Berry Phases and Drift in the KdV Equation

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Based on arXiv 1703.06142 (*JHEP*)
1907.01438 (*J Math Fluid Mech*)
2002.01780 (*Chaos*) w/ G. Kozyreff

Intro

(From Gravity to Fluids)

Motivation

Diffeomorphisms (cts deformations) are crucial in physics :

- General relativity (gauge + asymptotic symmetries)
- Hydrodynamics (fluid flows)
- Topological phases of matter (robust to deformations)

Question :
Observables sensitive to geometry of groups of diffeos ?
...i.e. **infinite-dimensional geometry** ?

THIS TALK IN TWO STATEMENTS

This talk is about **Berry phases** due to diffeos in **fluids**.

More precisely :

- Adiabatic diffeos produce Berry phases [B.O. 2017]
- These affect **drift velocity** of fluid parcels [B.O.-Kozyreff 2020]

1. Berry phases of diffeos

Reminder on Berry phases

Quantum system with cts parameters

- Vary parameters adiabatically and periodically
- $|\text{final}\rangle = e^{i\,(\text{Dynamical})\,+\,i\,\textbf{(Berry)}}\,|\text{initial}\rangle$

Example : "Parameters" = choice of **ref. frame**

- Change frames adiabatically = time-dependent diffeos g_t
- Berry $\sim \int \mathrm{d}t\,\langle\psi|\,g^{-1}\,\dot{g}\,|\psi\rangle$

Adiabatic diffeos

Wavefct $\psi(x)$ on circle

- Act with diffeos according to $g^{-1} \cdot \psi \equiv \sqrt{g'}\, \psi \circ g$

Eigenstates $\psi(x) \sim e^{ihx}$

- Choose adiabatic diffeos $g_t(x)$
- Berry $= \int \mathrm{d}t \, \langle \psi | \, g^{-1} \dot{g} \, | \psi \rangle = \int \mathrm{d}t \, \mathrm{d}x \, \frac{\dot{g}}{g'} \, h$
- Explicit despite ∞ parameters !

Virasoro Berry phases

For diffeos, Berry $= \int \mathrm{d}t\, \mathrm{d}x\, \frac{\dot{g}}{g'}\, h$

Now consider **CFT_2** with **central charge c**

- Diffeos $x \mapsto g(x)$ are conformal tsfs
- Let $|\psi\rangle = |h\rangle$ primary
- Berry $= \int \mathrm{d}t\, \langle\psi|\, g^{-1}\dot{g}\, |\psi\rangle \underset{\text{Virasoro rep}}{=} \int \mathrm{d}t\, \mathrm{d}x\, \frac{\dot{g}}{g'} \left[h + c \left(\frac{g''}{g'} \right)' \right]$
- 3D gravity symplectic flux [Barnich *et al.* '17, B.O. '17, Cotler-Jensen '18]

Dynamics exhibiting such phases ?

2. Drift velocity in KdV

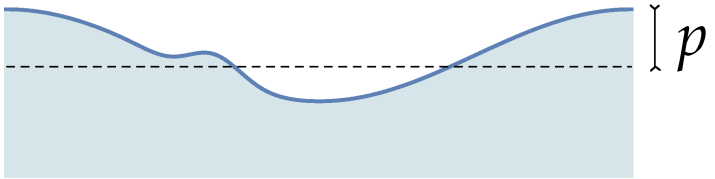
KdV flow

$p(x,t)$: wave on a circle

- **Korteweg-de Vries (KdV)** :

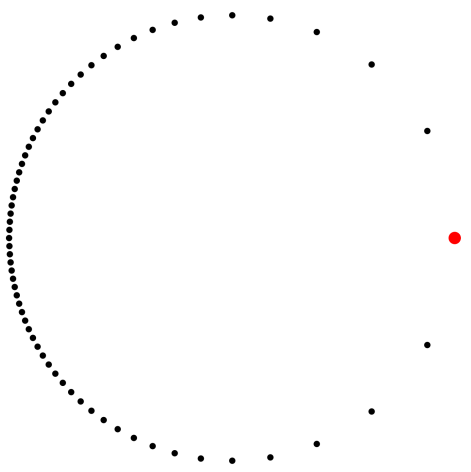
  $\dot{p} = -3pp' + c\,p'''$

- $p(x,t)$ **sources diffeos** $g_t(x)$



$g_t(x)$ = position of fluid parcel initially at x

- Fluid flow $\dot{g}_t \circ g_t^{-1}(x) = p(x,t)$



$p(x,t) = \infty$ time-dependent parameters

- What if p is periodic ?

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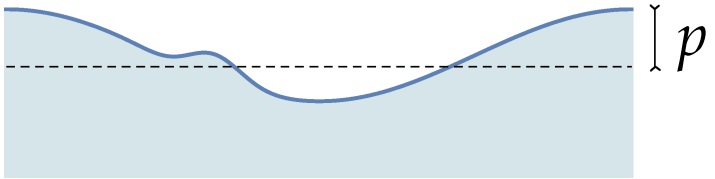

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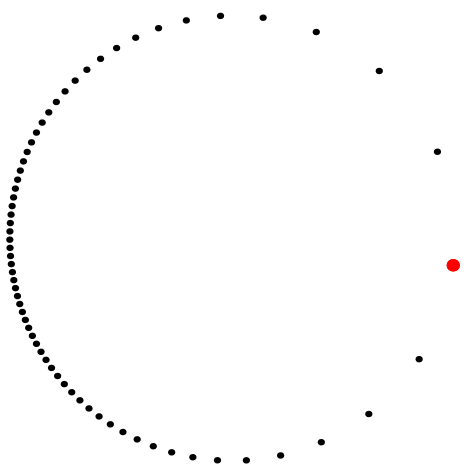

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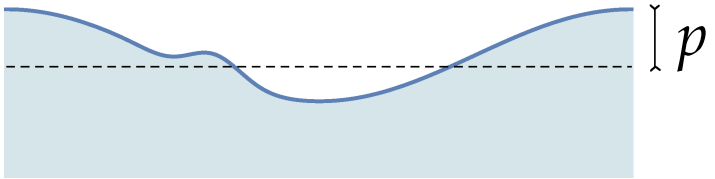

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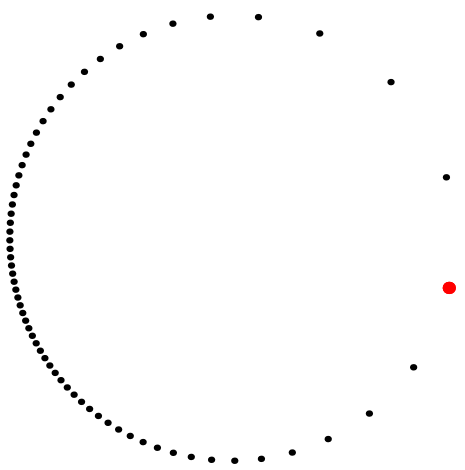

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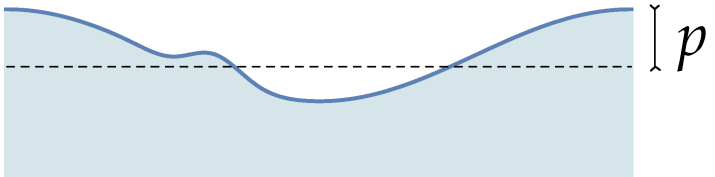

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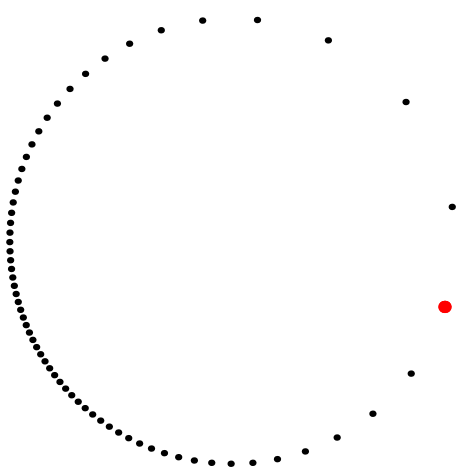

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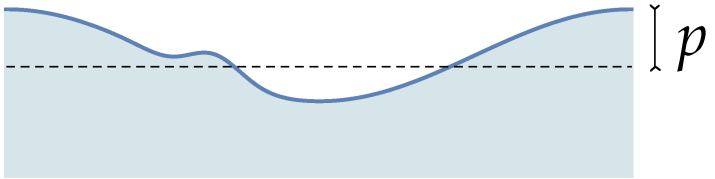

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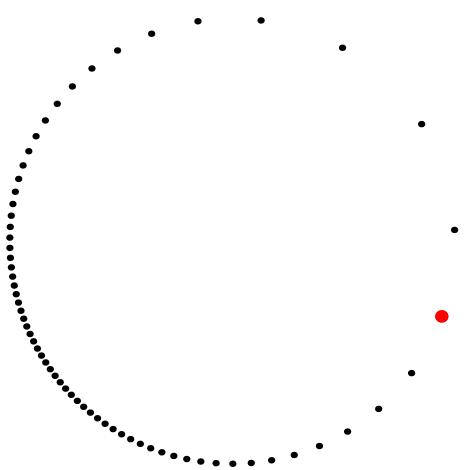

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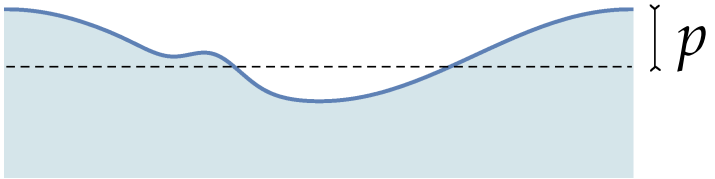

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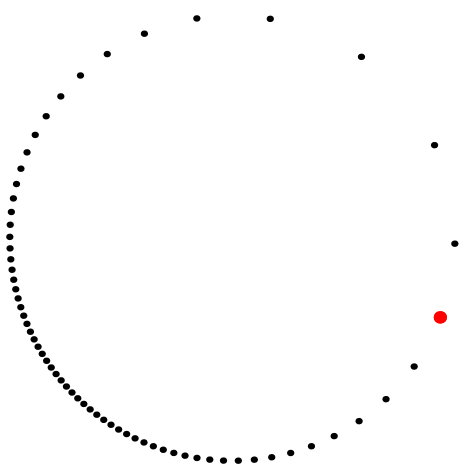

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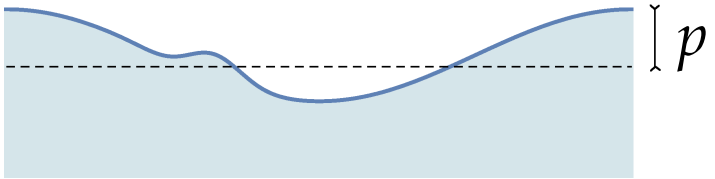

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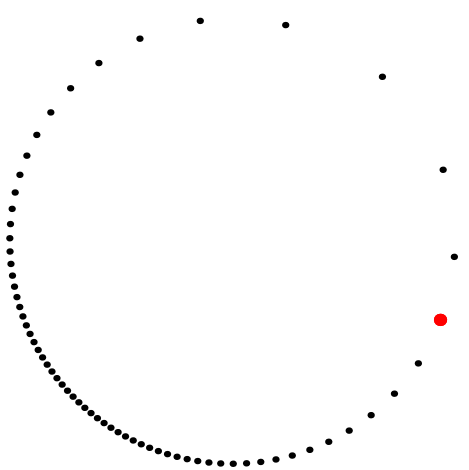

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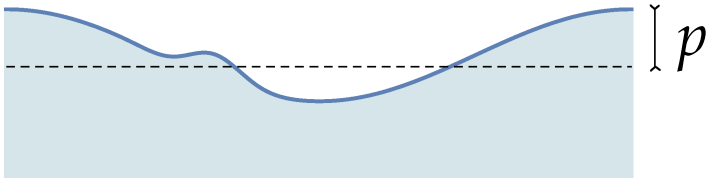

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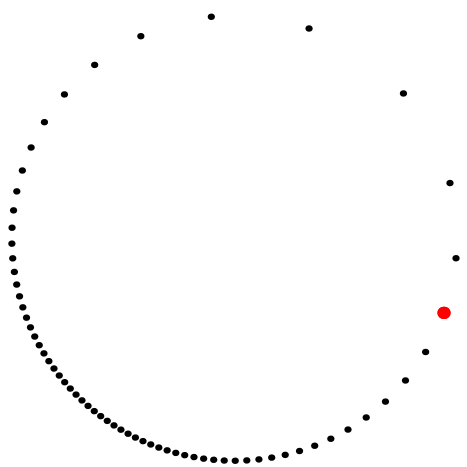

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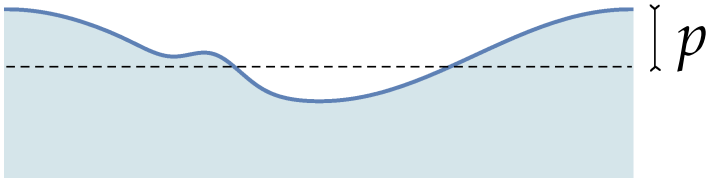

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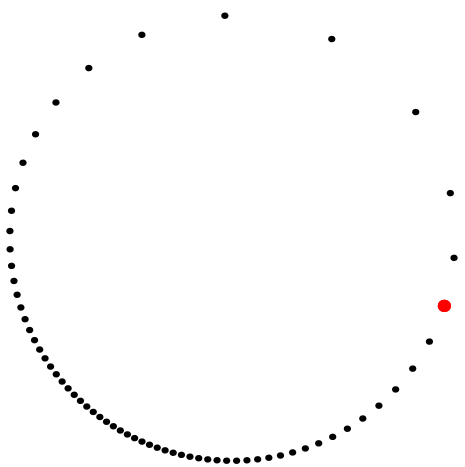

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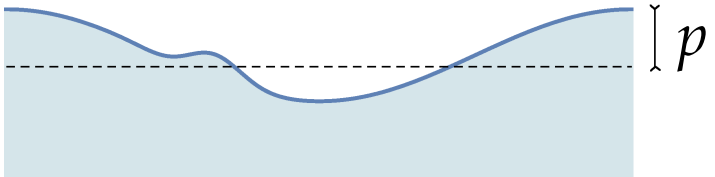

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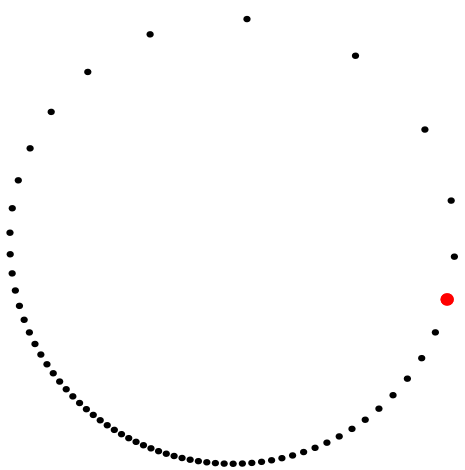

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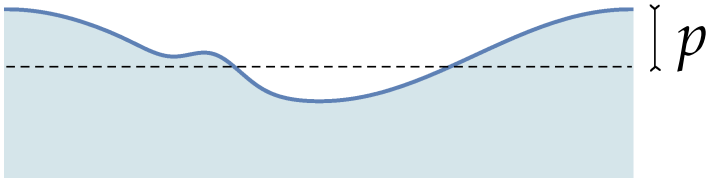

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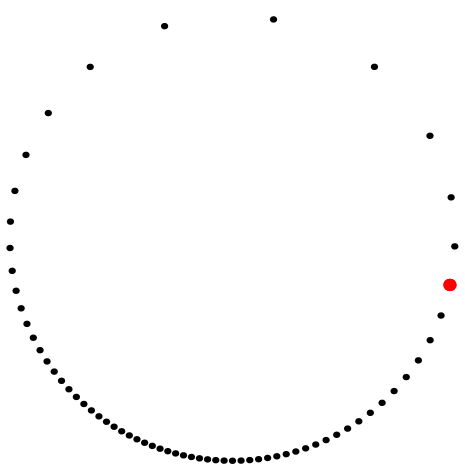

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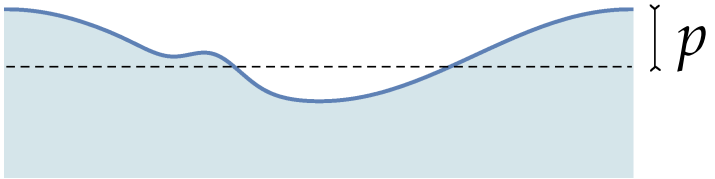

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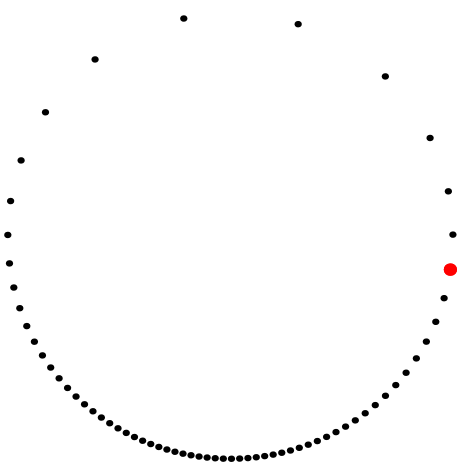

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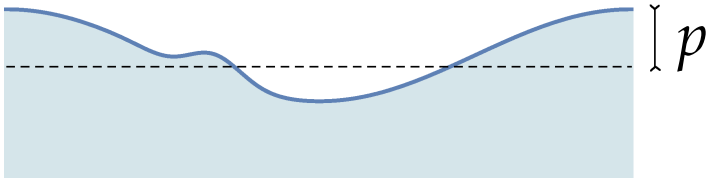

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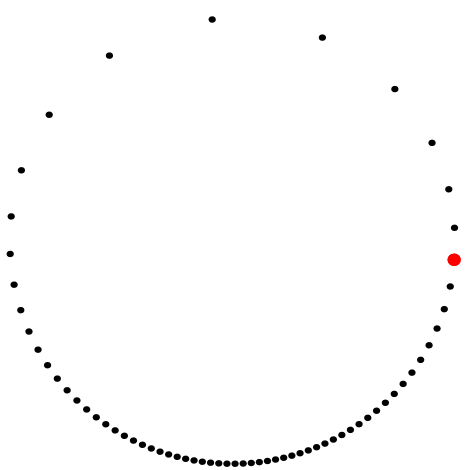

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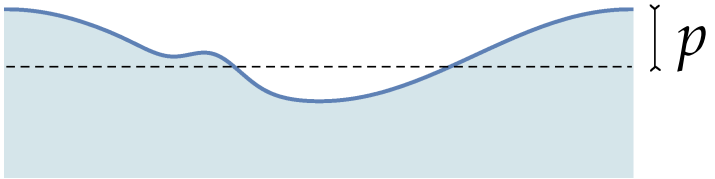

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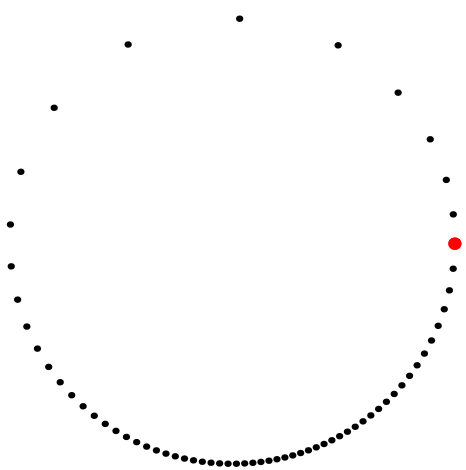

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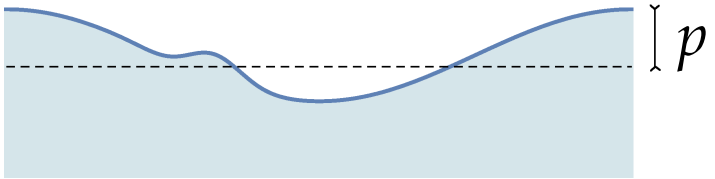

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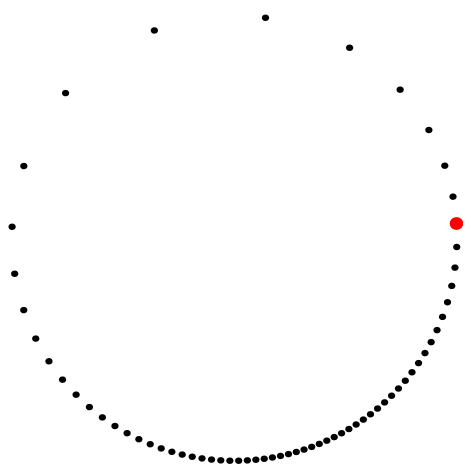

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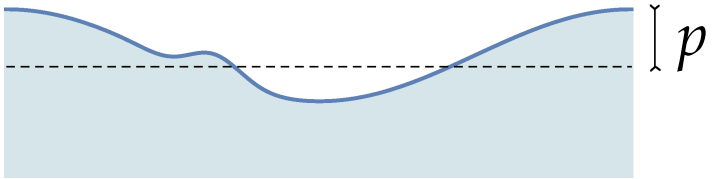

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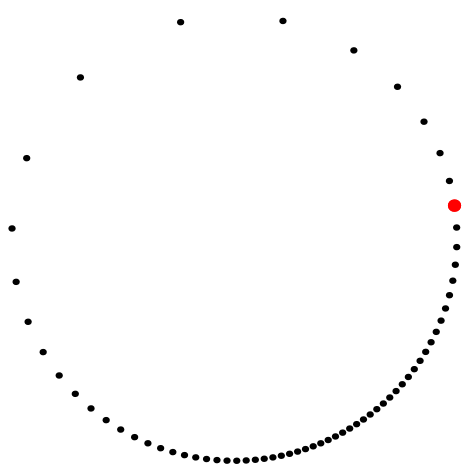

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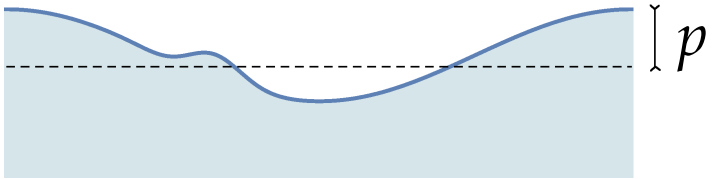

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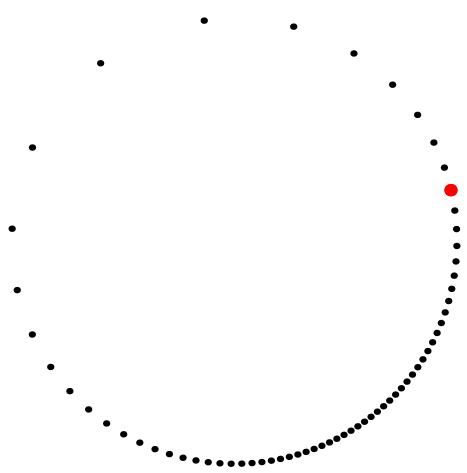


$p(x,t) = \infty$ time-dependent parameters

- What if p is periodic ?

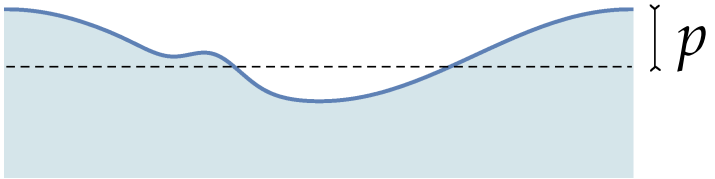
KdV flow

$p(x,t)$: wave on a circle

- **Korteweg-de Vries (KdV)** :

  $\dot{p} = -3pp' + c\,p'''$

- $p(x,t)$ **sources diffeos** $g_t(x)$



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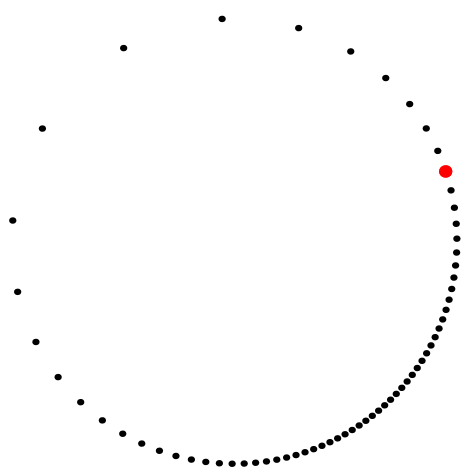

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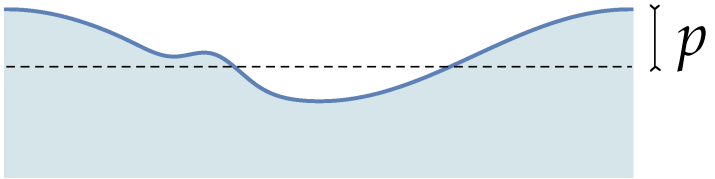

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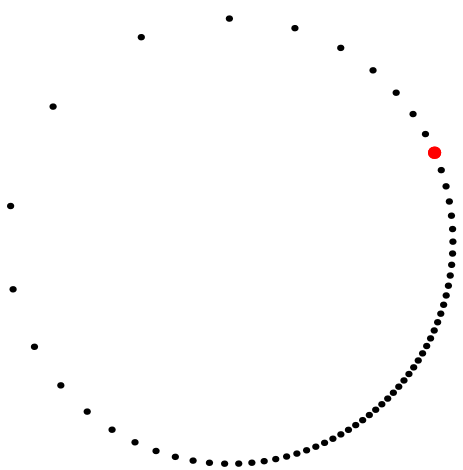

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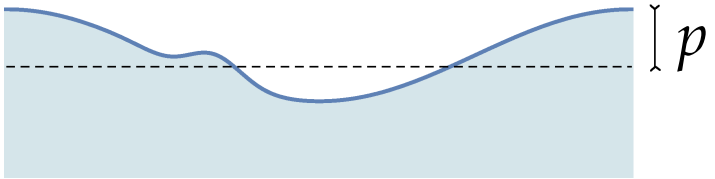

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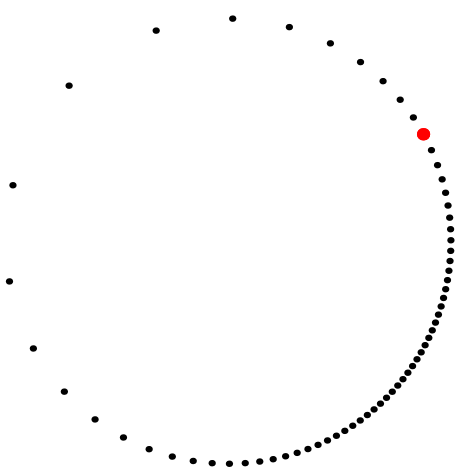

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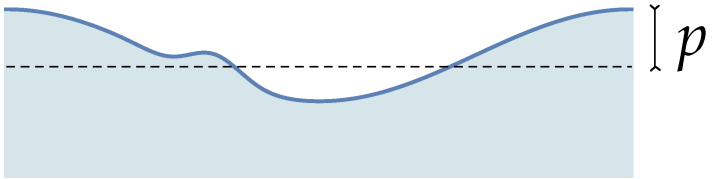

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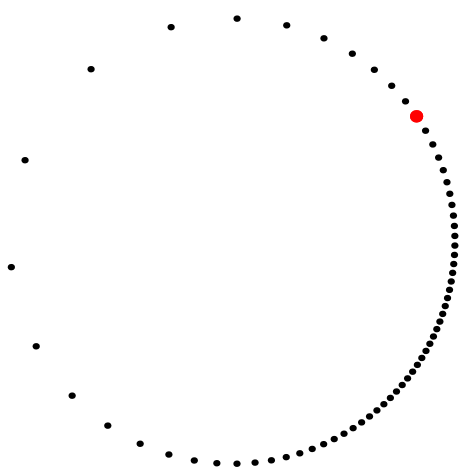

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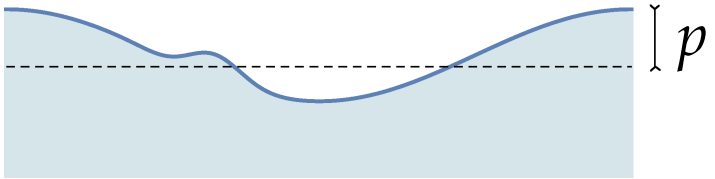

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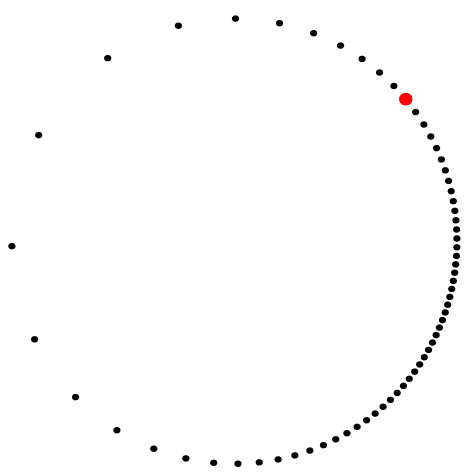

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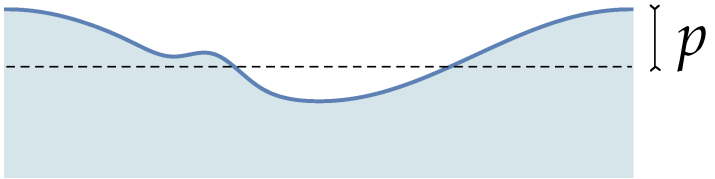

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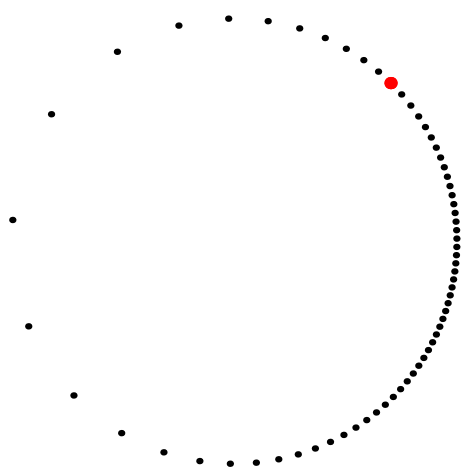

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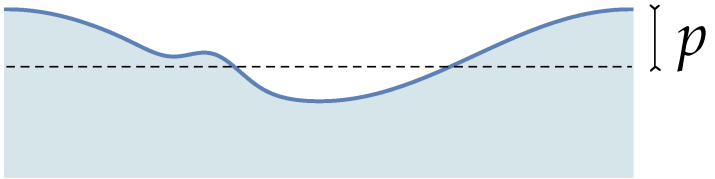

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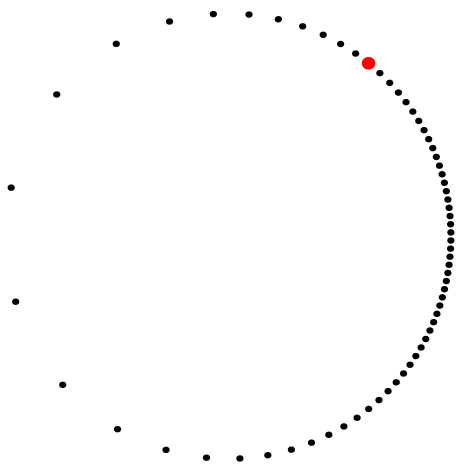

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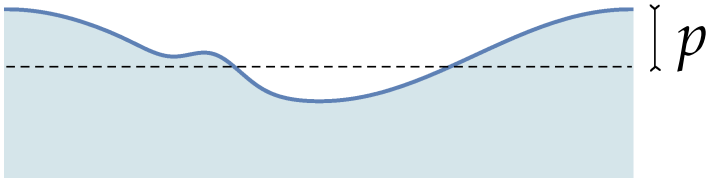

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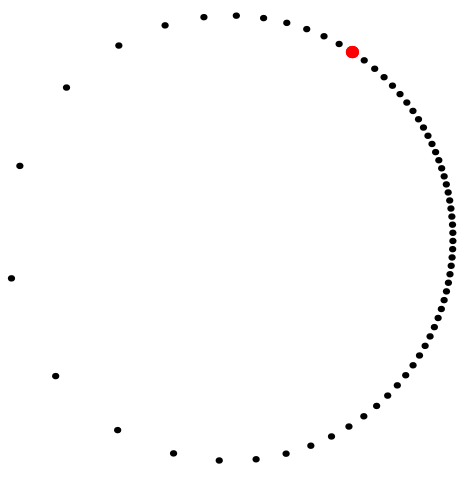

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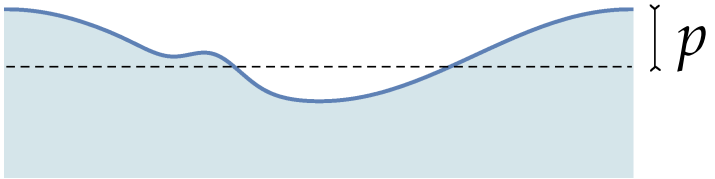

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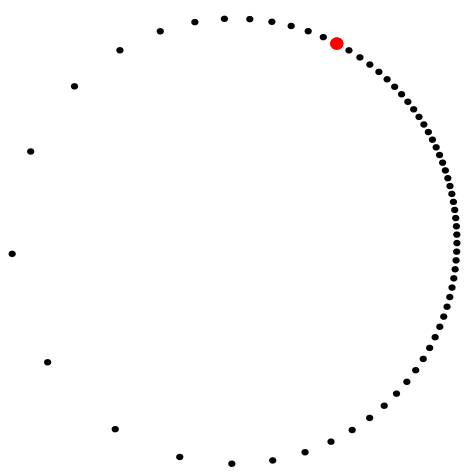

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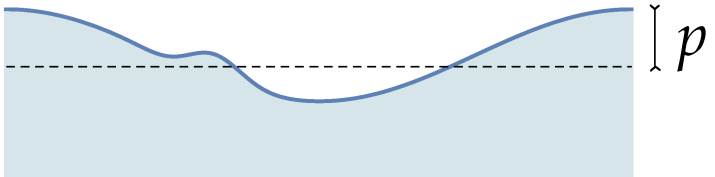

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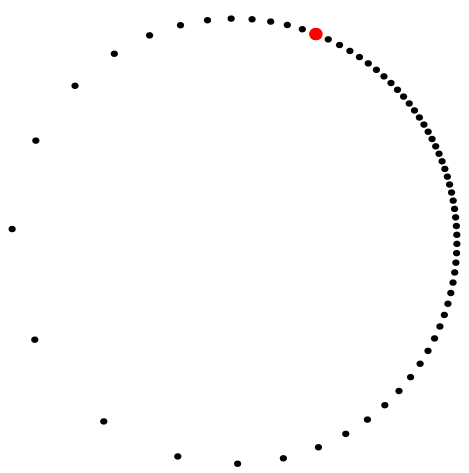

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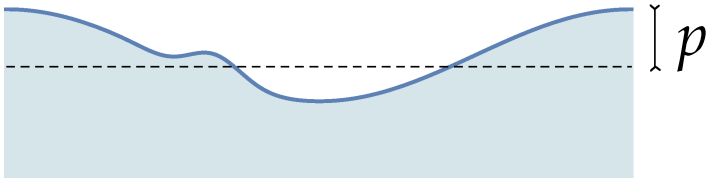

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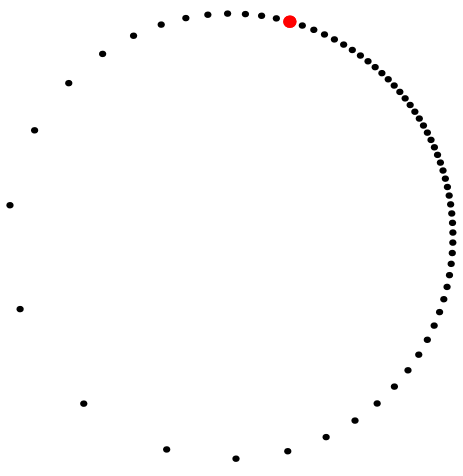

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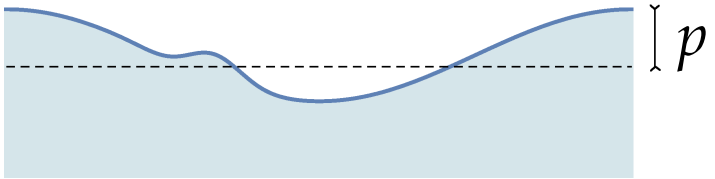

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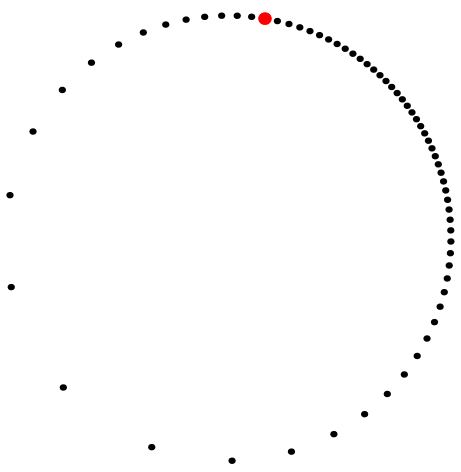

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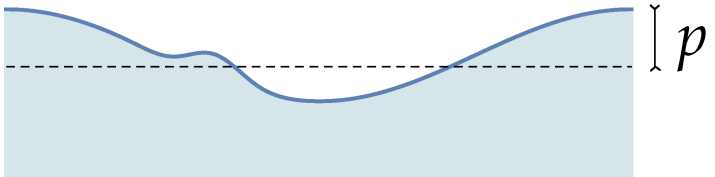

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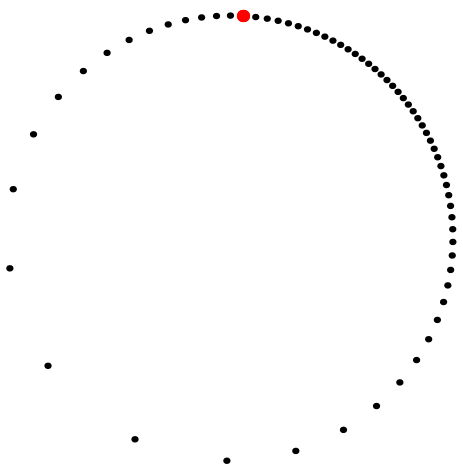

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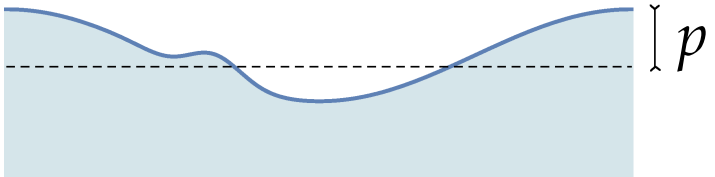

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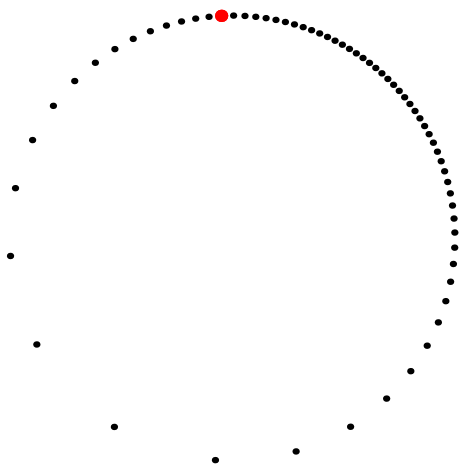

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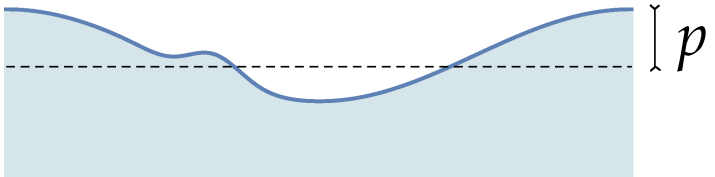

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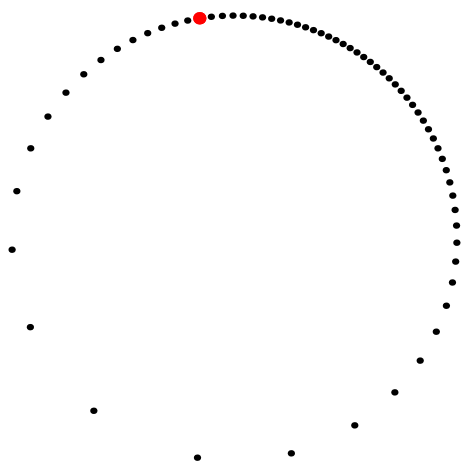

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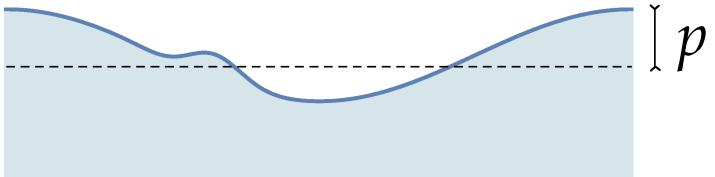

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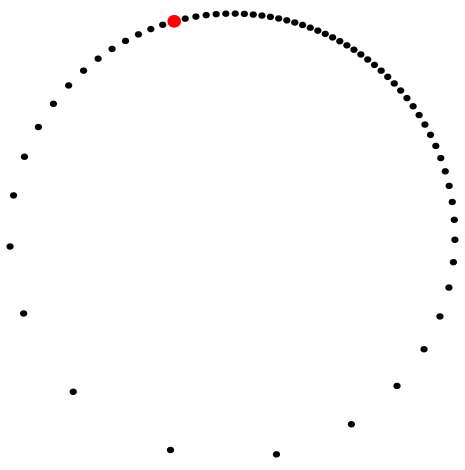

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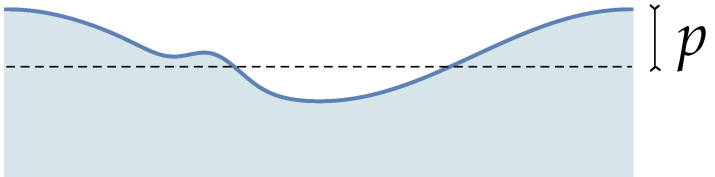

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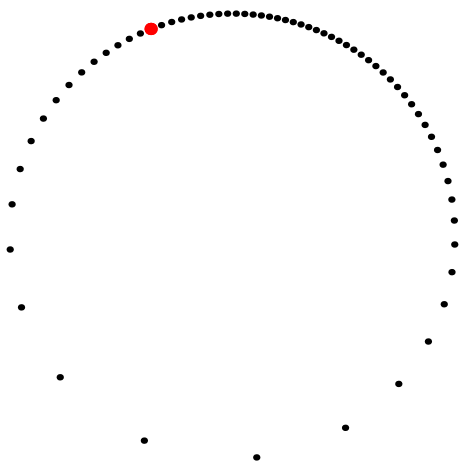

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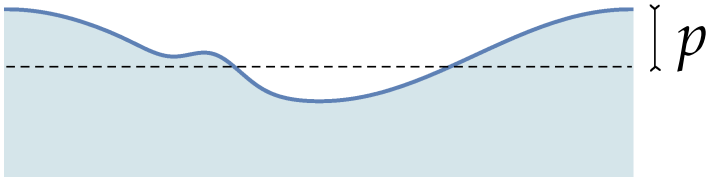

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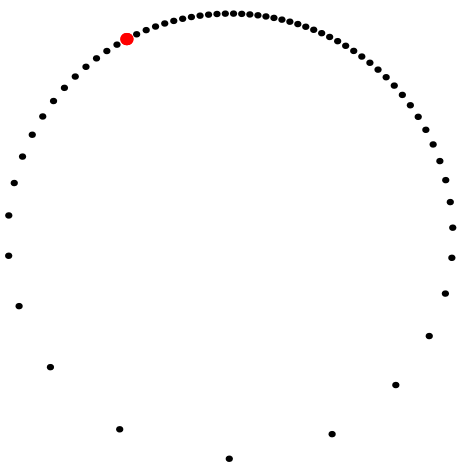

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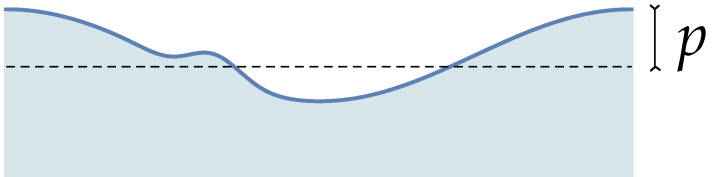

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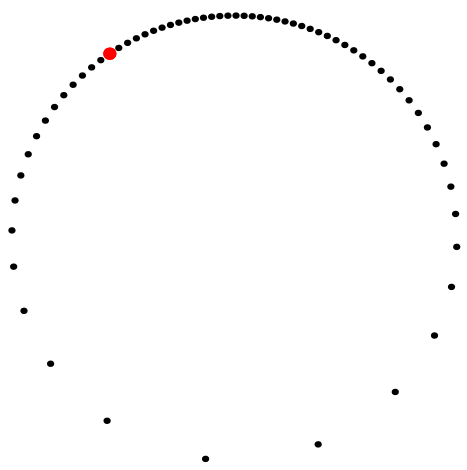

$p(x,t) = \infty$ time-dependent parameters

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KdV flow

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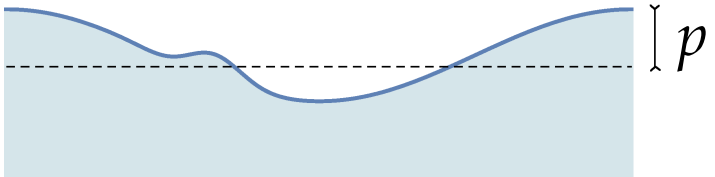

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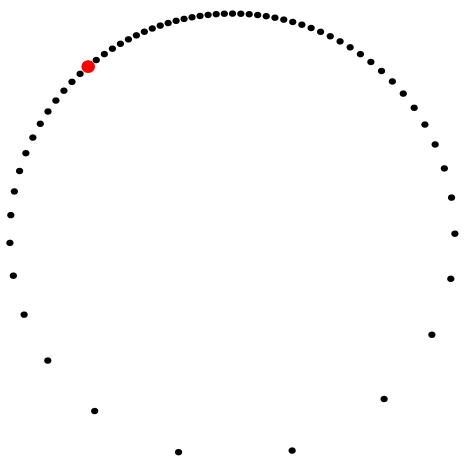

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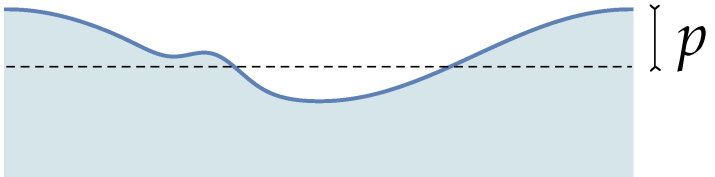

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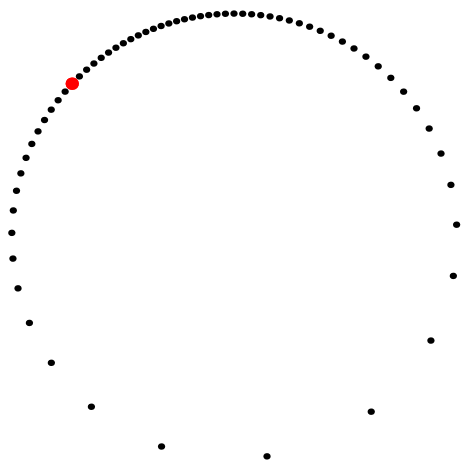

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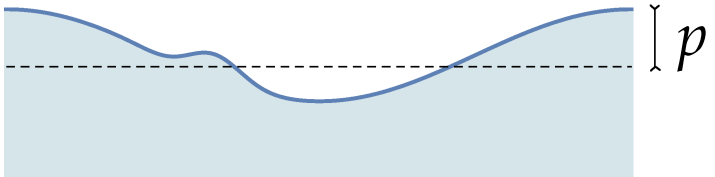

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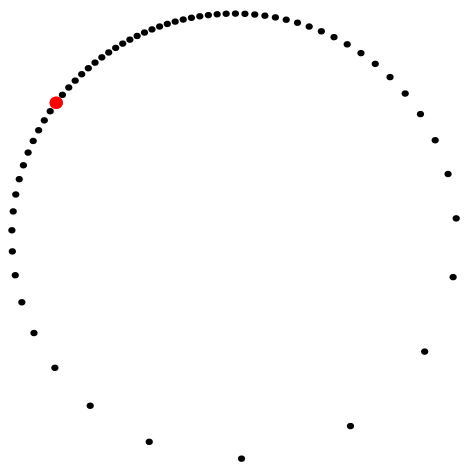

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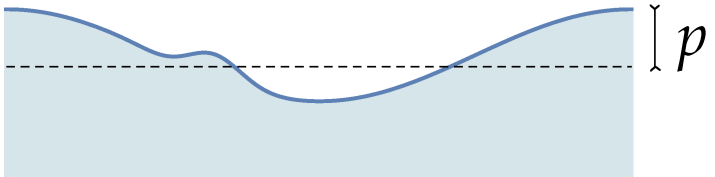

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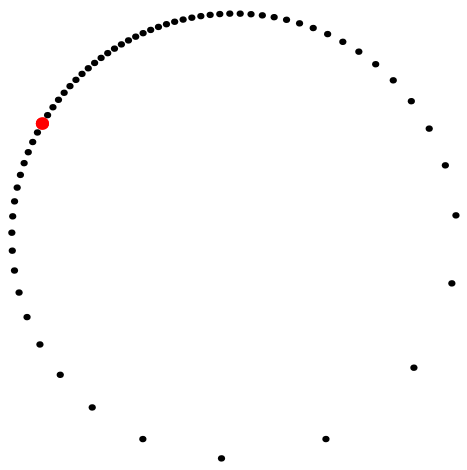

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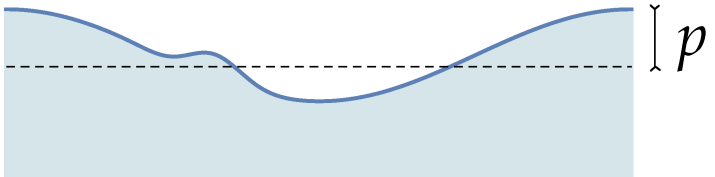

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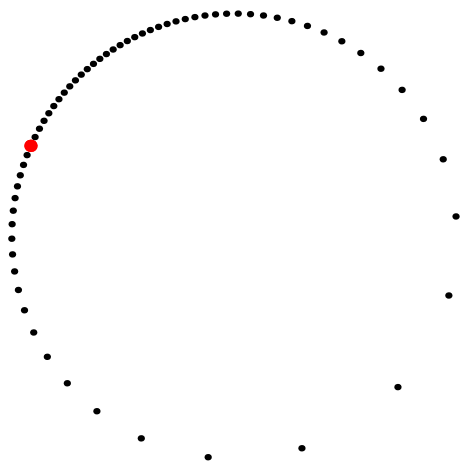

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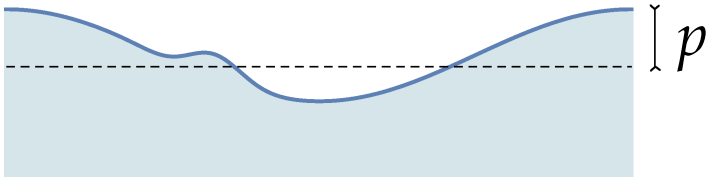

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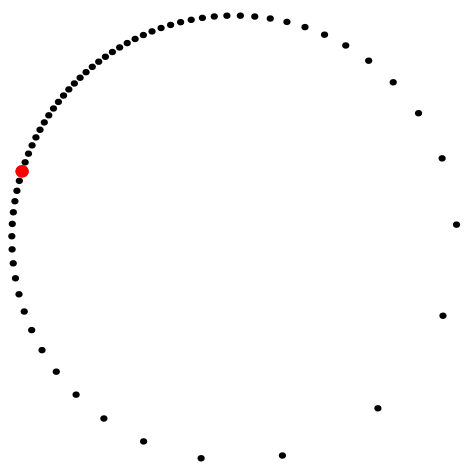

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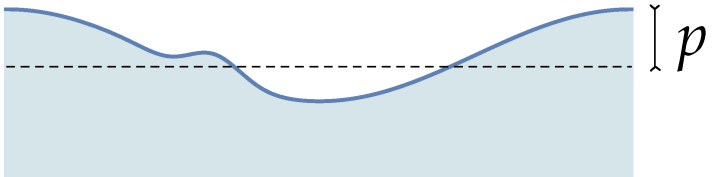

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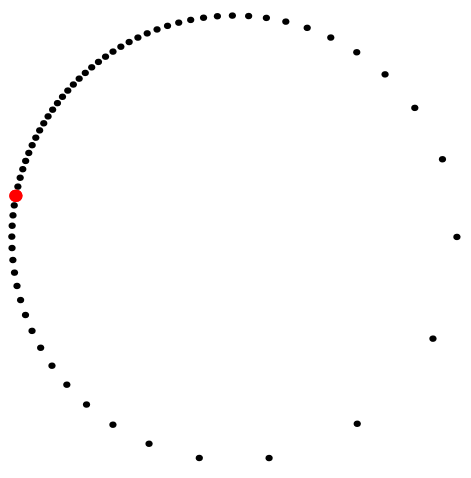

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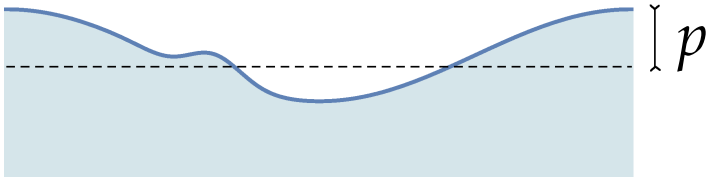

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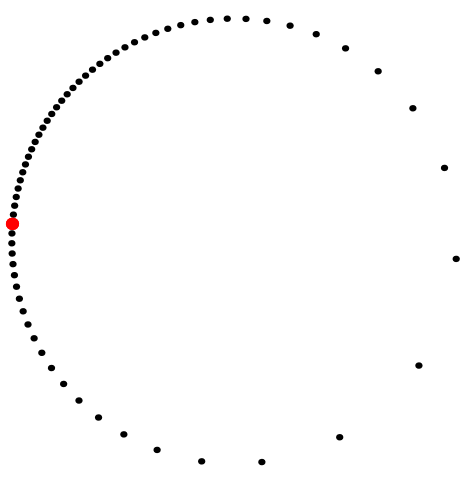

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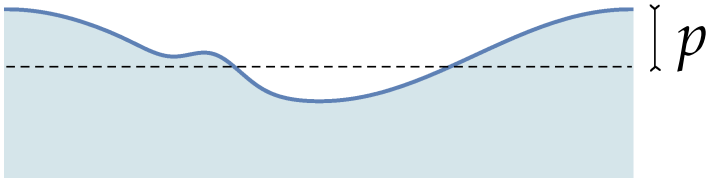

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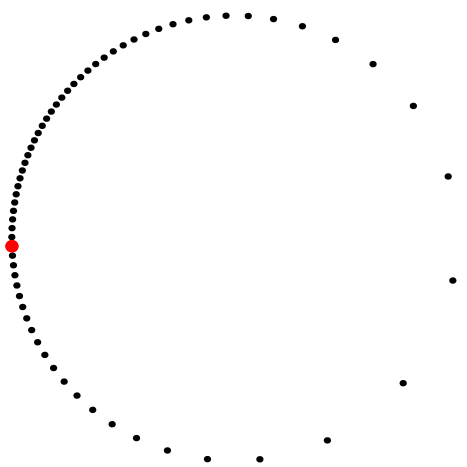

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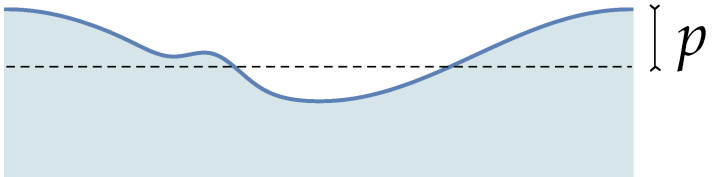

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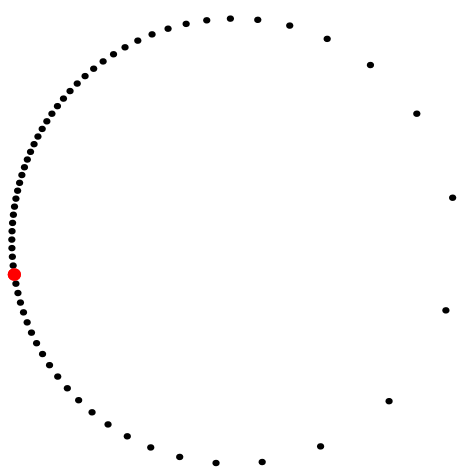

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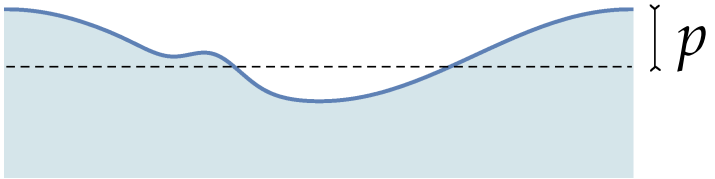

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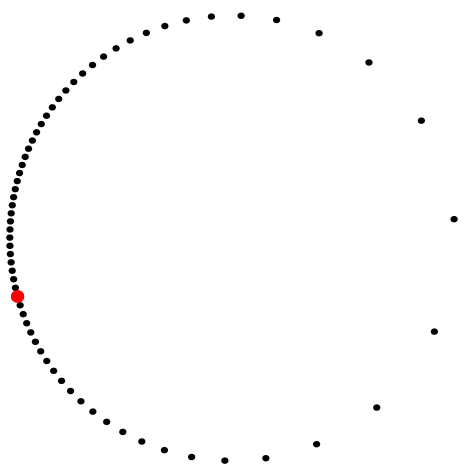

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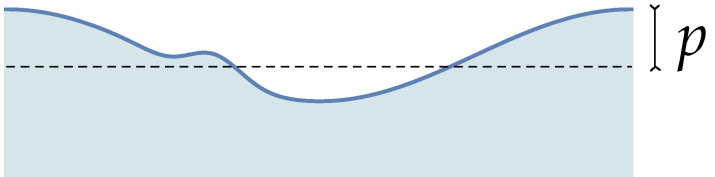

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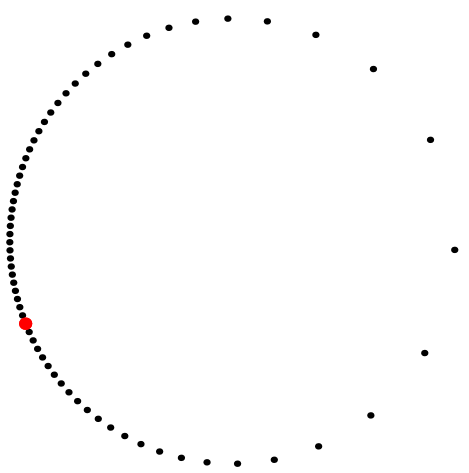

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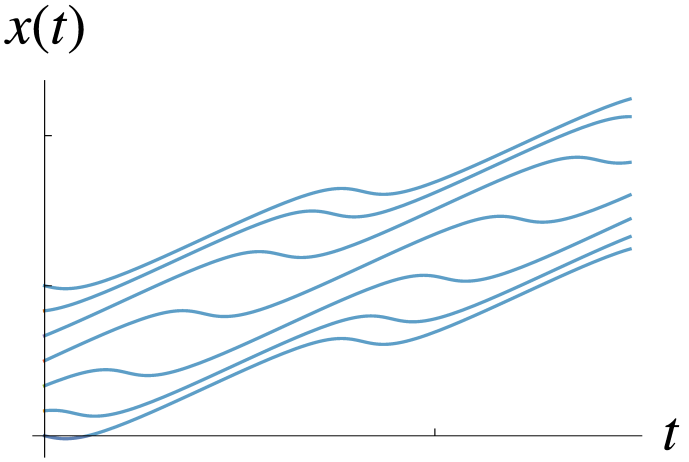
Periodic waves

Let $p(x, t+T) = p(x,t)$

- Solve $\dot{x} = p(x,t)$ numerically
- $x(t) \propto t$ "seen from far away"
- **Drift**



Rotation nbr of $g_T\, g_0^{-1}$

$$v_{\text{Drift}} = \lim_{t\to\infty} \frac{x(t)}{t} = \frac{1}{T} \lim_{n\to\infty} \frac{\left(g_T\, g_0^{-1}\right)^n (x_0)}{n} \equiv \frac{\Delta\phi}{T}$$

Claim : $\boxed{\Delta\phi \sim \text{Dynamical } \phi + \textbf{Berry } \phi}$ [B.O.-Kozyreff '20]

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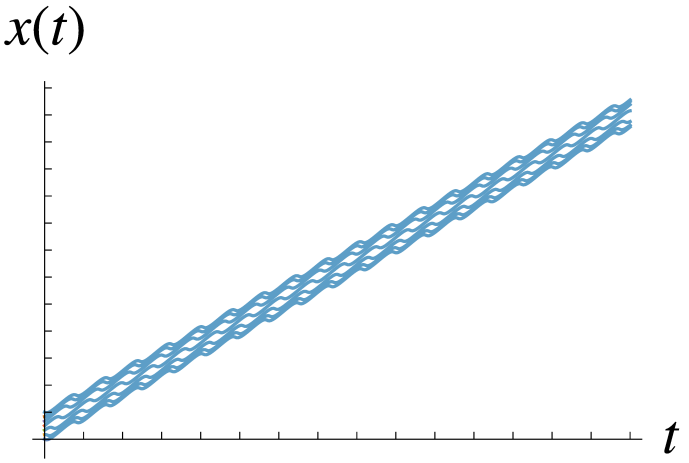

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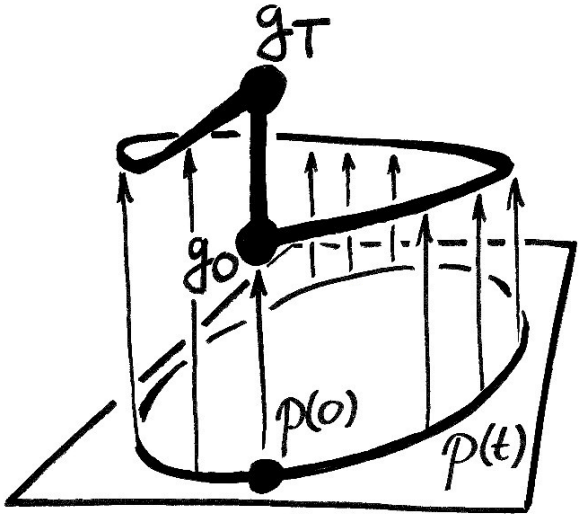
Drift and phases

$\dot{g}\,g^{-1} = p$ and $\dot{p} = -3pp' + cp'''$

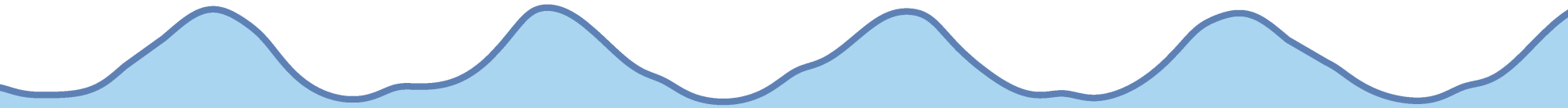
- $p =$ **stress tensor sourcing its own conf. tsfs**
- $p = g_t \cdot h$ $\qquad g^{-1}\cdot h \equiv h(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$

Integrate Berry connection !

- $\oint \mathrm{d}t\,\langle g^{-1}\dot{g}\rangle = \int_{\text{helix}} + \int_{\text{straight}}$
- Berry = Dynamical + $\Delta\phi$...
  + anomalous phase !
- Predicts v_{Drift}



- **Perfect match** with "observations" ! [B.O.-Kozyreff 2020]



Thanks for watching !

See also longer talk on YouTube. Comments welcome !