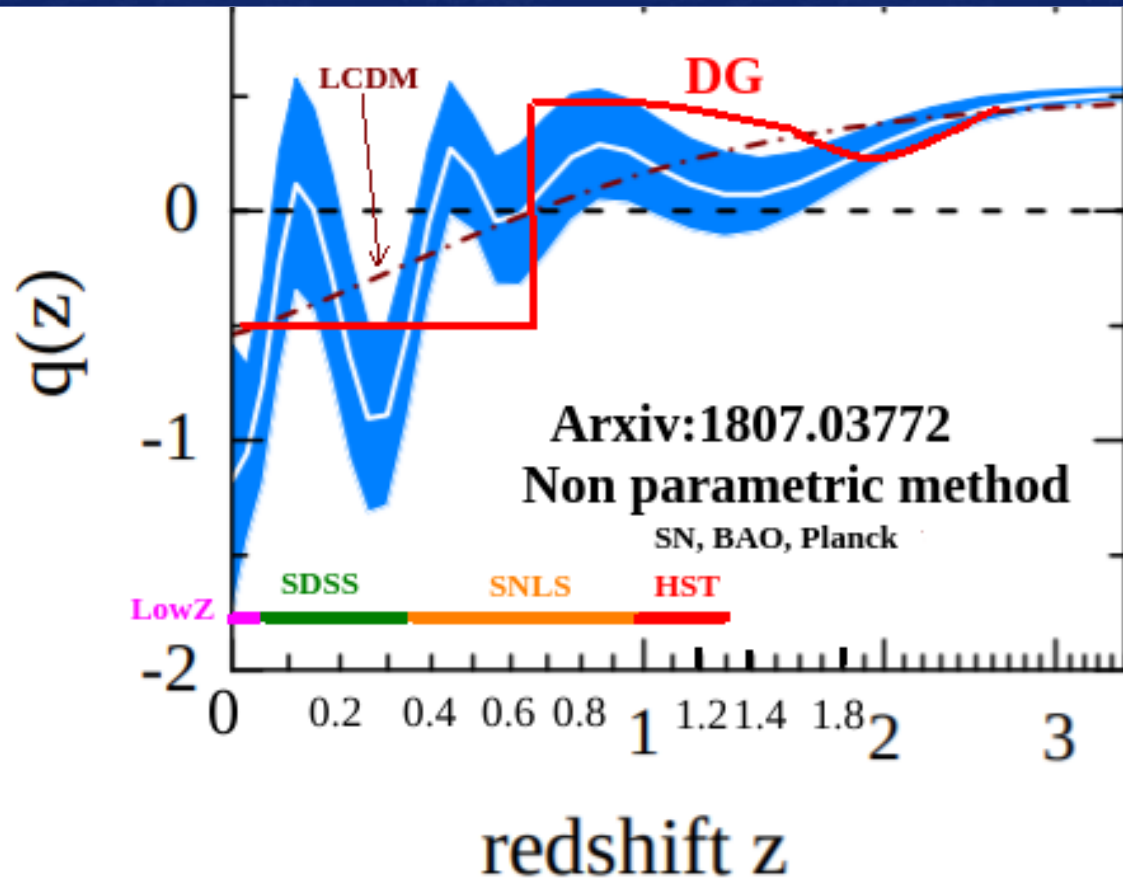
$$\chi_{DG}^2 = 740.8$$
$$\chi_{DG}^2 - \chi_{LCDM}^2 = +1.4$$

From JLA to Pantheon



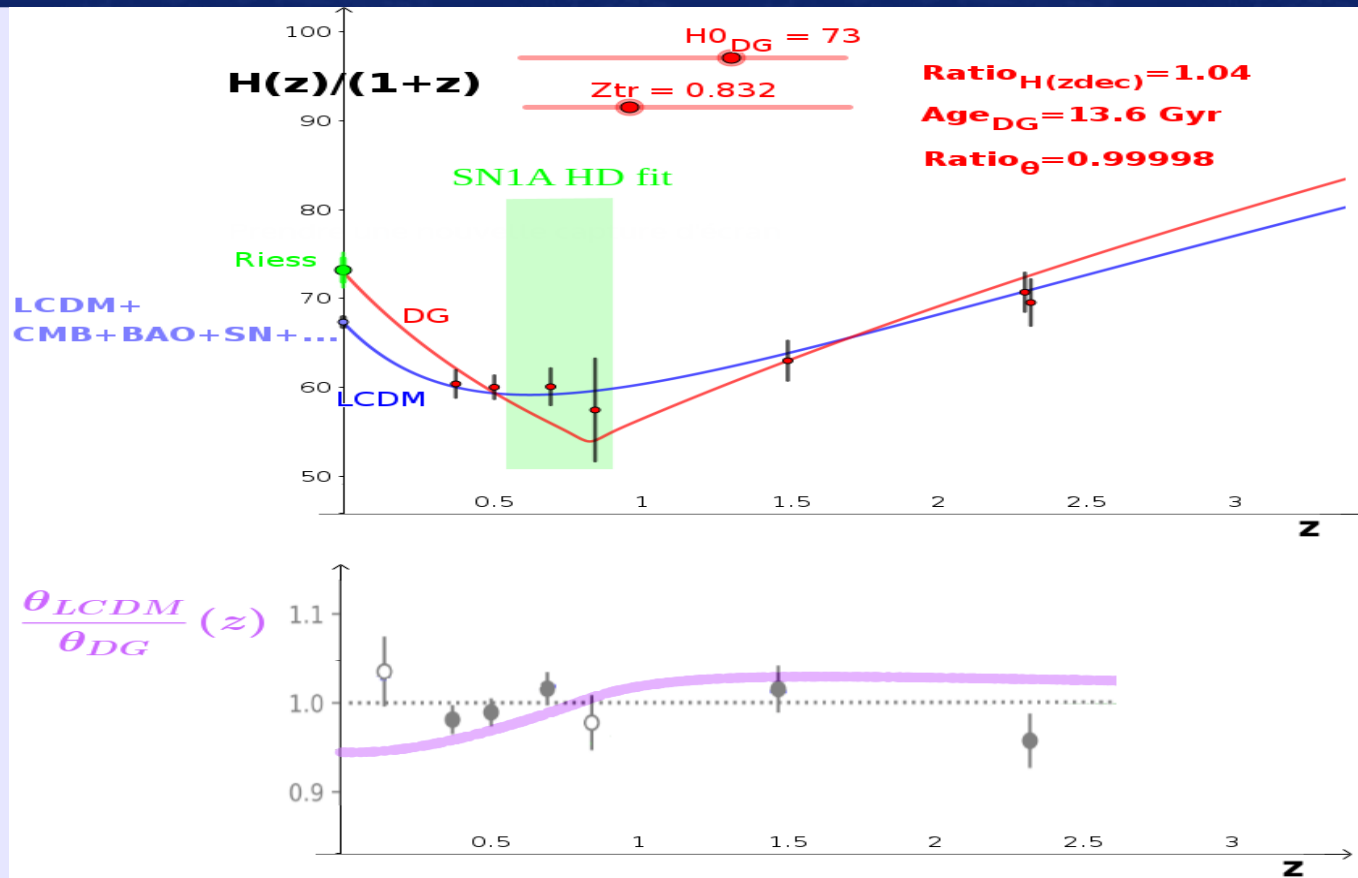
- A new 2σ wiggle effect \Rightarrow Z_{tr} very sensitive to HST magnitudes

$q(z)$ non parametric reconstruction

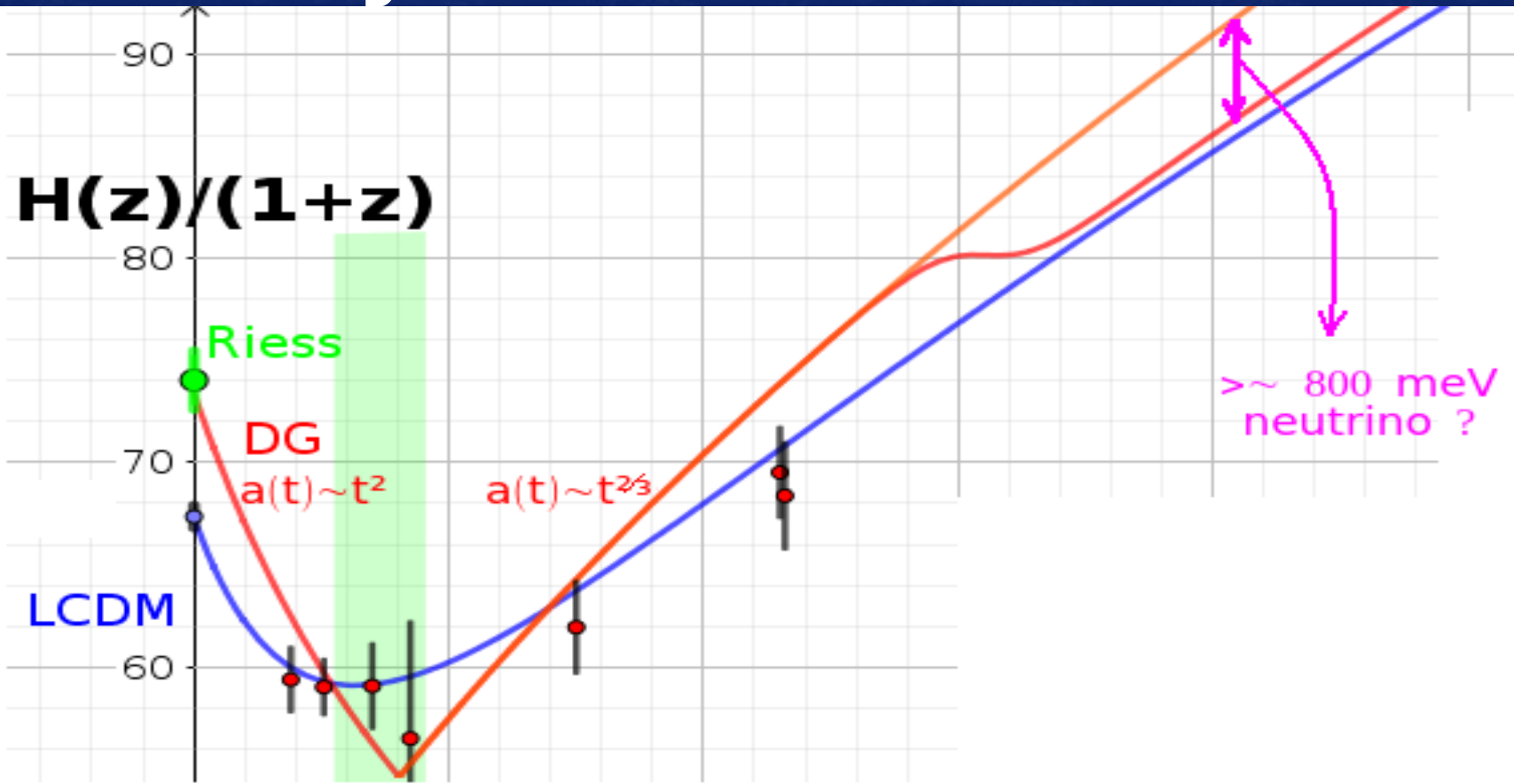


BAO, SN & CMB test of DG

SDSS 07/2020



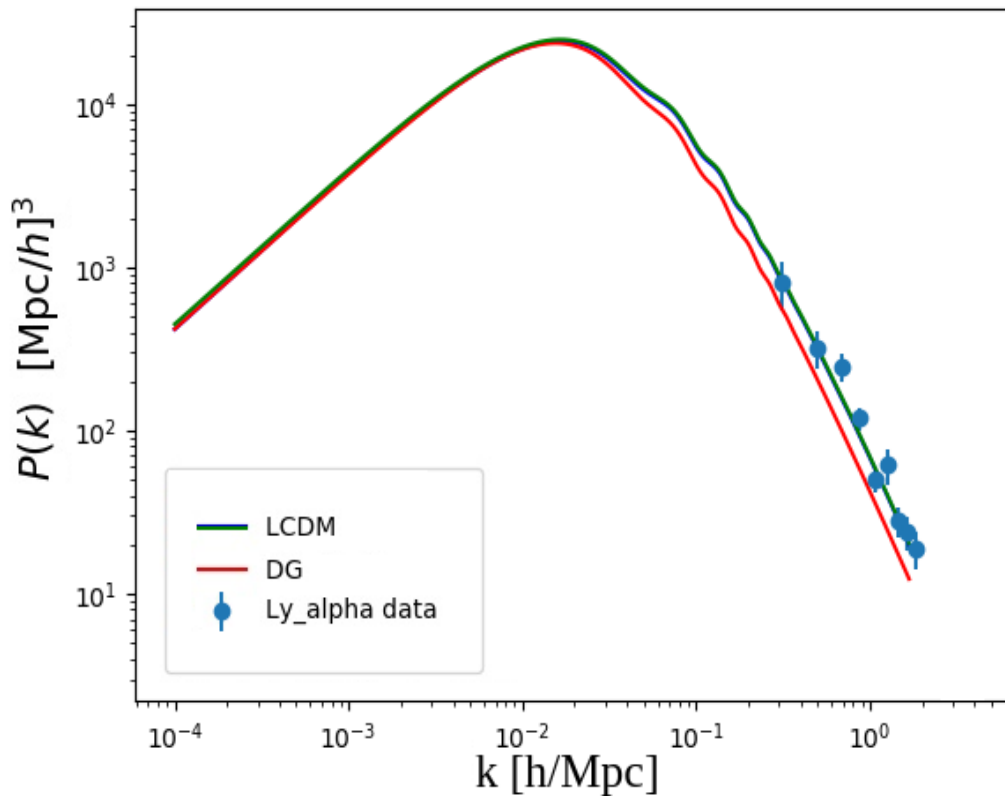
Can we improve the fit by a density increase effect ?



Matter power spectrum

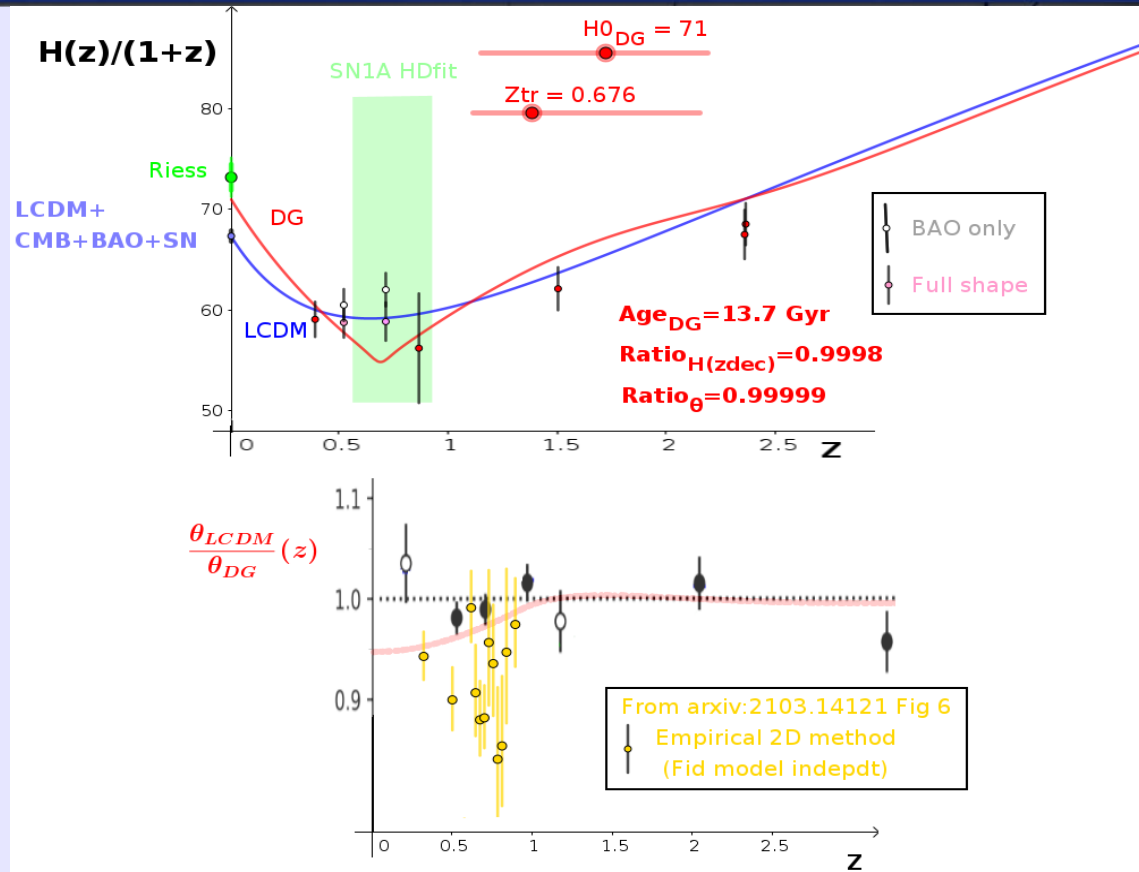
Ly- α data exclude DG(800meV) !

- $2 < z < 4 \Rightarrow$ very small non linearities
- bias(z,k) effects under control



Ad Hoc density increase near Z_{tr}

- Good agreement with BAO transverse, CMB and SN data !
- Tension in BAO LoS for BAO only method
But BAO only is Fid Model Biased according [arxiv 1811.12312](#) (Anselmi et al.)



First test with Class

- DG background (and fluctuations before z_{tr}) simulation straightforward with Class !
- But harder for fluctuations after z_{tr} \Rightarrow simplified (first step) methodology needed :



Assume homogeneous DE fluid giving same $H(z)$ as DG ($\Rightarrow w_{\text{eff}} = -2/3$)

Beware actual $z < z_{\text{tr}}$ lensing and LISW effects will need simulation of Dark Side !

We expect from these :

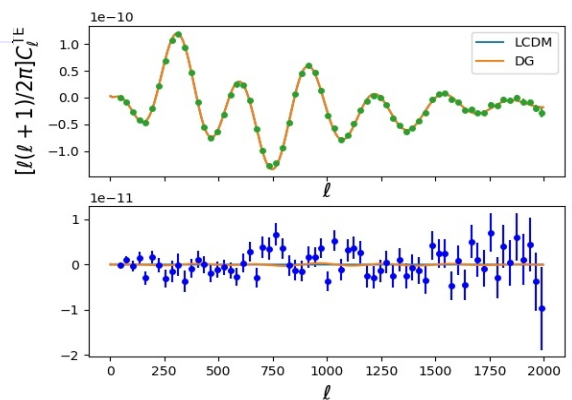
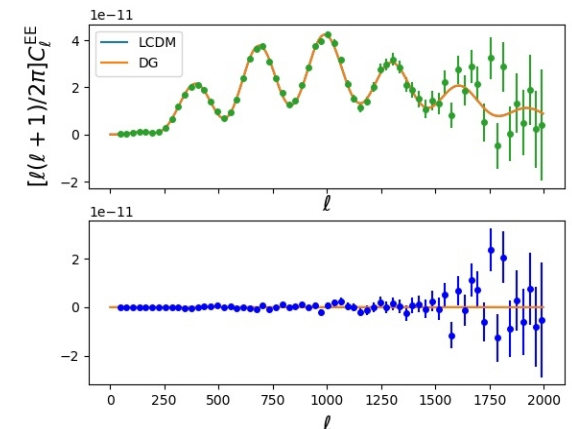
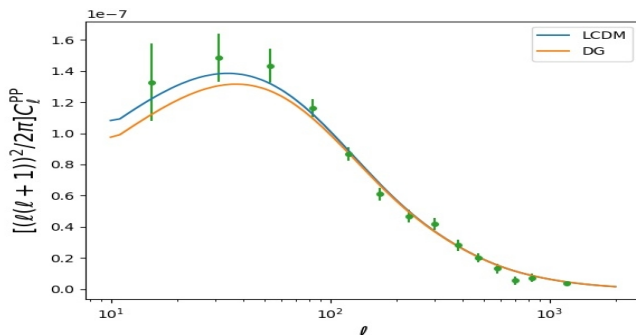
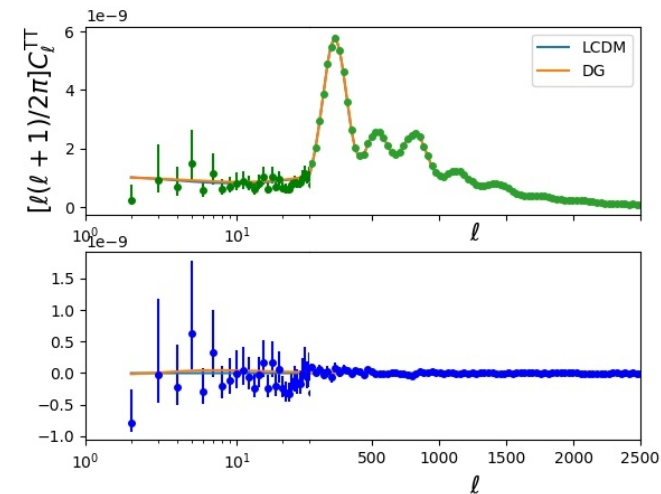
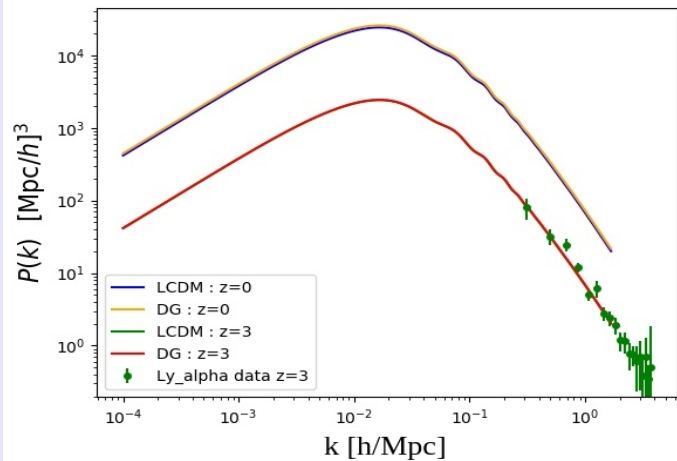
- Effects on Planck TT and $\Phi\Phi$ on the largest scales
- New possible inverse lenses from voids
- Effects on RSD, non linearities, ...

Planck and matter power spectra

Planck [arxiv:1807.06209](https://arxiv.org/abs/1807.06209)
Class software [arxiv:1104.2933](https://arxiv.org/abs/1104.2933)

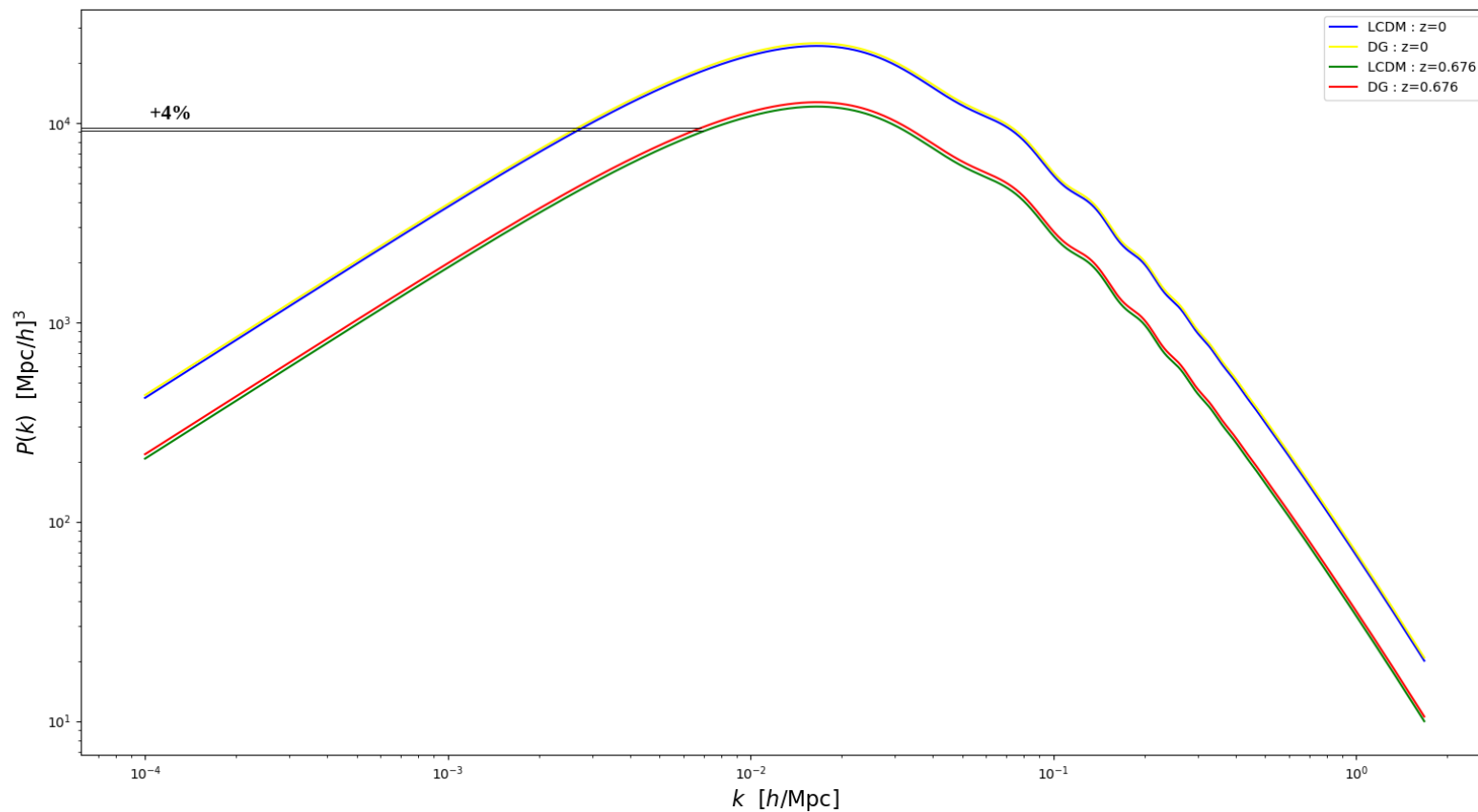
- Planck TT+TE+EE

$$\chi_{DG}^2 = 6701.9$$
$$\chi_{DG}^2 - \chi_{\Lambda\text{CDM}}^2 = +4.4$$



Matter Power spectra @Ztr

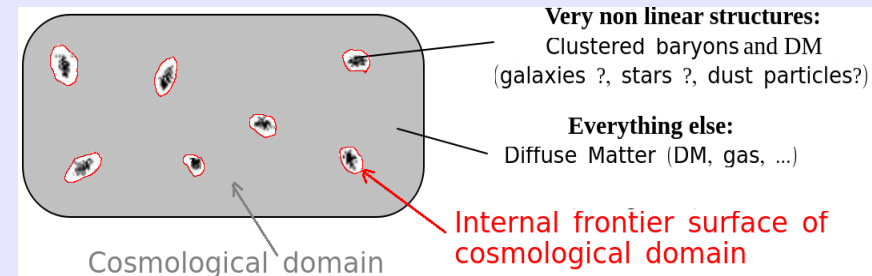
Class software



Neutrino mass generation near Z_{tr} or DG specific mechanism ?

- As in [arxiv:2102.13618](#) : Active and sterile neutrinos mass generation @ low z
i.e. @ Temperature $\ll m_\nu$: supercool transition ! But needs an energy source.
Or,
- Specific DG mechanism :
 - Discontinuities delimiting spatial domains with Minkowski background metric which g_{00} may not always synchronize on cosmological metric

⇒ Wiggles in $H(z)$, Pioneer effect !?



Conclusion and outlooks

- DG theory (not just a model) avoids Big-Bang singularity and BH horizon very naturally
- Spatial flatness and acceleration mechanism without new arbitrary actions and parameters
- Also an ideal framework to hopefully :
 - solve the old cosmological constant problem
 - explain matter-antimatter asymmetry
 - account for the initial large scale homogeneity
- Outlook :

New rich phenomenology from field discontinuities

Updated Review not on arXiv (too many versions !) but here :
www.darksideofgravity.com/DG.pdf

Other considerations


- Referees don't read beyond the abstract when it's too away from the mainstream
⇒ no feedback ! Confrontation never allowed.
- No tool available to test models different from Λ CDMs
(common belief is that any model close to Λ CDM for the background should also be close to Λ CDM for the fluctuations)
- Lack of confrontation between authors of conflicting observational results
- A too common « 5 sigma a priori pro Λ CDM » (Riess 2021)

How far could we go ?


Background dependent \Rightarrow  Generic huge EP violations


+ Ghost \Rightarrow OK* \Rightarrow  Quantum unstable

* EP violations (η effects) negligible far from $t=0$

+ Semiclassical \Rightarrow OK \Rightarrow OK** \Rightarrow  Static background

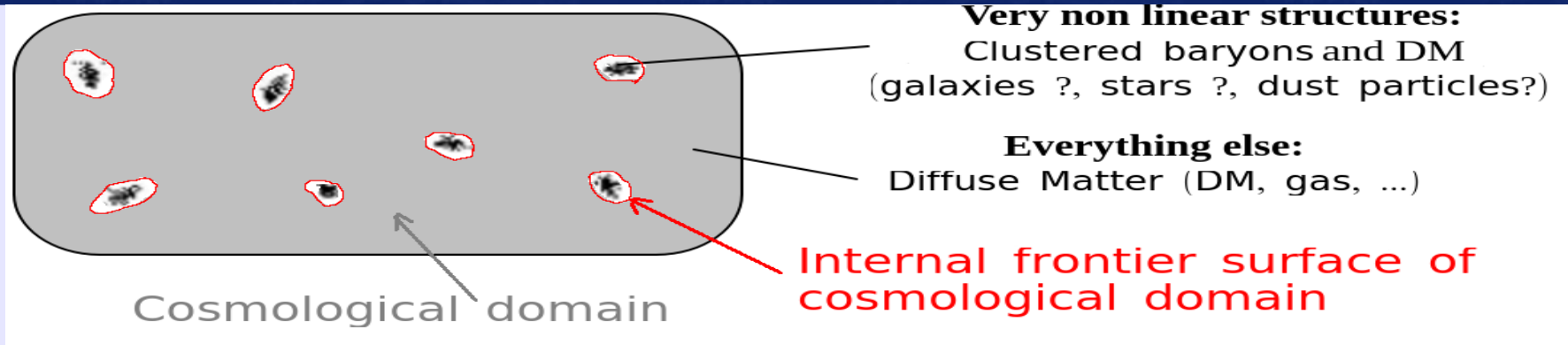
** harmless classical instabilities far from $t=0$

+ Offshell $\Gamma(t)$ \Rightarrow OK \Rightarrow OK \Rightarrow OK \Rightarrow  Unbounded evolution
No alternative to Λ

+ Time discontinuity \Rightarrow OK \Rightarrow OK \Rightarrow OK \Rightarrow OK \Rightarrow  Gravity switched off

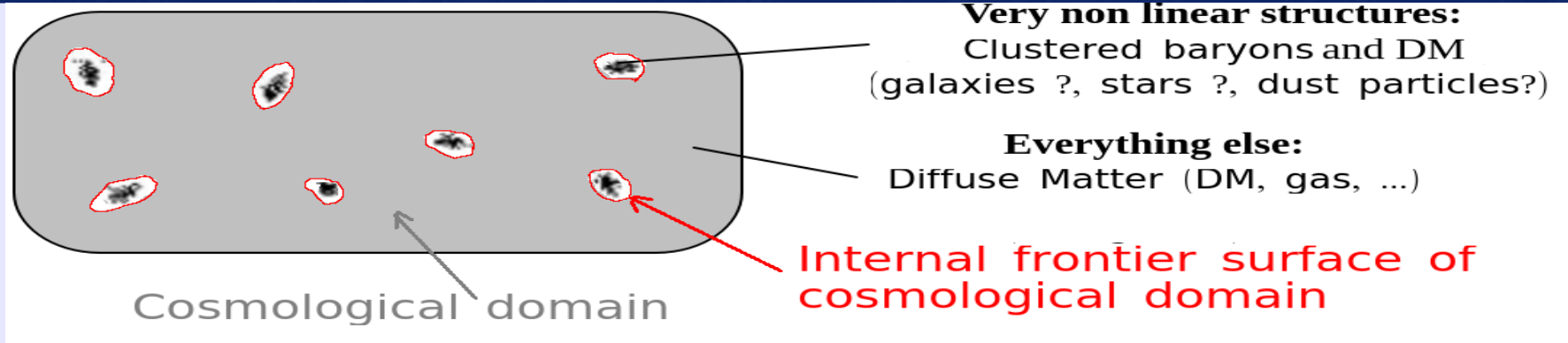
+ Spatial discontinuities \Rightarrow OK \Rightarrow OK \Rightarrow OK \Rightarrow OK \Rightarrow OK

Static bounded domains needed



- To avoid transition switching off gravity of clustered structures
- (?) To allow Matter-Radiation exchange (crossing metrics)
 - ⇒ avoid BH central singularities, allow light from GW170817 to propagate on the Dark Side
- (?) Might help simultaneous crossing of densities and pressures

Static bounded domains needed



- Bounded domains give no contribution to cosmological background

Cosmo metric on our side : $d\tau^2 = a^2(t)(dt^2 - d\sigma^2) = dt'^2 - a'^2(t')d\sigma^2$

- Bounded domains have a Minkowskian background (\sim GR)

Bounded domains back_metric on our side : $d\tau^2 = a^2(t)dt^2 - C_{frozen}^2 d\sigma^2 = dt'^2 - C_{frozen}^2 d\sigma^2$

- Simple extention of perturbation solutions from both sides of domain frontiers (continuous before z_{tr} , $1/C^2$ factor after z_{tr})

Linear conjugate perturbations (subhorizon)

Before transition

$$\frac{d^2\delta}{da^2} + \frac{3}{2a} \frac{d\delta}{da} \approx \frac{3}{2} \frac{\delta}{a^2}$$

$$\frac{d^2\tilde{\delta}}{d\tilde{a}^2} + \frac{3}{2\tilde{a}} \frac{d\tilde{\delta}}{d\tilde{a}} \approx -\frac{3}{2} \frac{\tilde{\delta}}{\tilde{a}^2}$$

$$\tilde{\delta} = -3\delta \propto a$$

$$\begin{aligned} \delta_{\min} = -1 &\Rightarrow \tilde{\delta}_{\max} = 3 \\ \delta \geq \frac{1}{3} &\Rightarrow \tilde{\delta} = \tilde{\delta}_{\min} = -1 \end{aligned}$$

After transition

$$\frac{d^2\delta}{da^2} + \frac{3}{2a} \frac{d\delta}{da} \approx -\frac{3}{2} \frac{\tilde{\delta}}{a^2}$$

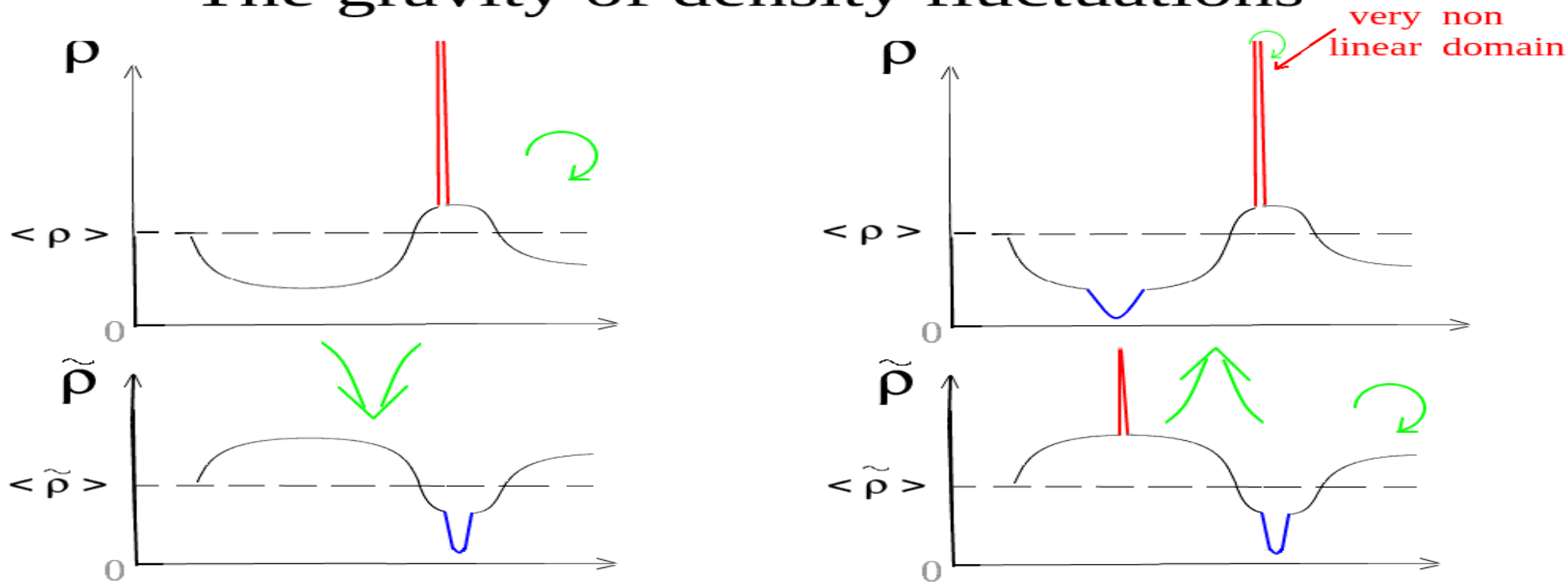
$$\frac{d^2\tilde{\delta}}{d\tilde{a}^2} + \frac{3}{2\tilde{a}} \frac{d\tilde{\delta}}{d\tilde{a}} \approx \frac{3}{2} \frac{\tilde{\delta}}{\tilde{a}^2}$$

$$\delta = -\frac{\tilde{\delta}}{2} \propto a^{3/2}$$

$$\begin{aligned} \tilde{\delta} \geq 2 &\Rightarrow \delta = \delta_{\min} = -1 \\ \tilde{\delta}_{\min} = -1 &\Rightarrow \delta_{\max} = \frac{1}{2} \end{aligned}$$

Phenomenology of conjugate perturbations

The gravity of density fluctuations



Before transition

After transition

Growth after DG transition

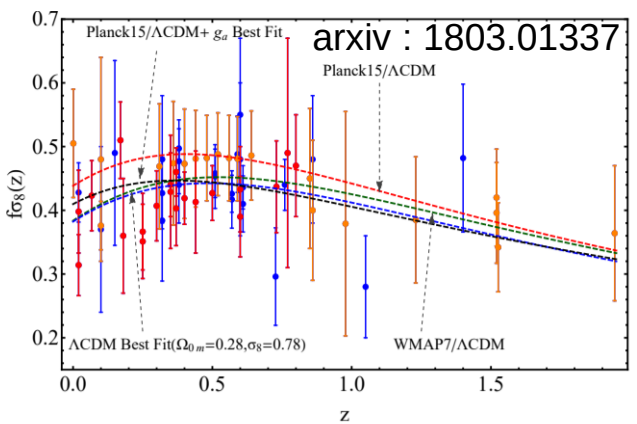
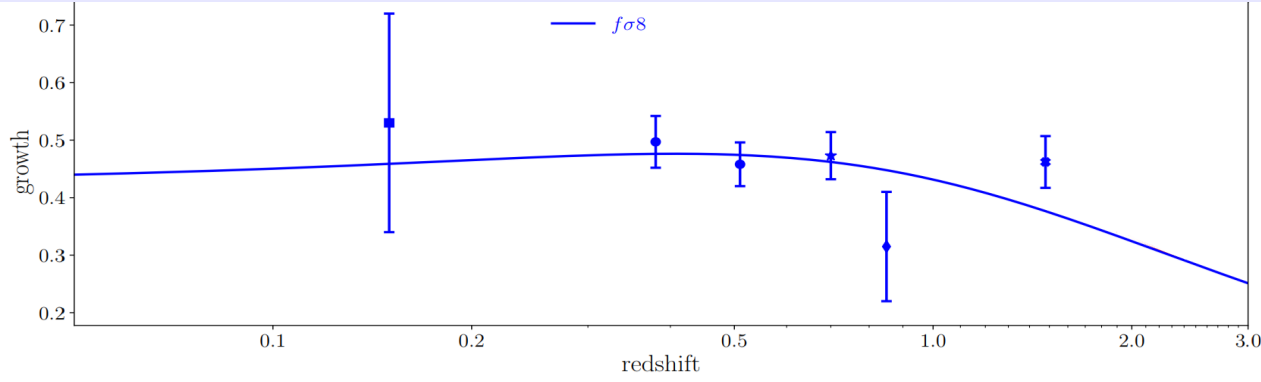
Highly non trivial !

Complete simulations needed !

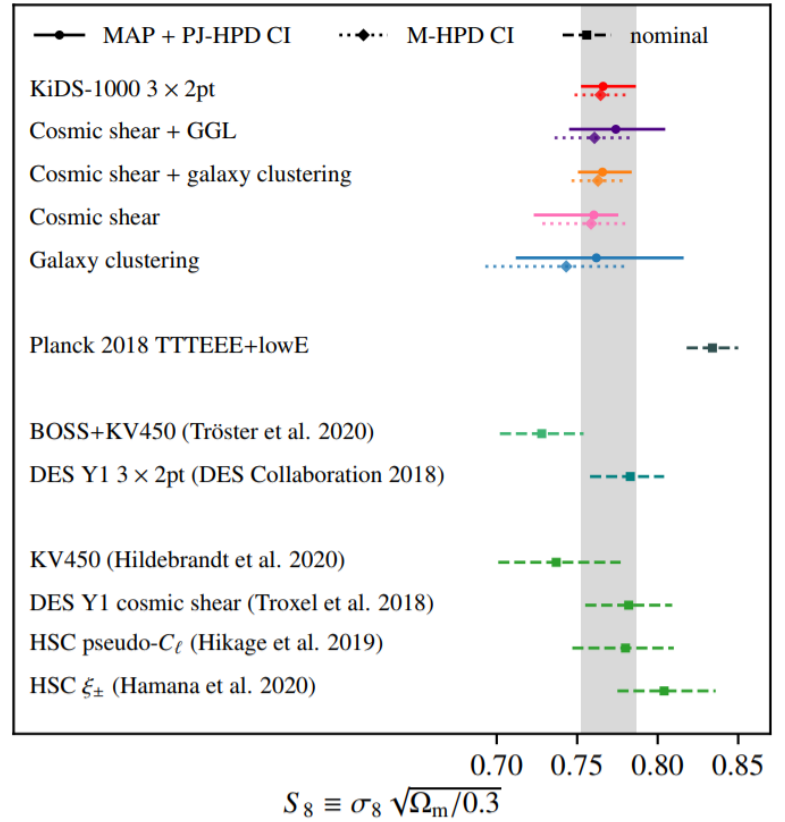
- Dark side fluctuations lead the game on linear scales only !
- There is a transient regime following z_{tr}
- $f(z)$ and $\sigma_8(z)$ strongly z , $|\delta|$ and $\text{sign}(\delta)$ dependent : f between negative and 1.5
 - Larger dark side linear fluctuations but
 - Less growth on the mean for our side overdensities @ $z < z_{\text{tr}}$: $\delta^+ \Rightarrow 1/2$
- Biases and non linear effects must be reconsidered, vs LCDM we expect :
 - Less gravity & lensing from our side Halos @ $z < z_{\text{tr}}$: $\Rightarrow \sigma_8$ anomaly !

Growth data

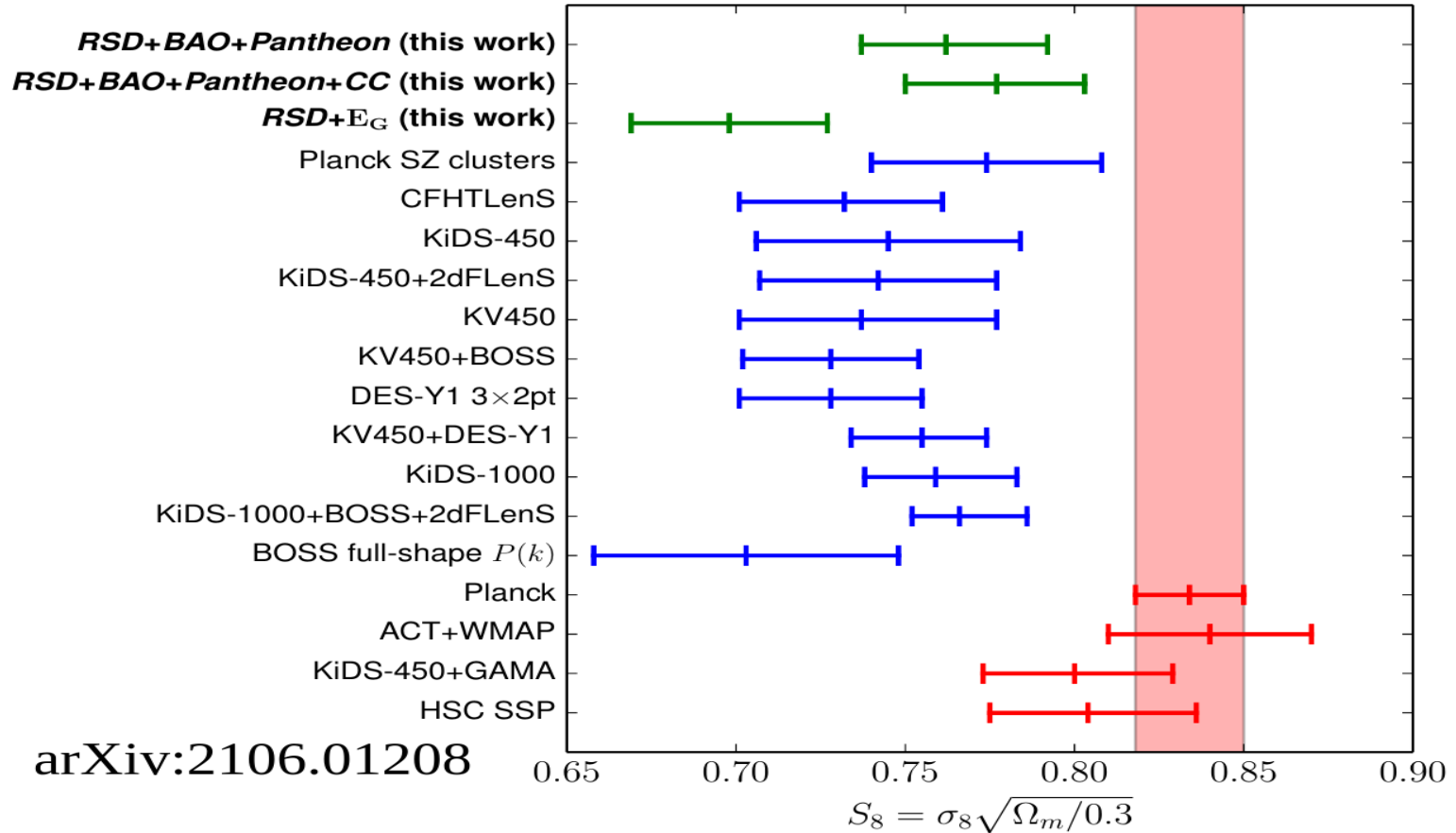
Data from <https://www.sdss.org/science/final-bao-and-rsd-measurements/>



New : σ_8 @ $z < 1$ from Kids (weak lensing) in 3 sigma tension (8% lower) with Planck. arxiv : 2007.15632



S_8 including RSD data



$S_8(z)$ tension

arXiv:2105.12108

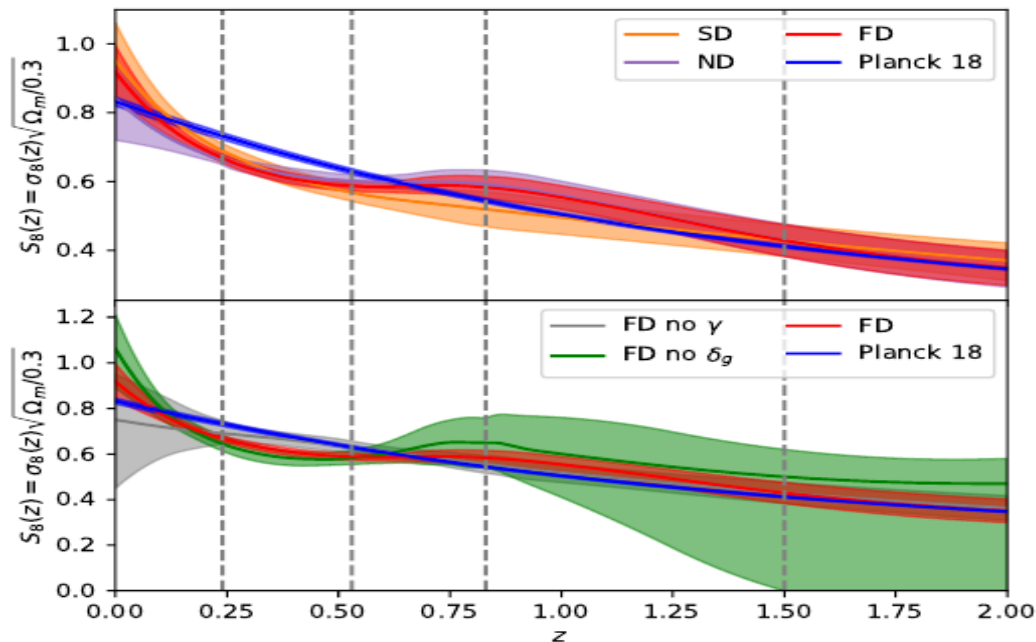


Figure 8: Reconstructed growth history. Each band shows the 68% C.L. constraints from different data combinations. The top panel shows the constraints from the ND and SD data sets (purple and orange respectively), as well as the full combination FD (red), and the Λ CDM constraints from *Planck* (blue). The bottom panel additionally shows results for the FD data excluding galaxy clustering (green) or cosmic shear (gray). The vertical dashed lines show the position of the redshift nodes used here to generate the growth factor spline. The S_8 tension can be seen at $z \sim 0.4$.

Theoretical $f\sigma_8$ recovering

arxiv: 1803.01337

Scale
dependent
(modified
gravity)

- Diff eq solved numerically

$$\delta''(z) + \left(\frac{H'(z)}{H(z)} - \frac{1}{1+z} \right) \delta'(z) \approx \frac{3}{2} (1+z) \frac{H_0^2}{H^2(z)} \Omega_{0M} \delta(z) \frac{G_{\text{eff}}(z, k)}{G_N}$$

scale independence \Rightarrow

$$\sigma_8(z) = \sigma_8(z=0) \frac{\delta(z)}{\delta(z=0)} \quad [f\sigma_8](z) = -(1+z)\sigma_8'(z)$$

- Fit result by approx analytical expression

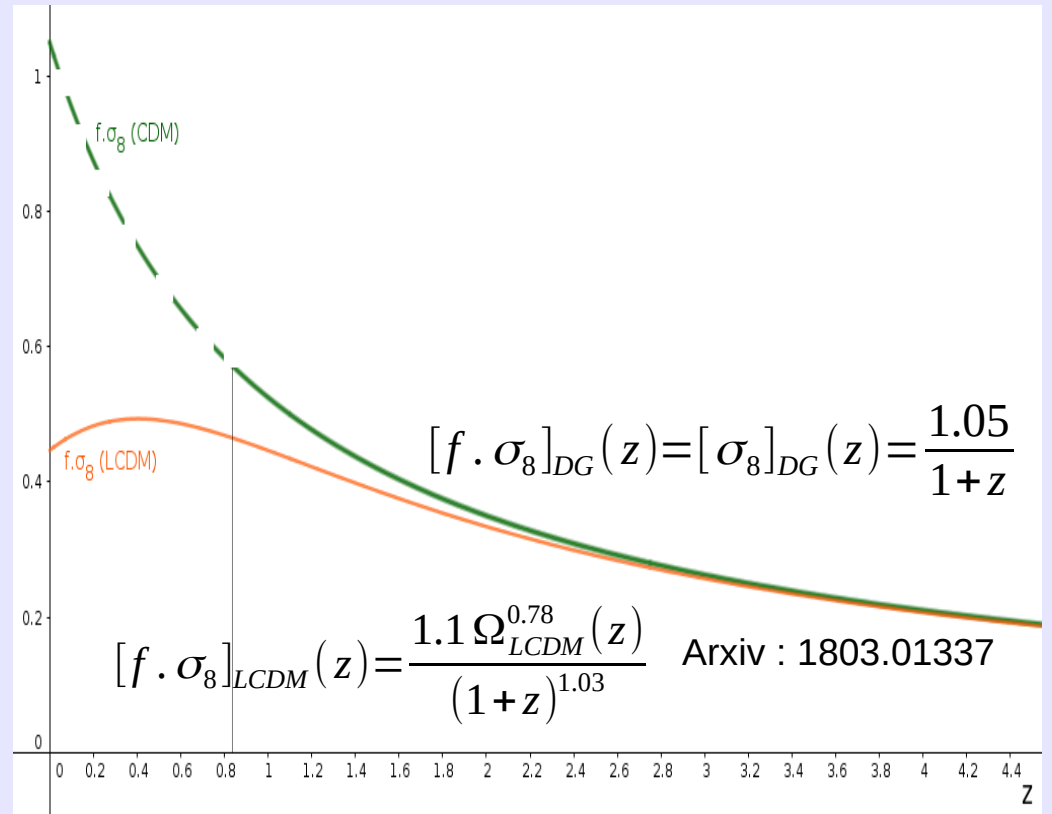
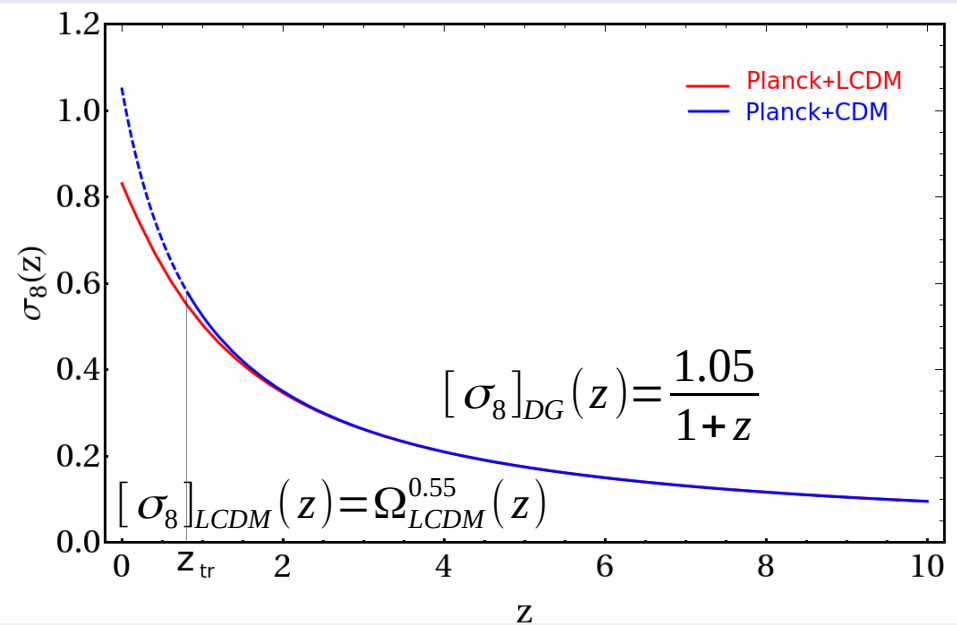
$$[\sigma_8](z) = \Omega_m^\alpha(z)$$

$$f\sigma_8(z) = \lambda\sigma_8 \frac{\Omega_m(z)^\gamma}{(1+z)^\beta}$$

works for LCDM and most modified gravity theories

but not for DG

Growth before DG transition (trivial)

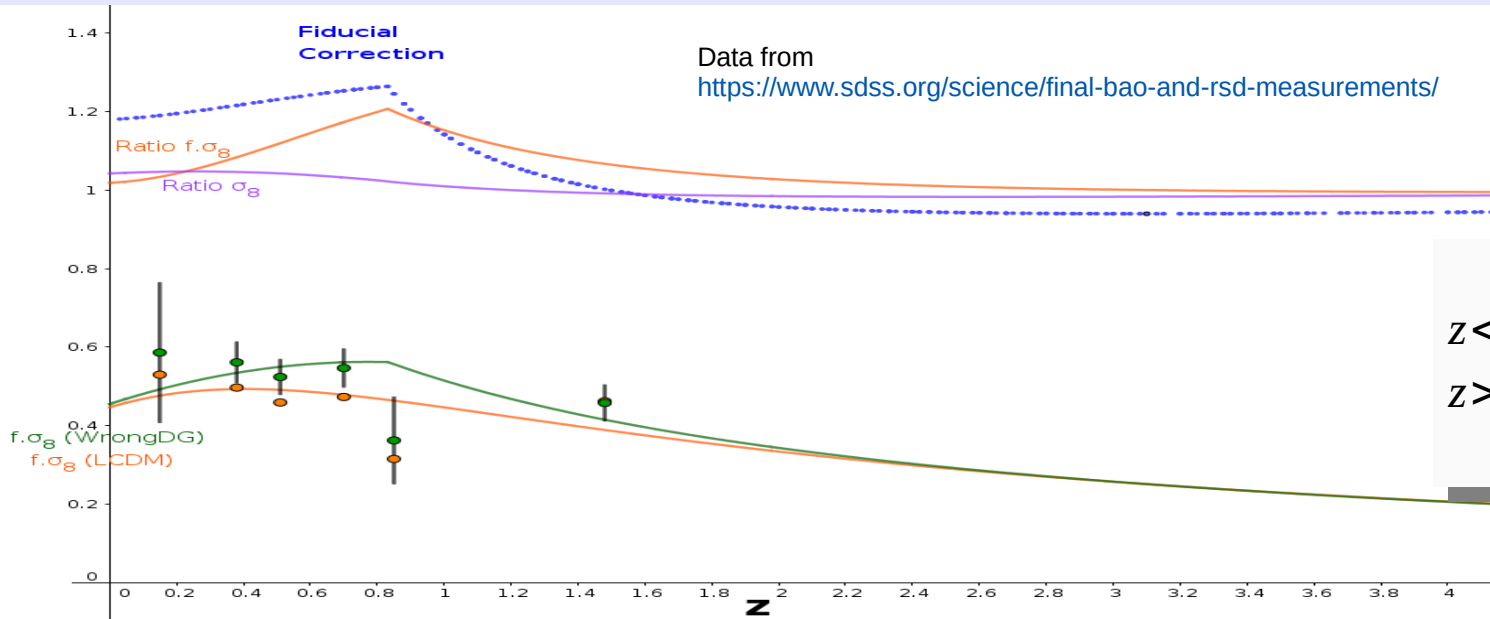


Growth vs wrong DG

Wrong DG : assume a dark energy fluid that would produce the same $H(z)$ as DG

Approx fiducial model correction (arxiv: 1509.05034) :

$$\left\{ \frac{[f \sigma_8]_{WrongDG}(z)}{[f \sigma_8]_{Planck}(z)} \right\}_{Data} = \left(\frac{[\sigma_8]_{WrongDG}(z)}{[\sigma_8]_{Planck}(z)} \right)^2 \left(\frac{H_{LCDM}(z)}{H_{DG}(z)} \frac{(D_A)_{LCDM}^2(z)}{(D_A)_{DG}^2(z)} \right)^{\frac{3}{2}} C(z)$$



$$[\sigma_8]_{WrongDG}(z):$$

$$z < z_{tr} \rightarrow 0.092 z^2 - 0.469 z + 0.906$$

$$z > z_{tr} \rightarrow 1.06 / (1+z)$$

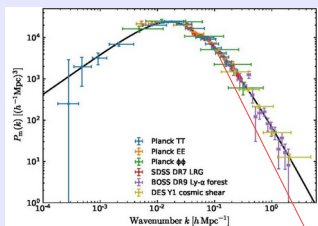
$$[f \sigma_8](z) = -(1+z) \sigma_8'(z)$$

Fiducial model correction

arxiv: 1509.05034

- Assume correction is small and bias $\propto \sigma_8$ (formula tested from Wmap_LCDM to Planck_LCDM as fiducial models)

$$\left\{ \frac{[f \sigma_8]_{fid}(z)}{[f \sigma_8]_{Planck}(z)} \right\}_{Data} = \left(\frac{[\sigma_8]_{fid}(z)}{[\sigma_8]_{Planck}(z)} \right)^2 \left(\frac{H_{Planck}(z) (D_A)_{Planck}^2(z)}{H_{fid}(z) (D_A)_{fid}^2(z)} \right)^{\frac{3}{2}} C(z)$$



with

$$C(z) = \frac{\int d^3 k \sqrt{\frac{P(k_{Planck})}{P(k_{fid})}}}{\int d^3 k} \simeq 1 + \frac{d \ln P(k)}{2 d \ln(k)} \left(\pi \frac{\sqrt{2}}{3} \left(\frac{(D_A)_{Planck}(z)}{(D_A)_{fid}(z)} - 1 \right) + \left(1 - \pi \frac{\sqrt{2}}{3} \right) \left(\frac{H_{fid}(z)}{H_{Planck}(z)} - 1 \right) \right)$$

Conclusion for Growth

- Models very significantly deviating from Λ CDM @ $z > 0.4$ can still fit growth data
- And may be fit better than Λ CDM @ low z (where are the tensions !)
- Then correction of RSD effects, reconstruction in BAO measurements might produce significant differences ($> 20\%$) for another fiducial cosmology than Λ CDM
- Tools should be provided to help people test any kind of model (not only Λ CDM like)

Fundamental discontinuities from discrete symmetries

Discontinuous
rules

T

- Different domains for the continuous and discontinuous : $]\dots, T - [$, $]\ T + , \dots [$

Continuous
laws

Continuous
laws

• A natural idea

- Discontinuous rules (just as continuous laws) must be generated by discrete (continuous) symmetries

- Unifying : symmetries bridge the gap between the continuous and discontinuous

• A necessary and not so new idea :

- Anyway QM rules need it ! Also omnipresent (but burried in formalism) in QFT

Criteria for a theory

- Criteria for a good theory :

not only :

1- Agrees with observations

but also :

2- Self consistent

and :

3- Economic = unifying = predictive

String Theory, Loop Quantum Gravity, ... try to address 1), 2) and 3)

- Usual bad criteria for a good theory :

- Simple

- Common sense compatible

Unification perspective ?

$$H^{-1} \sim 10^{17} \text{ sec}$$

$$\sqrt{\frac{G\hbar}{c^5}} \sim 10^{-43} \text{ sec}$$

But now we have an initial privileged $1/H$ at $t=0$, is it identical to

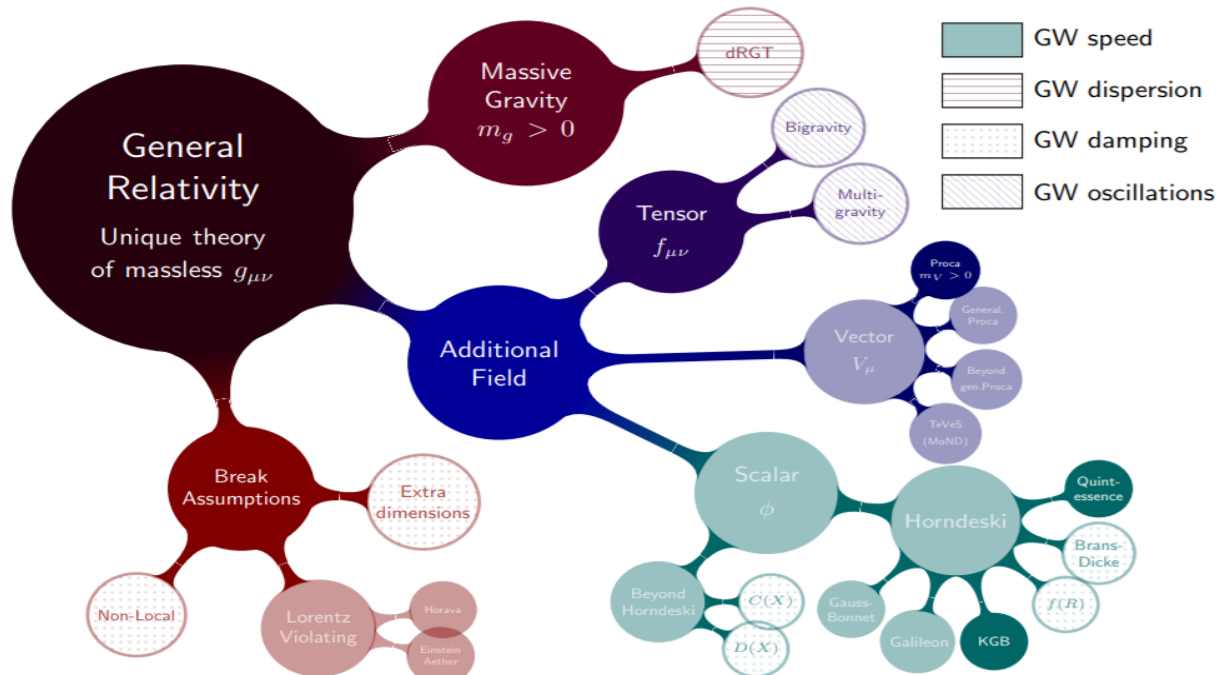
$$\sqrt{\frac{G\hbar}{c^5}} \sim 10^{-43} \text{ sec}$$

Modified gravity road map

from arxiv : 1807.09241

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \dots = \frac{8\pi G}{c^4} T_{\mu\nu} + \dots$$

Modified gravity roadmap



Most of them are not theories but models that could be low energy approximations of genuine theories !

DG Gravitational Waves

neglecting potentials

$$g_{\mu\nu} = C(\eta_{\mu\nu} + h_{\mu\nu}) \quad \tilde{g}_{\mu\nu} = C^{-1}(\eta_{\mu\nu} + \tilde{h}_{\mu\nu})$$

$$\tilde{h}_{\mu\nu} = -h_{\mu\nu}$$

$$(C + C^{-1}) \left(R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R_{\lambda}^{(1)\lambda} \right) = -8\pi G (C T_{\mu\nu}^{(0)} - C^{-1} \tilde{T}_{\mu\nu}^{(0)})$$

in vacuum

Gravitational Waves

$$\square h_{\mu\nu} = 0$$

Light Waves

$$\square A_{\mu} = 0$$

with potentials

$$g_{\mu\nu} = C(\text{diag}(-c_0, c_1, c_1, c_1) + h_{\mu\nu}) \quad \tilde{g}_{\mu\nu} = C^{-1}(\text{diag}(-c_0^{-1}, c_1^{-1}, c_1^{-1}, c_1^{-1}) + \tilde{h}_{\mu\nu})$$

Gravitational Waves

$$C \gg 1 \Rightarrow (c_0/c_1)\ddot{h}_{\mu\nu} - \nabla^2 h_{\mu\nu} = 0$$

$$C \ll 1 \Rightarrow (c_1/c_0)\ddot{h}_{\mu\nu} - \nabla^2 h_{\mu\nu} = 0$$

Light Waves

$$\text{Our side} \Rightarrow (c_0/c_1)\ddot{A}_{\mu} - \nabla^2 A_{\mu} = 0$$

$$\text{Dark side} \Rightarrow (c_1/c_0)\ddot{A}_{\mu} - \nabla^2 A_{\mu} = 0$$



Unify through symmetries

DG: Discrete and continuous symmetries unified:

Hidden Global Lorentz Invariance of background metric : structure of spacetime

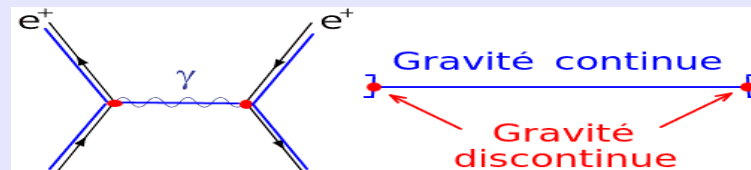
⇒ Induced manifest symmetries (local or Gauge ~ RG) are degraded

⇒ Induced permutation symmetry X

⇒ X:= induced discrete symmetry T with privileged frame (origin of time)

⇒ Discontinuous (non local) and continuous processes unified

Most natural way toward a more fundamental understanding (more unified) of QM
 discontinuities and non locality : allow discontinuous fields !...



Dynamical discrete symmetries

- Standard view :

Symmetries (cont & disc) \Rightarrow Action

Extreme action principle \Rightarrow Eoms & conservation equations

No dynamical processes associate with discrete symmetries

- Extended view :

Symmetries (cont & disc) \Rightarrow Action

Extreme action principle \Rightarrow Eoms & conservation equations

Discrete symmetries \Rightarrow Discontinuous processes

Dynamical discrete symmetries

- 1) Discrete (permutation) symmetry and continuous symmetries already unified in DG framework
 - 2) Just as discrete (T&P) and continuous spacetime symmetries already unified in the Lorentz group
- 1) and 2) turn out to be related : global T symmetry is permutation symmetry !

Dynamical discrete symmetries \Rightarrow discontinuous transitions in addition to usual continuous evolution processes deduced from differential eoms.

\Rightarrow Fills the gap between the discrete and the continuous

\Rightarrow Hopefully opens the way to a genuine unification (understanding) of QM discrete and non local laws to the rest of physics !

Field Discontinuities

- $a(t)$ discontinuities in time

- Different domains for the continuous and discontinuous : $]$... , $T - [$, $]$ $T +$, ... $[$
- Impossible discontinuity in GR, possible in DG thanks to permutation symmetry

- $a(t)$ discontinuities in space

- Save gravity of stars
- Allow exchanges between 2 sides (crossing metrics)
- If necessary, allow simultaneous crossing of densities and pressures

$$\rho(P, T) = \tilde{\rho}(P, T); p(P, T) = \tilde{p}(P, T)$$

⇒ Dark side voids can mimick dark matter

Not fundamental discontinuities (but can mimic them)

- Square potential wells and barriers
 - Approximate models for interfaces between different media
 - Continuity conditions for waves (fields and derivatives) at barrier
- Shock waves
- Topological defects (domain walls...)
 - Phase transitions with SSB
- BH singularities
 - discontinuous metric solutions but GR equations defined everywhere

a(t) discontinuous : GR

$$a^2(t)(dt^2-dx^2-dy^2-dz^2)$$

GR

$$H_-^2 = \frac{8\pi G}{3} \rho_- a_-^2 \quad H_+^2 = \frac{8\pi G}{3} \rho_+ a_+^2 \quad (1)$$

$$\frac{\dot{\rho}_-}{\rho_-} = -3H_- \quad \frac{\dot{\rho}_+}{\rho_+} = -3H_+ \quad (2)$$

T_- T_+

t

$$a_+(T_+) = C a_-(T_-)$$

$$H_+(T_+) = H_-(T_-)$$

$$\frac{c_+}{a_+^3(T_+)} = \rho_+(T_+) = \rho_-(T_-) = \frac{c_-}{a_-^3(T_-)}$$

(1) ~~OK~~
(2) OK

A piecewise GR ? (the flat homogeneous case)

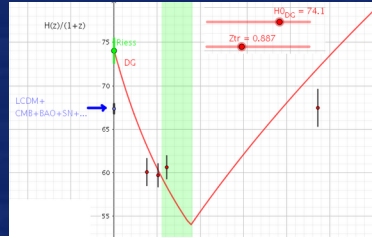
$$d\tau^2 = a^2(t)(-dt^2 + (dx^2 + dy^2 + dz^2))$$

Discontinuous scale factor and derivatives with continuous densities and Hubble rates ?

⇒ Not possible !

a(t) discontinuous : DG

$$a^2(t)(dt^2-dx^2-dy^2-dz^2)$$



$$a_-^2 H_-^2 - \tilde{a}_-^2 \tilde{H}_-^2 = \frac{8\pi G}{3} (\rho_- a_-^4 - \tilde{\rho}_- \tilde{a}_-^4) \quad \Big| \quad \text{DG} \quad a_+^2 H_+^2 - \tilde{a}_+^2 \tilde{H}_+^2 = \frac{8\pi G}{3} (\rho_+ a_+^4 - \tilde{\rho}_+ \tilde{a}_+^4) \quad (1)$$

$$\frac{\dot{\rho}_-}{\rho_-} = -3H_- \quad \frac{\dot{\tilde{\rho}}_-}{\tilde{\rho}_-} = -3\tilde{H}_- \quad \Big| \quad \frac{\dot{\rho}_+}{\rho_+} = -3H_+ \quad \frac{\dot{\tilde{\rho}}_+}{\tilde{\rho}_+} = -3\tilde{H}_+ \quad (2)$$

$T_- \quad T_+ \quad t$

$$\left. \begin{aligned} \left\{ \begin{aligned} a_+(T_+) &= C a_-(T_-) \\ \tilde{a}_+(T_+) &= C^{-1} \tilde{a}_-(T_-) \end{aligned} \right. \\ \left\{ \begin{aligned} H_+(T_+) &= H_-(T_-) \\ \tilde{H}_+(T_+) &= \tilde{H}_-(T_-) \end{aligned} \right. \\ \left\{ \begin{aligned} \rho_+(T_+) &= \rho_-(T_-) \\ \tilde{\rho}_+(T_+) &= \tilde{\rho}_-(T_-) \end{aligned} \right. \end{aligned} \right\} (2) \text{ OK}$$

$$\begin{aligned} a_+(T_+) &= \tilde{a}_-(T_-) \\ \tilde{a}_+(T_+) &= a_-(T_-) \end{aligned} \Rightarrow C = \frac{a_+(T_+)}{\tilde{a}_+(T_+)} = \frac{\tilde{a}_-(T_-)}{a_-(T_-)} \quad (1) \text{ OK}$$

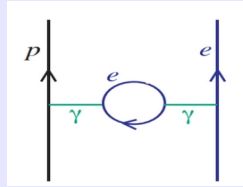
A piecewise DG ? (the flat homogeneous case)

$$d\tau^2 = a^2(t)(-dt^2 + (dx^2 + dy^2 + dz^2))$$

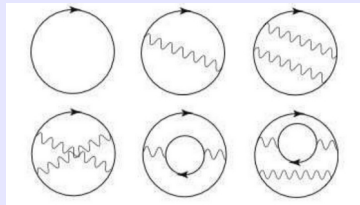
- Discontinuous scale factor with continuous Hubble rates and densities ?
 - Possible but only when conjugate densities are equal : the sought discontinuity triggering criterion !
 - Should both ρ and p cross at transition ? \Rightarrow If yes it's unlikely for the whole universe but likely for a part of the universe

Vacuum energy terms if gravity is classical

- Evidence for Vacuum energy Feynman graphs through Casimir and Lambshift effects : actually these graphs have quanta external legs.

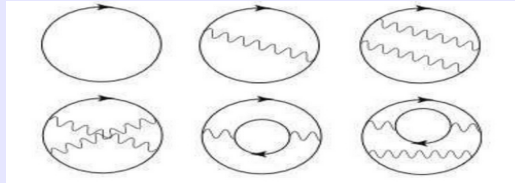


- In Quantum Gravity a cosmological constant is expected from the same kind of graphs in which the external legs are replaced by gravitons.
- If gravity is classical the true vacuum graphs (without quanta but rather external legs from classical external field) matter. But we have no evidence for the existence of such graphs so far !



Vacuum energy in DG equations

For a graph with quanta external legs, these correspond to particles coupled (classically) to one side of Janus field hence internal propagators must also be coupled to the same metric but such constraint no longer applies for a graph without quanta external legs !



⇒ Instead, it might be that the above true vacuum graphs belong to the background metric $\eta_{\mu\nu}$ and in this case :

a) No effect on Janus field as long as $g_{\mu\nu} \neq \tilde{g}_{\mu\nu} \neq \eta_{\mu\nu}$

b) $g_{\mu\nu} = \tilde{g}_{\mu\nu} = \eta_{\mu\nu} \Rightarrow$ DG vacuum source term is :

$$(\sqrt{g}\Lambda - \sqrt{\tilde{g}}\tilde{\Lambda}) g^{\mu\nu} \text{ vanishes because } \Lambda = \tilde{\Lambda}$$

SR vs QM

- Requirements for a « good » theory :

- | | | |
|------------------------------------|----------|-------------|
| - Self consistent | SR : OK | QM : OK |
| - Agrees with observations | SR : OK | QM : OK |
| - Economic = unifying = predictive | SR : OK* | QM : ??!!** |

* : SR unifies space and time, no other fundamental speed than c

** : QM has arbitrary weird postulates i.e. not based on symmetry principles,

QM constant h defines an additional energy scale (others already exist !)

⇒ unification required ! Would explain

$$\frac{e^2}{c} = \frac{\hbar}{137, \dots}$$

Stability issues

- Semiclassical gravity \Rightarrow no quantum instability
- Background : No Big Rip ! Moreover background remains bounded thanks to global time reversal !
- Fluctuations growth rate is not problematic :
 - Before transition : would expect dark side perturbations to be unstable in contracting phase but gravity from these is negligible : suppressed by C^4 factor ($\sim \text{scale_factor}^8$) .
 - After transition : dark side perturbations can start to grow under their own gravity.
However gravity is periodically switched off near $t=0 \Rightarrow$ Homogeneous universe !
- Existence of any gravitationally bound system requires $C > 1$ (resp $C < 1$) on our (resp dark) side
 \Rightarrow only the non-ghost interaction remains because the ghost interaction decouples

Energy conditions

$$T_{ab} X^a X^b \geq 0$$

$$\left(T_{ab} - \frac{1}{2}Tg_{ab}\right) X^a X^b \geq 0$$

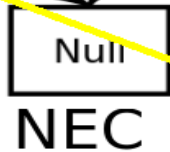
WEC +
No tachyons



NEC +
 $\rho + 3p \geq 0$

e.g. $\rho \geq 0 \Rightarrow w \geq -\frac{1}{3}$

Attractive gravity



$\rho + p \geq 0$

$\rho \geq 0$
 $w \geq -1$

$\rho \leq 0$
 $w \leq -1$

Photons feel attractive gravity

$\ddot{a} \leq 0$
Universe accélération
Inflation
($w \sim -1 < -1/3$)

- Required for:
- Stability of almost all scalar field theories
 - Singularity theorems
 - Against Closed Timelike Curves

Quantum
Phenomena



May be we can require
conditions to be valid on
Average only: ANEC, AWEC...

NEC and it's violation

arxiv:1401.4024

- NEC $\Rightarrow \dot{H} = -4\pi G(\rho + p) < 0 \Rightarrow H < 0$ (contraction) remains < 0 forever
 \Rightarrow No bounce \Rightarrow Big Bang Singularity.

- Scalar π theory with classical background π_c and perturbation χ $\pi = \pi_c + \chi$

- Lagrangian of perturbation (from linearized EoM)

sign(V)=sign($\rho+p$)
for theories with first
derivative Lagrangians

$$L_{\chi}^{(2)} = \frac{1}{2}U\dot{\chi}^2 - \frac{1}{2}V(\partial_i\chi)^2 - \frac{1}{2}W\chi^2$$

- Dispersion relation of waves $e^{i(\omega t - \mathbf{p}\cdot\mathbf{r})}$

$$U\omega^2 = V\mathbf{p}^2 + W$$

- Energy density

$$T_{00}^{(2)} = \frac{1}{2}U\dot{\chi}^2 + \frac{1}{2}V(\partial_i\chi)^2 + \frac{1}{2}W\chi^2$$

$U>0, V>0, W>0 \Rightarrow$
stable background

$U>0, V>0, W<0 \Rightarrow$ Tachyonic
instability on time scale $W^{-0.5}$

$U>0, V<0$ or $U<0, V>0 \Rightarrow$
Gradient instability at all
scales

$U<0, V<0 \Rightarrow$ classically stable
background but Quantum
ghost instability

Phantom Quintessence fields (treated as classical)

$$\ddot{\delta\phi}_k + 2\frac{\dot{a}}{a}\dot{\delta\phi}_k + a^2 \left(k^2 - \frac{\partial^2 V}{\partial\phi^2} \right) \delta\phi_k = -\frac{1}{2}\dot{h}\dot{\phi}$$

-m²

- arxiv:0104112 The Tensor to Scalar Ratio of Phantom Dark Energy Models A. E. Schulz, M. White
« we are certain that these perturbations are stable, because $\partial^2 V / \partial\phi^2$ is negative »
- arxiv:0301273 Can the dark energy equation-of-state parameter w be less than -1 ? Sean M. Carroll, Mark Hoffman, Mark Trodden
« our phantom model does not predict any significant departures from conventional dark-energy scenarios; in particular, there is no evidence of dramatic instabilities distorting the power spectrum »

Pros and Cons Quantum Gravity

- Pros

- Unification (gravity and EM are similar)
- Avoid GR singularities
- Technical difficulties (ambiguous regularization, instabilities ...) in semiclassical approach
- Required by no collapse interpretation of QM while collapse interpretation problematic for Bianchi identities and allows faster than light signalling.

- Cons

- True Unification : more fundamental level of description and understanding of QM itself (required by $1/137...$)
EM and DG gravity not anymore so similar
- DG already avoid singularities at classical level
- Huge technical difficulties for QG, Vacuum energy regularization might be avoided in DG
- Collapse interpretation not conflicting with Bianchi identities in piecewise gravity (discontinuous rather than quantum). Privileged frame ready to host faster than light signals without menacing causality.

Problems with semiclassical Gravity ?

arxiv:0802.1978

- Case 1 : Classical gravity triggers quantum collapses \Rightarrow no Energy-momentum conservation violation, nor violation of uncertainty relations contrary to popular argument by Eppley & Hannah ...

otherwise :

- Case 2A : No collapse interpretation of QM (MWI, decoherence ...) ruled out because classical gravity would see the uncollapsed superpositions
- Case 2B : Realistic collapse interpretation of QM leads to possible faster than light signaling. Either specific more local model of quantum collapse can solve this or ...

DG : instantaneous signaling is not anymore a menace to causality as soon as there exists a unic privileged instantaneity frame for any collapse !

Tension on H0

- Tension (4.4σ) between H0 low z (SN + cepheids) and H0 (CMB, BAO, SN, lensing)
- Reminder: Planck measures first peak angle ~ 0.001041 @ 0.03 %
 - \Rightarrow constraint on $f(\Omega_M, \Omega_b, \Omega_r, H0)$ @ 0.03 % (flat LCDM hypothesis)
 - \Rightarrow constraint on $F(\Omega_M, H0)$ @ 0.3 % and highly degenerate
 - $\Rightarrow \Omega_M$ & H0 @ % + low z constraints (SN, BAO, ...) @ 0.8%

Theory and experiment in gravitational physics

REVISED EDITION

Rosen's bimetric theory

(a) *Principal references*: Rosen (1973, 1974, 1977, 1978), Rosen and Rosen (1975), Lee et al. (1976).

(b) *Gravitational fields present*: the metric \mathbf{g} , a flat, nondynamical metric $\boldsymbol{\eta}$.

(c) *Arbitrary parameters and functions*: None.

(d) *Cosmological matching parameters*: c_0, c_1 .

(e) *Field equations*: The field equations are derived from the action

$$I = (64\pi G)^{-1} \int \eta^{\mu\nu} g^{\alpha\beta} g^{\gamma\delta} (g_{\alpha\gamma|\mu} g_{\beta\delta|\nu} - \frac{1}{2} g_{\alpha\beta|\mu} g_{\gamma\delta|\nu}) \times (-\eta)^{1/2} d^4x + I_{\text{NG}}(q_A, g_{\mu\nu}) \quad (5.67)$$

where the vertical line “|” denotes covariant derivative with respect to $\boldsymbol{\eta}$. The field equations may be written in the form

$$\square_{\boldsymbol{\eta}} g_{\mu\nu} - g^{\alpha\beta} \eta^{\gamma\delta} g_{\mu\alpha|\gamma} g_{\nu\beta|\delta} = -16\pi G (g/\eta)^{1/2} (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T), \quad \mathbf{Riem}(\boldsymbol{\eta}) = 0 \quad (5.68)$$

where $\square_{\boldsymbol{\eta}}$ is the d'Alembertian with respect to $\boldsymbol{\eta}$, and $T \equiv T_{\mu\nu} g^{\mu\nu}$.

(f) *Post-Newtonian limit*: We choose coordinates in which $\boldsymbol{\eta}$ has the form $\text{diag}(-1, 1, 1, 1)$ everywhere. In the universe rest frame, \mathbf{g} then has the asymptotic form $\text{diag}(-c_0, c_1, c_1, c_1)$ [see Equation (5.1)], where c_0 and c_1 may vary on a Hubble timescale. Following the method of Section 5.1, we obtain for the PPN parameters (Lee et al., 1976)

$$\begin{aligned} \gamma = \beta = 1, \quad \xi = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0, \\ \alpha_1 = 0, \quad \alpha_2 = (c_0/c_1) - 1 \end{aligned} \quad (5.69)$$

with

$$G_{\text{today}} \equiv G(c_0 c_1)^{1/2} = 1 \quad (5.70)$$

(g) *Discussion*: The PPN parameters are identical to those of general relativity except for α_2 , which may be nonzero if $c_0 \neq c_1$. Notice that the ratio c_1/c_0 is equal to the square of the velocity of weak gravitational waves, in units in which the speed of light is unity. This can be seen as follows. In a quasi-Cartesian coordinate system, in which $g_{\mu\nu}^{(0)} = \text{diag}(-1, 1, 1, 1)$, $\eta_{\mu\nu}$ must have the form

$$\eta_{\mu\nu} = \text{diag}(-c_0^{-1}, c_1^{-1}, c_1^{-1}, c_1^{-1})$$

and the vacuum, linearized field equations for $g_{\mu\nu}$ (wave equations for weak gravitational waves) take the form

$$(c_0/c_1) g_{\mu\nu,00} - \nabla^2 g_{\mu\nu} = 0 \quad (5.71)$$

whose solution is a wave propagating with speed $v_g = (c_1/c_0)^{1/2}$. Thus, in Rosen's theory, the PPN parameter α_2 measures the relative difference in speed (as measured by an observer at rest in the universe rest frame) between electromagnetic and gravitational waves. The values of c_0 and c_1 are determined by a solution of the cosmological problem. They can also be related to the covariant expressions

$$c_0 + 3c_1 = \eta^{\mu\nu} g_{\mu\nu}^{(0)}, \quad c_0^{-1} + 3c_1^{-1} = \eta_{\mu\nu} g^{(0)\mu\nu}$$

Theory and experiment in gravitational physics

REVISED EDITION

Consider a local, freely falling frame in any metric theory of gravity. Let this frame be small enough that inhomogeneities in the external gravitational fields can be neglected throughout its volume. However, let the frame be large enough to encompass a system of gravitating matter and its associated gravitational fields. The system could be a star, a black hole, the solar system, or a Cavendish experiment. Call this frame a “quasilocal Lorentz frame”. To determine the behavior of the system we must calculate the metric. The computation proceeds in two stages. First, we determine the external behavior of the metric and gravitational fields, thereby establishing boundary values for the fields generated by the local system, at a boundary of the quasilocal frame “far” from the local system. Second, we solve for the fields generated by the local system. But because the metric is coupled directly or indirectly to the other fields of the theory, its structure and evolution will be influenced by those fields, particularly by the boundary values taken on by those fields far from the local system. This will be true even if we work in a coordinate system in which the asymptotic form of $g_{\mu\nu}$ in the boundary region between the local system and the external world is that of the Minkowski metric. Thus, the gravitational environment in which the local gravitating system resides can influence the metric generated by the local system via the boundary values of the auxiliary fields. Consequently, the results of local gravitational experiments may depend

on the location and velocity of the frame relative to the external environment. Of course, local *nongravitational* experiments are unaffected ...

(c) A theory that contains the metric \mathbf{g} and additional dynamical vector or tensor fields or prior-geometric fields yields local gravitational physics that may have both location- and velocity-dependent effects. This will be true, for example, even if the auxiliary field is a flat background metric η . The background solutions for \mathbf{g} and η will in general be different, and therefore in a frame in which $g_{\mu\nu}$ takes the asymptotic form $\text{diag}(-1, 1, 1, 1)$, $\eta_{\mu\nu}$ will in general have a form that depends on location and that is not Lorentz invariant (although it will still have vanishing curvature). The resulting location and velocity dependence in η will act back on the local gravitational problem. (For a clear example of this, see Rosen’s theory in Chapter 5.) Be reminded that these effects are a consequence of the coupling of auxiliary gravitational fields to the metric and to each other, not to the matter and nongravitational fields. For metric theories of gravity, only $g_{\mu\nu}$ couples to the latter.

DG violates SEP : both LLI and LPI violation effects ! WEP is valid on each side.

GRAVITATION

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Mathematics was not sufficiently refined in 1917 to cleave apart the demands for "no prior geometry" and for a geometric, coordinate-independent formulation of physics. Einstein described both demands by a single phrase, "general covariance". The "no prior geometry" demand actually fathered general relativity, but by doing so anonymously, disguised as "general covariance", it also fathered half a century of confusion.

Active diffeomorphism

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General relativity is distinguished from other dynamical field theories by its invariance under *active* diffeomorphisms. Any theory can be made invariant under *passive* diffeomorphisms. Passive diffeomorphism invariance is a property of the *formulation* of a dynamical theory, while active diffeomorphism invariance is a property of the dynamical theory *itself*. Invariance under smooth displacements of the *dynamical* fields holds only in general relativity and in any general relativistic theory. It does not hold in QED, QCD, or any other theory on a fixed (flat or curved) background. Rovelli.

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Active diff invariance should not be confused with passive diff invariance, or invariance under change of coordinates. GR can be formulated in a coordinate free manner, where there are no coordinates, and no changes of coordinates. In this formulation, there field equations are *still* invariant under active diffs. Passive diff invariance is a property of a formulation of a dynamical theory, while active diff invariance is a property of the dynamical theory itself. A field theory is formulated in manner invariant under passive diffs (or change of coordinates), if we can change the coordinates of the manifold, re-express all the geometric quantities (dynamical *and non-dynamical*) in the new coordinates, and the form of the equations of motion does not change. A theory is invariant under active diffs, when a smooth displacement of the dynamical fields (*the dynamical fields alone*) over the manifold, sends solutions of the equations of motion into solutions of the equations of motion. Distinguishing a truly dynamical field, namely a field with independent degrees of freedom, from a nondynamical field disguised as dynamical (such as a metric field g with the equations of motion $\text{Riemann}[g]=0$) might require a detailed analysis (for instance, hamiltonian) of the theory.

(Rovelli, 2001, 122)

Diffeomorphism invariance is the key property of the mathematical language used to express the key conceptual shift introduced with GR: the world is not formed by a fixed non-dynamical spacetime structure, which defines localization and on which the dynamical fields live. Rather, it is formed solely by dynamical fields in interactions with one another. Localization is only defined, relationally, with respect to the fields themselves. (Rovelli, 2007, 1312)