



**CLUSTER OF EXCELLENCE**  
**QUANTUM UNIVERSE**

# **On the EFT description of ALPs coupled to heavy chiral matter**

Quentin Bonnefoy  
(DESY Hamburg)

LIO international conference on "Future colliders and the origin of mass"

IP2I - Lyon  
22/06/2021

Based on 2011.10025 [hep-ph]  
with L. Di Luzio, C. Grojean, A. Paul, A. Rossia

No new particle at the LHC, no clear deviation from SM couplings :  
renewed interest in **effective field theories** (EFTs)

$$\mathcal{L} = \mathcal{L}_{d \leq 4} + \sum_i \frac{c_i}{\Lambda^{d_i - 4}} \mathcal{O}_i$$

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Slightly less discussed aspect : **different types of EFTs** for different types of BSM states

Which EFTs ?

## **The SMEFT**

(linear EW sym.)

## **The HEFT**

(non-linear EW sym.)

Which EFTs ?

## The SMEFT (linear EW sym.)

$$A_\mu^{a,i,Y} \quad \psi_{i,L/R} \quad H$$

$$SU(3) \times SU(2) \times U(1)$$

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[Buchmüller/Wyler '85, Grzadkowski et al '10]

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Use of the SMEFT : assumption about the nature of EWSB and about the origin of the UV states mass

**[see e.g. Cohen/Craig/Lu/Sutherland '20 + refs therein]**

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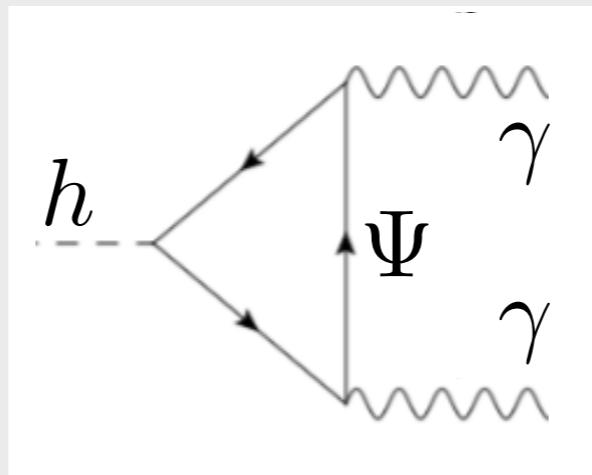
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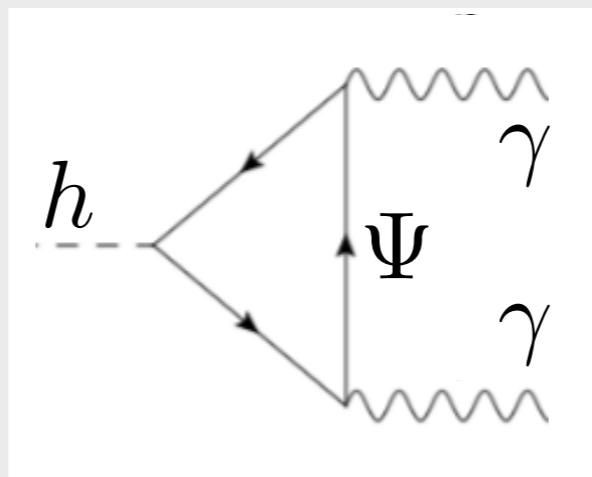
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Extension to **ALP EFTs**:

$$\begin{aligned} & \text{SM d.o.f.s} + a \\ & a \rightarrow a + \epsilon_{\text{PQ}} f \end{aligned}$$

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Focus on the **couplings to gauge fields** :

$$\mathcal{L} \supset -\frac{g^2 C_{WW}}{16\pi^2} \frac{a}{f} W^a \tilde{W}^a - \frac{g'^2 C_{BB}}{16\pi^2} \frac{a}{f} B \tilde{B}$$

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[Brivio/Gavela/Merlo/Mimasu/  
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modify  
 $a\gamma Z, aZZ$   
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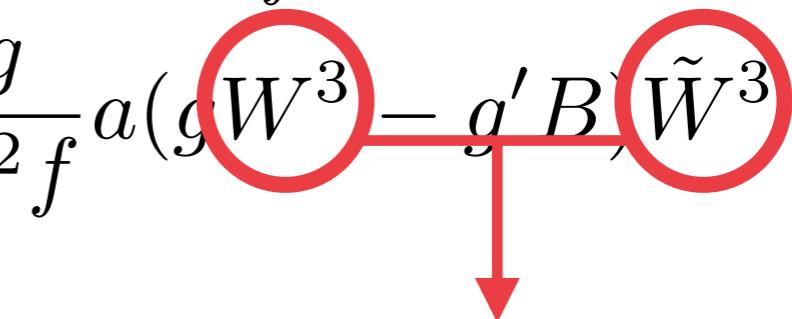
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New structures in **axion couplings** :

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Pheno impact : **breakdown of sum rules**

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 $aZZ, aWW$

$$\left[ \frac{\Gamma(a \rightarrow ZZ)}{\Gamma(a \rightarrow \gamma\gamma)} - 1 - \frac{(t_W^2 - 1)^2}{2t_W^2} \frac{\Gamma(a \rightarrow Z\gamma)}{\Gamma(a \rightarrow \gamma\gamma)} \right]^2 - \frac{2(t_W^2 - 1)^2}{t_W^2} \frac{\Gamma(a \rightarrow Z\gamma)}{\Gamma(a \rightarrow \gamma\gamma)} = 0$$

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With a linear EW sym.:  $\partial_\mu a \left( H^\dagger \overleftrightarrow{D}_\nu H \right) \tilde{B}^{\mu\nu}$

$$\partial_\mu a \left( D_\nu H^\dagger \tilde{W}^{\mu\nu} H - H^\dagger \tilde{W}^{\mu\nu} D_\nu H \right)$$

$$\partial_\mu a \left( H^\dagger \overleftrightarrow{D}_\nu H \right) \left( H^\dagger \tilde{W}^{\mu\nu} H \right)$$

Suppressed by some  
additional  $\left(\frac{v}{\Lambda}\right)^n$

UV realization :

	$SU(3)$	$SU(2)$	$U(1)$
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$E_1$	1	1	$Y - \frac{1}{2}$
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$$\begin{aligned} \mathcal{L} \supset & \bar{L}_1 N_1 H, \quad \bar{L}_1 E_1 \tilde{H}, \\ & \bar{L}_2 E_2 H, \quad \bar{L}_2 N_2 \tilde{H} \end{aligned}$$

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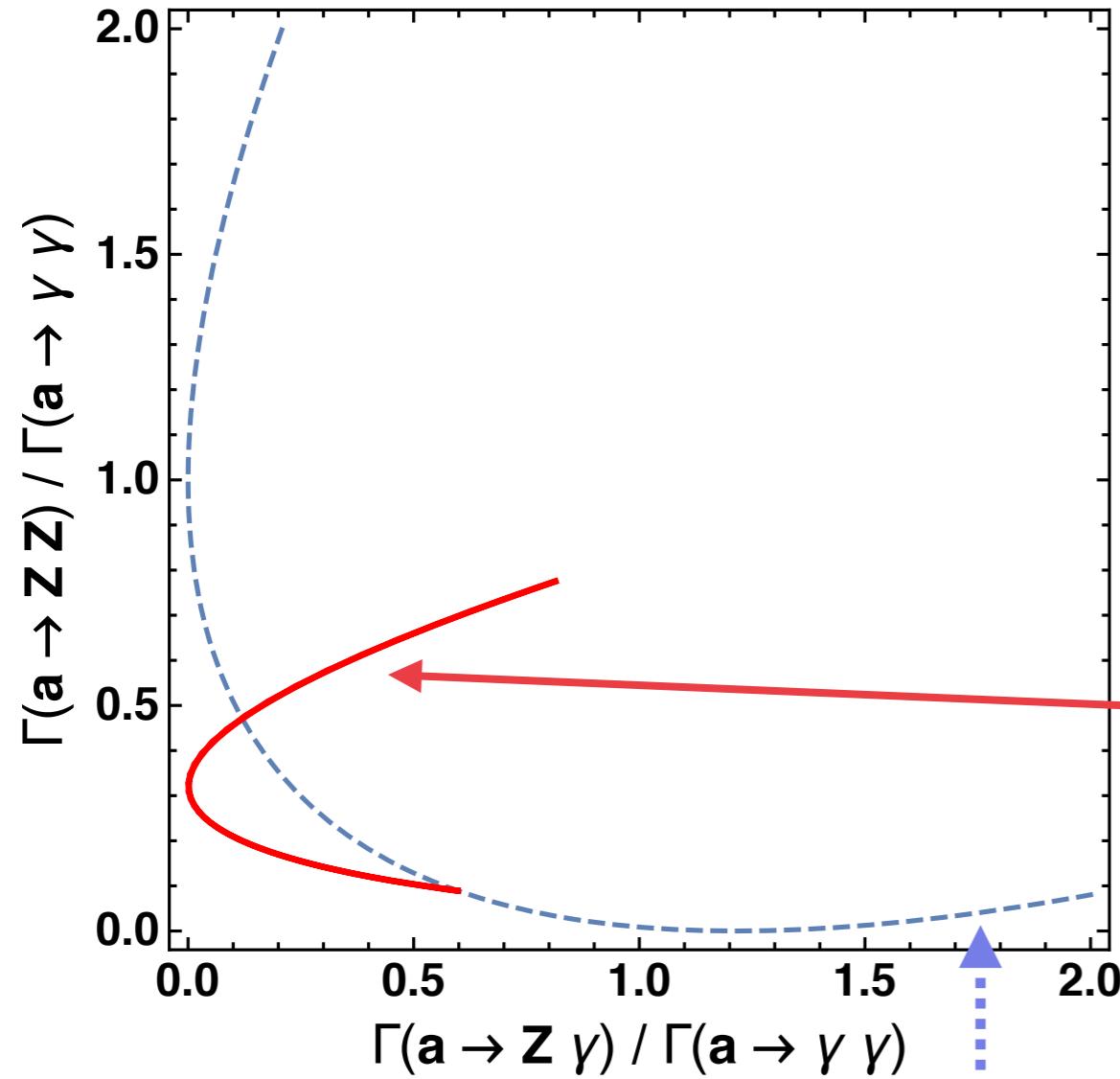
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$$V = V(|H_1|^2, |H_2|^2, |\Phi|^2) + \lambda \Phi^2 H_1 H_2$$

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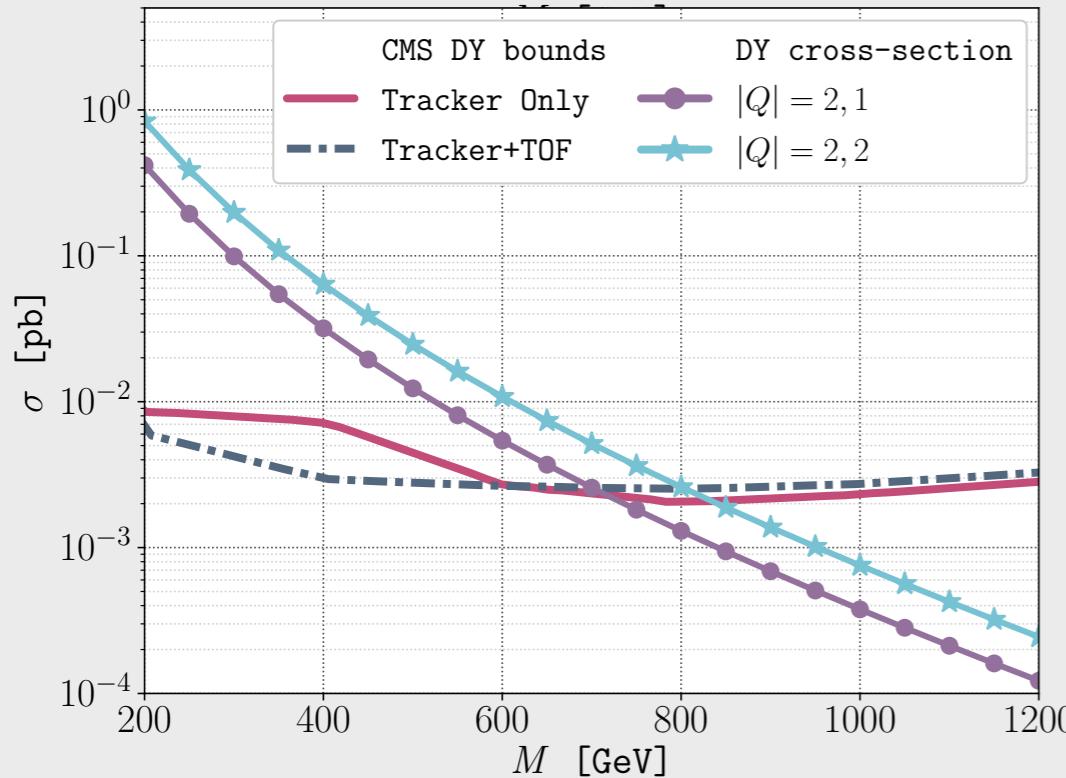
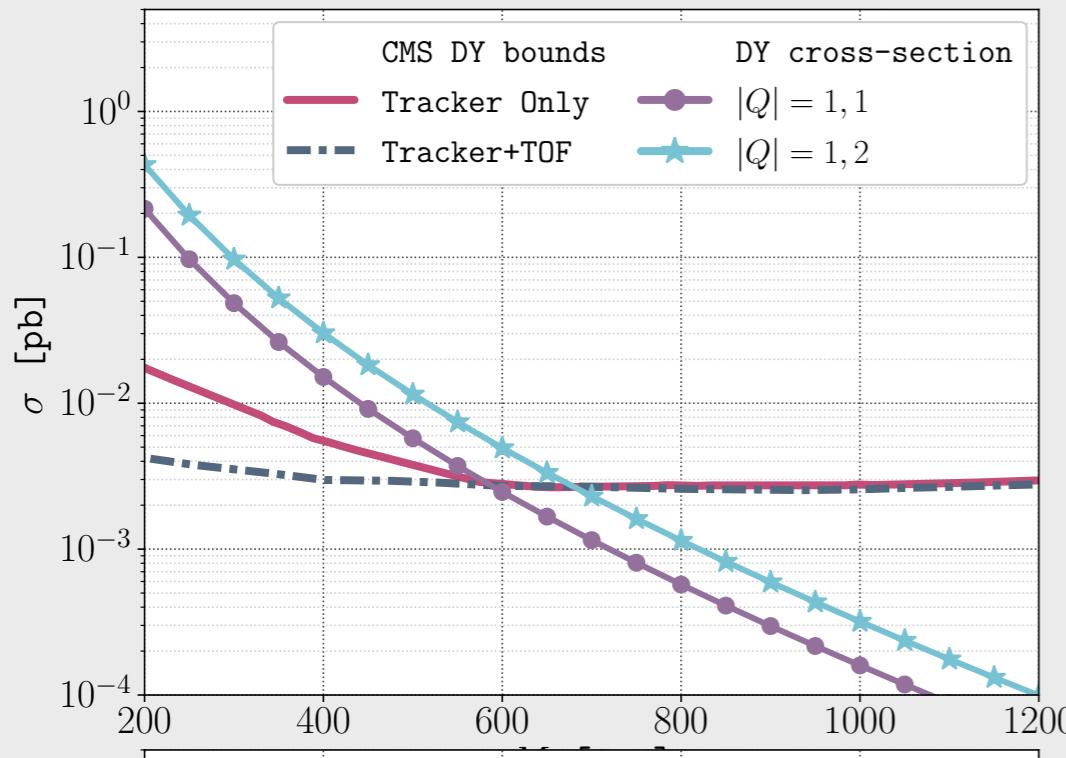
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Large  $y_L$  : linear EW  
sym. limit

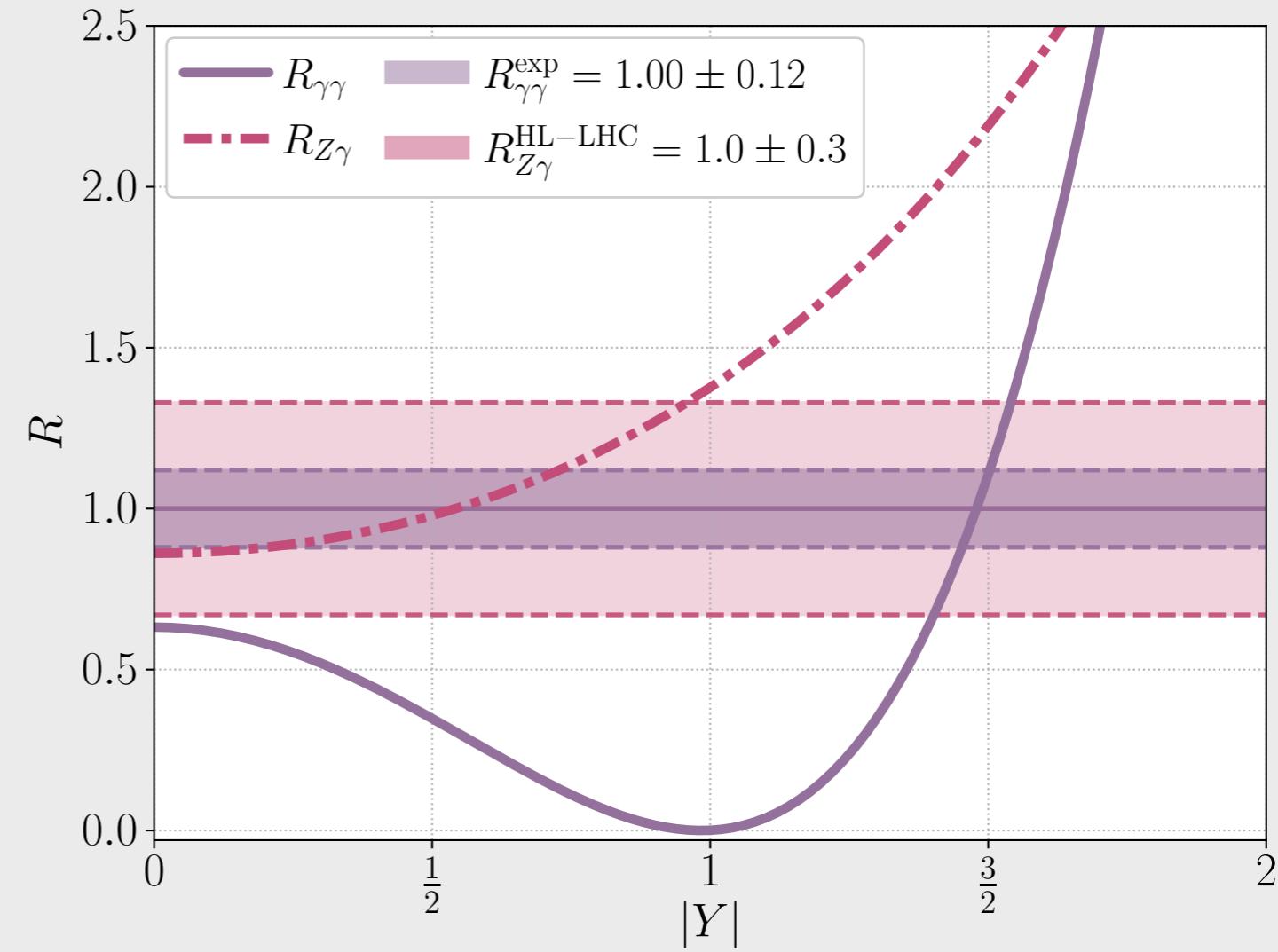
Vanishing  $y_L$ :  $\ll$  IR »  
chirality

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- **electroweak precision tests** : satisfied in the custodial limit
- **direct searches** for stable charged particles :



- **Higgs couplings**  
(in the alignment limit) :



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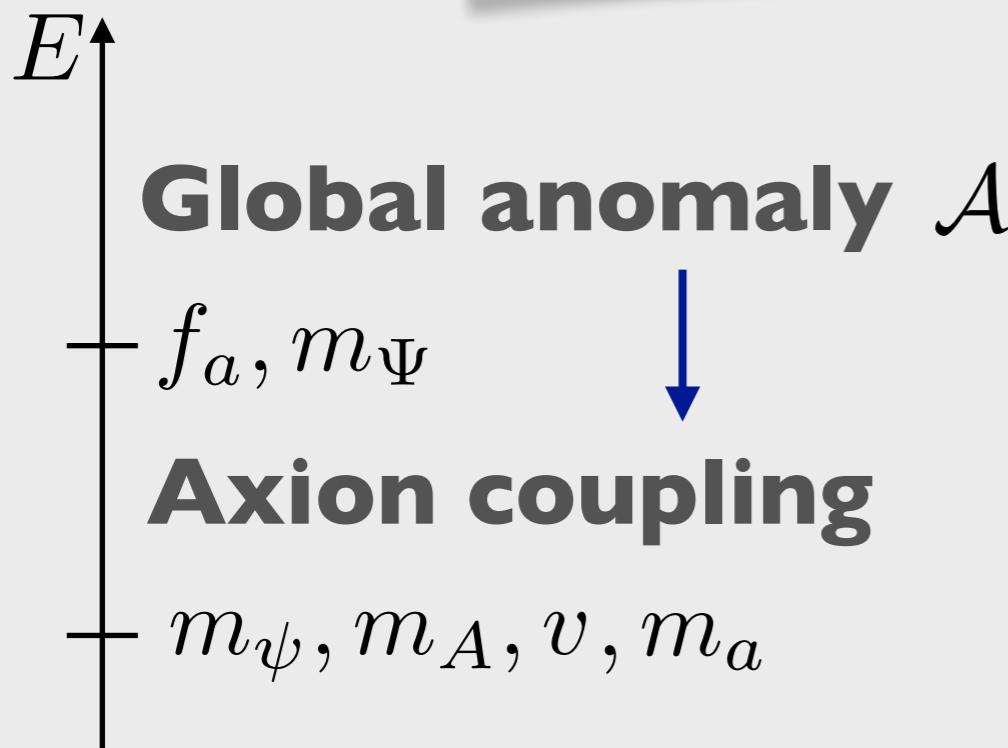
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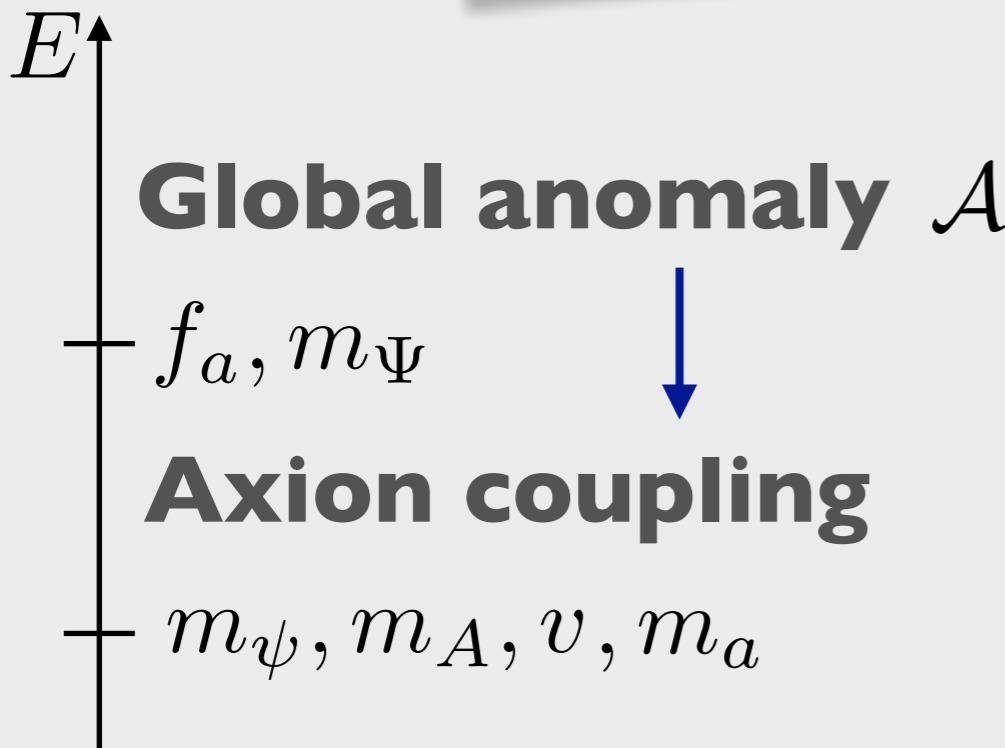
**The simplest model** which realizes this **is not yet excluded**

Thank you !

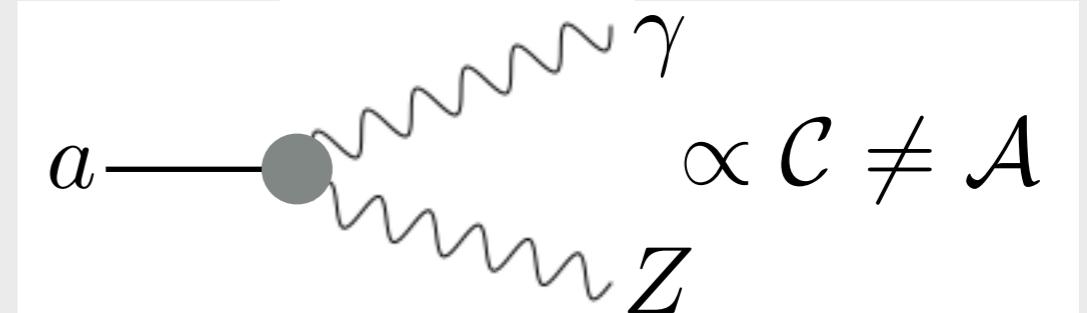
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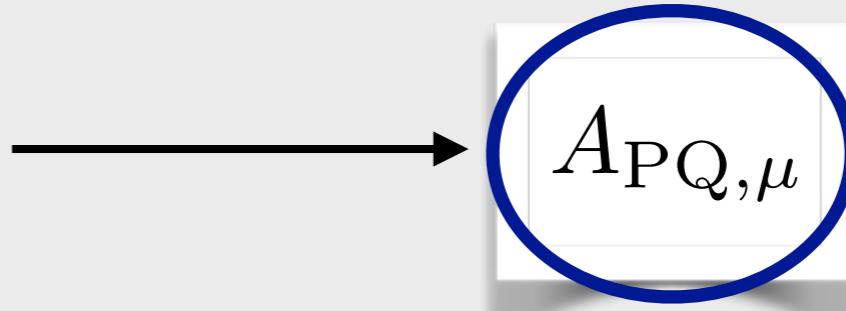
**Wrong conclusion** e.g. in  
DFSZ model [Quevillon, Smith '19]



$$\delta_{PQ} \left( -\frac{\mathcal{A}}{16\pi^2 f_a} a F \tilde{F} \right) = -\frac{\mathcal{A}}{16\pi^2} \epsilon_{PQ} F \tilde{F} = 0$$

since boundary term...  
better to deal with  
« bulk » objects

**Anomalies MUST match  
for a GAUGE symmetry**



$$-\frac{\mathcal{C}}{16\pi^2f_a}aF\tilde{F}=-\frac{\mathcal{A}}{16\pi^2f_a}aF\tilde{F}+\frac{\mathcal{C}-\mathcal{A}}{8\pi^2f_a}\partial_{\mu}aA_{\nu}\tilde{F}^{\mu\nu}$$

$$-\frac{\mathcal{C}}{16\pi^2 f_a} a F \tilde{F} = -\frac{\mathcal{A}}{16\pi^2 f_a} a F \tilde{F} + \frac{\mathcal{C} - \mathcal{A}}{8\pi^2 f_a} \partial_\mu a A_\nu \tilde{F}^{\mu\nu}$$

$\partial_\mu a - f_a A_{PQ,\mu}$  

$A_\nu - \frac{\partial_\nu \theta_A}{m_A}$  

We obtain that

for a massless gauge field :  $\mathcal{A} = \mathcal{C}$

for a massive (chiral) gauge field :  $\mathcal{A} \neq \mathcal{C}$

[Anastasopoulos/Bianchi/Dudas/Kiritsis '06,  
Dudas/Mambrini/Pokorski/Romagnoni '09, ...]