

$\text{AdS}_5 \times S^5$ Amplitudes and their flat-space limit

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Work based on 2010.12557 and 2002.04604
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IRN:QFS June 10th 2021



- Understand better the unitarity structure of $\text{AdS}_5 \times S^5$ amplitudes.
- Apply bootstrap methods to amplitude computations in curved space-time as their are usually complicated to study.
- Driven from the great success of the one loop case, where it was possible to reconstruct the full correlator, we hope to extract further information on the higher loops terms.

4d $SU(N)$ $\mathcal{N} = 4$ SYM

at large N and large $\lambda = g_{YM}^2 N$

Type IIB SUGRA

on $AdS_5 \times S^5$ with g_s coupling

CFT

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle = \mathcal{G}^{(0)}(U, V) + \frac{1}{N^2} \mathcal{G}^{(1)}(U, V) + \frac{1}{N^4} \mathcal{G}^{(2)}(U, V) + \frac{1}{N^6} \mathcal{G}^{(3)}(U, V) + \dots$$

dDisc

$$\{\gamma_{n,l}^{(1)}, a_{n,l}^{(0)}\}$$

Flat
Space

Amplitude

$$\mathcal{A}_{10}^{sugra} = G_N \text{ (triangle)} + G_N^2 \text{ (square)} + G_N^3 \text{ (pentagon)} + \dots$$

Cuts & Discontinuities

4d $SU(N)$ $\mathcal{N} = 4$ SYM

at large N and large $\lambda = g_{YM}^2 N$

$\frac{1}{2}$ BPS scalar
 $[0, 2, 0]_R \Delta = 2$

$T_{\mu\nu}$ supermultiplet

CFT

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dDisc

$$\{\gamma_{n,l}^{(1)}, a_{n,l}^{(0)}\}$$

Type IIB SUGRA

on \mathbb{R}^{10} with G_N coupling

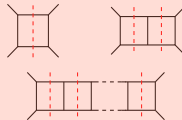
$$G_N = \frac{\pi^4 L^8}{2(N^2 - 1)}$$

Graviton multiplet

Amplitude

$$\mathcal{A}_{10}^{sugra} = G_N \text{ (triangle)} + G_N^2 \text{ (square)} + G_N^3 \text{ (pentagon)} + \dots$$

Cuts & Discontinuities



Flat
Space

Examples: Two Loops

The two loop correlator can be written in terms of OPE data as:

$$\begin{aligned}\mathcal{G}^{(3)}(z, \bar{z}) \propto & \frac{1}{48} (z\bar{z})^{n+2} \log^3(z\bar{z}) a_{n,\ell}^{(0)} \left(\gamma_{n,\ell}^{(1)} \right)^3 g_{2n+8,\ell} \\ & + \frac{1}{8} (z\bar{z})^{n+2} \log^2(z\bar{z}) \gamma_{n,\ell}^{(1)} \left(2a_{n,\ell}^{(0)} \gamma_{n,\ell}^{(2)} + a_{n,\ell}^{(1)} \gamma_{n,\ell}^{(1)} \right) \dots\end{aligned}$$

Unfortunately as the mixing at the previous order has not been solved and as higher traces contributions appear we can only construct the higher logarithmic contributions, which is build up of $\gamma_{n,\ell}^{(1)}$ contributions, for which the mixing is solved.

- We have a four point correlator in the strong coupling limit which is dual to a graviton amplitude.
- We want to study the $1/N$ corrections that map to SUGRA loop corrections on $\text{AdS}_5 \times S^5$.
- As we have information only on $\gamma_{n,\ell}^{(1)}$ we can construct only the higher logarithmic term of the correlator.
- We can take the flat space/bulk-point limit to map the dDisc of the correlator to a flat-space quantity.

- We want to study the scattering of 4 gravitons in 10D SUGRA.
- We will start by considering the two-loop amplitude, the integrand has been known in the literature for a long time.
- The \mathcal{A}_{10}^{sugra} is function of the s, t, u Mandelstam invariants having defined $p_i^2 \rightarrow 0$.

10D SUGRA

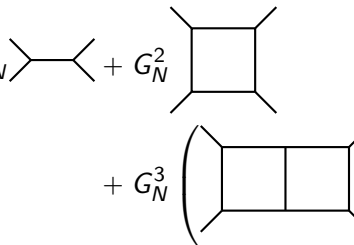
The two loop amplitude takes schematically the form:

$$\mathcal{A}_{10}^{sugra} = G_N \text{ (tree diagram)} + G_N^2 \text{ (one-loop diagram)} + G_N^3 \left(\text{two-loop diagrams} \right) + \dots$$

The equation shows the schematic form of the two-loop amplitude \mathcal{A}_{10}^{sugra} in 10D SUGRA. It is a sum of terms with increasing powers of G_N . The first term is G_N multiplied by a tree-level diagram consisting of two vertices connected by a single line. The second term is G_N^2 multiplied by a one-loop diagram, which is a square with four external lines. The third term is G_N^3 multiplied by a pair of two-loop diagrams enclosed in large parentheses. The first diagram in the parentheses is a rectangle divided into two squares by a vertical line, with four external lines. The second diagram is a rectangle with two diagonal lines crossing in the center, also with four external lines. The entire expression ends with $+\dots$.

10D SUGRA

The two loop amplitude takes schematically the form:

$$\mathcal{A}_{10}^{sugra} = G_N \text{ (tree diagram)} + G_N^2 \text{ (one-loop diagram)} + G_N^3 \left(\text{two-loop diagram 1} + \text{two-loop diagram 2} \right) + \dots$$


The integrals were obtained in 4 dimensions using the differential equations method and then uplifted to 10 d using DRR.

10D SUGRA

The two loop amplitude takes schematically the form:

$$\mathcal{A}_{10}^{sugra} = G_N \text{ (tree diagram)} + G_N^2 \text{ (square diagram)} + G_N^3 \left(\text{two squares} + \text{crossed rectangle} \right) + \dots$$

The integrals were obtained in 4 dimensions using the differential equations method and then uplifted to 10 d using DRR.

To map with the CFT we want to extract an observable that matches the one obtained from the Bulk-point limit. This is the discontinuity of the Amplitude.

Discontinuities & Cuts

Discontinuities are defined as:

$$\text{disc}_{x>1} F_3(x) \equiv \frac{1}{2\pi i} (F_3(x + i0) - F_3(x - i0)) .$$

and they can be resolved as

$$F_3(x) = \int_1^\infty dx' \frac{\text{disc}_{x'>1} F_3(x')}{x' - x} .$$

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They can analogously be extracted from cuts, for example at two loops:

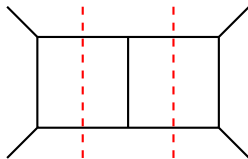
The diagram shows an equation for two-loop discontinuities. On the left is the expression $2\pi i \text{ disc}_s$ followed by a diagram of a two-loop box with four external lines. This is set equal to the sum of two diagrams. The first diagram, labeled c_1 with a brace underneath, shows a vertical dashed red line representing a cut. The second diagram, labeled c_2 with a brace underneath, shows a diagonal dashed red line representing a cut. Both diagrams on the right have the same box structure as the one on the left.

$$2\pi i \text{ disc}_s = \underbrace{\text{diagram with vertical cut}}_{c_1} + \underbrace{\text{diagram with diagonal cut}}_{c_2} .$$

Example: Two Loops

As we have seen from the CFT we can construct only the part of the correlator proportional to $\gamma^{(1)}$. In the amplitude we have control over the complete result but we can extract some information:

- The type of cut corresponds to the type of operators exchanged. So c_1 is the double trace contribution while c_2 is the triple trace.
- The contributions from the CFT match the functions appearing in c_1 , the mismatch can be understood from the missing contributions of $\text{Log}^2(1 - \bar{z})$ (in CFT picture contributions connected to $a_{n,\ell}^{(0)}, \gamma_{n,\ell}^{(2)}, a_{n,\ell}^{(1)}$).
- We can construct a cut that directly matches the higher log contribution



- We are able to match the $(\gamma_1^{(1)})^\kappa$ contribution to a specific iterated cut of the amplitude, in the flat-space limit.
- We observe an interesting match between the maximal-cut of ladders and the Mellin amplitude coefficient, in the flat-space limit.
- By using iterated discontinuities we can reconstruct more contribution to the double-trace/cut, we are hopeful that the same type of relations should hold also for iterated dDiscs.

Thank You!