Consistent truncations around half-maximal AdS vacua from Exceptional Field Theory

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Based on:

- AdS_6 and AdS_7: [1808.05597], [1901.11039] with E. Malek and H.Samtleben
- AdS_5: [2012.15601] with E. Malek

Consistent truncations

- **Compactification**: $AdS_D \times M_{int.}$ with $M_{int.}$ compact
  - Lower dim. (gauged) supergravity $\overset{?}{\longleftrightarrow}$ 11/10-d theory
  - Domain walls, AdS-black holes,... easier in D-dim. gSUGRA
- **Consistent truncation**. Set higher KK-modes to zero

$$\begin{tikzcd} \boxed{\text{10/11-dim supergravity}} \arrow[d, shift right, "\text{Truncation to zero modes}"'] \\ \boxed{\text{D-dim. gSUGRA}} \arrow[u, shift right, "\text{Uplift solution to solution}"'] \end{tikzcd}$$

  - Not an effective action (no scale separation in AdS)
  - In general, hard to construct (non-trivial mixing of metric and flux,...)
  - Exceptional Field Theory: provides tools for a systematic analysis.
    - Maximal case: CT for $\mathrm{AdS}_5 \times S^5$, other sphere reductions, hyperboloids, 4d dyonic gaugings,...
      [Hohm, Samtleben, Baguet, Malek, Inverso, Trigiante,....]
    - Tools also from Exceptional Generalise Geometry.
      [Waldram, Lee, Strickland-Constable, Ashmore, Petrini, Cassani, Josse, Graña, Gabella....]
  - **Today**: Half-maximal consistent truncations in ExFT.

Exceptional Field Theory

Exceptional Field Theory

ExFT is a reformulation of supergravity adapted to a split $11 = D_{ext} + d_{int}$ where metric and flux are treated in the same footing.

11d supergravity on a torus $T^d \longrightarrow E_{d(d)}$ global symmetry.

- Purely internal degrees of freedom (scalars)

$$\left\{ g_{mn}, C^{(3)}_{mnp}, C^{(6)}_{mnpqrs} \right\} \to \mathcal{M}_{M_1 N_1} \in \frac{E_{d(d)}}{K(E_d)} \quad \text{(generalised metric)}$$

- Mixed component fields

$$\left\{ g_{\mu n}, C^{(3)}_{\mu np}, C^{(6)}_{\mu npqrs} \right\} \to \mathcal{A}_\mu{}^{N_1} \in \mathcal{R}(E_{d(d)})$$

$$\left\{ C^{(3)}_{\mu\nu p}, C^{(6)}_{\mu\nu pqrs} \right\} \to \mathcal{B}_{\mu\nu}{}^{N_2} \in \tilde{\mathcal{R}}(E_{d(d)})$$

$$\cdots$$

ExFT: Use covariant objects to rewrite 11d supergravity **before** the truncation

Exceptional Field Theory

- Embed internal coordinates y^m into $Y^{M_1} = (y^m, \dots) \in \mathcal{R}(E_{d(d)})$
- Fields: $\mathcal{M}_{M_1 N_1}(x, Y)$, $\mathcal{A}_\mu(x, Y)$, $\mathcal{B}_{\mu\nu}(x, Y)$,...
- Action: [Hohm, Samtleben]

$$\mathcal{L}_{ExFT} \sim \mathcal{R}_{g_{ext.}} + \mathfrak{D}_\mu \mathcal{M}_{M_1 N_1} \mathfrak{D}^\mu \mathcal{M}^{M_1 N_1} + \mathcal{F}_{\mu\nu}{}^{M_1} \mathcal{F}^{\mu\nu\, N_1} \mathcal{M}_{M_1 N_1} + \dots$$
$$+ \mathcal{L}_{top.} + \mathcal{L}_{scalar\ pot.}$$

- Section constraint:

$$Y^{M_1 N_1}_{P_1 Q_1} \partial_{M_1} \otimes \partial_{N_1} = 0 \,, \qquad Y^{M_1 N_1}_{P_1 Q_1} \to E_{d(d)} \text{ invariant tensor}$$

  - Required by closure of the algebra of generalised diffeomorphisms.
  - Solved by restricting coordinate dependence to a subset of Y^{M_1}
  - Inequivalent solutions to the SC reduce $\mathcal{L}_{ExFT}$ to 11d supergravity or IIB

Supersymmetric flux backgrounds

Compactifications with *Fluxes* $= 0$ (Special holonomy manifolds)

1. $\exists$ globally defined spinor $\epsilon \Rightarrow$ Structure group $G \subset SO(d)$
2. $\nabla\epsilon = 0 \;\Rightarrow$ Covariantly constant spinors

From $\epsilon \rightarrow$ construct bosonic bilinears

e.g. : SU(3)$\subset$SO(6)

1. $\Rightarrow$ globally defined $\omega_{(2)}$ and $\Omega_{(3)}$ satisfying

$$\omega_{(2)} \wedge \Omega_{(3)} = 0 \,, \qquad \omega_{(2)} \wedge \omega_{(2)} \wedge \omega_{(2)} \sim \Omega_{(3)} \wedge \bar{\Omega}_{(3)} \sim Vol_6$$

2. $\Rightarrow d\omega_{(2)} = d\Omega_{(3)} = 0$

Compactifications with *Fluxes* $\neq 0$

1. Same as in the fluxless case
2. $(\nabla + \text{"}\textit{Flux terms}\text{"})\,\epsilon = 0$ (In general hard to solve)

Supersymmetric flux backgrounds and ExFT

- $11/10$-dim spinors ϵ organise into rep's of $K(E_{d(d)})$ max. compact subgroup of $E_{d(d)}$.
- Then, [Coimbra, Strickland-Constable, Waldram]

$$(\nabla + \text{"} \mathit{Flux\ terms} \text{"})\,\epsilon = 0 \longrightarrow \nabla_{ExFT}\, \varepsilon = 0$$

with $\varepsilon \in K(E_{d(d)})$.

Supersymmetric flux backgrounds in ExFT

1. $\Rightarrow$ Generalised structure. Half-maximal case: $Spin(d-1) \subset E_{d(d)}$.
2. $\Rightarrow$ (Weak) integrability for the generalised structures

All internal d.o.f. are encoded into the structures: structures $\Rightarrow \mathcal{M}_{M_1 M_2}$

Generalised structures for AdS vacua

Half-maximal AdS_7, AdS_6, AdS_5 vacua [Malek]

Generalised $Spin(d-1) \subset E_{d(d)}$ **structures:** $J_u \in \mathcal{R}(E_{d(d)})$, $\hat{K} \in \tilde{\mathcal{R}}(E_{d(d)})$

$$J_u \curlywedge J_v = \frac{1}{d-1}\delta_{uv}\, J_w \curlywedge J^w \,, \qquad \hat{K} \curlywedge J_w \curlywedge J^w > 0 \,, \qquad \big[u \to Spin(d-1)\big]$$

Weakly integrability ($\Lambda_{uvw} \leftrightarrow$ cosmological constant)

$$\mathcal{L}_{J_u} J_v = \Lambda_{uvw} J^w$$

As well as

$$\mathfrak{d}\hat{K} = \epsilon^{uvw}\Lambda_{uvw} J_z \curlywedge J^z \qquad (\text{for } AdS_7)$$

$$\mathfrak{d}\hat{K} = \epsilon^{uvwz}\Lambda_{uvw} J_z \qquad (\text{for } AdS_6)$$

$$\mathcal{L}_{\hat{K}} J_u = \epsilon_{uvwxz}\Lambda^{vwx} J^z \qquad (\text{for } AdS_5)$$

Break $Spin(d-1) \to G_R \subset Spin(d-1)$ ($G_R \to$ R-symmetry)

$\curlywedge \to$ ExFT "wedge product", $\mathcal{L}, \mathfrak{d} \to$ ExFT "differential operators".

Minimal consistent truncations

Conjecture [Gauntlett, Varela]

For any SUSY solution of 11/10-d SUGRA of the form $\mathrm{AdS}_D \times_w M$, there is a consistent truncation to a D-dimensional gauged SUGRA keeping only the gravitational supermultiplet.

Half-maximal gravitational multiplet: 1 scalar $X(x)$.
Truncation Ansatz [Malek]

$$\mathcal{J}_u = X^{-1}(x)\, J_u(Y)\,, \qquad \hat{\mathcal{K}} = X^2(x)\, \hat{K}(Y)$$

as well as

$$\mathrm{AdS}_7: \quad \mathcal{A}_\mu = A_\mu(x)^u\, J_u,, \qquad \mathcal{B}_{\mu\nu} = -B_{\mu\nu}(x)\, J_w \curlywedge J^w\,, \quad \ldots$$

$$\mathrm{AdS}_6: \quad \mathcal{A}_\mu = A_\mu(x)^u\, J_u\,, \qquad \mathcal{B}_{\mu\nu} = -B_{\mu\nu}(x)\, J_w \curlywedge J^w + \tilde{B}_{\mu\nu}(x)\hat{K}\,, \quad \ldots$$

$$\mathrm{AdS}_5: \quad \mathcal{A}_\mu = A_\mu(x)^u\, J_u + A_\mu^{(0)}\hat{K}(x)\,,\ \mathcal{B}_{\mu\nu} = B_{\mu\nu}(x)\, J_w \curlywedge J^w + B_{\mu\nu}{}^u(x)\, J_u \curlywedge \hat{K}, \ldots$$

- Consistency follows from integrability conditions.
- 11/10d fields can be obtained from $\mathcal{M}_{MN}(\mathcal{J}, \hat{\mathcal{K}})$ and universal dictionaries between SUGRA and ExFT.

Consistent truncations with vector multiplets

- To accept truncations keeping N vector multiplets, the vacuum must have extra structures
- $J_u \longrightarrow J_A = (J_u, \bar{J}_{\bar{u}})$ with $\bar{u} = 1, \dots N$

$Spin(d-1-N)$ structure [Malek]

Generalised $Spin(d-1-N) \subset Spin(d-1)$ **structure**: J_A

$$J_A \curlywedge J_B = \frac{1}{d-1-N} \eta_{AB} \, J_C \curlywedge J^C \,, \quad \eta_{AB} \to SO(d-1,N) \text{ metric}$$

Differential conditions

$$\mathcal{L}_{J_A} J_B = f_{AB}{}^C J_C \,, \qquad f_{AB}{}^C \text{constant}$$

- $N \leq d-1$ and new J's transform as reps of the R-symmetry group
- Truncation Ansatz:

$$\mathcal{J}_u = X^{-1} b_u{}^A J_A \,, \quad \hat{\mathcal{K}} = X^2 \hat{K} \,, \quad \mathcal{A}_\mu = A_\mu{}^A J_A \,\dots$$

with $b_u{}^A b_v{}^B \eta_{AB} = \delta_{uv}$. $\quad \Rightarrow \quad \{b_u{}^A, X\} \in \frac{SO(d-1,N)}{SO(d-1)\times SO(N)} \times \mathbb{R}^+$

The method

1. Construct general classes of half maximal structures.
   - Construct general Ansätze for the structures J_u and $\hat{K}$ compatible with the R-symmetry G_R.
   - Solve algebraic structure conditions and weakly integrability conditions.
   - Read off the background fields from the structures and compare with known classes of vacua.
2. Construct minimal consistent truncations.
   - Take the truncation Ansatz and read off the uplift formulae.
3. Study truncations keeping vector multiplets
   - Construct Ansätze for $\bar{J}_{\bar{u}}$ and try to solve the structure conditions and differential conditions given the vacua structures.
   - These conditions turn out to be very restrictive $\Rightarrow$ Many No-Go theorems
   - For those cases where $\bar{J}_{\bar{u}}$ exist, construct the truncation.

Results I: $\mathrm{AdS}_7 \times I \times S^2$ in mIIA [1808.05597], [1901.11039]

- R-symmetry group: $SU(2)$.
- AdS_7 half-maximal structures classified in terms of a function t on the interval I satisfying:
$$\dddot{t} = -\frac{m_{IIA}}{2}\,, \qquad t(z) \geq 0$$
with equality on the endpoints.
Same class as [Apruzzi, Fazzi, Rossa, Tomasiello]
- Vector multiplets? **Not possible for general** m_{IIA}
Only when $m_{IIA} = 0$: Truncation with a single vector multiplet (sub-truncation of maximal CT around $\mathrm{AdS}_7 \times S^4$)

Results II: AdS$_6 \times \Sigma_2 \times S^2$ in IIB [1808.05597], [1901.11039]

- R-symmetry group: $SU(2)$.
- AdS$_6$ half-maximal structures classified in terms of two holomorphic functions f^α $(\alpha = 1, 2)$ on the Riemann surface Σ_2 satisfying:

$$\bar{\partial} f^\alpha = 0 \,, \qquad i\, \partial f^\alpha \bar{\partial} \bar{f}_\alpha \geq 0$$

  Same class as [D'Hoker, Gutperle, Karch, Uhlemann]
- Vector multiplets?

N	$SU(2)_R$ rep	Consistent truncation?	Gauging
1	$\mathbf{1}$	**Only if** $\exists\chi$: $\partial(e^{i\chi}\partial f^\alpha) \in$ Real function on Σ	SU(2)×U(1)
2	$\mathbf{1} \oplus \mathbf{1}$	**NO**	N/A
3	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$	**NO**	N/A
3	$\mathbf{3}$	**Only if** $r\, d\Pi^\alpha = p^\alpha \Pi^\beta \wedge \Pi_\beta$	ISO(3)
4	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$	**NO**	N/A
4	$\mathbf{3} \oplus \mathbf{1}$	**Only if** $\exists\,\mathbf{3}$ and $\exists\,\mathbf{1}$ with $\chi = \frac{1}{2}\left(\frac{p_\alpha \bar{\partial} \bar{f}^\alpha}{p_\beta \partial f^\beta}\right)$	ISO(3)×U(1)

with Π^α a 1-form constructed from f^α.

Results III: AdS$_5 \times M_3 \times S^2 \times S^1$ in Mtheory [2012.15601]

- R-symmetry group: $SU(2) \times U(1)$
- AdS$_5$ vacua in terms of a function D on M_3 satisfying

$$(\partial_1^2 + \partial_1^2)D + \partial_3^2 e^D = 0$$

[Lin, Lunin, Maldacena]

- Vector multiplets? The possibilities that are not ruled out are:

N	$SU(2) \times U(1)$ rep.	Conditions to be satisfied	Gauging
1	$\mathbf{1}_0$	$d\bar{\nu} = d\bar{\Phi} = 0$	SU(2)×U(1)
2	$\mathbf{1}_0 \oplus \mathbf{1}_0$	Two different "compatible" $\mathbf{1}_0$'s	SU(2)×U(1)
2	$\mathbf{1}_q \oplus \mathbf{1}_{-q}$	$\bar{\Phi}_1 = \frac{1}{q}\, d\bar{\nu}_2 \quad \bar{\Phi}_2 = -\frac{1}{q}\, d\bar{\nu}_1$	SU(2)×U(1)′
3	$\mathbf{1}_0 \oplus \mathbf{1}_0 \oplus \mathbf{1}_0$	Three different "compatible" $\mathbf{1}_0$'s	SU(2)×U(1)
3	$\mathbf{1}_0 \oplus \mathbf{1}_q \oplus \mathbf{1}_{-q}$	"compatible" $\mathbf{1}_0$ and $\mathbf{1}_q \oplus \mathbf{1}_{-q}$	SU(2)×U(1)′
3	$\mathbf{3}_0$	S^4 fibred over Riemann surface	ISO(3)×U(1)

Where $\bar{\nu}$ a 1-form and $\bar{\Phi}$ 2-form constructed from $\bar{\nu}$ and D. See details in [2012.15601]

Conclusions

- ExFT gives a natural language to study consistent truncations
- We used it to systematically construct and classify truncations around half-maximal AdS vacua
- Minimal consistent truncations can always be constructed but truncations keeping vector multiplets are very rare.

Outlook:

- Find solutions to the differential conditions of vector multiplets.
- **KK spectrum?**
  - In [Malek, Samtleben '19] a new method to compute the whole KK spectrum using ExFT was developed.
  - It can be used to compute not only the KK spectrum of the maximally supersymmetric vacuum but also for any other (less supersymmetric) vacua inside the consistent truncation.
  - Work in progress: adapt this method to the half-maximal case. Formulation of the consistent truncation in terms of structures plays a crucial role.