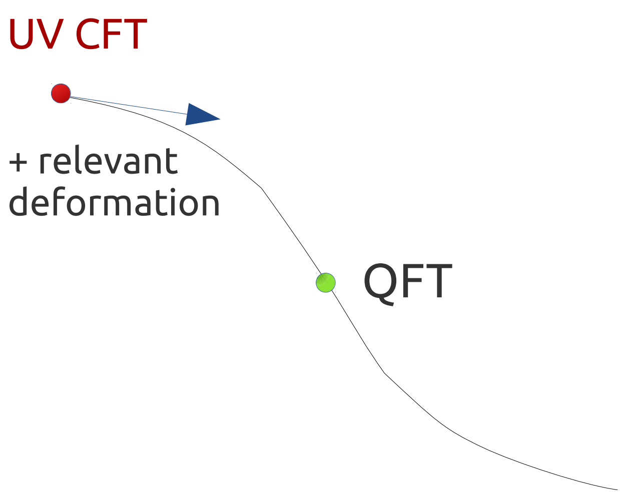
$T\bar{T}$, $J\bar{T}$, holography and all that

Monica Guica

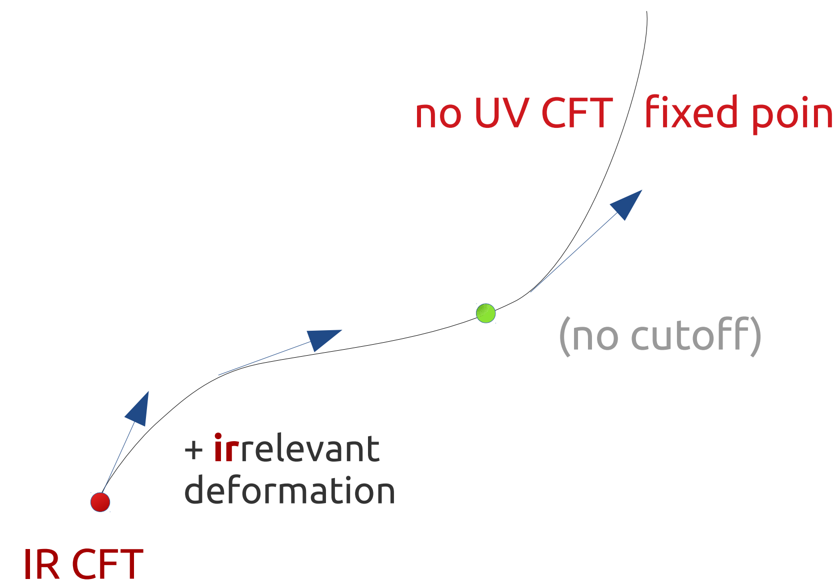
IphT, CEA Saclay

In a nutshell

Usual framework: local, UV complete QFTs

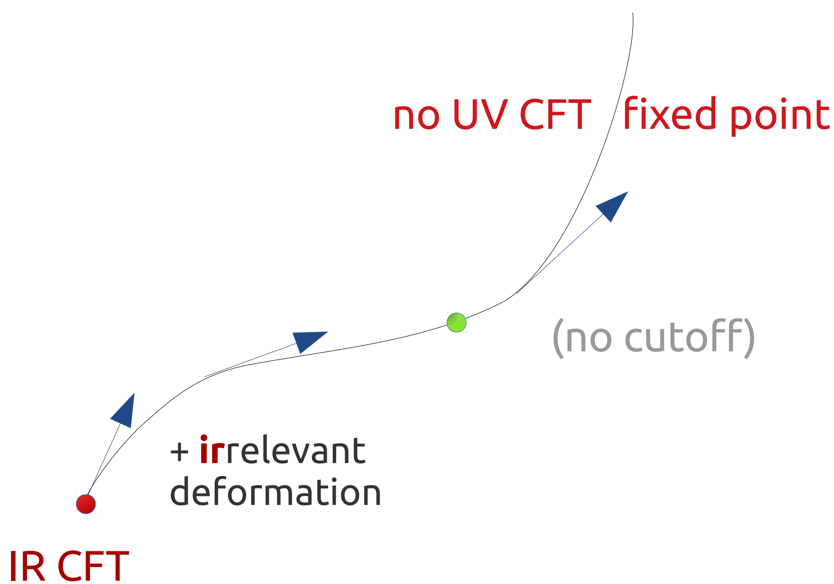


Examples of non-local, UV complete QFTs ?



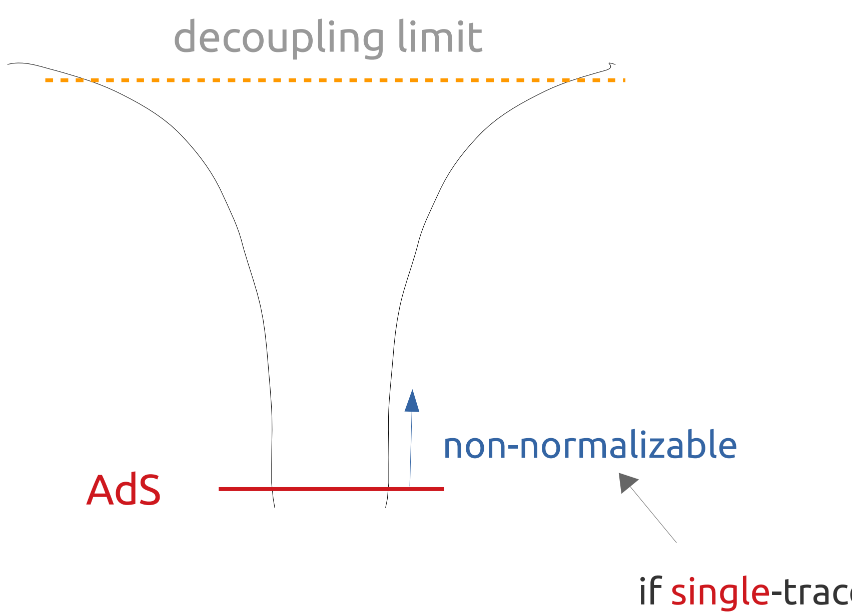
Why interesting?

Examples of non-local, UV complete QFTs ?



- novel UV behaviour in (integrable) QFT

Holography



- non-AdS holography?

Smirnov-Zamolodchikov deformations

- irrelevant deformations of 2d QFTs → bilinears of two (higher spin) conserved currents $J^A,\ J^B$

- define $\mathcal{O}_{J^A J^B}$:

$$\boxed{\lim_{y\to x} \epsilon^{\alpha\beta} J^A_\alpha(x) J^B_\beta(y) = \mathcal{O}_{J^A J^B} +}$$ derivative terms

Zamolodchikov '04

SZ '16

nice factorization properties

- deformation : $$\boxed{\frac{\partial S(\mu)}{\partial \mu} = \int d^2x\, \mathcal{O}_{J^A J^B}(\mu)}$$

SZ, Cavaglia et al. '16

- examples:

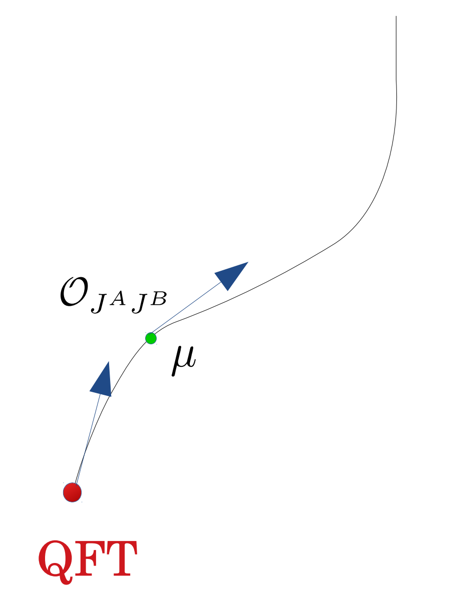
  universal

$$\begin{cases} T\bar{T}: \quad J^A_\alpha = T_\alpha{}^A\,, \quad J^B_\beta = T_\beta{}^B \quad (\times \epsilon_{AB}) \\ J\bar{T}: \quad J^A_\alpha = J_\alpha\,, \quad J^B_\beta = T_{\beta\bar{z}} \end{cases}$$

~~Lorentz~~

- highly tractable : exact finite -size spectrum, S-matrix, preserves integrability

- deformed theory non-local (scale $\mu^{\#}$) but argued UV complete

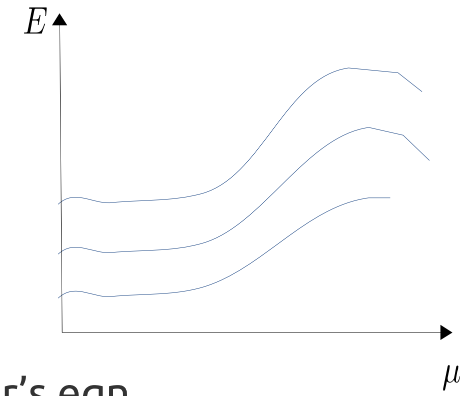


Sample results in TT̄

- universal deformation of 2d QFTs $\quad \frac{\partial S}{\partial \mu} = \int d^2 z \, \underbrace{(T_{zz}T_{\bar{z}\bar{z}} - T_{z\bar{z}}^2)_\mu}_{\text{“}T\bar{T}\text{”}}$

$$[\mu] = (length)^2$$

- in compact space ( ) → energy levels continuously deformed



- deformed energies $E_\mu(R)$ determined solely by initial spectrum via Burger’s eqn

e.g. seed CFT

$$E_\mu(R) = \frac{R}{2\mu}\left(\sqrt{1 + \frac{4\mu E_0}{R} + \frac{4\mu^2 P^2}{R^2}} - 1\right)$$

$\mu > 0$: ground state energy $E_0 = -\frac{c}{12R}$ becomes complex for $R < R_{min} = \#\sqrt{\mu c}$

- Hagedorn behaviour $S \propto E$ at high energy $\quad T_H = R_{min}^{-1}$

$\mu < 0$: all states with $E_0 > \frac{R}{4|\mu|}$ acquire imaginary energies → no sense in compact space

Cooper, Dubovsky, Moshen

- S-matrix : $\quad \mathcal{S}_\mu = e^{i\mu \sum_{i,j} \epsilon_{\alpha\beta} p_i^\alpha p_j^\beta} \mathcal{S}_0 \quad$ Dubovsky et al.

Sample results in $T\bar{T}$

- connection to the worldsheet theory of the bosonic string

$n\,T\bar{T}$ - deformed free bosons **=** Nambu-Goto action for a string in $n+2$ target space dimensions in static gauge

$T\bar{T}$ deformation = change of gauge in the NG action (conformal → static)

deformed and undef. theories related by a field-dependent coordinate transformation

- non-perturbative definition of the $T\bar{T}$ deformation in terms of coupling to topological (JT) gravity

- status of off-shell observables unclear

minimum length → theory of 2d quantum gravity ? Dubovsky et al.

flow equation for correlation functions → ≈ non-local QFT Cardy

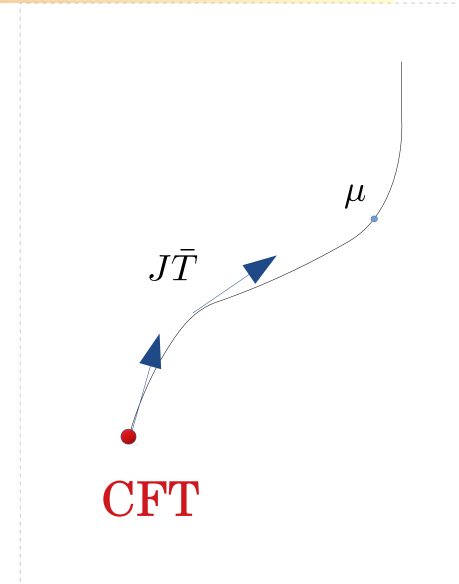
Sample results in J$\bar{\text{T}}$ - deformed CFTs

- universal deformation of 2d QFTs/CFTs with a $U(1)$ current

$$\frac{\partial S_{J\bar{T}}}{\partial \lambda} = \int d^2 z \, \underbrace{(J_z T_{\bar{z}\bar{z}} - J_{\bar{z}} T_{z\bar{z}})}_{\text{“}J\bar{T}\text{”}} \lambda \quad (1,2)$$

$$[\lambda] = length$$

$$\lambda^\mu \partial_\mu \propto \partial_{\bar{z}}$$



- breaks Lorentz invariance $T_{z\bar{z}} \neq T_{\bar{z}z} (= 0)$

- preserves $\underbrace{SL(2,\mathbb{R})_L}_{\text{local \& conformal}} \times \underbrace{U(1)_R}_{\text{non-local!}}$ ← simpler than T$\bar{\text{T}}$

$\boxed{CFT_1!}$

Im!

$$E_{L,R} = \frac{1}{2}(E \pm P)$$

- finite-size spectrum $E_R = \frac{4\pi}{\lambda^2 k}\left(R - \lambda Q_0 + \sqrt{(R-\lambda Q_0)^2 - \lambda^2 k R \, E_R^{(0)}}\right)$

$$Q(\lambda) = Q_0 + \frac{\lambda k}{2} E_R$$

- off-shell observables: correlation functions of operators in mixed basis $\mathcal{O}(z, \bar{p})$ CFT1 correlators w/

non-local QFT

$$h(\lambda) = h + \lambda q \bar{p} + \frac{k}{4}\lambda^2 \bar{p}^2 \qquad q(\lambda) = q + \frac{k}{2}\lambda \bar{p}$$

MG’19

→ spectral flow with momentum-dependent parameter

Holographic interpretation

- in AdS/CFT parlance, the Smirnov-Zamolodchikov deformations are double-trace

A. of the SZ deformations themselves

B. of a single-trace analogue of the SZ deformations

Double-trace deformations in AdS/CFT

- mixed boundary conditions for dual bulk fields

- e.g. scalar $$\Phi = \phi_{(0)}\, z^{d-\Delta} + \ldots + \phi_{(\Delta)}\, z^{\Delta} + \ldots$$

- undeformed CFT : source $\mathcal{J}$ (fixed) vev $\langle \mathcal{O} \rangle$ (fluctuates)

- $I_\mu = I_{CFT} + \mu \int \mathcal{O}^2$

only uses large N field theory

1. variational principle (equivalent to Hubbard-Stratonovich at large N)

$$\delta S_\mu = \delta S_{CFT} - \delta\,(\mu \int \mathcal{O}^2) = \int \mathcal{O}\delta\mathcal{J} - \mu \int \delta\mathcal{O}^2 = \int \mathcal{O}\delta(\underbrace{\mathcal{J} - 2\mu\mathcal{O}}_{\text{new source } \tilde{\mathcal{J}}})$$

new vev $\tilde{\mathcal{O}}$ new source $\tilde{\mathcal{J}}$

2. interpret result in terms of bulk field data

$$\tilde{\mathcal{J}} = \phi_{(0)} - 2\mu\phi_{(\Delta)} \ = \ \text{fixed (mixed b.c.)} \qquad \langle \tilde{\mathcal{O}} \rangle = \phi_{(\Delta)}$$

Holographic dictionary for $T\bar{T}$ - deformed CFTs

- variational principle (incrementally in μ) → relation between new and old sources and vevs

$$\boxed{\begin{aligned}\gamma_{\alpha\beta}(\mu) &= \gamma_{\alpha\beta}(0) - \mu \hat{T}_{\alpha\beta}(0) + \mu^2 \hat{T}_\alpha{}^\gamma \hat{T}_{\gamma\beta}(0) \\ \hat{T}_{\alpha\beta}(\mu) &= \hat{T}_{\alpha\beta}(0) - \mu\, \hat{T}_\alpha{}^\gamma \hat{T}_{\gamma\beta}(0)\end{aligned}}$$

$$T_{\alpha\beta} - \gamma_{\alpha\beta} T$$

- both signs of μ
- other (matter) vevs can be on
- only uses large N field theory

- holographic interpretation (large N, large gap) → Fefferman Graham expansion for AdS3 metric

$$ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \left(\underbrace{\frac{g^{(0)}_{\alpha\beta}}{\rho} + g^{(2)}_{\alpha\beta}}_{\text{universal}} + \ldots \right) dx^\alpha dx^\beta$$

non-univ

$$g^{(0)}_{\alpha\beta} \leftrightarrow \gamma_{\alpha\beta}(0) \, , \qquad g^{(2)}_{\alpha\beta} \leftrightarrow 8\pi G \ell\, \hat{T}_{\alpha\beta}(0)$$

- $\gamma_{\alpha\beta}(\mu)$ fixed ↔ mixed non-linear boundary conditions for the AdS3 metric

$$\gamma_{\alpha\beta}(\mu) = g^{(0)}_{\alpha\beta} - \frac{\mu}{4\pi G\ell} g^{(2)}_{\alpha\beta} + \frac{\mu^2}{(8\pi G\ell)^2} g^{(2)}_{\alpha\gamma} g^{(0)\gamma\delta} g^{(2)}_{\delta\beta}$$

- only depend on asymptotics

- $\langle T_{\alpha\beta}(\mu) \rangle$ depends non-linearly on $g^{(0)}, \, g^{(2)}$

Comments

- the above holographic dictionary can be used to compute the deformed energy spectrum →

  → perfect match to field-theory formula (both signs of μ, matter field vevs on → universal!)

- precision holography, despite the deformation being irrelevant

Pure gravity

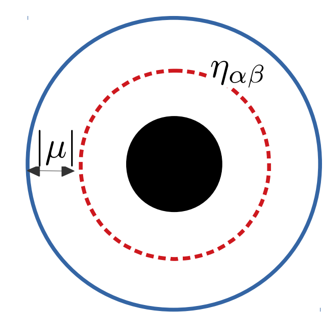
- the FG expansion terminates $$ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \frac{g^{(0)}_{\alpha\beta} + \rho\, g^{(2)}_{\alpha\beta} + \rho^2\, g^{(2)}_{\alpha}{}^{\gamma} g^{(2)}_{\gamma\beta}}{\rho}\, dx^\alpha dx^\beta$$

- $\gamma_{\alpha\beta}(\mu)$ coincides with the induced metric at $\boxed{\rho_c = -\frac{\mu}{4\pi G\ell}}$ $\mu < 0$

- $\langle T_{\alpha\beta}(\mu)\rangle$ coincides with the Brown-York stress tensor at ρ_c

- in agreement with observation that $T\bar{T}$ – deformed energies

  = energy of ``black hole in a box''
$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu M}{R} + \frac{4\mu^2 J^2}{R^2}}\right)$$



McGough, Mezei, Verlinde

Asymptotic symmetries

- large diffeomorphisms that preserve the mixed bnd. conditions $\leftrightarrow$ symmetries of dual field theory

- expect : $T\bar{T}$ deformation breaks conformal symmetries to $U(1)_L \times U(1)_R$ and makes theory non-local

- asymptotic symmetry group: $\boxed{Virasoro(u) \times Virasoro(v)}$ with same **c** as in CFT

  u, v $\rightarrow$ field-dependent coordinates

- similar results for $J\bar{T}$: dual to AdS3 with mixed bnd. conditions between metric and U(1) CS gauge field

- asymptotic symmetry group

  J non-local

  $$\underbrace{SL(2,\mathbb{R})_L \times U(1)_L}_{\text{Virasoro - Kac-Moody}} \times U(1)_R$$

  Virasoro - Kac-Moody $\times$ Virasoro$_R$ $\leftarrow$ $f(x^- - \lambda \int J)$

- suggests $T\bar{T}$, $J\bar{T}$ - deformed CFTs correspond to non-local generalizations of 2d CFTs

- note that different bnd conditons on AdS3 = radical modifications of the dual theory, as compared to naive ASG suggestion

Partial conclusions

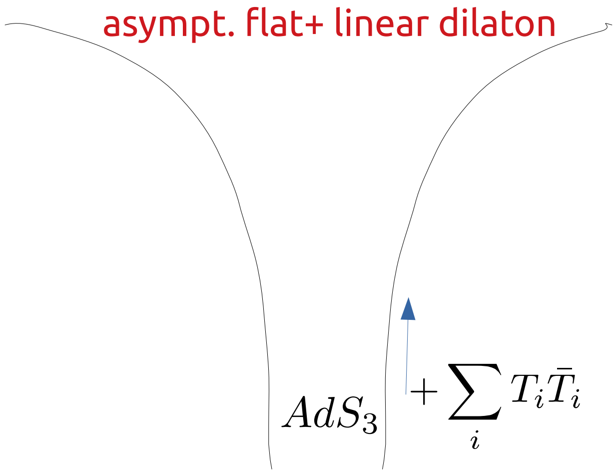
- holographic duals of the $T\bar{T}$, $J\bar{T}$ deformations of holographic CFTs are highly tractable (at sugra level)

  $\rightarrow$ precision holography

- however $\rightarrow$ boring, because dual spacetime is always asymptotically locally AdS

- will now discuss holography for the single-trace analogues of the SZ deformations

The NS5 - F1 system



N_5 NS5 and N_1 F1 strings in the NS5 decoupling limit

$$g_s \to 0\,, \quad \alpha' \qquad \text{fixed}$$

UV: Little String Theory

non-gravitational, non-local theory with Hagedorn growth

IR: AdS_3 dual to $(\mathcal{M}_{6N_5})^{N_1}/S_{N_1}$ symmetric orbifold CFT

- can be obtained via TsT of near norizon AdS

- worldsheet σ - model : exactly marginal deformation of the $SL(2,\mathbb{R})\times SU(2)\times U(1)^4$ WZW model that describes the near-horizon AdS_3 by $J^-\bar{J}^-$

- expand infinitesimally around IR AdS_3 → source for (2,2) single-trace operator $\sum_i T_i\bar{T}_i$

Proposed holographic duality

$$Z_{string}[\mathrm{NS5\text{-}\ F1}] = Z\left[(T\bar{T} - \mathrm{def.\ \ CFT}_{6N_5})^{N_1}/S_{N_1}\right]$$

Giveon, Itzhaki, Kutasov

+

- RHS is well-defined at finite deformation
- spectrum of string excitations exactly matches $T\bar{T}$ spectrum
- black hole entropy (Hagedorn)
- correlation functions $\langle O(p)O(-p)\rangle$ using worldsheet

−

- uses free product structure in an essential way
- not clear how to deform away from this (singular) point in moduli space
- naively different behaviour from $T\bar{T}$ correlator

more checks?

- similar story holds for J$\bar{\mathrm{T}}$: pure NS-NS string background obtained from $AdS_3 \times S^3 \times T^4$ + TsT on one AdS and one angular direction → warped AdS_3

field-dependent?

- universal near-horizon geometry of extremal black holes, with Virasoro x Virasoro ASG

To sum up...

- $T\bar{T}$, $J\bar{T}$ deformations are highly tractable : spectrum, S-matrix, ~ correlators

- their holographic duals are also highly tractable , though slightly boring

- single-trace analogues of $T\bar{T}$, $J\bar{T}$ are conjecturally dual to non- asymptotically AdS spacetimes

    → possibly tractable instances of non-AdS holography

    → directly relevant for understanding the near-horizon dynamics of (extremal) black holes

- in both single/double-trace case, the ASG analyses indicate existence of Virasoro x Virasoro symmetry

Q: can we use the high degree of (concrete) solvability of the $T\bar{T}$, $J\bar{T}$ deformations to learn more about the field-dependent symmetries that appeared in the ASG analyses?

Field – dependent symmetries of $T\bar{T}$, $J\bar{T}$

- what are they ?
- how do they act ?
- do they survive quantization ?
- how do they constrain observables?

Field-dependent symmetries

- consider a 2d classical field theory with null coordinates $U, V = \sigma \pm t$

- consider the coordinate shifts: $U \to U + \epsilon\, f(u)$ $V \to V - \epsilon\, \bar{f}(v)$ where $u(U,V),\, v(U,V)$ are some possibly field-dependent coordinates

- the variation of the action is $\delta_f S = -\int dU dV \epsilon\, f'(u)\; (T_{UU}\partial_V u + T_{VU}\partial_U u)$

- in a 2d CFT $T_{VU} = 0$ off-shell → for $u = U$, $\delta_f S = 0\;\; \forall f(u)$ → infinite conformal symmetries

- in $T\bar{T}$, $J\bar{T}$ – deformed CFTs , still only two independent components of the stress tensor off-shell

  → choose $u(U,V)\;\; \ni\;\; T_{UU}\partial_V u + T_{VU}\partial_U u = 0 \;\;\Rightarrow$ infinite field-dependent symmetries

- special structure of $T\bar{T}$, $J\bar{T}$ → universal form for $u(U,V),\, v(U,V)$
  = coordinates in terms of which the deformed dynamics trivializes to that of the original CFT
  → field-dependent symmetries = original CFT symmetries (∞) seen through the prism of these coord.

Example: classical JT̄ - deformed CFTs

- work in Hamiltonian formalism $\partial_\lambda \mathcal{H} = \epsilon^{\alpha\beta} J^\alpha T_{\beta V}$ $J \,:\, U(1)$ shift current for ϕ

- deformed Hamiltonian density

$$\mathcal{H}_{L,R} = \frac{\mathcal{H} \pm \mathcal{P}}{2}$$

$$\boxed{\mathcal{H}_R = \frac{2}{\lambda^2}\left(1 - \lambda\mathcal{J}_+ - \sqrt{(1-\lambda\mathcal{J}_+)^2 - \lambda^2 \mathcal{H}_R^{(0)}}\right)}$$

undeformed

$$\mathcal{J}_\pm = \frac{\pi \pm \phi'}{2}$$

- can show that for such JT – deformed CFTs, $\boxed{u = U}$ (due to $SL(2,\mathbb{R})$), $\boxed{v = V - \lambda\phi}$

- conserved charges

$$Q_f = \int d\sigma f(U)\mathcal{H}_L \qquad \bar{Q}_{\bar{f}} = \int d\sigma \bar{f}(v)\mathcal{H}_R$$

+ affine U(1)

- charge algebra $\{Q_f, Q_g\} = Q_{fg'-f'g}$ $\{\bar{Q}_{\bar{f}}, \bar{Q}_{\bar{g}}\} = \bar{Q}_{\bar{f}'\bar{g}-\bar{f}\bar{g}'}$ $\{Q_f, \bar{Q}_{\bar{f}}\} = 0$

→ two commuting copies of the functional (‘= ∂_σ) Witt – Kac-Moody algebra

→ similar results for TT̄ and JTa

J$\bar{\text{T}}$ - charge algebra in compact space

- compact space → use Fourier basis of functions $f_n(U) = e^{inU/R}$, $\bar{f}_n(v) = e^{-inv/R_v}$ → $Q_n, \bar{Q}_n$

  $R_v = R - \lambda w$ is the field-dependent radius of v $w = J_0 - \bar{J}_0 = \text{winding of } \phi$

- in term of these Fourier modes, the charge algebra is

$$\{Q_m, Q_n\} = -i\frac{(m-n)}{R} Q_{m+n} \qquad \{\bar{Q}_m, \bar{Q}_n\} = -i\frac{(m-n)}{R_v} \bar{Q}_{m+n}$$

not the usual Witt algebra!

Two problems with quantization

1. $\{\bar{Q}_{\bar{f}}, J_0\} = -\frac{\lambda}{2R_v} \bar{Q}_{\bar{f}'}$, $\{\bar{Q}_{\bar{f}}, P\} = -\frac{1}{R_v} \bar{Q}_{\bar{f}}$ → acting with the corresp. quantum generator $\bar{L}_{-n}$ will not respect charge/momentum quantization

2. expected Virasoro symmetry is in tension with the J$\bar{\text{T}}$ - deformed finite size spectrum

$$E_R = \frac{4\pi}{\lambda^2 k}\left(R - \lambda Q_0 + \sqrt{(R-\lambda Q_0)^2 - \lambda^2 k R\, E_R^{(0)}}\right)$$

Resolution

1. Solution for v determined up to a constant $\rightarrow$ fix such that charge quantization is respected

$$v_{new} = \sigma - \lambda\phi + \frac{\lambda R_v}{R - \lambda Q_K}\widetilde{\phi}_0 \qquad \widetilde{\phi}_0 = \phi_0 - \frac{\lambda}{R_v}\int d\sigma\hat{\phi}(\mathcal{J}_- + \frac{\lambda}{2}\mathcal{H}_R)$$

generator of spectral flow in J$\bar{\text{T}}$

- modified charges $\bar{\mathcal{Q}}_n = \int d\sigma e^{-inv_{new}/R_v}\mathcal{H}_R$ are conserved and have Poisson brackets that are consistent with semiclassical quantization
- new charge algebra has quadratic terms on the RHS
- the combinations

$$\left\{\begin{array}{ll} \tilde{Q}_n = R\,Q_n - \lambda E_R K_n + \dfrac{\lambda^2 E_R^2}{4}\delta_{n,0}\,, & K_n - \dfrac{\lambda E_R}{2}\delta_{n,0} \\ \tilde{\bar{\mathcal{Q}}}_n = R_v\bar{\mathcal{Q}}_n - \lambda E_R\bar{\mathcal{K}}_n + \dfrac{\lambda^2 E_R^2}{4}\delta_{n,0}\,, & \bar{\mathcal{K}}_n - \dfrac{\lambda E_R}{2}\delta_{n,0} \end{array}\right.$$

do satisfy Witt-Kac-Moody^2

2. $\tilde{Q}_0,\ \tilde{\bar{\mathcal{Q}}}_0$ coincide with the undeformed CFT energies $E_{L,R}^{(0)}$ $\leftarrow$ integer-spaced spectrum

$\rightarrow$ not the left/right energies in the JT – deformed CFT!

Partial summary

- there exists an infinite set of classical conserved charges in JT – deformed CFTs (compact or not)
- these charges are consistent with semiclassical quantization in compact space
- there exists a non-linear combination of these charges whose Poisson bracket algebra is two copies of Witt- Kac-Moody at classical full non-linear level
- tension with the JT – deformed spectrum is resolved because the zero mode of the Witt algebra does not coincide with the energy
- quantization → resolve normal ordering issues (e.g. order by order in λ) → no problem of principle

Conclusions

- $T\bar{T}$, $J\bar{T}$ are a set of well-defined and highly tractable irrelevant deformations of 2d QFTs

  → deformed spectrum, S-matrix, ~ correlators, precise holographic dictionary

  → UV complete non-local QFTs

- there exist closely related single-trace analogues of $T\bar{T}$, $J\bar{T}$

  → relevant for non-AdS holography (near-horizon dynamics of general back holes)

  → suggest larger set of theories similar to $T\bar{T}$, $J\bar{T}$ - UV completeness, symmetries?

  but more general ? - ~~spectrum~~

- do $T\bar{T}$, $J\bar{T}$ - deformed CFTs correspond to non-local 2d CFTs → field-dependent Virasoro symmetries?

  → classically: yes in non-compact space, also in compact space for $J\bar{T}$

  → $J\bar{T}$: obstacle to quantization removed → quantum algebra? central extension?

Thank you !