Jackiw-Teitelboim gravity with defects and the Aharonov-Bohm effect

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(w/ K. Suzuki)

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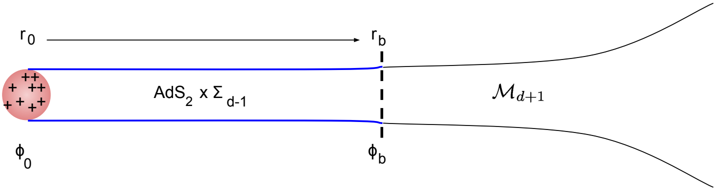
Euclidean JT gravity is a solvable theory of quantum gravity:

$$I = -\frac{\phi_0}{2}\left(\int_M d^2x\sqrt{g}R + 2\int_{\partial M}\sqrt{h}K\right)$$
$$-\frac{1}{2}\left(\int_M d^2x\sqrt{g}\left[\phi R + W(\phi)\right] + 2\int_{\partial M}\sqrt{h}\phi_b(K-1)\right)$$

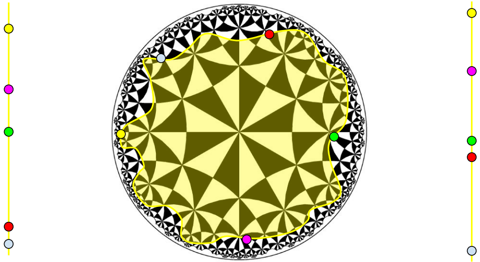
and appears in the dimensional reduction of the near-horizon limit of near-extremal black holes ($W(\phi) = 2\phi + \ldots$)



[Jackiw; Teitelboim; Bardeen and Horowitz; Almheiri and Polchinski; Maldacena, Stanford and Yang; Saad, Shenker, Stanford; Engelsoy, Mertens, and Verlinde; Trivedi et al., Sachdev]

Features

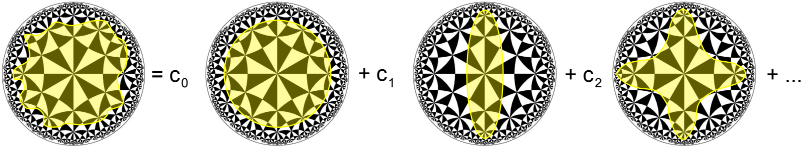
- two-dimensional gravity $\Rightarrow$ no propagating d.o.f.
- couple to a neutral scalar dilaton ϕ via $(R+2)\phi$ + a cutoff gives dynamics $\leftrightarrow$ embeddings in $\mathcal{H}_2$
- the quantized theory is exactly solvable
- in the limit $\phi_b \to \infty$, there is an emergent SL(2,$\mathbb{R}$) symmetry described by a Schwarzian action $\leftrightarrow$ SYK model



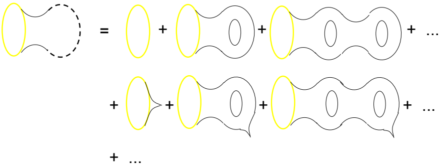
Two different parametrizations of a cutoff surface

Quantization

To quantize, we find eigenfunctions of the graviton Hamiltonian in ambient AdS_2 (perturbation theory on fixed M)



However, this is a theory of quantum *gravity* so we would like to include other geometries which locally solve the equations of motion subject to a particular boundary condition (perturb M)

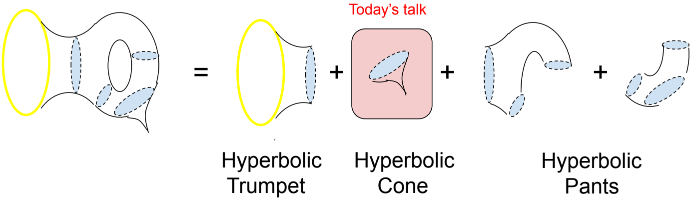


The genus expansion involves surfaces which are only locally AdS. These can be included in JT gravity by introducing *defects*:

$$W(\phi) = 2\phi + 2\epsilon e^{-\alpha\phi} \Rightarrow I = I_{(0)} + \epsilon I_{(1)} + \mathcal{O}(\epsilon^2)$$

$$I_{(1)} = -\frac{1}{2}\left(\int_M \phi(R+2) - \alpha\phi(x_1)\right) - \int_{\partial M} \phi_b(K-1)$$

and we must integrate over the defect location x_1



A defect corresponds to one of a few building block geometries (there are also pants with cusps)

[Witten; Maxfield and Turiaci]

Solving JT for the hyperbolic disk: classically

An intuitive method is to map the problem to the quantum mechanics of a charged particle on the hyperbolic disk,

(1) Fix the ambient geometry: $ds^2 = d\rho^2 + \sinh^2\rho \, d\varphi^2$, $\phi = \gamma_D \cosh\rho$ with boundary at $\rho = \rho_B$.

(2) On-shell, the action becomes:
$I_D = -\phi_b \int_{\partial_M} d\varphi\sqrt{h}(K-1) = -2\pi q\chi(D) - qA_D + qL_D$

(3) Consider $\mathbf{A}_D = (\cosh\rho - 1)d\varphi$. Then $B = d\mathbf{A} = \sqrt{g} d\rho \wedge d\phi$ is a constant magnetic field

(4) Statistical mechanics fixes $\varphi(\tau_E + \beta) = \varphi(\tau_E) + 2\pi$ which up to an overall constant fixes L_D and serves as a constraint:
$\beta \equiv C \lim_{\rho_B \to \infty} \frac{L_D}{\phi_B}$

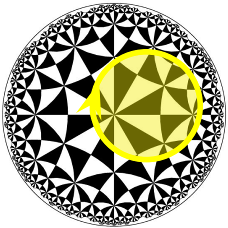
[Yang; Kitaev and Suh; Comtet and Houston; Comtet; Pioline and Troost]

Solving JT for the hyperbolic disk: quantization

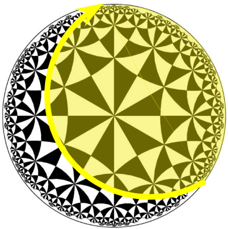
Quantizing the constrained problem is surprisingly simple: we solve the Schrodinger equation

$$(\partial_{\tau_E} + \hat{\mathcal{H}})G(\tau_E; \rho, \varphi; \rho', \varphi') = 0$$

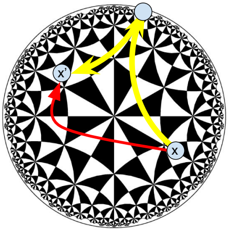
where $2\hat{\mathcal{H}}|j,k\rangle = (j[1-j] - q^2)|j,k\rangle$ is the SL(2,$\mathbb{R}$) Casimir.



j ϵ reals



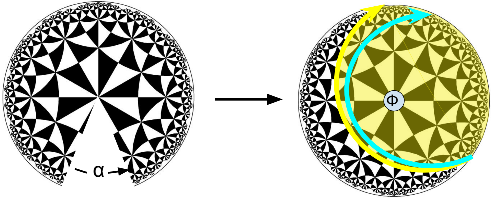
j =½+is



Gs(τ; x; x')

What about defects?

Work on the disk instead of cone, deficit → Aharonov-Bohm field!



- $I_{D_\alpha} = -q_\alpha A_{D_\alpha} + q_\alpha L_{D_\alpha} = -q_\alpha A_D + q_\alpha L_D - q_\alpha \int \mathbf{A}^{AB}$
- $\mathbf{A}^{AB} = -\frac{\alpha}{2\pi}(\cosh\rho_0 - 1 - \sinh\rho_0)\, d\varphi \xrightarrow{\rho_0\to\infty} \frac{\alpha}{2\pi} d\varphi$
- Why? In first-order formalism, $A_M = -\int d^2x\sqrt{g}\frac{R}{2} = -\int d\omega$, so $\omega \sim \mathbf{A}$. But, $R \to -2 + 2\alpha\delta(x_1)$, hence $\omega/\mathbf{A}$ must include a topological piece.

[Grosche; Lisovyy; Kitaev]

The inclusion of an AB field has surprising effects. In the Schwarzian limit,

Poincaré disk

$$Z_D(\beta) = \frac{\exp(\frac{2\pi^2}{\beta})}{\sqrt{2\pi}\beta^{3/2}}$$

$$\rho_D(s) = \frac{s}{2\pi^2}\sinh(2\pi s)$$

punctured disk

$$Z_{D_\alpha}(\beta) = \frac{\exp(\frac{(2\pi-\alpha)^2}{2\beta})}{\sqrt{2\pi}\beta^{1/2}}$$

$$\rho_{D_\alpha}(s) = \frac{1}{\pi}\cosh[(2\pi-\alpha)s]$$

Note that $\alpha \to 0$ does not match! The reason is that $q \to \infty$ and $\alpha \to 0$ do not commute ($q_\alpha \propto \alpha$). However, an unwritten constant B-field piece in the punctured disk that is suppressed by q^{-1} which does match as $\alpha \to 0$. The mismatch is related to an enhanced symmetry for $\alpha = 0$.

Summary, advertisements, and future directions

- JT gravity is an interesting and popular avenue of research because it is an example of an exactly solvable theory of quantum gravity
- Quantum gravity includes sums over higher genus surfaces with boundaries whose wavefunctions can be exactly found in JT gravity
- A fruitful and intuitive way to do so is to map to a charged particle on the hyperbolic plane with a magnetic field + an Aharonov-Bohm field
- *With the wavefunctions/propagators, we can solve for the Réyni and von Neumann entropies. At low T, a topological contribution dominates over the classical Hartle-Hawking result*
- *We can solve for correlation functions of matter fields including quantum gravity effects, for instance OTOCs*
- *We can match to dimensionally-reduced SL(2,$\mathbb{Z}$) black holes*
- *Complex geometries? Trumpet with length $b \leftrightarrow$ cone with deficit $-i(2\pi - \alpha)$*
- Probe the fine grain structure of QG, for instance graviton fluctuation contributions to the double trumpet and "half-wormholes" [Saad, Shenker, Stanford, Yao]?