

Exploring the space of QFTs

IRN: QFS
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Outline

1. S-matrix Bootstrap — Monolith
[w/ Yifei He, Martin Kruczenski, Pedro Vieira 1909.06495]
2. Elliptic sinh-Gordon TBA
[work in progress w/ Stefano Negro & Fidel Schaposnik]
3. S-matrices from Conformal correlators
[work in progress w/ Yifei He & Miguel Paulos]

S-matrix Bootstrap

- Non-perturbative S-matrix from $\left\{ \begin{array}{l} \bullet \text{ unitarity} \\ \bullet \text{ crossing} \\ \bullet \text{ analyticity} \end{array} \right.$

- S-matrix program 1960s

Revival [2016 Paulos, Penedones, Toledo, van Rees, Vieira]

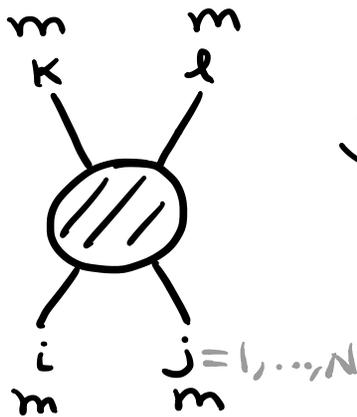
constrain space of QFTs \rightarrow bounds

- This talk: 2D QFTs $\left\{ \begin{array}{l} \text{easier :) } \\ \text{connection to integrability} \end{array} \right.$

S-matrix Bootstrap

- 2D, $O(N)$ global symmetry, no bound states

[w/ Yifei He, Martin Kruczenski, Pedro Vieira 1909.06495]



$$S_{ij}^{kl}(\tau) = \sum_a S_a(\tau) (P_a)_{ij}^{kl}$$

↳ singlet, antisym, sym

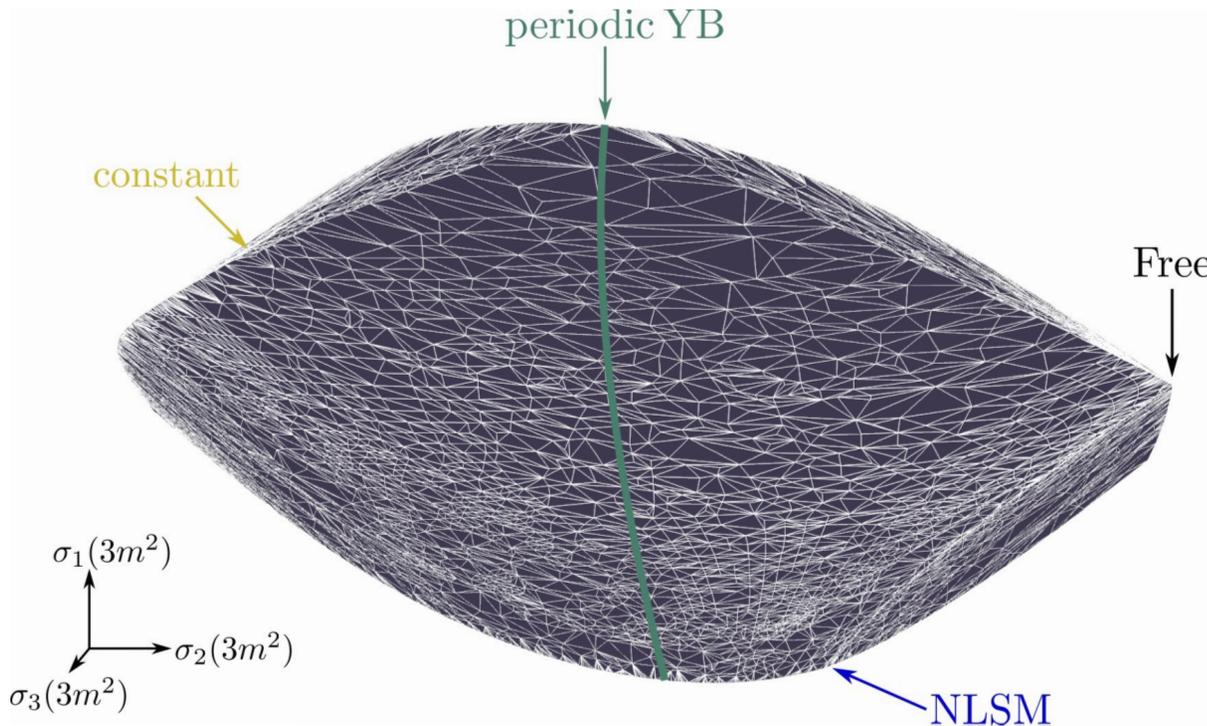
$$\tau = (p_1 + p_2)^2$$

- What is the space of allowed $S_a(\tau)$?

↳ plot 3D subsections : Monolith
 $S_a(\tau_*)$

S-matrix Bootstrap

- Integrable theories & exact S-matrices



- Free (vertex)

$$S_a = 1$$

- NLSM (vertex)

$$S_a(\theta) = \frac{\Gamma(\theta + \frac{1}{2} + \frac{1}{N-2}) \Gamma \Gamma \Gamma}{\Gamma \dots \Gamma}$$

$(s = 4m^2 \cosh^2 \frac{\theta}{2})$

[Zamolodchikov², 1978]

- pYB (pre-vertex) ?

$$S_a(\theta) = \prod_{\infty} \frac{\Gamma(\theta + \dots) \dots \Gamma}{\Gamma \dots \Gamma}$$

[Hortacsu et al 1979]

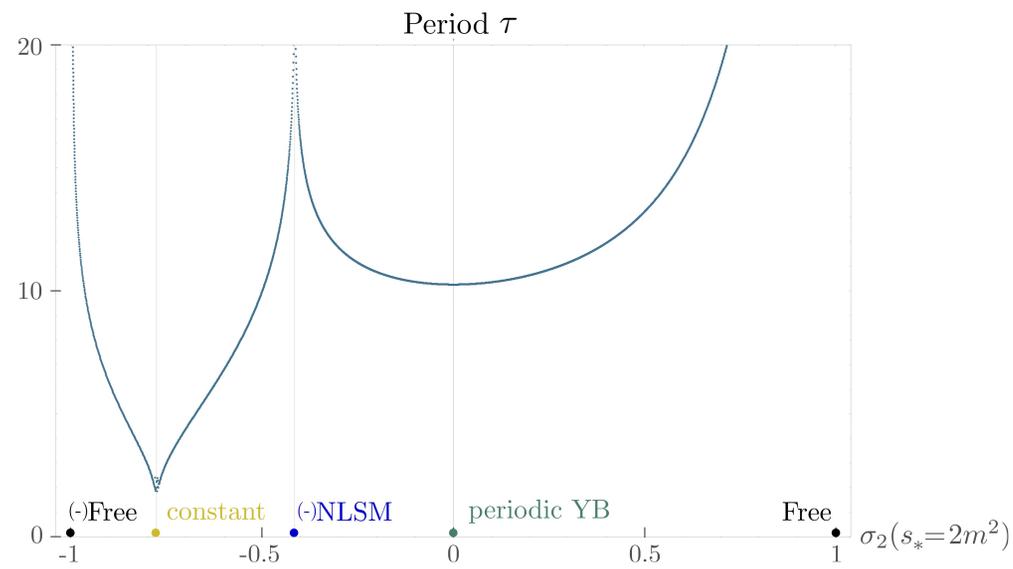
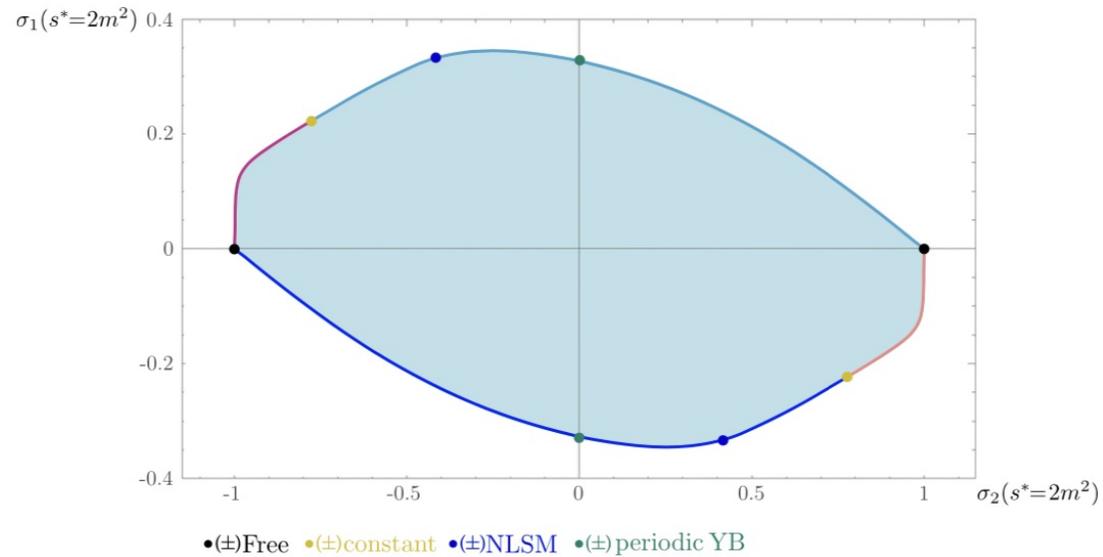
- constant

$$S_a = \left(1, -1, \frac{N-2}{N+2} \right)$$

S-matrix Bootstrap

• Emergent periodicity $S_a(\theta) = S_a(\theta + \tau)$

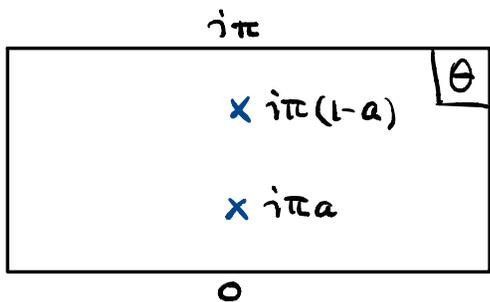
$$(\tau = 4m^2 \cosh^2 \frac{\theta}{2})$$



Elliptic sinh-Gordon TBA

- Simplest integrable S-mat w/ $S(\theta + \tau_\ell) = S(\theta)$. $\tau_\ell = \pi \frac{K_\ell}{K\sqrt{1-\ell^2}}$
[Munardo, Penati '99]

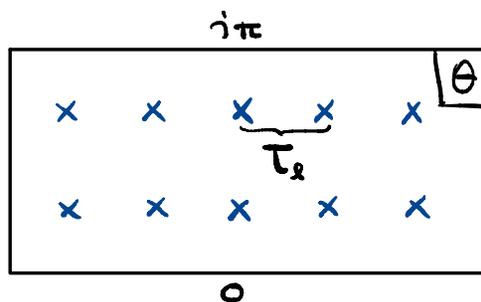
$$S_{\text{shG}}(\theta) = \frac{\text{sh } \theta - i \sin \pi a}{\text{sh } \theta + i \sin \pi a}$$



$\tau_\ell \rightarrow \infty$
($\ell \rightarrow 0$)

$$S_{\text{ellshG}}(\theta)$$

$$= \frac{\text{sn}(2K_\ell i\theta/\pi) + \text{sn}(2K_\ell a)}{\text{sn}(2K_\ell i\theta/\pi) - \text{sn}(2K_\ell a)}$$



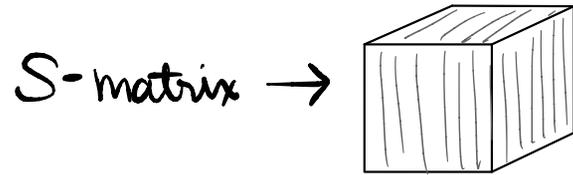
$$\mathcal{L}_{\text{shG}} = \frac{1}{2} (\partial\phi)^2 - \frac{m^2}{b^2} \text{ch}\left(\frac{b}{8\pi} \phi\right)$$

$$a = \frac{b^2}{1+b^2}$$

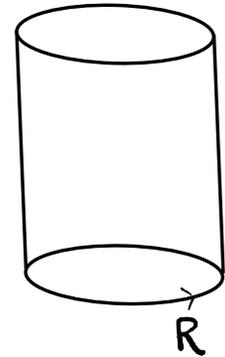
?

Elliptic sinh-Gordon TBA

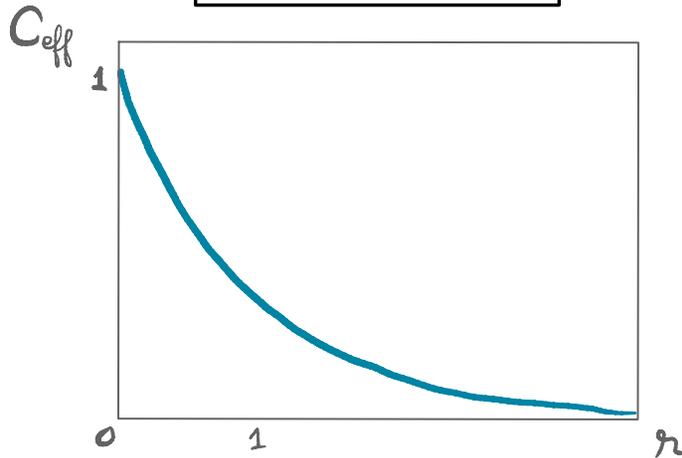
- Thermodynamic Bethe Ansatz (TBA)



$$\rightarrow E_0(R) = -\frac{\pi}{6R} C_{\text{eff}}(\underbrace{mR}_R)$$



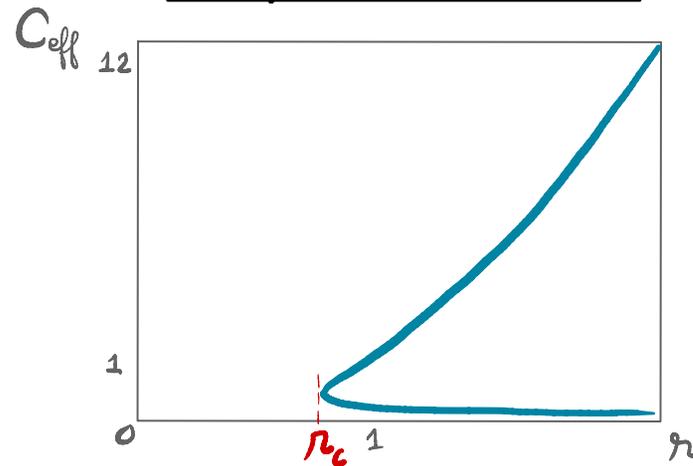
sinh-Gordon



$$C_{UV} = 1, \quad C_{IR} = 0$$

$$r \rightarrow 0 \quad r \rightarrow \infty$$

elliptic sinh-Gordon



- $r_c \rightarrow 0$ as $l \rightarrow 0$
- $\sqrt{r-r_c}$ singularity
[Camilo, Fleury, Lencsés, Negro, Zamolodchikov]
- $(\overline{T\overline{T}})_S: S(\theta) = \exp\left\{i \sum_s \alpha_s \text{sh}(s\theta)\right\}$
[Smirnov, Zamolodchikov '16]

S-matrices_{2D} from CFTs_{1D}

- Observation:

dual S-mat problem \longleftrightarrow CFT analytic functionals
 [Mazac, Paulos]

- Natural

QFT in AdS₂ \longleftrightarrow Conformal theory 1D

[Paulos, Penedones, Toledo, van Rees, Vieira '16]

$$m_i^2 R^2 = \Delta_i(\Delta_i - d)$$

flat space $R \rightarrow \infty$

$$\Delta_i \rightarrow \infty$$

$$\left(\frac{m_i}{m} = \lim_{\Delta_i \rightarrow \infty} \frac{\Delta_i}{\Delta} \right)$$

$$S(s) = \lim_{\Delta_i \rightarrow \infty} \mathcal{G}(z = s/4)$$

[Komatsu, Paulos, van Rees, Zhao '20]

- Idea: Recover $\left\{ \begin{array}{l} \text{crossing} \\ \text{analyticity} \\ \text{unitarity} \end{array} \right.$ of $S(s)$ from conformal correlator.

Thank you !