

RANDOM FIELD THEORIES
&
PARISI - SOURLAS SUSY

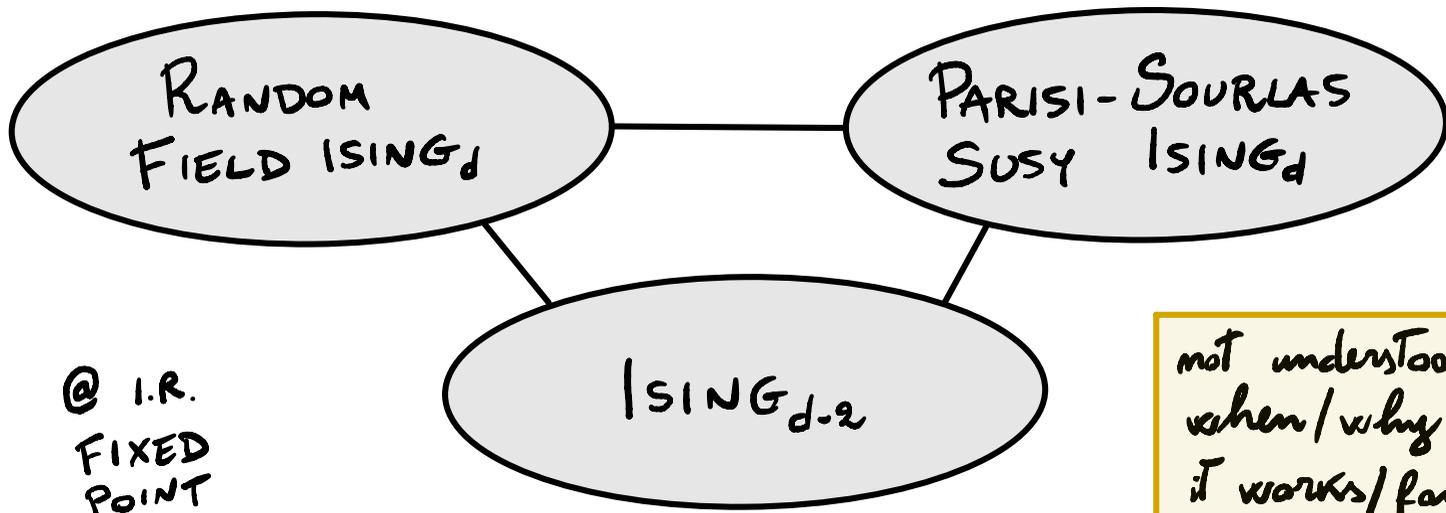
IRN:QFS Kick-off meeting
11 June 2021

EMILIO TREVISANI

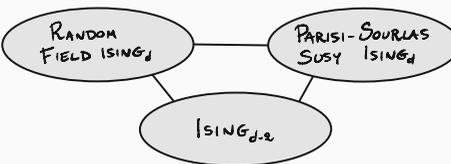
with: APRATIM KAVIRAJ [1912.01617] + [To appear]
 SLAVA RYCHKOV [2009.10087]

MOTIVATIONS

- Models with impurities (w/ long standing questions)
- Beautiful story relating 3 different models



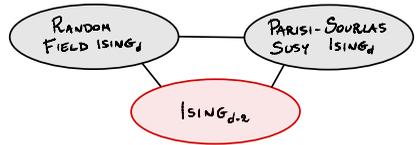
INDICATIONS FROM MONTE-CARLO

DIMENSION d	
≥ 6	✓
5	✓ (?)
4	✗
3	✗

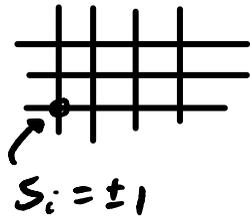
Trivial
(mean field)
Theory

M.C.

ISING MODEL



$$H = -J \sum_{\langle i,j \rangle} s_i s_j$$

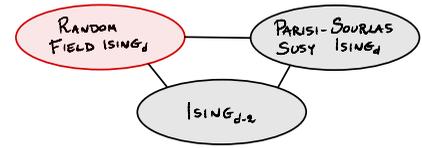


Continuous description (close to the F.P.)

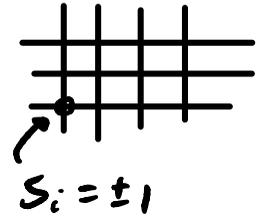
$$S = \int d^d x \left(\partial_\mu \phi \right)^2 + m^2 \phi^2 + \lambda \phi^4$$

- upper critical dimension $d_c = 4$

RANDOM FIELD ISING



$$H = -J \sum_{\langle i,j \rangle} S_i S_j + \sum_i h_i S_i$$



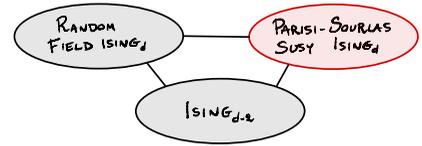
Continuous description (close to the F.P.)

$$S = \int d^d x \left(\partial_\mu \phi \right)^2 + m^2 \phi^2 + \lambda \phi^4 + h(x) \phi(x)$$



- h gaussian $\overline{h(x)} = 0$ $\overline{h(x)h(y)} = \delta(x-y)$
- upper critical dimension $d_c = 6$

PARISI - SOURLAS ISING



$$S = \int d^d x d\theta d\bar{\theta} (D_a \Phi)^2 + m^2 \Phi^2 + \lambda \Phi^4$$

$$\begin{cases} \Phi(x, \theta, \bar{\theta}) = \varphi(x) + \theta \bar{\psi}(x) + \bar{\theta} \psi(x) + \theta \bar{\theta} w(x) \\ D_a = (\partial_\mu, \partial_\theta, \partial_{\bar{\theta}}) \end{cases}$$

anticommuting scalars

$$S = \int d^d x (\partial\varphi\partial w - w^2 + \partial\psi\partial\bar{\psi}) + m^2(\varphi w + \psi\bar{\psi}) + \lambda(\varphi^3 w + 3\varphi^2\psi\bar{\psi})$$

• NON UNITARY

• upper critical dimension $d_c = 6$

II

PERTURBATIVE
R.G.

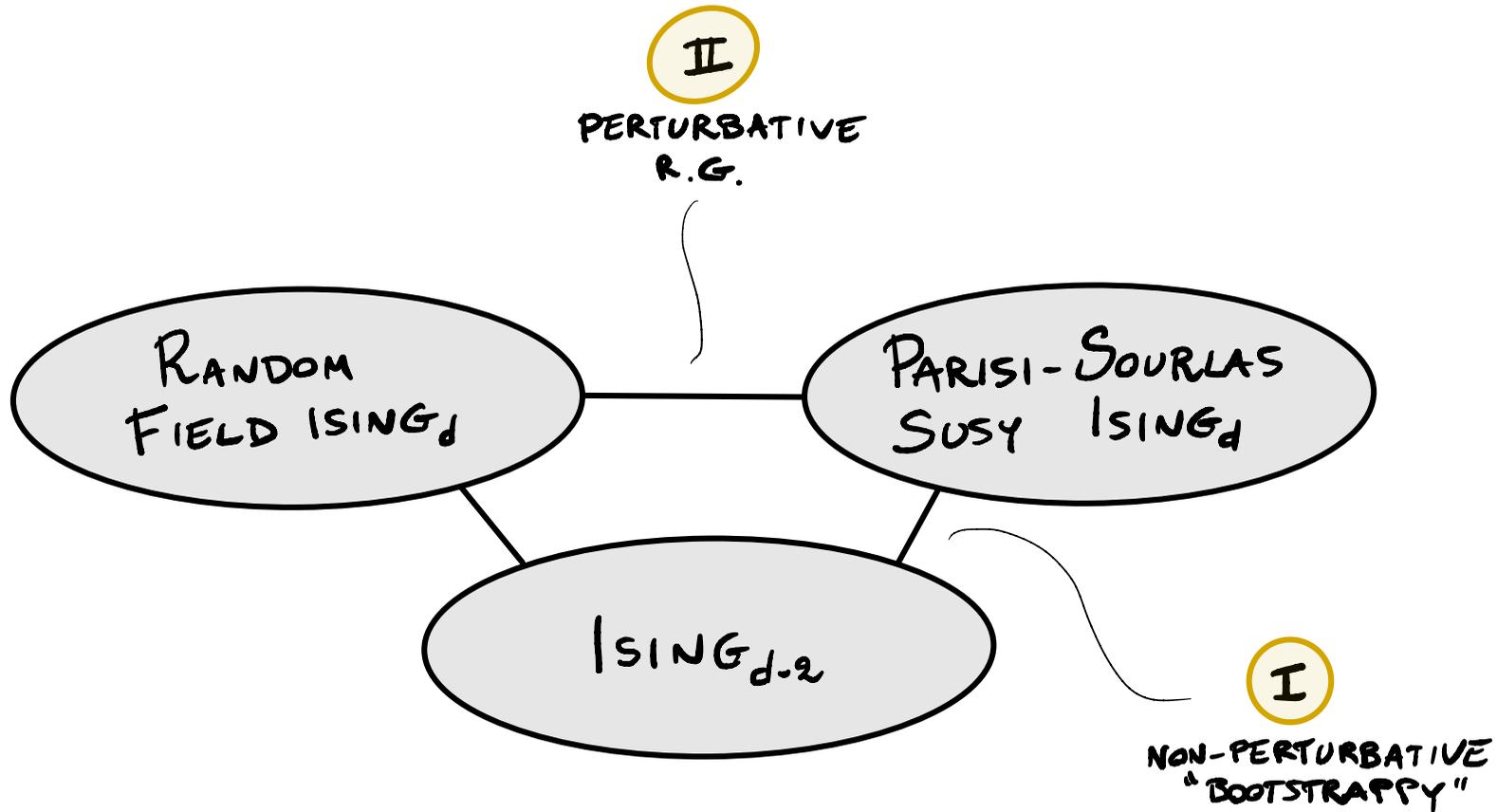
RANDOM
FIELD ISING_d

PARISI-SOURLAS
SUSY ISING_d

ISING_{d-2}

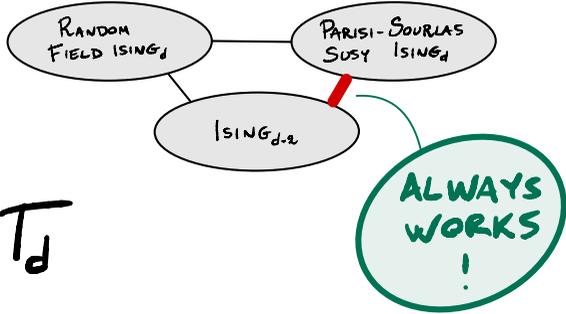
I

NON-PERTURBATIVE
"BOOTSTRAPPY"



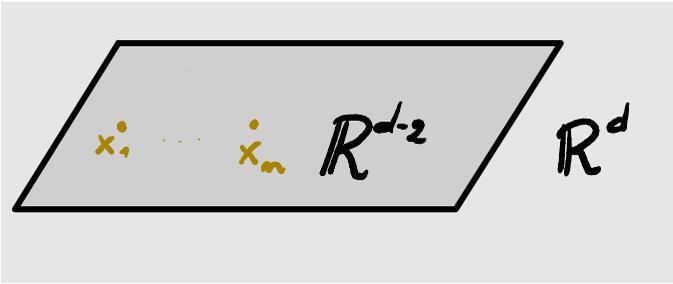
I

DIMENSIONAL REDUCTION



- Define PARISI-SOURIAS SCFT_d
- Understand DIMENS. REDUCTION to CFT_{d-2}

$$\langle U_1(x_1) \dots U_m(x_m) \rangle_{\text{SCFT}_d} = \langle \hat{U}_1(x_1) \dots \hat{U}_m(x_m) \rangle_{\text{CFT}_{d-2}}$$



DECOUPLING
of ∞ -many
OPERATORS!

DIMENSIONAL REDUCTION

OPERATORS

TMD

OPE

CONFORMAL BLOCKS

SUPER C.B.

$$G_{\Delta, \ell}^{(d)} = g_{\Delta, \ell}^{(d-2)}$$

RELATION of CONFORMAL BLOCKS across dimensions

$$g_{\Delta, \ell}^{(d-2)} = g_{\Delta, \ell}^{(d)} + c_{2,0} g_{\Delta+2, \ell}^{(d)} + c_{1,-1} g_{\Delta+1, \ell-1}^{(d)} + c_{0,-2} g_{\Delta, \ell-2}^{(d)} + c_{2,-2} g_{\Delta+2, \ell-2}^{(d)}$$

$$c_{2,0} = -\frac{(\Delta-1)\Delta(\Delta-\Delta_{12}+\ell)(\Delta+\Delta_{12}+\ell)(\Delta-\Delta_{34}+\ell)(\Delta+\Delta_{34}+\ell)}{4(d-2\Delta-4)(d-2\Delta-2)(\Delta+\ell-1)(\Delta+\ell)^2(\Delta+\ell+1)},$$

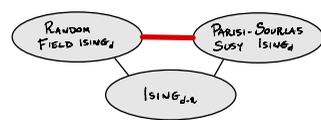
$$c_{1,-1} = -\frac{(\Delta-1)\Delta_{12}\Delta_{34}\ell}{(\Delta+\ell-2)(\Delta+\ell)(d-\Delta+\ell-4)(d-\Delta+\ell-2)},$$

$$c_{0,-2} = -\frac{(\ell-1)\ell}{(d+2\ell-6)(d+2\ell-4)},$$

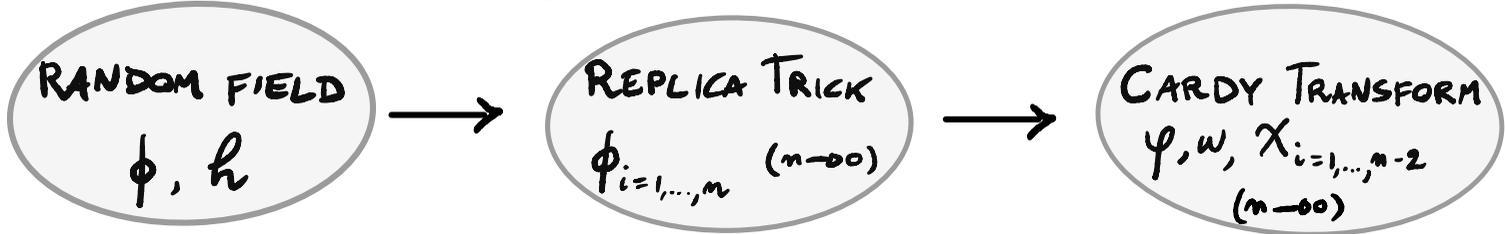
$$c_{2,-2} = \frac{(\Delta-1)\Delta(\ell-1)\ell(d-\Delta-\Delta_{12}+\ell-4)(d-\Delta+\Delta_{12}+\ell-4)(d-\Delta-\Delta_{34}+\ell-4)(d-\Delta+\Delta_{34}+\ell-4)}{4(d-2\Delta-4)(d-2\Delta-2)(d+2\ell-6)(d+2\ell-4)(d-\Delta+\ell-5)(d-\Delta+\ell-4)^2(d-\Delta+\ell-3)}.$$

II

EMERGENCE of SUSY

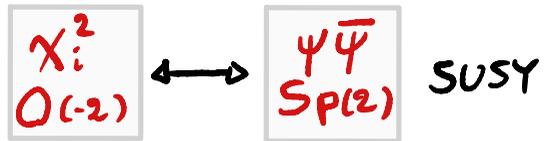


o DEFINE R.G. for RANDOM FIELD THEORY

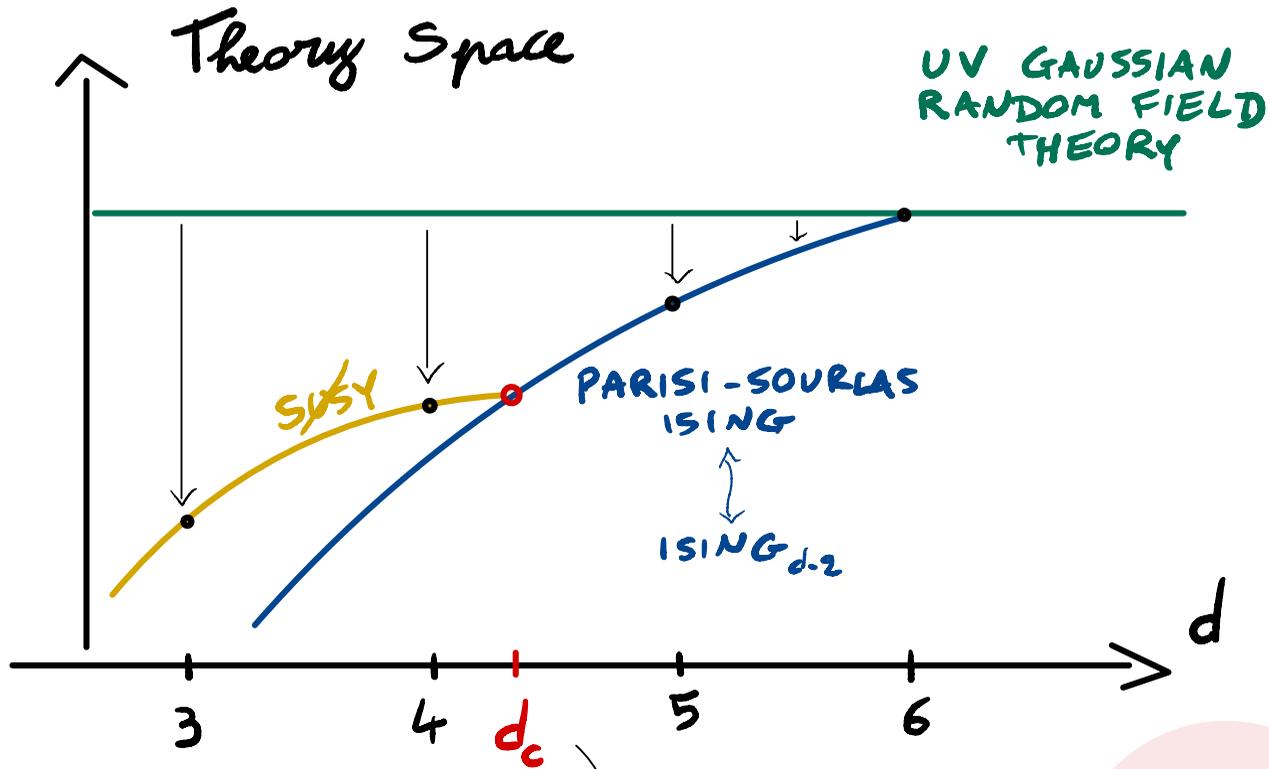


o STUDY R.G. for R.F. ISING in $d = 6 - \epsilon$

$S = \int d^d x (\partial \varphi \partial w - w^2 + \partial \chi_i \partial \chi_i) + m^2 (\varphi w + \chi_i^2) + \lambda (\varphi^3 w + 3 \varphi^2 \chi_i^2)$



study perturbations in ϵ -expansion { irrelevant \rightarrow SUSY
 relevant \rightarrow SUSY



operators of the form $X_i^{k>2}$ become relevant

AGREES WITH MONTE CARLO!

