$AdS_5 \times S^5$ Amplitudes

and their flat-space limit

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Motivation

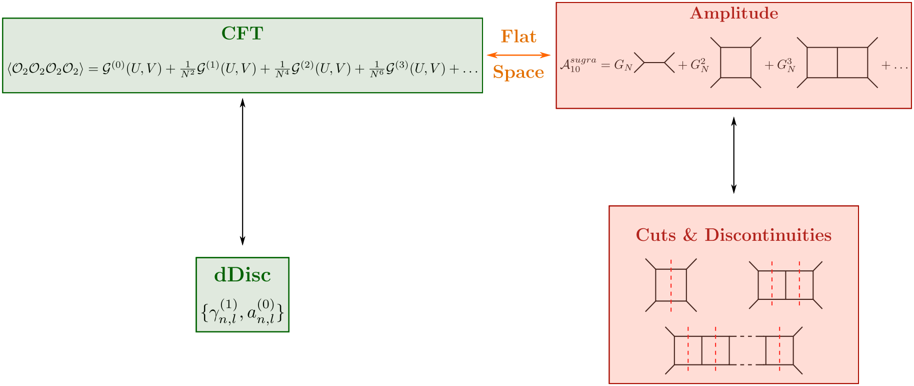
- Understand better the unitarity structure of $\mathrm{AdS}_5 \times \mathrm{S}^5$ amplitudes.
- Apply bootstrap methods to amplitude computations in curved space-time as their are usually complicated to study.
- Driven from the great success of the one loop case, where it was possible to reconstruct the full correlator, we hope to extract further information on the higher loops terms.

4d $SU(N)\ \mathcal{N}=4$ SYM

at large N and large $\lambda = g_{YM}^2 N$

Type IIB SUGRA

on $\mathrm{AdS}_5 \times S^5$ with g_s coupling



4d $SU(N)\,\mathcal{N}=4$ SYM
at large N and large $\lambda = g_{YM}^2 N$

Type IIB SUGRA
on $\mathbb{R}^{10}$ with G_N coupling

$\frac{1}{2}$BPS scalar
$[0,2,0]_R\ \Delta = 2$

$T_{\mu\nu}$ supermultiplet

$G_N = \frac{\pi^4 L^8}{2(N^2-1)}$

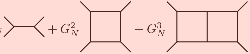
Graviton multiplet

CFT

$$\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\rangle = \mathcal{G}^{(0)}(U,V) + \frac{1}{N^2}\mathcal{G}^{(1)}(U,V) + \frac{1}{N^4}\mathcal{G}^{(2)}(U,V) + \frac{1}{N^6}\mathcal{G}^{(3)}(U,V) + \dots$$

Flat Space

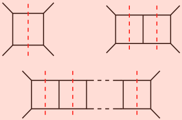
Amplitude

$\mathcal{A}_{10}^{sugra} = G_N$  $+\dots$

dDisc

$\{\gamma_{n,l}^{(1)}, a_{n,l}^{(0)}\}$

Cuts & Discontinuities



Examples: Two Loops

The two loop correlator can be written in terms of OPE data as:

$$\mathcal{G}^{(3)}(z,\bar{z}) \propto \frac{1}{48}(z\bar{z})^{n+2}\log^3(z\bar{z})a^{(0)}_{n,\ell}\left(\gamma^{(1)}_{n,\ell}\right)^3 g_{2n+8,\ell} \\ + \frac{1}{8}(z\bar{z})^{n+2}\log^2(z\bar{z})\gamma^{(1)}_{n,\ell}\left(2a^{(0)}_{n,\ell}\gamma^{(2)}_{n,\ell}+a^{(1)}_{n,\ell}\gamma^{(1)}_{n,\ell}\right)\ldots$$

Unfortunately as the mixing at the previous order has not been solved and as higher traces contributions appear we can only construct the higher logarithmic contributions, which is build up of $\gamma^{(1)}_{n,\ell}$ contributions, for which the mixing is solved.

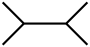
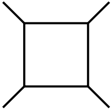
CFT How

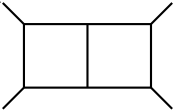
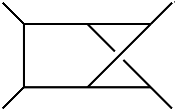
- We have a four point correlator in the strong coupling limit which is dual to a graviton amplitude.
- We want to study the $1/N$ corrections that map to SUGRA loop corrections on $\mathrm{AdS}_5 \times \mathrm{S}^5$.
- As we have information only on $\gamma^{(1)}_{n,\ell}$ we can construct only the higher logarithmic term of the correlator.
- We can take the flat space/bulk-point limit to map the dDisc of the correlator to a flat-space quantity.

10D SUGRA

- We want to study the scattering of 4 gravitons in 10D SUGRA.
- We will start by considering the two-loop amplitude, the integrand has been known in the literature for a long time.
- The $\mathcal{A}_{10}^{sugra}$ is function of the s, t, u Mandelstam invariants having defined $p_i^2 \to 0$.

The two loop amplitude takes schematically the form:

$\mathcal{A}_{10}^{sugra} = G_N$  $+ G_N^2$ 

$+ G_N^3 \Bigg($  $+$  $\Bigg) + \dots$

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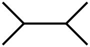
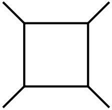
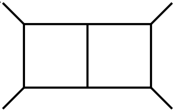
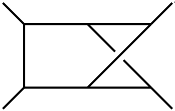
$\mathcal{A}_{10}^{sugra} = G_N$  $+ \, G_N^2$ 

$+ \, G_N^3 \Big($  $+$  $\Big) + \dots$

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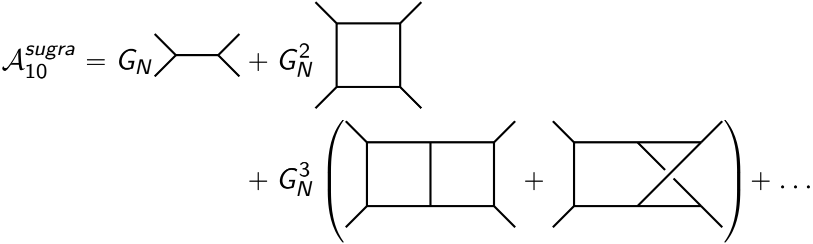

The integrals were obtained in 4 dimensions using the differential equations method and then uplifted to 10 d using DRR.
To map with the CFT we want to extract an observable that matches the one obtained from the Bulk-point limit. This is the discontinuity of the Amplitude.

Discontinuities & Cuts

Discontinuities are defined as:

$$\mathrm{disc}_{x>1} F_3(x) \equiv \frac{1}{2\pi i} \left(F_3(x + i0) - F_3(x - i0) \right) .$$

and they can be resolved as

$$F_3(x) = \int_1^\infty dx' \frac{\mathrm{disc}_{x'>1} F_3(x')}{x' - x} .$$

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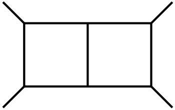
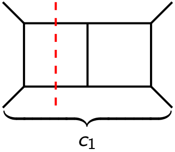
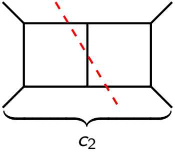
They can analogously be extracted from cuts, for example at two loops:

$2\pi i \ \mathrm{disc}_s$  $=$ 

$+$ 

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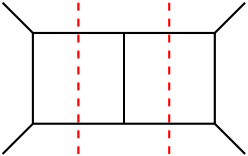
Example: Two Loops

As we have seen from the CFT we can construct only the part of the correlator proportional to $\gamma^{(1)}$. In the amplitude we have control over the complete result but we can extract some information:

- The type of cut corresponds to the type of operators exchanged. So c_1 is the double trace contribution while c_2 is the triple trace.
- The contributions from the CFT match the functions appearing in c_1, the mismatch can be understood from the missing contributions of $\mathrm{Log}^2(1-\bar{z})$ (in CFT picture contributions connected to $a^{(0)}_{n,\ell}, \gamma^{(2)}_{n,\ell}, a^{(1)}_{n,\ell}$).
- We can construct a cut that directly matches the higher log contribution



Conclusions

- We are able to match the $(\gamma_1^{(1)})^\kappa$ contribution to a specific iterated cut of the amplitude, in the flat-space limit.
- We observe an interesting match between the maximal-cut of ladders and the Mellin amplitude coefficient, in the flat-space limit.
- By using iterated discontinuities we can reconstruct more contribution to the double-trace/cut, we are hopeful that the same type of relations should hold also for iterated dDiscs.

Thank You!