Infinite distances and the Kawamata-Morrison conjecture

Callum Brodie

Institut de Physique Théorique, Université Paris Saclay

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with Andrei Constantin, Andre Lukas, and Fabian Rühle

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The infinite distance conjecture

Growing number of properties conjectured to hold of any effective theory from string theory or quantum gravity

We focus on one prominent conjectured property:

Infinite distance conjecture

At any infinite distance in moduli space, an infinite tower of states becomes exponentially massless

Occurs in compactification due to link of geometry and physics

E.g. compactify on circle: as $R \to \infty$ all KK-modes come down

(Exists also a finite version, but we won't consider it)

The Kawamata-Morrison conjecture

In mathematics, exists Kawamata-Morrison conjecture (unproven) on the structure of Calabi-Yau manifolds (CYs)

Key part of the conjecture for three-folds (CY_3s):

Kawamata-Morrison conjecture

Num. of non-isomorphic CY_3s in extended Kähler cone is finite

(Below we recall definition of extended Kähler cone)

This conjecture is important for minimal model program

In this talk, argue:

Kawamata-Morrison implied by infinite distance conjecture

The extended Kähler cone

Kähler cone $\mathcal{K}(X)$

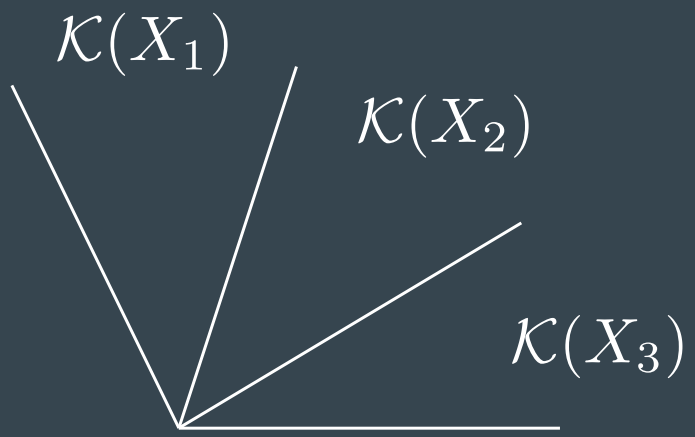
Choices of Kähler form s.t. vol$(C) > 0$ for all submanifolds C

Flop: geometric transition

$\mathrm{CY}_3 \to \mathrm{CY}_3$, changes $\{d_{ijk}, c_2(X)\}$ but not $\{h^{1,1}(X), h^{2,1}(X)\}$

Extended Kähler cone

Union of Kähler cones of CY_3s connected to original by flops



Kähler moduli space for M-theory on CY_3

Consider infinite distance conjecture in particular context:

Kähler moduli space of M-theory on CY_3 to 5d

Kähler moduli space not complexified

$\Rightarrow$ moduli space is just extended Kähler cone

(And no non-geometrical phases)
(Moduli space does not extend beyond extended Kähler cone)

Geodesic motion

Moduli space splits into ‘angular’ part (vector multiplets), plus one overall volume modulus (hyper multiplet)

Metric on angular part independent of volume
Given by classical expression $G \sim \partial\partial \ln \left(d_{ijk} \theta^i \theta^j \theta^k \right)$

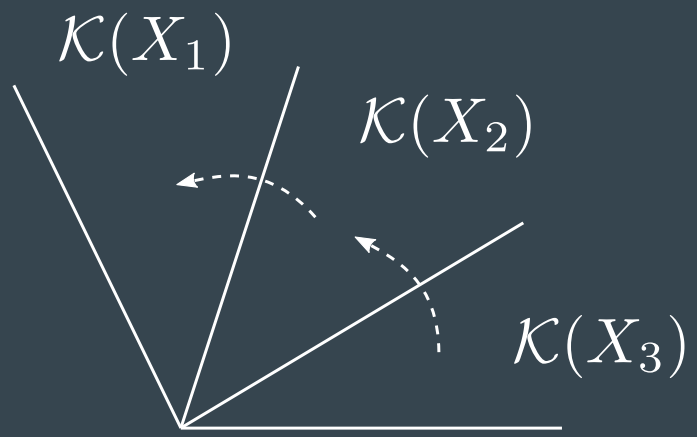
$\Rightarrow$ Straightforward to compute distances across flops

(Consider geodesics at fixed volume)

(Motion in volume direction would only increase distance)

Flops and distances

Importantly, flop process takes finite distance



In accordance with infinite distance conjecture:
no infinite tower of states

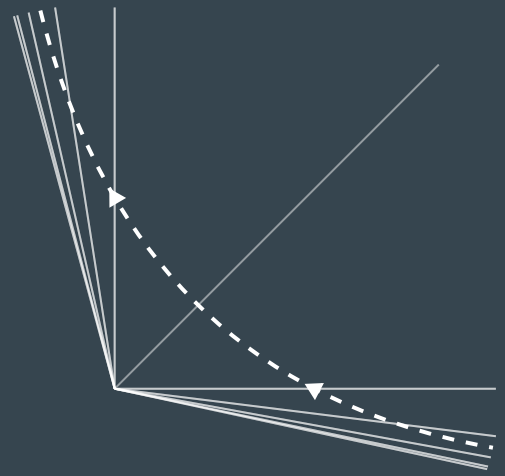
(Rather, one hypermultiplet is massless exactly at transition)

Similarly true for any finite chain of flops

Infinite flop chains

But, there exist CY_3s with interesting phenomenon:

∞ many Kähler cones $\Rightarrow$ infinite chain of flops

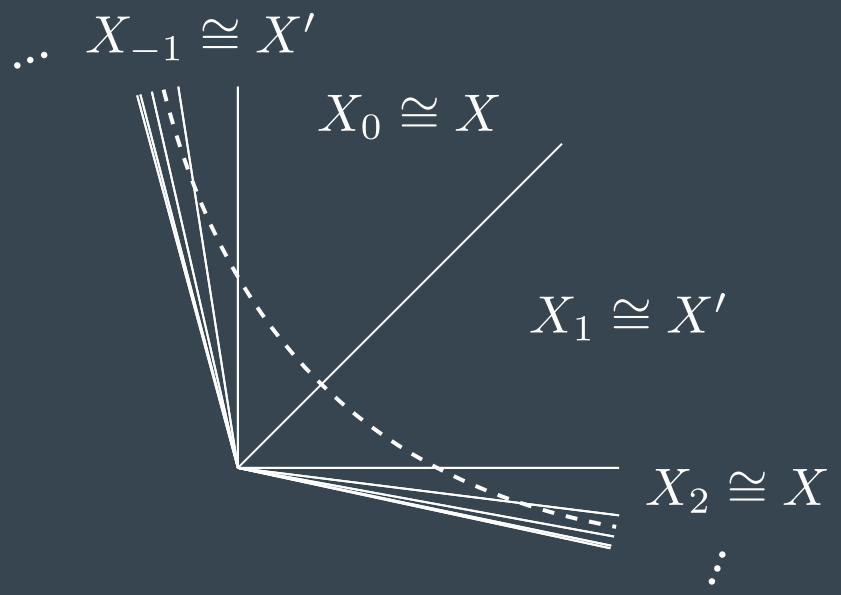


(Quite common: occurs e.g. for $\sim 1/10$ of CICYs)

Seems to lead to ∞ distance, but no tower of states

Symmetries of the Kähler moduli space

But: for known examples, many CYs in chain are isomorphic



$\Rightarrow$ These CYs related by diffeomorphism
$\Rightarrow$ Related by remnant of 11d diffeomorphism symmetry
$\Rightarrow$ Related by gauge symmetry

Need to divide symmetry out
$\Rightarrow$ In known cases, no infinite distance

The need for the Kawamata-Morrison conjecture

$\Rightarrow$ Problem for infinite distance conjecture
iff ∞ many non-isomorphic CYs in extended Kähler cone

Negative existence of CYs with this property
is subject of Kawamata-Morrison conjecture

$\Rightarrow$ Infinite distance conjecture
implies Kawamata-Morrison conjecture

And in this context, also the converse implication

Summary

- Considered swampland conjecture - infinite distance conjecture - in context of M-theory compactified on CY_3.
- Infinite distance conjecture would be false if CY_3s exist with certain property: ∞ non-isomorphic flops
- Avoidance of these requires correctness of maths conjecture: Kawamata-Morrison conjecture

Thank you