

# Branes, Fermions, & Superspace Dualities

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## Motivation

- Gaugino condensates in supergravity theories [Dine,Rohm,Seiberg,Witten] .  
Action structure because of SUSY: (Heterotic, Type I, (4D) EFTs...)

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- What if gaugino lives in hypersurface of  $\text{codim} \neq 0$ ? [Horava,Witten] and brought to KKLT context in [Hamada,Hebecker,Shiu,Soler]

$$\mathcal{S} \supset \int (F^2 - 2(\lambda\lambda F)\delta + ???)$$

where  $(\lambda\lambda F)$  is a coupling on brane worldvolume

- Lead to speculation about  $\mathcal{O}(\lambda^4)$  terms on brane actions

**Today:** approach to obtain fermion couplings on branes from first principles

## Branes and fermions in superspace formulation I

- [Bergshoeff,Sezgin,Townsend] In **superspace formulation** of supergravity:  
M2-brane action obtained by considering it is a 3D hypersurface in  
 $(11|32)$  superspace  $(Z^M = (x^m, \theta^\mu) \quad m = 0, 1, \dots, 10 \text{ & } \mu = 1, \dots, 32)$

$$S_{M2} = -T_{M2} \int d^3\zeta \sqrt{-\det(P[\mathbf{G}(Z)])} + \mu_{M2} \int P[\mathbf{A}_3(Z)]$$

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$$\mathbf{S}_{M2} = -T_{M2} \int d^3\zeta \sqrt{-\det(P[\mathbf{G}(Z)])} + \mu_{M2} \int P[\mathbf{A}_3(Z)]$$

- Same for any other brane, it lives in  $(10|32)$  superspace

$$\mathbf{S}_{Dp} = -T_{Dp} \int d^{p+1}\zeta \sqrt{-\det(P[\mathbf{G} + \mathbf{B}] + F)} + \mu_{Dp} \int P \left[ \sum_q \mathbf{C}_q e^{-\mathbf{B}} \right] e^{-F}$$

## Branes and fermions in superspace formulation II

Consider a brane-only solution for simplicity

**Bulk:**  $1/2$  supercharges ( $Q_\alpha \ \alpha = 1, \dots, 32$ ) spontaneously broken

**Brane worldvolume:**  $\mathbf{S}(Z)$  built as product of *off-shell superfields*: it is a superfield and it has a  $\theta$  expansion up to order  $(\theta)^{32}$

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- Bulk on-shell: 1/2 of  $\theta \Rightarrow$  Goldstinos = fermions on brane
  - Fermions on brane action  $\Leftrightarrow$  superfield  $\theta$  expansions
- The other half are redundancies:  $\kappa$ -symmetry  
[Bergshoeff,Sezgin,Townsend] (for M2-case, but it's general)

$$\delta_\kappa Z^M \text{ (at } \theta = 0) = \begin{cases} \delta_\kappa x = 0 \\ \delta_\kappa \theta = \frac{1}{2}(1 + \Gamma_{M2(Dp)})\kappa \end{cases}, \quad \delta_\kappa \mathbf{S} = 0$$

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- Extends to all Dp-brane only backgrounds
- And also to all general backgrounds including branes
- To compute brane fermion couplings:  $\theta$  **expansions of superfields**

## $\theta$ expansion of superfields & superspace dualities

We want superfield  $\theta$ -expansions in M-theory, and Type II supergravities, but getting them is not easy...

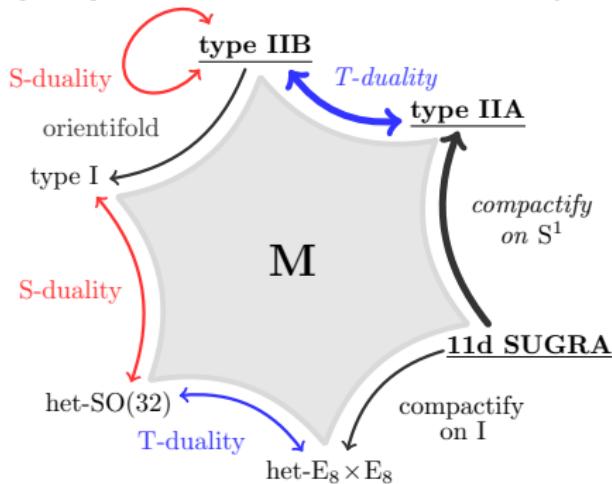
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## $\theta$ expansion of superfields & superspace dualities

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**Strategy:** obtain superfields in 11D SUGRA (less fields, less computations, more clean, and easier to capture any structure), then use

### Superspace Generalization of Duality web



## Superfields in 11D SUGRA

**Goal:**  $\theta$  expansion of  $\mathbf{S}_{M2}(Z) : \mathbf{G}_{MN}(Z)$  and  $\mathbf{A}_{MNP}(Z)$

We use normal coordinate method (bosonic background &  $\delta_\epsilon \psi_m = D_m \epsilon$ )

[Alvarez-Gaume,Freedmann,Mukhi; McArthur; Atick,Dhar; Grisaru,Knutt; Tsimpis; ...]

$$\mathbf{G}_{mn}(Z) = g_{mn} - i(\bar{\theta} \Gamma_{(m} D_{n)} \theta)$$

$$\mathbf{A}_{mnp}(Z) = A_{mnp} - \frac{3i}{2}(\bar{\theta} \Gamma_{[mn} D_{p]} \theta)$$

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[Alvarez-Gaume, Freedmann, Mukhi; McArthur; Atick, Dhar; Grisaru, Knutt; Tsimpis; ...]

$$\begin{aligned} \mathbf{G}_{mn}(Z) &= g_{mn} - i(\bar{\theta}\Gamma_{(m}D_{n)}\theta) - \frac{1}{4}(\bar{\theta}\Gamma_a D_{(m}\theta)(\bar{\theta}\Gamma^a D_{n)}\theta) \\ &\quad + \frac{1}{12}(\bar{\theta}\Gamma_{(m|}\mathcal{T}_b{}^{dfgh}\theta)(\bar{\theta}\mathcal{H}^b{}_{|n)d}{}^{fgh}\theta) + \frac{1}{12}(\bar{\theta}\Gamma_{(m}\Gamma^{bc}\theta)(\bar{\theta}\mathcal{W}_{|n)bc}\theta) + \dots \\ \mathbf{A}_{mnp}(Z) &= A_{mnp} - \frac{3i}{2}(\bar{\theta}\Gamma_{[mn}D_{p]}\theta) - \frac{3}{4}(\bar{\theta}\Gamma_{a[m}D_{n}\theta)(\bar{\theta}\Gamma^a D_{p]}\theta) \\ &\quad + \frac{1}{8}(\bar{\theta}\Gamma_{[mn|}\mathcal{T}_b{}^{dfgh}\theta)(\bar{\theta}\mathcal{H}^b{}_{|p)d}{}^{fgh}\theta) + \frac{1}{8}(\bar{\theta}\Gamma_{[mn}\Gamma^{bc}\theta)(\bar{\theta}\mathcal{W}_{|p]bc}\theta) + \dots \end{aligned}$$

where

$$\begin{aligned} \mathcal{H}^b{}_{mdfgh} &= \Gamma^b H_{dfgh} D_m - 6e_m^b \Gamma_{df} e_g^p e_h^q [D_p, D_q] \\ \mathcal{W}_{mbc} &= \Sigma_{bc}^{dfgh} H_{dfgh} D_m + \frac{1}{8} \Gamma_f e_m^f e_b^p e_c^q [D_p, D_q] + \frac{1}{4} \Gamma_b e_c^q [D_m, D_q] \\ \Sigma_{bc}^{dfgh} &= \frac{1}{576} \left( \Gamma_{bc} \Gamma^{dfgh} - 8\delta_{[c}^{[d} \Gamma_{b]} \Gamma^{fgh]} - 12\delta_{[c}^{[d} \delta_{b]}^f \Gamma^{gh]} \right) \\ \mathcal{T}_c^{dfgh} &= \frac{1}{288} (\Gamma_c \Gamma^{dfgh} - 12\delta_c^{[d} \Gamma^{fgh]}) \end{aligned}$$

Complications appear at order  $(\theta)^4 \dots$

## Superspace dimensional reduction: idea

M2-brane lives in (11|32) superspace, D2 brane in (10|32) superspace

**Superspace compactification** relates them

- $S^1$  (bosonic) compactification Ansatz in superspace

$$\hat{\mathbf{G}}_{\hat{m}\hat{n}} = \begin{pmatrix} e^{-2\Phi/3}(\mathbf{G}_{mn} + e^{2\Phi} \mathbf{C}_m \mathbf{C}_n) & e^{4\Phi/3} \mathbf{C}_m \\ e^{4\Phi/3} \mathbf{C}_n & e^{4\Phi/3} \end{pmatrix}, \quad \hat{\mathbf{A}}_{mnp} = \mathbf{C}_{mnp}, \quad \hat{\mathbf{A}}_{mn\ 10} = \mathbf{B}_{mn}$$

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- To obtain fermion couplings on D2-brane we need dimensional reduction Ansatz of fermions & SUSY operators

## Superspace dimensional reduction: an example

The dilaton superfield using the previous approach

$$\begin{aligned}\hat{\mathbf{G}}_{10 \ 10} &= e^{4\Phi/3} = e^{4(\phi+\rho^{(2)}+\dots)/3} \\ \hat{G}_{10 \ 10} - i\bar{\theta}\hat{\Gamma}_{10}\hat{D}_{10}\hat{\theta} + \dots &= e^{4\phi/3} \left( 1 + \frac{4\rho^{(2)}}{3} + \dots \right)\end{aligned}$$

Dimensional reduction of 11D supercovariant derivative requires dim reduction of 11D gravitino ( $\delta_\epsilon \lambda = \Delta \epsilon$ )

$$\hat{D}_{10} = \frac{e^\phi}{3} \Gamma_* \Delta \quad , \quad \hat{\Gamma}_{10} = e^{2\phi/3} \Gamma_* \quad , \quad \hat{\theta} = e^{-\phi/6} \theta = e^{-\phi/6} (\theta_+ + \theta_-)$$

So

$$\rho^{(2)} = \frac{-i}{4} \bar{\theta} \Delta \theta \quad \Rightarrow \quad \Phi = \phi - \frac{i}{4} \bar{\theta} \Delta \theta + \dots$$

## Fermion couplings on the D2-brane

So the D2-brane action is

$$S_{D2} = - \int d^3\zeta e^{-\Phi} \sqrt{-\det(\mathbf{g}_{ij} + \mathbf{B}_{ij} + f_{ij})} + \int (\mathbf{C}_3 - \mathbf{C}_1 \wedge (\mathbf{B}_2 + f_2))$$

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$$\mathbf{B}_{ij} = B_{ij} - i \bar{\theta} \Gamma_* \Gamma_{[i} D_{j]} \theta + \dots$$

$$\mathbf{C}_i = C_i - \frac{i}{2} e^{-\phi} \bar{\theta} \Gamma_* \left( D_i - \frac{1}{2} \Gamma_i \Delta \right) \theta + \dots$$

$$\mathbf{C}_{ijk} = C_{ijk} - \frac{3i}{2} e^{-\phi} \bar{\theta} \left( \Gamma_{[ij} D_{k]} - \frac{1}{6} \Gamma_{ijk} \Delta \right) \theta - 3i C_{[i} \bar{\theta} \Gamma_* \Gamma_j D_{k]} \theta + \dots$$

- Full agreement with [Marolf, Martucci, Silva] at order  $(\theta)^2$

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- Full agreement with [Marolf, Martucci, Silva] at order  $(\theta)^2$
- Approach works at all orders and we computed  $(\theta)^4$  terms (huge formulae)

## **Superspace T-duality: idea**

D $p$ -brane lives in certain  $(10|32)$  superspace, D $(p \pm 1)$  brane in a related  $(10|32)$  superspace

**Superspace T-duality** relates these superspaces

Approach is like in dim reduction:

- Promote (bosonic) T-duality relations to superfield level
- Taylor expand and identify terms order by order
- Use T-duality rules for fermions & SUSY operators [Hassan]

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$$\begin{aligned}\tilde{\mathbf{G}}_{99} &= \frac{1}{\mathbf{G}_{99}} \quad , \quad \tilde{\Phi} = \Phi - \frac{1}{2} \log(\mathbf{G}_{99}) \quad , \quad \tilde{\mathbf{G}}_{9m} = -\frac{\mathbf{B}_{9m}}{\mathbf{G}_{99}} \quad , \quad \tilde{\mathbf{B}}_{9m} = -\frac{\mathbf{G}_{9m}}{\mathbf{G}_{99}} \\ \tilde{\mathbf{G}}_{mn} &= \mathbf{G}_{mn} - \frac{\mathbf{G}_{9m}\mathbf{G}_{9n} - \mathbf{B}_{9m}\mathbf{B}_{9n}}{\mathbf{G}_{99}} \quad , \quad \tilde{\mathbf{B}}_{mn} = \mathbf{B}_{mn} - \frac{\mathbf{G}_{9m}\mathbf{B}_{9n} - \mathbf{B}_{Bm}\mathbf{G}_{9n}}{\mathbf{G}_{99}} \\ &\left( \left( \sum_q \tilde{\mathbf{C}}_q \right) e^{-\tilde{\mathbf{B}}} \right)_{9m_1 \dots m_p} = \left( \left( \sum_q \mathbf{C}_q \right) e^{-\mathbf{B}} \right)_{m_1 \dots m_p}\end{aligned}$$

## Superspace T-duality: an example

An example in some detail (order  $(\theta)^2$ )

$$\begin{aligned} (\mathbf{G}_{99})^A &= \left( \frac{1}{\mathbf{G}_{99}} \right)^B \\ (G_{99} - i\bar{\theta}\Gamma_9 D_9\theta + \dots)^A &= \left( \frac{1}{G_{99} + \gamma_{mn}^{(2)} + \dots} \right)^B = \left( \frac{1}{G_{99}} - \frac{\gamma_{mn}^{(2)}}{(G_{99})^2} + \dots \right)^B \end{aligned}$$

At order  $(\theta)^2$ :

$$(-i\bar{\theta}\Gamma_9 D_9\theta)^A = \{[\text{Hassan}] \text{ rules}\} = - \left( \frac{-i\bar{\theta}\Gamma_9 D_9\theta}{(G_{99})^2} \right)^B = - \left( \frac{\gamma_{mn}^{(2)}}{(G_{99})^2} \right)^B$$

## Fermion couplings on the Dp-brane

With this we can compute the  $\theta$  expansion for any Dp-brane

$$S_{Dp} = - \int d^{p+1} \zeta e^{-\Phi} \sqrt{-\det(\mathbf{g}_{ij} + \mathbf{B}_{ij} + f_{ij})} + \int \left( \sum_n \mathbf{C}^{(n)} e^{-(\mathbf{B}_2 + f_2)} \right)$$

where

$$\begin{aligned}\Phi &= \phi - \frac{i}{4} \bar{\theta} \Delta \theta + \dots \\ \mathbf{g}_{ij} &= g_{ij} - i \bar{\theta} \Gamma_{(i} D_{j)} \theta + \dots \\ \mathbf{B}_{ij} &= B_{ij} - i \bar{\theta} \Gamma_* \Gamma_{[i} D_{j]} \theta + \dots \\ \mathbf{C}_{i_1 \dots i_n}^{(n)} &= C_{i_1 \dots i_n}^{(n)} - \frac{i}{2} e^{-\phi} \bar{\theta} \mathcal{P}_n \left( n \Gamma_{[i_1 \dots i_{n-1}} D_{i_n]} - \frac{1}{2} \Gamma_{i_1 \dots i_n} \Delta \right) \theta + \dots\end{aligned}$$

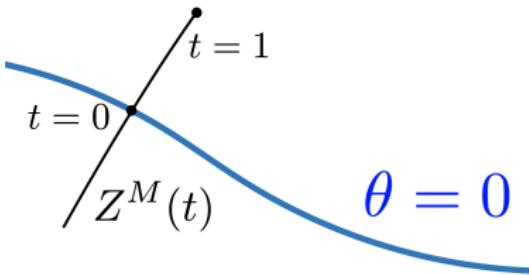
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## Summary

- Superspace formulation of supergravity for brane fermion couplings
- Goal: obtain superfield  $\theta$  expansions
- Best approach: obtain 11D superfields, then use superspace duality web
- Brane actions at order  $(\theta)^2$  ✓
- For order  $(\theta)^4$  limitations from 11D approach to write expansion nicely

Thank  
you

## NORCOR idea



$Z^M(t)$  is a particular geodesic in curved superspace satisfying:

$$v^B \nabla_B v^A = 0 \quad , \quad v^A(t = 0) = (0, y^\alpha)$$

$$v^A(t) = \frac{dZ^M}{dt}(Z(t)) E_M^A(Z(t))$$

$$\mathcal{S}(Z(t)) = \sum_n \frac{t^n}{n!} \left( \frac{\delta^n \mathcal{S}}{\delta t^n} \right)_{t=0} \quad \& \quad \frac{\delta \mathcal{S}}{\delta t} = \frac{\delta Z^M}{\delta t} \frac{\delta \mathcal{S}}{\delta Z^M} = \mathcal{L}_v \mathcal{S}$$

$$\text{So} \quad \mathcal{S}(Z(t=1)) = \sum_n \frac{1}{n!} ((\mathcal{L}_y)^n \mathcal{S})_{t=0} = (e^{\mathcal{L}_y} \mathcal{S})_{t=0}$$

## NORCOR in practice: supervielbein

$\mathcal{S}(Z(t=1)) = (e^{\mathcal{L}_y} \mathcal{S})_{t=0}$ : what do we do with it?

- ① Apply derivatives:  $\mathcal{L}_y E_M^A = \nabla_M y^A + y^C E_M^B T_{BC}^A$

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- ② Evaluate at  $\theta = 0$ : use tangent space props (e.g.  $\omega_{Ma}^\beta = 0$ ),  
WZ gauge  $E_M^A(\theta = 0) = \begin{pmatrix} e_m^a(x) & \psi_m^\alpha(x) \\ 0 & \delta_\mu^\alpha \end{pmatrix}, \quad y^A = (0, y^\alpha)$

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- ➌ Use superspace Bianchi IDs  $dH_4 = 0, \nabla T^A = E^B R_B^A, \nabla R_B^A = 0$  to write superspace objects into familiar spacetime fields

$$\mathcal{L}_y E_m^a = -iy^\alpha (\Gamma^a)_{\alpha\beta} \psi_m^\beta \quad , \quad \mathcal{L}_y E_m^\alpha = \nabla_m y^\alpha + e_m^b (T_b)_\beta^\alpha y^\beta$$

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- ➍ Replace tangent vector  $y^A$  for  $\theta$  and write superfield expansion:

$$E_m^a(Z) = e_m^a(x) - i\bar{\theta}\Gamma^a \psi_m(x) + \dots \quad , \quad E_m^\alpha(Z) = \psi_m^\alpha(x) + (D_m(x)\theta)^\alpha + \dots$$

- ➎ In bosonic backgrounds take  $\psi_m \rightarrow 0$