CORRELATORS FROM SEPARATION OF VARIABLES IN SPIN CHAINS AND CFT'S

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based on

2103.15800 [Cavaglia, Gromov, FLM]

2011.08229 [Gromov, FLM, Ryan]

2003.05811 [FLM, Preti]

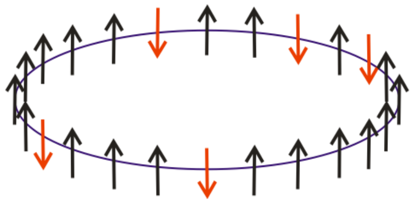
1910.13442 [Gromov, FLM, Ryan, Volin]

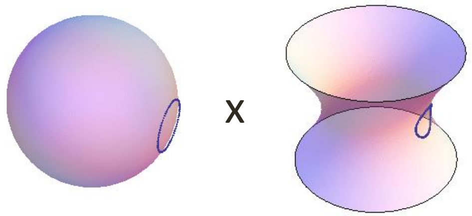
1907.03788 [Cavaglia, Gromov, FLM]

How to compute correlators $\langle\Psi|\hat{O}|\Phi\rangle$?

Hard even in integrable models

We study it for:

- Spin chains 

- N=4 super Yang-Mills in 4d / strings on AdS_5 x S_5 

Main motivation – solve an interacting 4d gauge theory for the first time

$\langle \Psi | \hat{O} | \Phi \rangle$ = ?

Idea: find a basis where wavefunctions factorize

$$\langle x | \Psi \rangle \sim Q(x_1) Q(x_2) \dots Q(x_N)$$

Separation of Variables (SoV)

like hydrogen atom $\Psi = F_1(r) F_2(\theta) F_3(\varphi)$

Should exist in any integrable model, expected to be powerful [Smirnov, Sklyanin, … 90's]
but understanding has been very limited

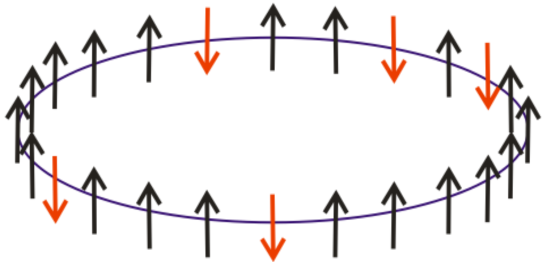
We developed SoV for SU(N) spin chains,
starting to explore similar structures in QFT

SPIN CHAINS

SU(2) spin chains

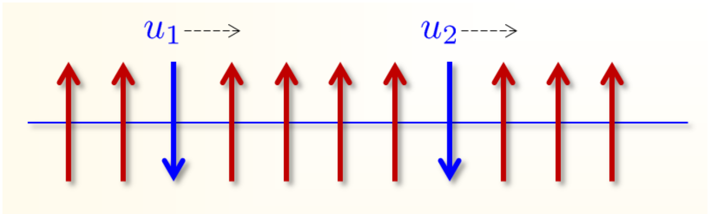
[Sklyanin 90]

$$H = \sum_{n=1}^{L} (1 - P_{n,n+1})$$



$$T(u) = R_{a1}(u-\theta_1)\dots R_{aL}(u-\theta_L)g$$

$$T(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}$$



The Bethe roots u_j are fixed by Bethe equations

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j} \frac{u_j - u_k + i}{u_j - u_k - i}$$

$$E = \sum_i \frac{1}{u_i^2 + 1/4}$$

$|\Psi\rangle = B(u_1)B(u_2)\dots B(u_M)|0\rangle$ built via 'creation operator'

$\langle x|$ = eigenstates of same operator $B(u) = \prod (u - x_k)$

So indeed wavefunctions factorize $\langle x|\Psi\rangle = \prod_k Q_1(x_k)$

$$Q(u) = \prod_i (u - u_i)$$

Q's are polynomials

$$\langle \mathbf{x}_1 \dots \mathbf{x}_L|\Psi\rangle = \prod_{k=1}^{L} \prod_{j=1}^{M} (u_j - \mathbf{x}_k) = \prod_{k=1}^{L} Q_1(\mathbf{x}_k)$$

SU(N) spin chains

$$|\Psi\rangle = B(u_1)B(u_2)\dots B(u_M)|0\rangle$$

still true? Yes!

[Gromov, **FLM**, Sizov 16]

Simple result, hard to prove

$$B(u) = T_{23}(u)T_{12}(u-i)T_{23}(u) - T_{23}(u)T_{13}(u-i)T_{22}(u) + T_{13}(u)T_{11}(u-i)T_{23}(u) - T_{13}(u)T_{13}(u-i)T_{21}(u) \,.$$

[Sklyanin 92, Smirnov 00]

Replaces huge 'nested' Bethe ansatz formula, new result in old field!

$$|\Psi\rangle = \sum_{a_i=2,3} F^{a_1 a_2 \dots a_M} T_{1a_1}(u_1)T_{1a_2}(u_2)\dots T_{1a_M}(u_M)|0\rangle$$

Sutherland; Kulish, Reshetikhin 83

sum of exponentially many terms

$\langle x|$ = eigenstate of B(u) $\Rightarrow$ $\langle x|\Psi\rangle = \prod_k Q_1(x_k)$ i.e. wavefunctions factorize so we have SoV

See also [Maillet, Niccoli 18]

Proofs: [Lyashik, Slavnov 18] [Maillet, Niccoli 18,19,20] [Ryan, Volin 18,19]

SoV measure

$$\langle x|\Psi\rangle \sim Q(x_1)Q(x_2)\dots Q(x_N)$$

For scalar products we also need measure
(analog of $r^2 \sin\theta dr d\varphi d\theta$ for hydrogen atom)

We found it for any SU(N)

Longstanding problem resolved !

[Cavaglia, Gromov, **FLM** 19] [Gromov, **FLM**, Ryan, Volin 19]
[Gromov, **FLM**, Ryan 20]

$$\langle\Psi_A|\Psi_B\rangle = \int d^L\mathbf{x} \underbrace{\left[\prod_i Q_A(x_i)\right]}_{\text{state A}} \underbrace{M(\mathrm{x})}_{\text{measure}} \underbrace{\left[\prod_i Q_B(x_i)\right]}_{\text{state B}}$$

$$\det\left|\underbrace{\left(\frac{\hat{x}^{j-1}}{1+e^{2\pi(\hat{x}-\theta_i)}}\right)}_{1\leqslant i,j\leqslant L}\otimes\underbrace{\begin{pmatrix}\mathcal{D}_x^{N-2} & \mathcal{D}_x^{N-4} & \dots & \mathcal{D}_x^{2-N}\\ \vdots & \vdots & \ddots & \vdots\\ \mathcal{D}_x^{N-2} & \mathcal{D}_x^{N-4} & \dots & \mathcal{D}_x^{2-N}\end{pmatrix}}_{(N-1)\times(N-1)}\right|$$

Gives new determinant (easy to evaluate) representations for many correlators!

$$\langle\Psi_B|\Psi_A\rangle \propto \begin{vmatrix} \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{2+} \right\rangle_j & \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{2-} \right\rangle_j \\ \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{3+} \right\rangle_j & \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{3-} \right\rangle_j \end{vmatrix}$$

Expect many applications

can insert operators like B(u), likely full set

APPLICATIONS FOR CFT'S

$$\langle x|\Psi\rangle \sim Q(x_1)Q(x_2)\dots Q(x_N)$$

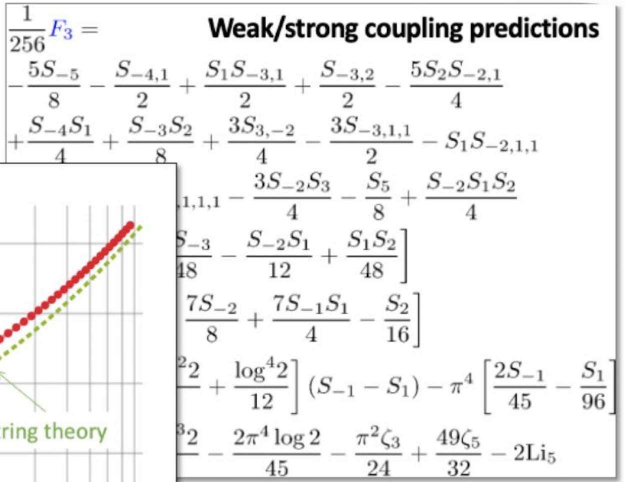
Still to be made precise in N=4 SYM

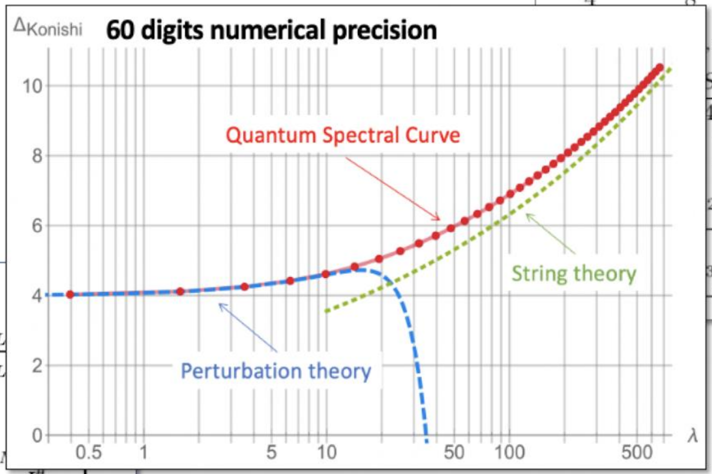
But we know the Q's at finite coupling! No longer polynomials

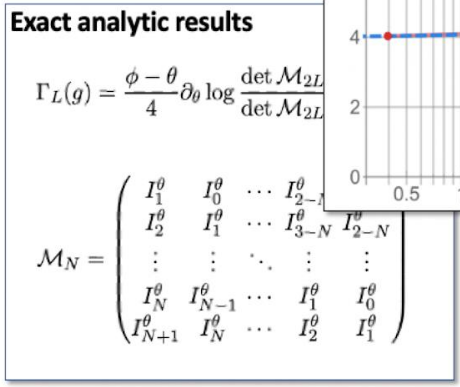
Fixed by Quantum Spectral Curve – set of functional equations

[Gromov, Kazakov, Leurent, Volin 13]

Give exact spectrum,
many applications







relevant for QCD

And many more...

[Gromov, **FLM**, Sizov 13,14] [Gromov, **FLM**, Sizov 15 x2]
[Gromov, **FLM** 15, 16] [Marboe,Volin 14-17] [Alfimov,Gromov,Kazakov 14]

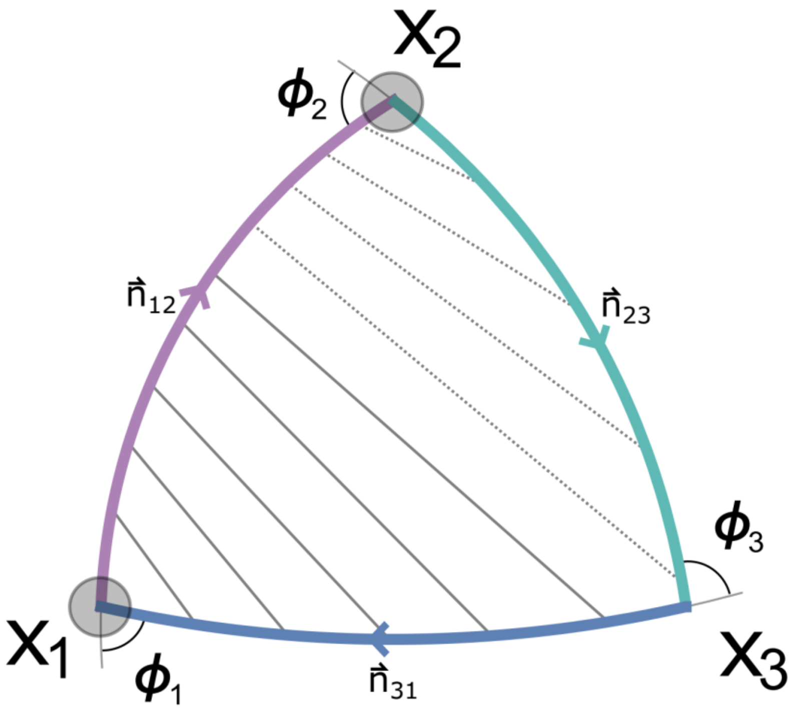
extends to non-susy deformations

[Kazakov, Leurent, Volin 15] [FLM, Preti 20]

Goal: use SoV to compute
3pt correlators

Key open problem

CORRELATORS IN N=4 SYM



[Cavaglia, Gromov, **FLM** 18]

Finally put the idea
on quantitative level in N=4 SYM

3 Wilson lines + local operators at cusps

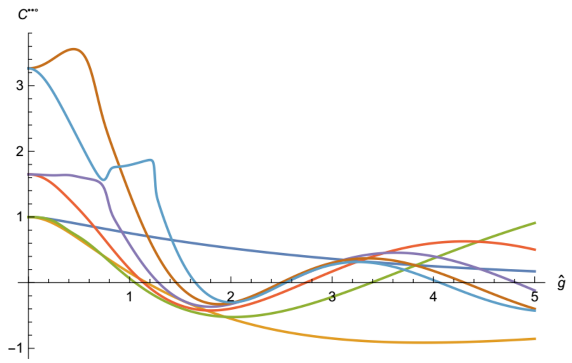
$$W = \mathrm{Tr}\,\mathcal{P}\exp\int dt\left[iA\cdot\dot{x} + \vec{\Phi}\cdot\vec{n}\,|\dot{x}|\right]$$
$$\vec{n}\cdot\vec{n}_\theta = \cos\theta \to \infty$$

Resum ∞ many ladder diagrams

Complicated result becomes
extremely simple in terms of QSC !

$$C_{123} = \frac{\langle Q_1 Q_2 Q_3\rangle}{\sqrt{\langle Q_1^2\rangle\langle Q_2^2\rangle\langle Q_3^2\rangle}}$$

$$\langle f(u)\rangle \equiv \left(2\sin\frac{\beta}{2}\right)^\alpha \int_{c-i\infty}^{c+i\infty} f(u)\frac{du}{2\pi i u}\ ,\quad c>0$$



Structure precisely as expected from
separation of variables

[Giombi, Komatsu 18] – similar structure in very different regime

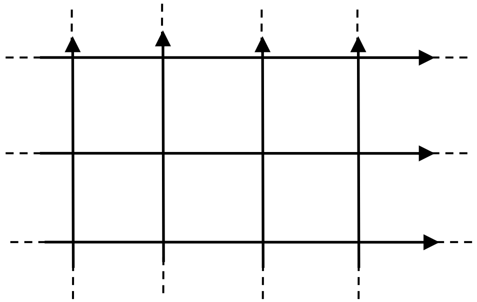
Extension to local operators

Gurdogan, Kazakov 2015

"fishnet CFT" $$S=\frac{N}{2}\int d^4x\,\mathrm{tr}\,\left(\partial^\mu\phi_1^\dagger\partial_\mu\phi_1+\partial^\mu\phi_2^\dagger\partial_\mu\phi_2+2\xi^2\,\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\right)$$

Baby version of N=4 SYM, no susy but inherits integrability, Q-functions etc

Integrability visible directly from Feynman graphs



We find very similar structures

$$C_{\mathcal{OOL}}\propto\frac{d\Delta}{d\xi^2}=\frac{\int_{|}\frac{q\bar{q}}{u}\frac{du}{2\pi i}}{\int_{|}i\left(q^+\bar{q}^- - q^-\bar{q}^+\right)\frac{du}{2\pi i}}$$

[Cavaglia, Gromov, **FLM** 21]
[+ with A. Sever to appear]

Holographic dual derived almost rigorously! Should give more data

Gromov, Sever 19

Correlators from SoV for fishnets

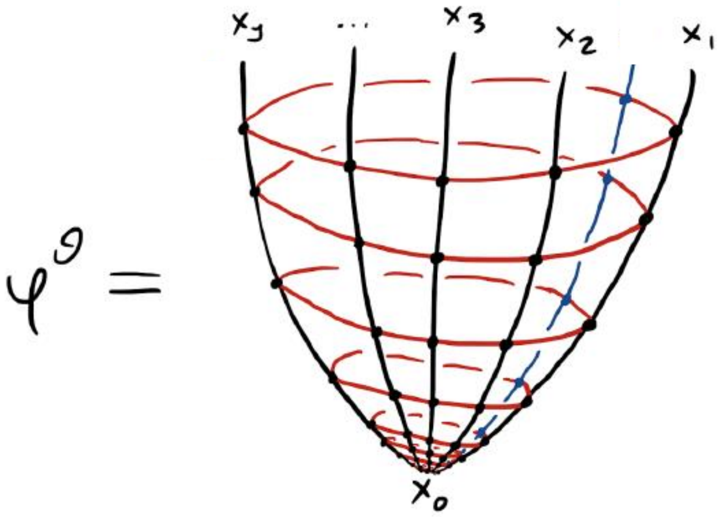
Get SO(4,2) spin chain in principal series rep

Wavefunction of spin chain = correlator in CFT

$$\varphi_{\mathcal{O}}(x_1,\ldots,x_J) = \langle \mathcal{O}(x_0)\, \mathrm{tr}\, [\phi_1^\dagger(x_1)\ldots\phi_J^\dagger(x_J)]\rangle \;.$$

[Gromov, Sever 19]

Tr $(\phi(x_0))^J$

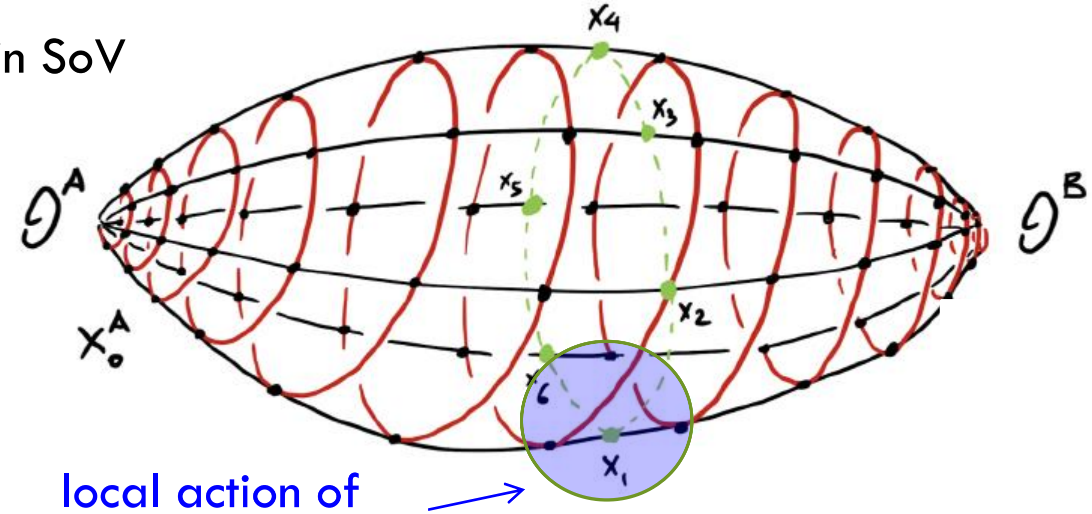


Can compute large set of correlators in det form, via spin chain SoV

[Cavaglia, Gromov, **FLM** 21]

E.g. 2pt function with local insertions to all loop orders

$$\frac{\partial \hat{H}}{\partial h_\alpha}\hat{H}^{-1} = -8\left[-\frac{x^2_{\alpha,\alpha-1}+x^2_{\alpha,\alpha+1}}{2}\left(1+x^\mu_\alpha\frac{\partial}{\partial x^\mu_\alpha}\right)+(x^2_{\alpha,\alpha-1}x^\mu_{\alpha+1}+x^2_{\alpha,\alpha+1}x^\mu_{\alpha-1})\frac{\partial}{\partial x^\mu_\alpha}\right]$$
$$\times\, \Box_\alpha^{-1}\frac{1}{x^2_{\alpha,\alpha-1}}\frac{1}{x^2_{\alpha,\alpha+1}}\;.$$



local action of differential operator

Conjecture similar structures for 1pt functions with defect

Can write formal det expressions for SYM, practical aspects are in progress…

SUMMARY

- SoV developed for higher rank SU(N) spin chains, obtained SoV measure, scalar products as det
- Works in very general situations when BA fails, including fishnet theory

- Spin chains – expect many applications, extension to super case, …
- Extension to N=4 SYM seems within reach, hints of hidden structures
- Correlators in matrix models of dually weighted graphs e.g. quadrangulations, insights into AdS_2 / CFT_1 ? [Kazakov, FLM in progress]