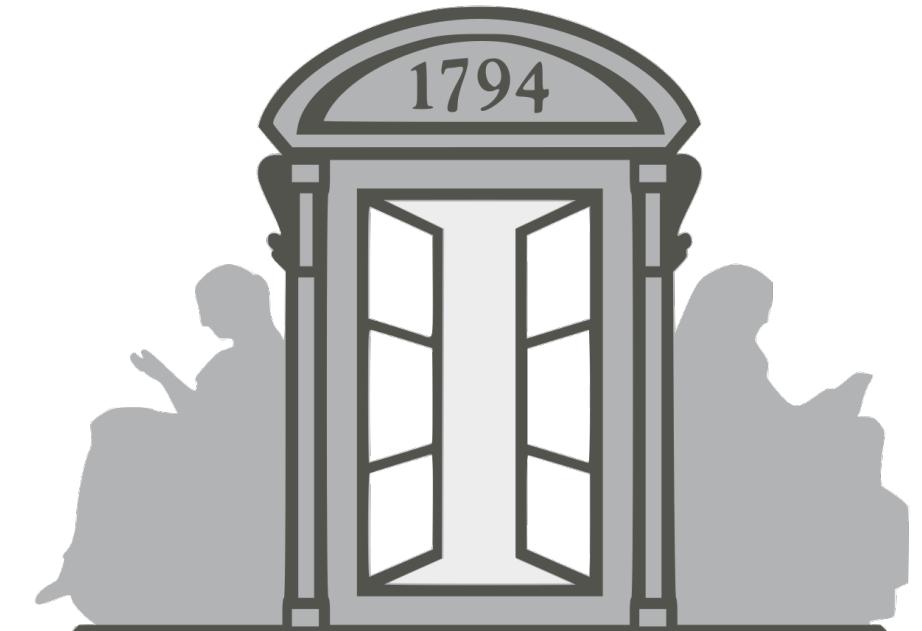


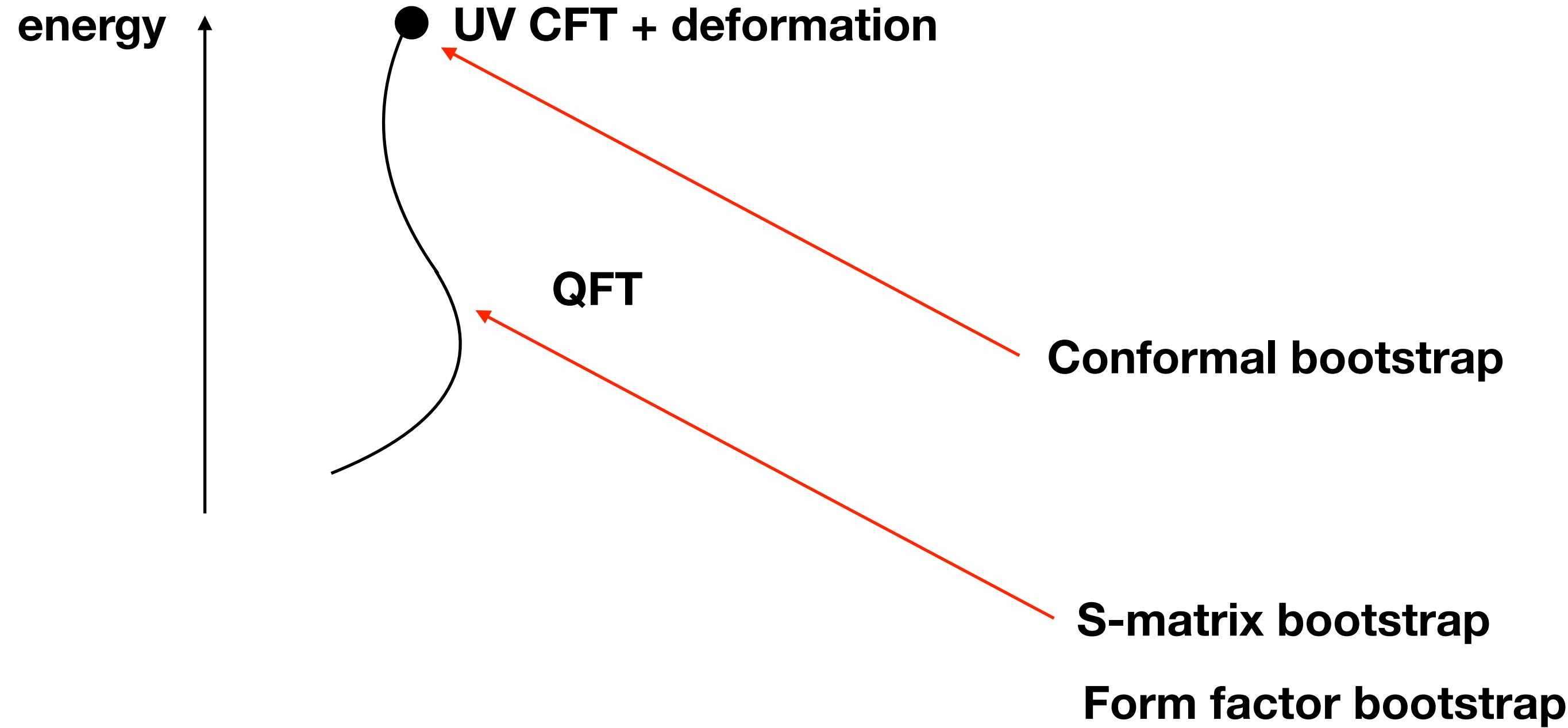
Form factor bootstrap in quantum field theories

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ÉCOLE NORMALE
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Bootstrap = a model independent way to put bounds on the space of theories
using first principles and symmetries only

Most general **QFT observables** = correlation functions of local operators

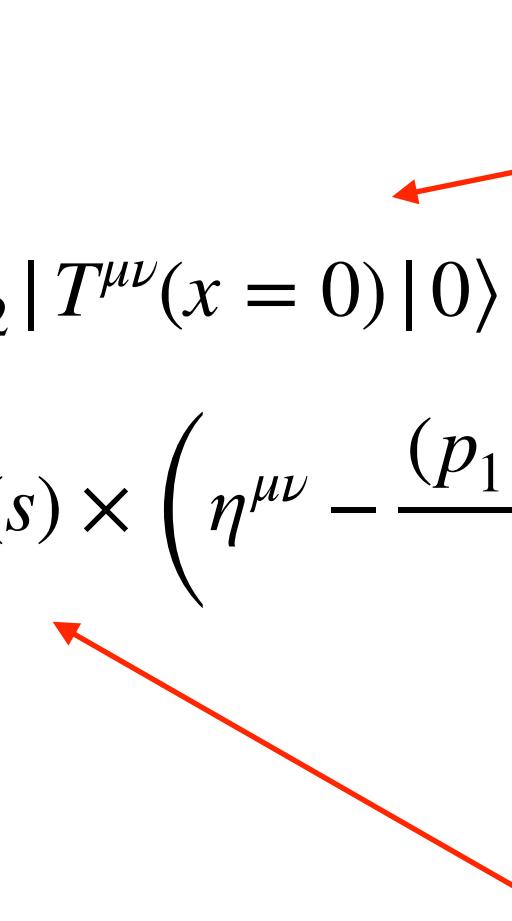
In QFTs with a mass gap one can define **asymptotic states (particles)**.
Other observables: **scattering amplitudes and form factors**

$${}_{out}\langle m, \vec{p}_1; m, \vec{p}_2 | T^{\mu\nu}(x=0) | 0 \rangle =$$

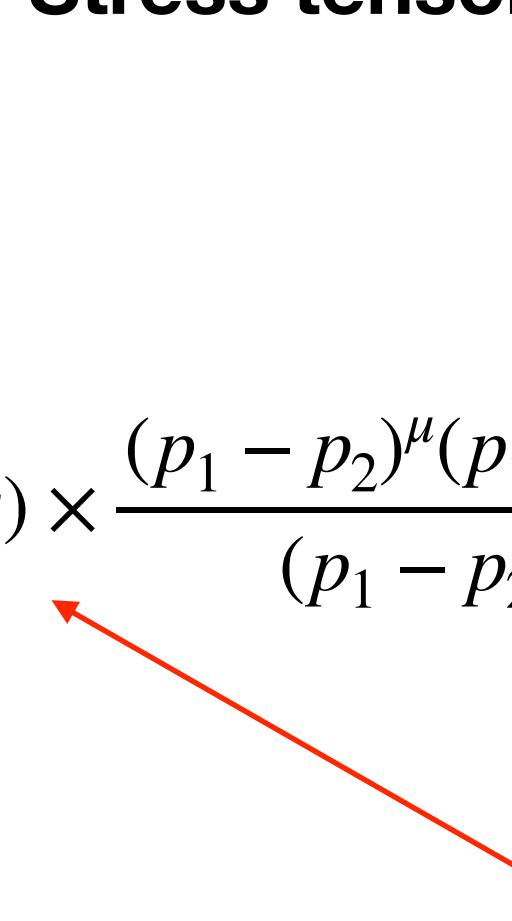
$$-\mathcal{F}_{(0)}(s) \times \left(\eta^{\mu\nu} - \frac{(p_1 + p_2)^\mu(p_1 + p_2)^\nu}{(p_1 + p_2)^2} \right) + \mathcal{F}_{(2)}(s) \times \frac{(p_1 - p_2)^\mu(p_1 - p_2)^\nu}{(p_1 - p_2)^2}.$$

$s \equiv -(p_1 + p_2)^2$

Spin 0 form factor component



Spin 2 form factor component



Stress tensor form factor

$$\Theta(x) \equiv \eta_{\mu\nu} T^{\mu\nu}(x)$$

$${}_{out}\langle m, \vec{p}_1; m, \vec{p}_2 | \Theta(x=0) | 0 \rangle \equiv \mathcal{F}_\Theta(s)$$

Trace of the stress tensor form factor



$$\mathcal{F}_\Theta(s) = \mathcal{F}_{(2)}(s) - (d-1) \mathcal{F}_{(0)}(s)$$

Normalisation conditions

$$\lim_{s \rightarrow 0} s^{-1} \mathcal{F}_{(0)}(s) = \mathbf{const}, \quad \lim_{s \rightarrow 0} \mathcal{F}_{(2)}(s) = -2m^2, \quad \lim_{s \rightarrow 0} \mathcal{F}_\Theta(s) = -2m^2.$$

Lorentz group: $SO(1, d-1)$

Little group: $SO(d-1)$

$$\langle 0 | T^{\mu\nu}(x) T^{\rho\sigma}(0) | 0 \rangle_T =$$

$$-\frac{i}{(d-1)^2} \int_0^\infty ds \rho_\Theta(s) \Delta_{F,\Theta}^{\mu\nu;\rho\sigma}(x; s) - i \int_0^\infty ds \rho_2(s) \Delta_{F,2}^{\mu\nu;\rho\sigma}(x; s)$$

Spin 2 spectral density component

Spin 0 spectral density component

Feynman propagators

$$\Delta_F(x; s) = \lim_{\epsilon \rightarrow 0^+} \int \frac{d^d p}{(2\pi)^d} e^{ip \cdot x} \frac{1}{p^2 + s - i\epsilon}$$

$$\Delta_{F,\Theta}^{\mu\nu;\rho\sigma}(x; s) = (s \eta^{\mu\nu} - \partial^\mu \partial^\nu)(s \eta^{\rho\sigma} - \partial^\rho \partial^\sigma) \Delta_F(x; s)$$

$$\Delta_{F,2}^{\mu\nu;\rho\sigma}(x; s) = s^2 \Pi_2^{\mu\nu;\rho\sigma}(p^\alpha \rightarrow i\partial^\alpha, p^2 \rightarrow -s) \Delta_F(x; s)$$

Relation with form factors

$$2\pi\rho_\Theta(s) = \omega^2 |\mathcal{F}_\Theta(s)|^2 + \dots$$

$$2\pi s^2 \rho_2(s) = \frac{2\omega^2}{d^2 - 1} |\mathcal{F}_{(2)}(s)|^2 + \dots$$

$\omega^2 = \frac{1}{\mathcal{N}_d} \frac{\Omega_{d-1}}{2(2\pi)^{d-2}}$

$\mathcal{N}_d \equiv 2^{d-1} \sqrt{s} (s - 4m^2)^{(3-d)/2}$

Mandelstam variables

$$s \equiv -(p_1 + p_2)^2$$

$$t \equiv -(p_1 - p_3)^2$$

$$u \equiv -(p_1 - p_4)^2$$

$$s + t + u = 4m^2$$

Scattering amplitude

$$\mathcal{S}(s, t, u) \times (2\pi)^2 \delta^d(p_1 + p_2 - p_3 - p_4) \equiv {}_{out}\langle m, \vec{p}_3; m, \vec{p}_4 | m, \vec{p}_1; m, \vec{p}_2 \rangle_{in}$$

$$\mathcal{S}_j(s) = \# \int_{-1}^{+1} d \cos \theta (1 - \cos^2 \theta)^{\frac{d-4}{2}} C_j^{\frac{d-3}{2}}(\cos \theta) S(s, t(s, \cos \theta), u(s, \cos \theta))$$

Spin j=0,2,4,.. partial amplitude

Gegenbauer polynomial

Unitarity

$$s \geq 4m^2$$

$$\begin{pmatrix} 1 & \mathcal{S}_j^*(s) \\ \mathcal{S}_j(s) & 1 \end{pmatrix} \succeq 0, \quad \forall j = 0, 2, 4, \dots$$

Numerical S-matrix bootstrap:

- Construct an ansatz for the S-matrix amplitud [using crossing and analyticity]
- Compute partial amplitudes
- Plug the partial amplitudes into the unitarity constraint
- Bound parameters of the ansatz by solving SDP optimisation problem

Miguel Paulos, Joao Penedones, Jonathan Toledo, Balt van Rees, Pedro Vieira; 16,17

$$\begin{pmatrix} 1 & \mathcal{S}_0^*(s) & \omega \mathcal{F}_\Theta^*(s) \\ \mathcal{S}_0(s) & 1 & \omega \mathcal{F}_\Theta(s) \\ \omega \mathcal{F}_\Theta(s) & \omega \mathcal{F}_\Theta^*(s) & 2\pi \rho_\Theta(s) \end{pmatrix} \succeq 0$$

Numerical form factor bootstrap:

$$\begin{pmatrix} 1 & \mathcal{S}_2^*(s) & \varepsilon \mathcal{F}_{(2)}^*(s) \\ \mathcal{S}_2(s) & 1 & \varepsilon \mathcal{F}_{(2)}(s) \\ \varepsilon \mathcal{F}_{(2)}(s) & \varepsilon \mathcal{F}_{(2)}^*(s) & 2\pi s^2 \rho_2(s) \end{pmatrix} \succeq 0$$

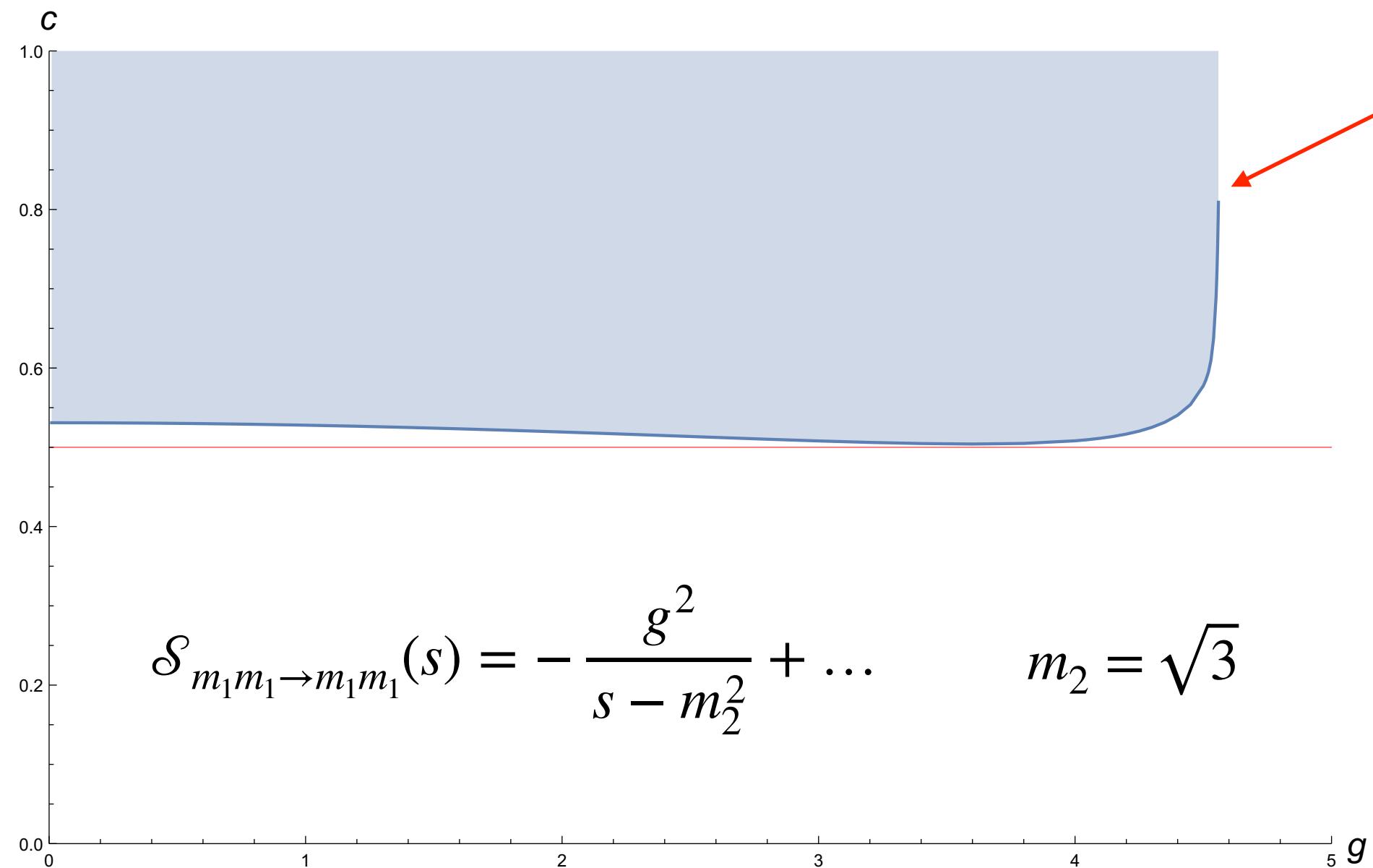
[DK, Simon Kuhn, Joao Penedones; 2019]
[DK; 2020]



$$\varepsilon \equiv \omega \sqrt{\frac{2}{d^2 - 1}}.$$

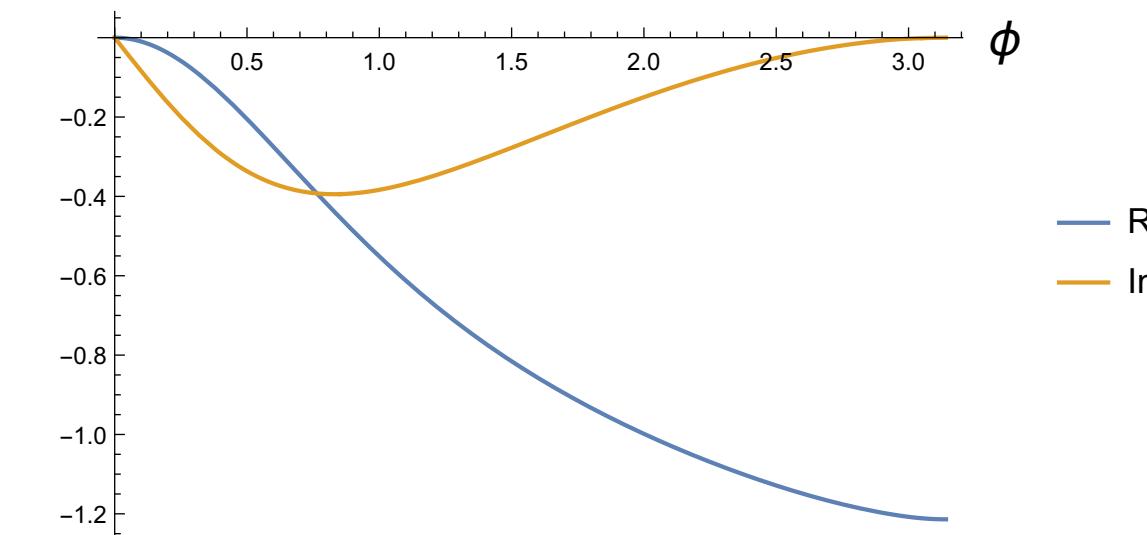
Hongbin Chen, Liam Fitzpatrick, DK; to appear

[DK, Simon Kuhn, Joao Penedones; 2019]

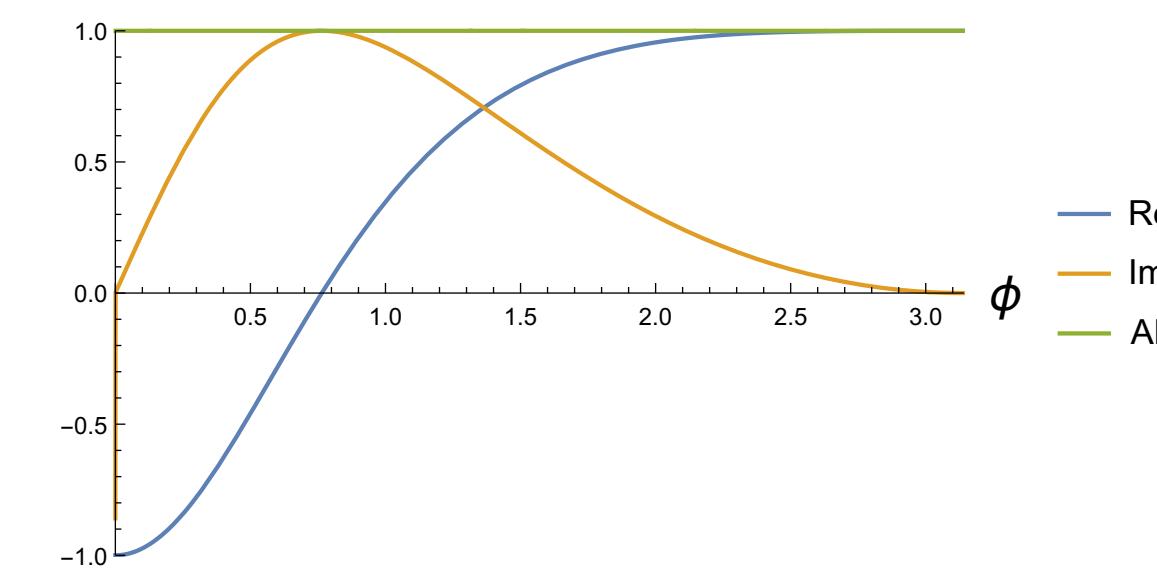


sine-Gordon model

Scattering amplitude



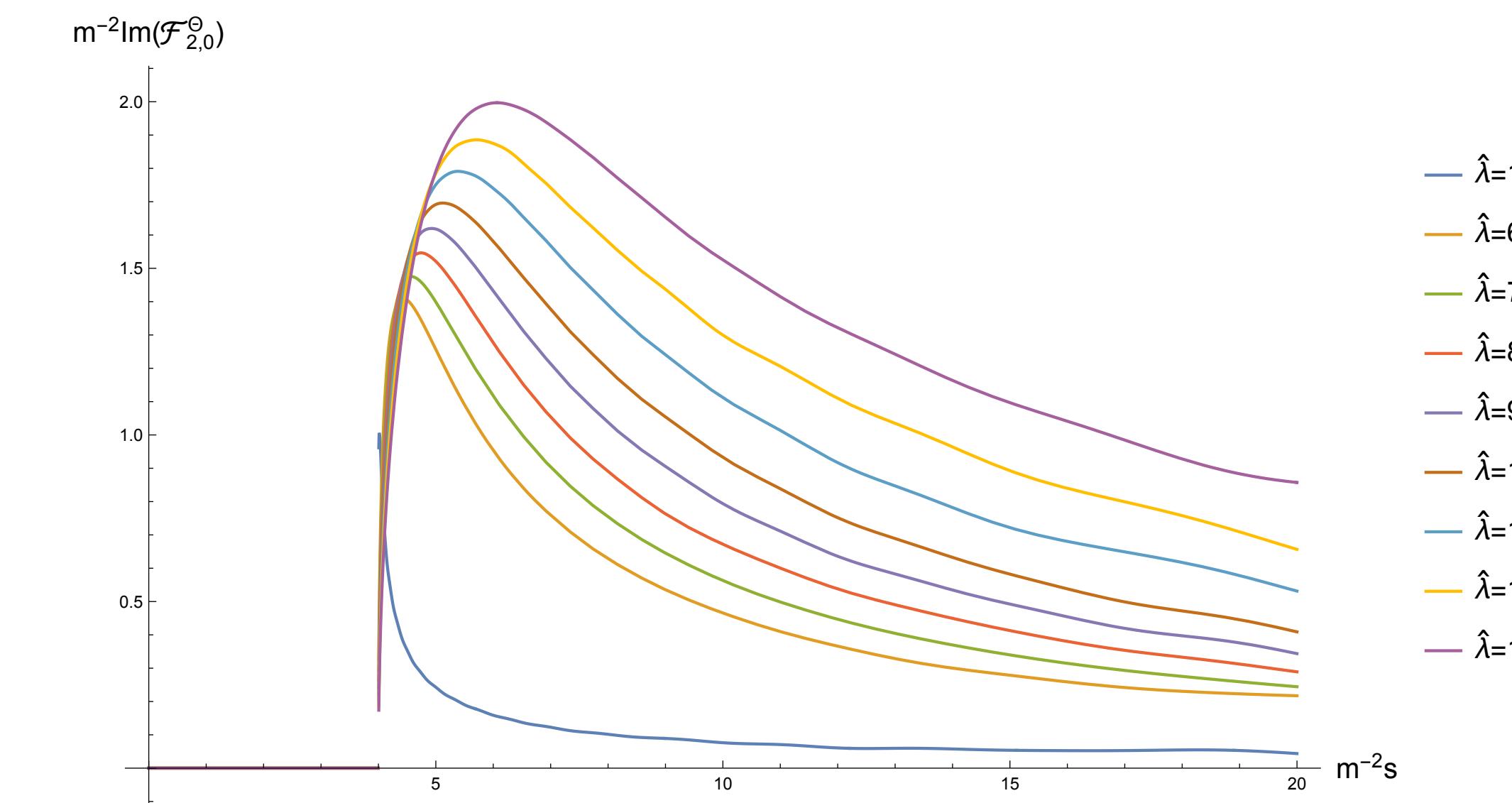
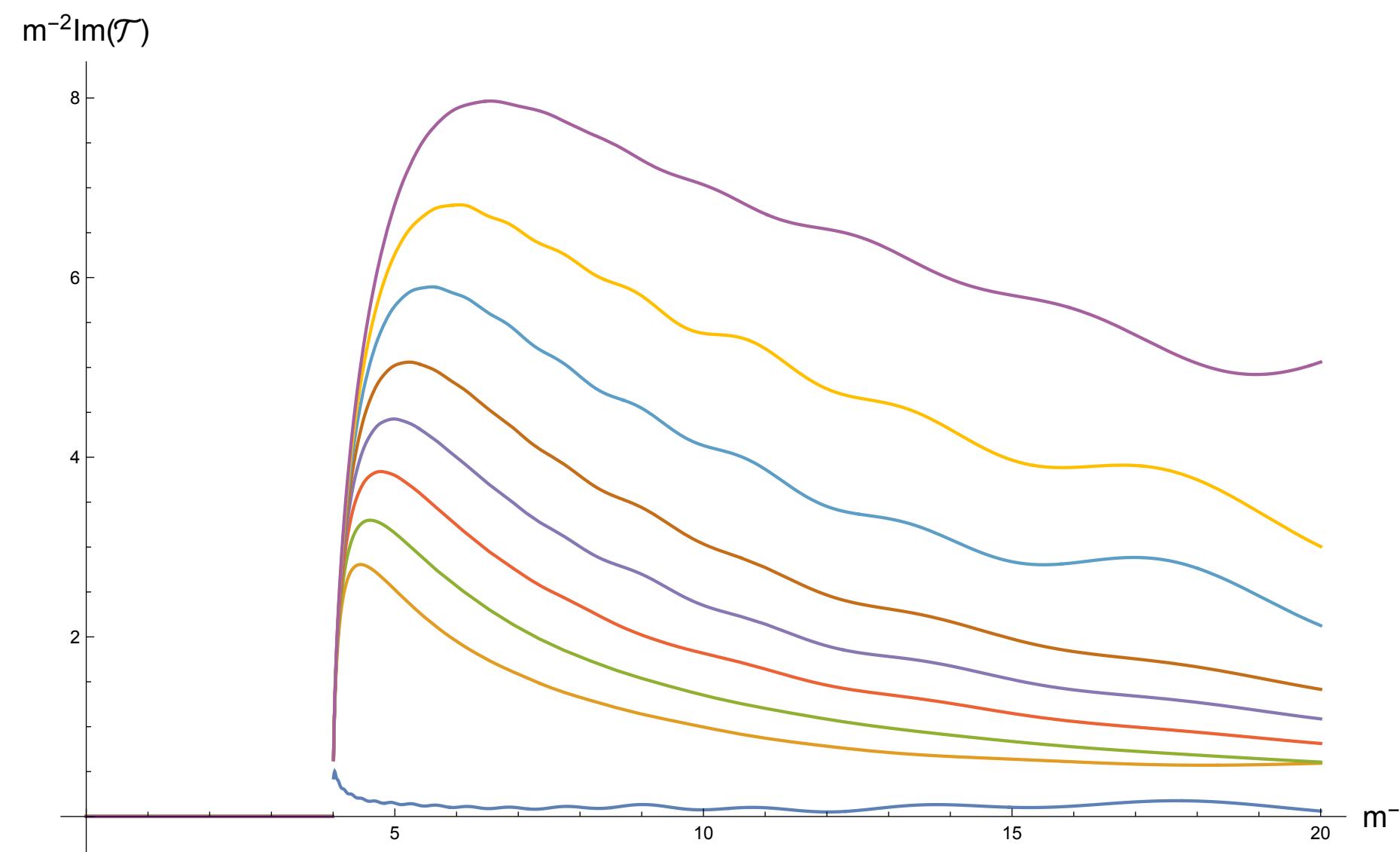
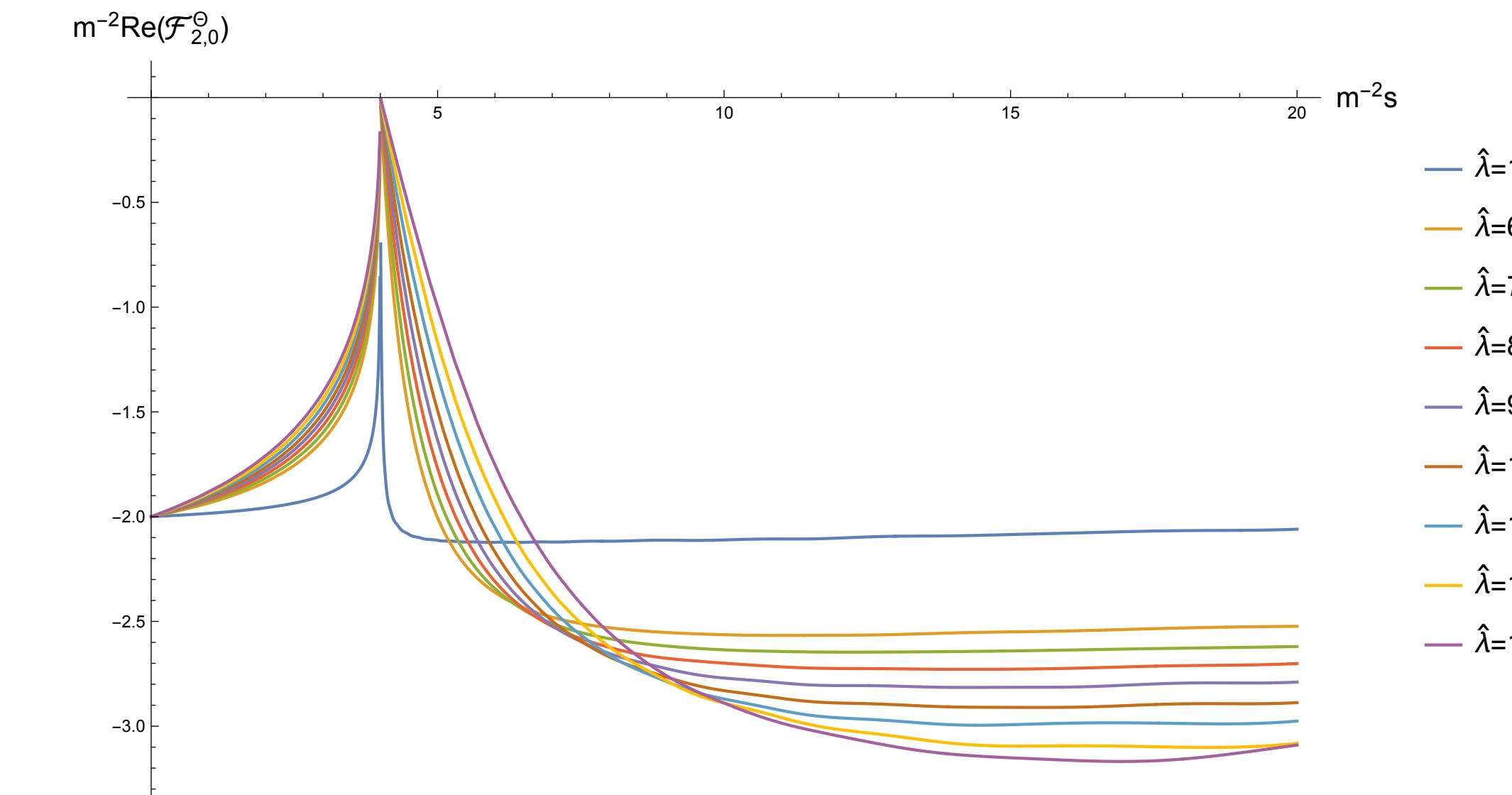
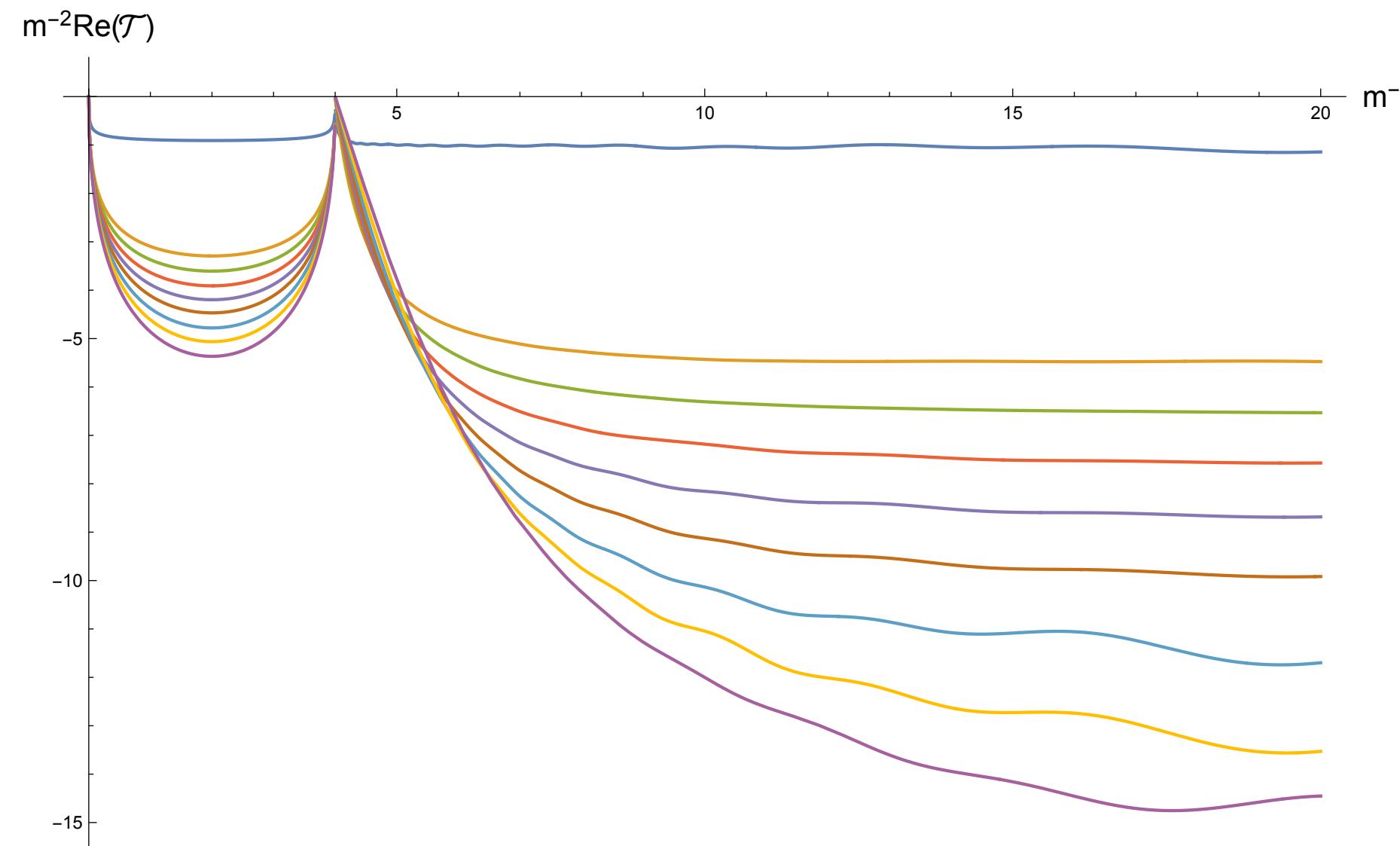
Form factor



$$c = 12\pi \int_0^\infty ds \frac{\rho_\Theta(s)}{s^2}$$

Preliminary results in the ϕ^4 model in 2d

Hongbin Chen, Liam Fitzpatrick, DK; to appear



$\hat{\lambda}=1$
 $\hat{\lambda}=6$
 $\hat{\lambda}=7$
 $\hat{\lambda}=8$
 $\hat{\lambda}=9$
 $\hat{\lambda}=10$
 $\hat{\lambda}=11$
 $\hat{\lambda}=12$
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Thank you!