

Incoherent hydrodynamics and density waves

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based on

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Motivation

- Hydrodynamics describes late-time, long wavelength behavior of systems with global $U(1)$ at finite temperature T , chemical potential μ , magnetic field B
⇒ Constitutive relations of conserved currents in gradient expansion, with transport coefficients determined by UV theory [Kovtun '12; ...]
- Physical systems can break translations explicitly through lattice or impurities. Momentum relaxes
 - QFT with background sources $\nabla_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu + \mathcal{O} \nabla^\nu \phi_s$
 - Modify conservation equation $\dot{P}^i + \nabla_j T^{ij} = -\Gamma^{ij} P_j$ [Davison, Goutéraux '14]
- Spontaneous breaking of translations below some critical temperature T_c ⇒ Goldstone modes. Observed in phase diagram of strange metals
- Pseudo-spontaneous regime when explicit deformations are perturbatively small ⇒ pinning of density waves

Motivation

2d hydrodynamic modes

- Normal phase
 - Longitudinal: sound + thermoelectric diffusion
 - Transverse: momentum diffusion
- Broken phase [Delacrétaz,Goutéraux,Hartnoll,Karlsson '17]
 - Longitudinal: sound + thermoelectric diffusion + internal/“crystal” diffusion
 - Transverse: sound
- Use holography to model strongly-coupled, non-quasiparticle quantum matter [Hartnoll,Lucas,Sachdev '16]
- Homogeneous models breaking translations in scalar sector (linear axions, Q-lattices, helical lattices, massive gravity)
[Andrade,Withers '13; Donos,Gauntlett '13; Donos,Goutéraux,Kiritsis '13;
Alberte,Ammon,Baggioli,Jiménez-Alba,Pujolàs '17,...]
- Derive hydrodynamic theory with strong explicit lattice and (pinned) density waves

Holographic Q-lattice

Bulk Q-lattice model with global $U(1)$

$$S = -\frac{1}{2} \int d^{d+1}x \sqrt{-g} G(|Z|^2) \partial_\mu Z \partial^\mu \bar{Z} + \dots,$$

Z : complex scalar dual to \mathcal{O}_Z of dimension Δ , with boundary expansion

$$Z = z_s r^{\Delta-d} + \dots + z_v r^{-\Delta} + \dots$$

- z_s : source of \mathcal{O}_Z
- z_v : VEV of \mathcal{O}_Z

Decomposition

$$Z = \Omega + iS = \phi e^{i\chi} \quad \Phi(\phi) \equiv G \phi^2$$

- $\delta\phi_s$ source Ω and pin the Goldstone modes
- ξ source S

Holographic Q-lattice

Homogeneous background

$$ds^2 = -U(r)dt^2 + \frac{1}{U(r)}dr^2 + g_{ij}(r)dx^i dx^j$$
$$\phi = \phi(r) \quad \chi = k_i x^i + c$$

- Dual QFT at finite T with explicit or spontaneous structure (below T_c) depending on asymptotics of ϕ
- Solutions labelled by k_i and c
- $k_i \neq 0 \Rightarrow$ breaking of translations + $U(1)$ down to diagonal subgroup
- $\delta\langle S \rangle = \langle \Omega \rangle \delta c$, with δc parametrising phase fluctuations \Rightarrow Goldstone modes

Holographic Q-lattice – diffusive mode

Focus on backgrounds with $k_i = 0 \Rightarrow$ equation for $\delta\chi$ decouples

Integrate over bulk and solve order by order in a hydrodynamic expansion

$$\delta\chi = e^{-i\omega t + i\varepsilon q_j x^j} [\delta c + \varepsilon \delta\chi_{[1]}(r) + \varepsilon^2 \delta\chi_{[2]}(r) + \dots], \quad \omega = \varepsilon\omega_{[1]} + \varepsilon^2\omega_{[2]} + \dots$$

Low energy Green's function [DMPZ '19; Amoretti, Areán, Goutéraux, Musso '18]

$$G_{SS} = \frac{\langle \Omega \rangle^2}{w^{ij} q_i q_j - i\omega \theta}, \quad \theta = \frac{s \Phi^{(0)}}{4\pi}$$

- θ is a new (horizon) transport coefficient
- $w^{ij} = \frac{\delta^2 w}{\delta k_i \delta k_j}$, where w is the free energy

Josephson-type equation for phase mode $\delta\hat{c}(t, x^i)$

$$(\theta \partial_t - w^{ij} \partial_i \partial_j) \delta\hat{c} = \langle \Omega \rangle \hat{\xi}$$

Crystal diffusion/phason

[Delacrétaz, Goutéraux, Hartnoll, Karlsson '17; Baggioli, Landry '20]

Holographic Q-lattice – gap

Introduce infinitesimal background source $\delta\phi_s \Rightarrow$ pinning the Goldstone mode

Then

$$G_{ss} = \frac{\langle \Omega \rangle^2}{\langle \Omega \rangle \delta\phi_s + w^{ij} q_i q_j - i\omega \theta}$$

and Josephson relation becomes

$$[\sigma_b (\partial_t + \delta\omega_g) - w^{ij} \partial_i \partial_j] \delta\hat{c} = \langle \Omega \rangle \hat{\xi}, \quad \delta\omega_g = \frac{\langle \Omega \rangle}{\theta} \delta\phi_s$$

$\delta\omega_g \sim \delta\phi_s$ relaxation time for the gapped mode

Full holographic setup

EMD with global $U(1)^2$ [DMPZ '19; Amoretti,Areán,Goutéraux,Musso '17; '19].

$$S = \int d^4x \sqrt{-g} \left[R - V - \frac{1}{2} (G\partial Z \partial \bar{Z} + W\partial Y \partial \bar{Y}) - \frac{\tau}{4} F^2 \right]$$

Z and Y complex scalars dual to \mathcal{O}_Z and \mathcal{O}_Y

- $Y \rightarrow$ explicit lattice with periods $\sim 1/k_{si}$ determined externally
- $Z \rightarrow$ density waves with periods $\sim 1/k_i$ determined thermodynamically

Polar decomposition

$$Y = \psi e^{i\sigma} \quad Z = \phi e^{i\chi} \quad \Psi \equiv W(\psi)^2 \quad \Phi \equiv G(\phi)^2$$

Background scalar sector

$$\psi = \psi(r) \quad \sigma = k_{si}x^i \quad \phi = \phi(r) \quad \chi = k_i x^i + c$$

- Asymptotic boundary behavior describing QFT at T, μ deformed by explicit lattice, with density waves

Hydrodynamic perturbations

Schematically:

- Hydrodynamic expansion of perturbations

$$\delta X = e^{-i\omega t + i\varepsilon q_j x^j} [\delta X_{[0]}(r) + \varepsilon \delta X_{[1]}(r) + \varepsilon^2 \delta X_{[2]}(r) + \dots]$$

- Induce thermal gradient, electric field, source for S : $\{\zeta_i, E_i, \xi\}$
- At each order split them as

$$\delta X_{[n]} = \delta \tilde{X}_{[n]} + \frac{\partial X_b}{\partial T} \delta T_{[n]} + \frac{\partial X_b}{\partial \mu} \delta \mu_{[n]} + \frac{\partial X_b}{\partial c} \delta c_{[n]}$$

- Order by order solution
 - $\omega_{[1]} = \delta T_{[0]} = \delta \mu_{[0]} = 0$
 - $\delta \tilde{X}_{[1]}$ is the change of the background solution under $\delta k_i = i \varepsilon q_i \delta c$

Hydrodynamic perturbations

- Hamiltonian decomposition of eoms \Rightarrow constraints on radial hypersurface + radial evolution
- Constraints at the boundary lead to the Ward identities of charge conservation, diffemorphism and Weyl invariance \Rightarrow current conservation
- Define bulk $U(1)$ and heat current, then use radial eoms to relate boundary currents $\delta J^i, \delta J_H^i$ to horizon currents
- Integrate out fluid velocity from horizon momentum constraint and derive field theory constitutive relations

The effective theory

Constitutive relations for charge and heat currents

$$\begin{aligned}\delta J^i &= \sigma_H^{ij} \left(\hat{E}_j - \partial_j \delta \hat{\mu} \right) + T \alpha_H^{ij} \left(\hat{\zeta}_j - T^{-1} \partial_j \delta \hat{T} \right) - \gamma^i \partial_t \delta \hat{c} \\ \delta J_H^i &= T \bar{\alpha}_H^{ij} \left(\hat{E}_j - \partial_j \delta \hat{\mu} \right) + T \bar{\kappa}_H^{ij} \left(\hat{\zeta}_j - T^{-1} \partial_j \delta \hat{T} \right) - \lambda^i \partial_t \delta \hat{c}\end{aligned}$$

Transport coefficients expressed in terms of horizon data and susceptibilities

$$\begin{aligned}\sigma_H^{ik} &= \frac{\tau^{(0)} s}{4\pi} g_{(0)}^{ik} + \frac{4\pi\rho^2}{s} (\mathcal{B}^{-1})^{ik} & \alpha_H^{ik} &= 4\pi\rho (\mathcal{B}^{-1})^{ik} \\ \bar{\kappa}_H^{ik} &= 4\pi Ts (\mathcal{B}^{-1})^{ik} & \gamma^i &= 4\pi T\rho (\mathcal{B}^{-1})^{ik} \quad \eta_k = \frac{\rho}{Ts} \lambda^i\end{aligned}$$

where

$$\mathcal{B}_{ij} = \Psi^{(0)} k_{si} k_{sj} + \Phi^{(0)} k_i k_j \quad \eta_i = \frac{\Phi^{(0)} k_i}{4\pi T}$$

- Coupling to massless modes $\delta \hat{c}$ determined by a single extra quantity η_i

The effective theory

Pin density waves with perturbative deformation $\delta\phi_s \Rightarrow$ Josephson relation

$$[\theta\partial_t + \langle\Omega\rangle\delta\phi_s]\delta\hat{c} + \eta_i\delta\hat{J}_H^i - \partial_i\hat{w}^i = \langle\Omega\rangle\hat{\xi}, \quad w^i = \frac{\delta w}{\delta k_i}$$

- Josephson relation only involves heat current J_H^i

Along with the current conservation equations

$$\partial_t\hat{\rho} + \partial_i\delta\hat{J}^i = 0$$

$$T\partial_t\hat{s} + \partial_i\delta\hat{J}_H^i = 0$$

define a closed system of equations for $\delta\hat{T}, \delta\hat{\mu}, \delta\hat{c}^l$

\Rightarrow Incoherent hydrodynamics with density waves

Pseudo-gapless modes

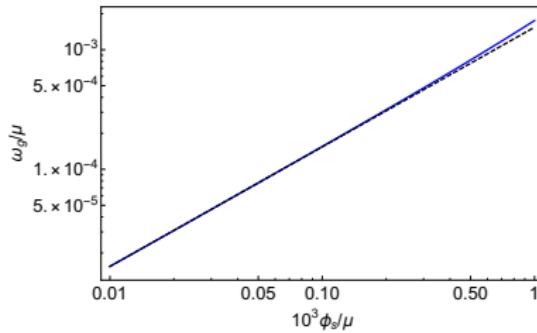
At zero wavevector in the Goldstone mode sector, Josephson relation leads to

$$[i \mathcal{M}^{-1} + \delta\omega_g] \delta c = 0, \quad \mathcal{M} = \frac{(\theta - \eta_i \lambda^i)}{\langle \Omega \rangle \delta\phi_s}$$

- Gap is given in terms of horizon quantities

$$\delta\omega_g = \frac{4\pi \langle \Omega \rangle \delta\phi_s}{s \Phi^{(0)}} \left(1 + \frac{k^2 \Phi^{(0)}}{k_s^2 \Psi^{(0)}} \right)$$

- Numerical confirmation



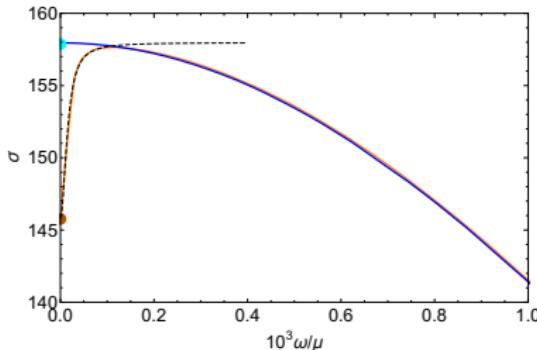
Finite frequency response

At zero wavevector, solve for $\delta\langle S \rangle, \delta J^i, \delta J_H^i$ in terms of the sources ζ_i, E_i, ζ_S
⇒ obtain retarded Green's functions, e.g.

$$\sigma^{ik} \equiv (i\omega)^{-1} G_{J^i J^k} = \sigma_H^{ik} + \omega \gamma^i \frac{(\theta - \eta_i \lambda^i)}{\omega + i\mathcal{M}^{-1}} \eta_j T \bar{\alpha}_H^{jk}$$

$$G_{SS} = i \langle \Omega \rangle^2 \frac{(\theta - \eta_i \lambda^i)}{\omega + i\mathcal{M}^{-1}}$$

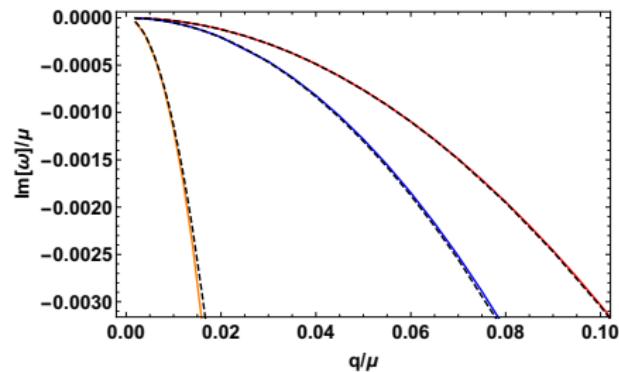
- Gaps determine poles
- $\omega \rightarrow 0, \delta\phi_s \rightarrow 0$ limits do not commute [Donos, Pantelidou '19; DMPZ '19]
- Numerical confirmation



Diffusive modes

Switch off sources and plug constitutive relations in conservation equations + Josephson relation \Rightarrow 3 purely diffusive modes with complicated analytic expressions involving transport coefficients and susceptibilities

- Numerical confirmation

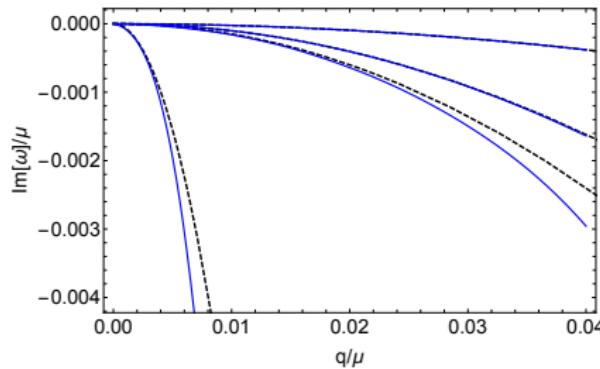


Finite magnetic field

Put the system at finite magnetic field B [Donos,Pantelidou,VZ '21]

Longitudinal + transverse sound \Rightarrow magnetophonon + (gapped) magnetoplasmon

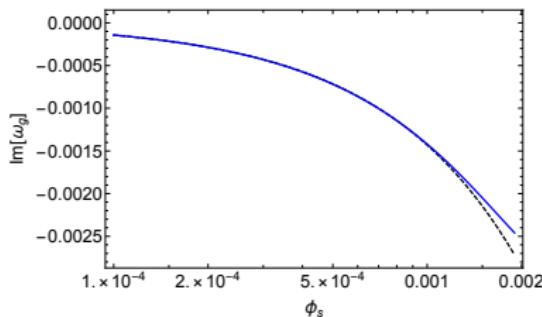
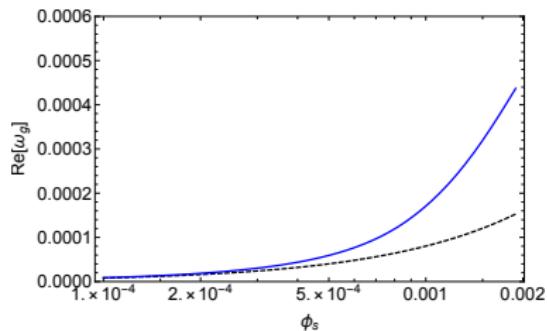
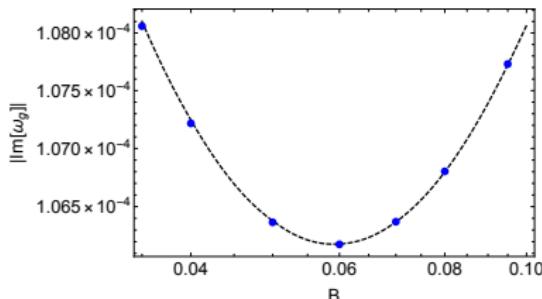
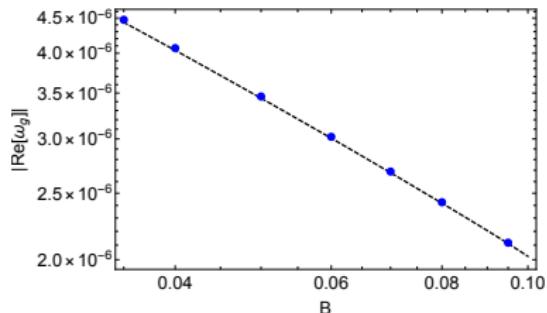
- Due to strong lattice, hydrodynamic modes are purely diffusive, in contrast to (pseudo)-spontaneous cases [Delacrétaz,Goutéraux,Hartnoll,Karlsson '19; Baggioli,Grienerger,Li '20]
- Analytic construction and numerical confirmation



Finite magnetic field

Pinning frequency has gap + resonance

- Results exact in B , perturbative in $\delta\phi_s$



Outlook

Summary

- Used holography to derive effective hydrodynamics in phases with density waves breaking translations spontaneously, in the presence of momentum relaxation

Future directions

- (Pseudo)-spontaneous regime [Amoretti,Areán,Goutéraux,Musso '19; Delacrétaz,Goutéraux,Hartnoll,Karlsson '19; Baglioli,Grienerger,Li '20]?
- Understand from field theory [Delacrétaz,Goutéraux,Hartnoll,Karlsson '17; Amoretti,Areán,Brattan,Magnoli '21]
- Coupling J^i to Josephson relation?
- Inhomogeneous models?
- Analytic construction of sound modes?

Thank You