On the dispersive evaluation of the HVP contribution to a_μ and α_{QED}, and implications for the EW fit

Bogdan MALAESCU

LPNHE, CNRS

Based on: 1908.00921(DHMZ), 2006.04822(WP Theory Initiative), 2008.08107(BM,MS)

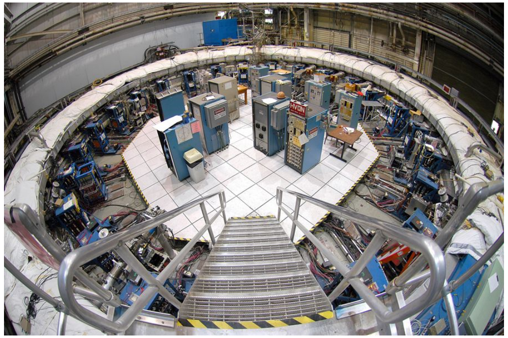
In collaboration & useful discussions with:
Michel DAVIER, Andreas HOECKER, Zhiqing ZHANG (DHMZ)
Laurent LELLOUCH, Matthias SCHOTT; the Gfitter group

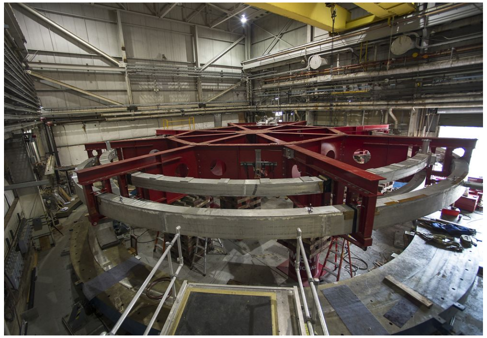
LIO international conference on Future colliders and the origin of mass

25/06/2021

How it all started (again)…

BNL → 1 month long trip for the g-2 storage ring





→ Fermilab
July 26, 2013

This is NOT an UFO !!! ;-)

Content of the talk

- Introduction: the $(g\text{-}2)_\mu$ experiment & theoretical prediction
- Data on $e^+e^- \rightarrow$ hadrons
- Combination of all e^+e^- data:

  focus on the combination procedure

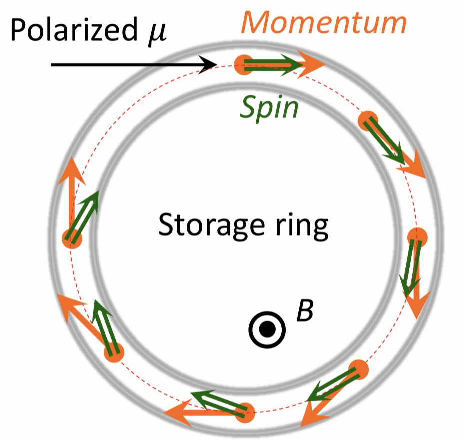
  (HVPTools and fit based on analyticity & unitarity)
- Indications of uncertainties on uncertainties and on correlations & their implications for combinations
- Results on a_μ
- Impact of correlations between a_μ and α_{QED} on the EW fit
- Conclusions

The $(g-2)_\mu$: definition & experimental measurement

- Magnetic dipole moment of a charged lepton: $\vec{\mu} = g\frac{e}{2m}\vec{s}$
- "anomaly" = deviation w.r.t. Dirac's prediction: $a = \frac{g-2}{2}$
- Experimental "ingredients" to measure a_μ:

→ Polarised muons from pion decays (parity violation)

$\bar{\nu}_\mu \leftarrow \pi^- \longrightarrow \mu^-_{\text{polarised}}$



→ "Anomalous frequency"

(difference between spin precession and cyclotron frequency)

proportional to a_μ for the "magic γ"

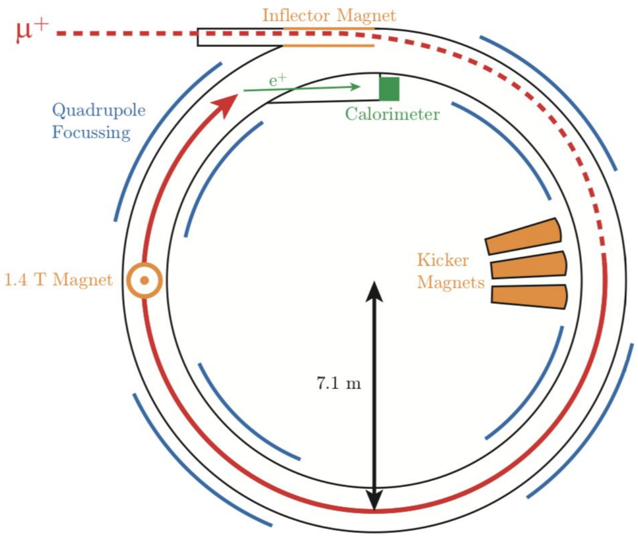
$$\vec{\omega}_a = \frac{e}{m_\mu c}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right] \approx \frac{e}{m_\mu c} a_\mu \vec{B}$$

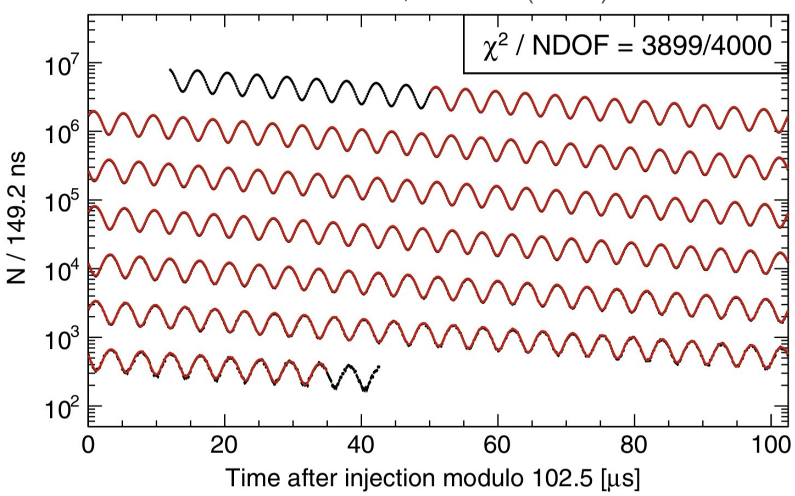
→ Parity violation in muon decays

(electron emitted in the direction opposite to the muon spin)

$\mu^-_{\text{polarised}} \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$

The (g-2)$_\mu$ experiment





a_μ^{Exp}(BNL): (11 659 208.9 ±6.3) 10^{-10}

→ Expected uncertainty reduction by a factor 4 with the experiment at Fermilab

- improved apparatus and enhanced statistics: more intense (x20) and pure muon beam; B-field mapped every 3 days with special trolley with probes pulled through beampipe (homogeneity ~ ppm); tracking system for electron detectors etc.
- first publication: similar precision & good agreement with BNL (7th of April 2021) PRL 126, 141801 (2021)

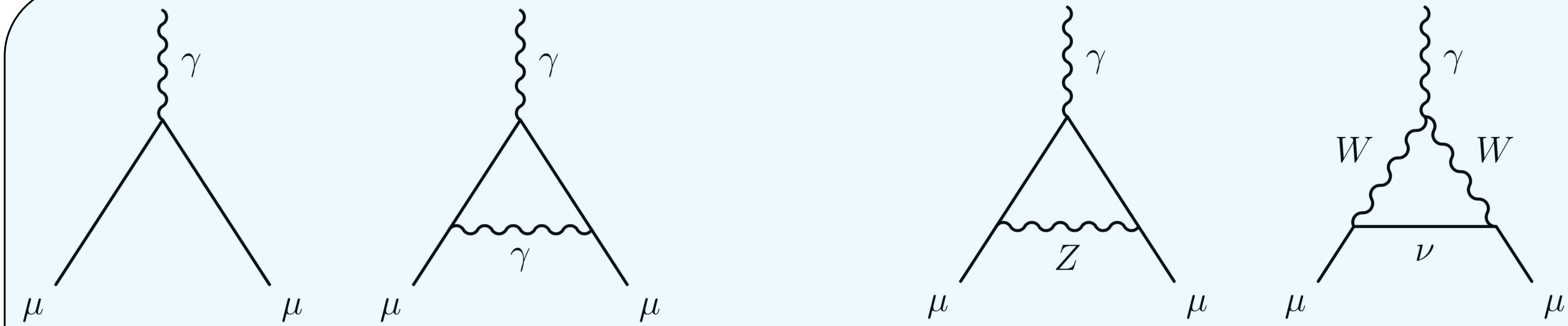
a_μ^{Exp}(Fermilab): (11 659 204.0 ±5.1 ±1.8) 10^{-10} → so far only 6% of the total data

a_μ^{Exp}(Fermilab + BNL): (11 659 206.1 ± 4.1) 10^{-10} (0.35 ppm) → One of the most precise quantities ever measured

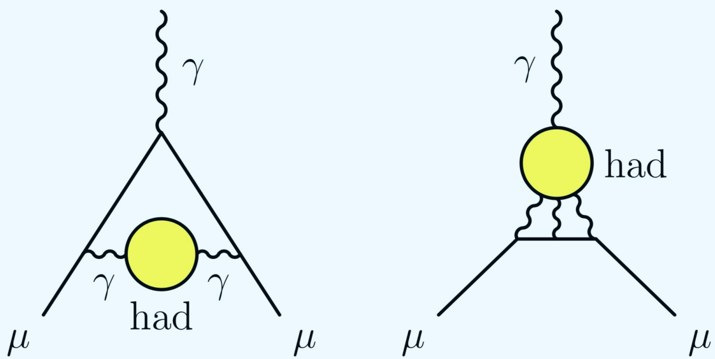
→ Initiative for a measurement using slow muons (KEK, Japan)

Theoretical prediction

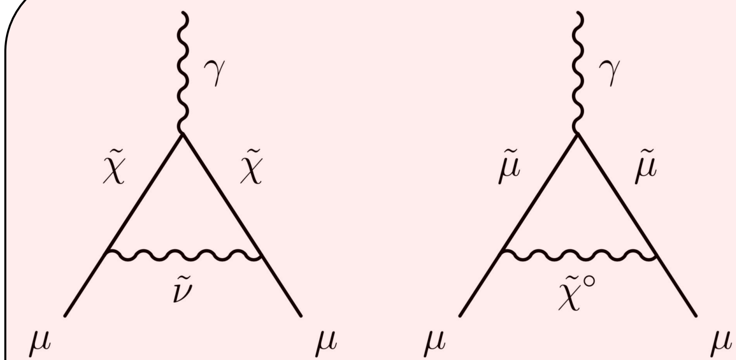
Why is it (so) complicated to compute one number ? (*very precisely*)



+ Many other diagrams at higher orders...



Dominant uncertainties: non-perturbative...



? ? ?

Hadronic Vacuum Polarization and Muon $(g\text{-}2)_\mu$

Dominant uncertainty for the theoretical prediction: from lowest-order HVP piece

Cannot be calculated from QCD (low mass scale), but one can use experimental data on $e^+e^- \to$ hadrons cross section

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

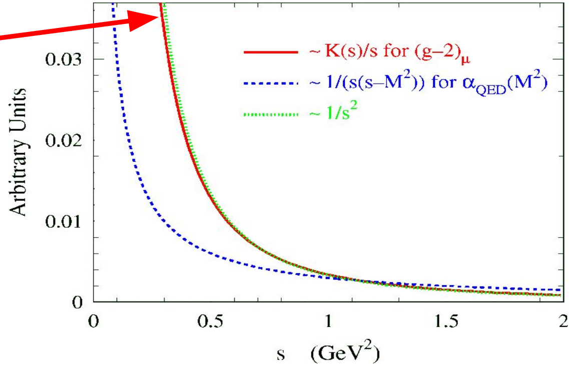
$$12\pi\ \mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0\ [e^+e^- \to \text{hadrons}\ (\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

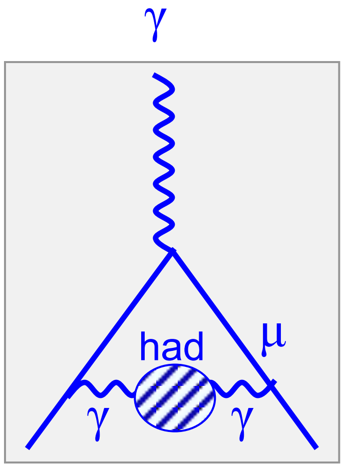
Im[ ] ∝ | hadrons $|^2$

$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\ \frac{K(s)}{s}\ R(s)$$

Dispersion relation

Bouchiat and Michel, 1961

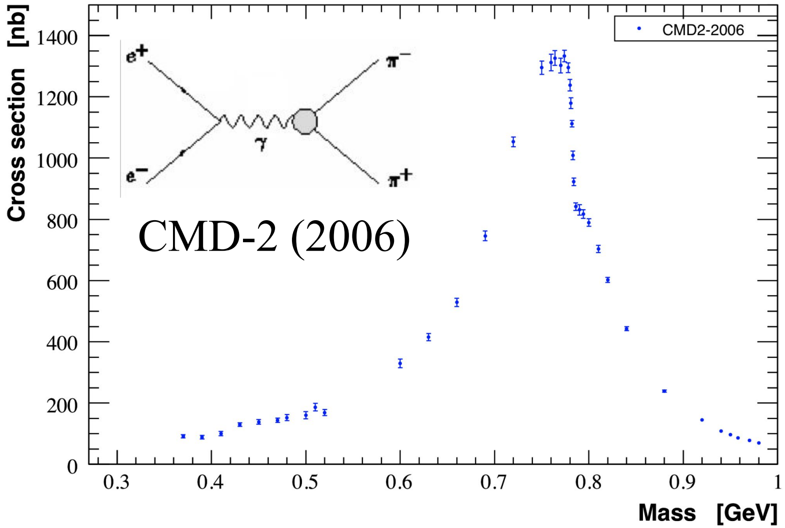


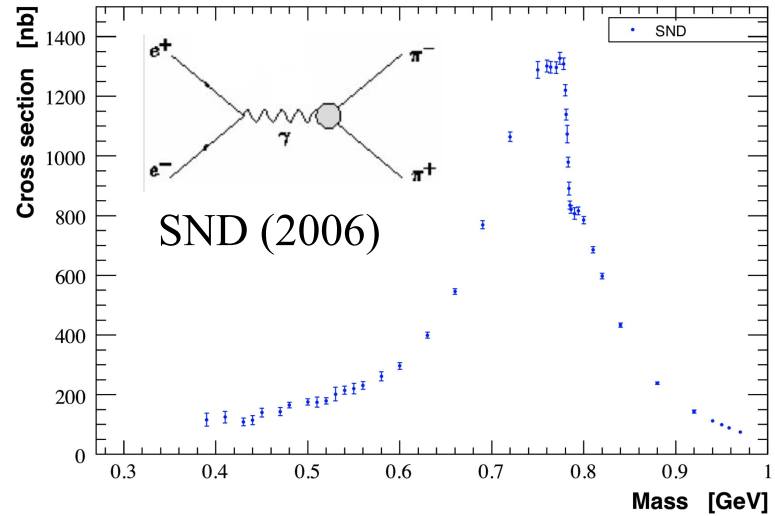


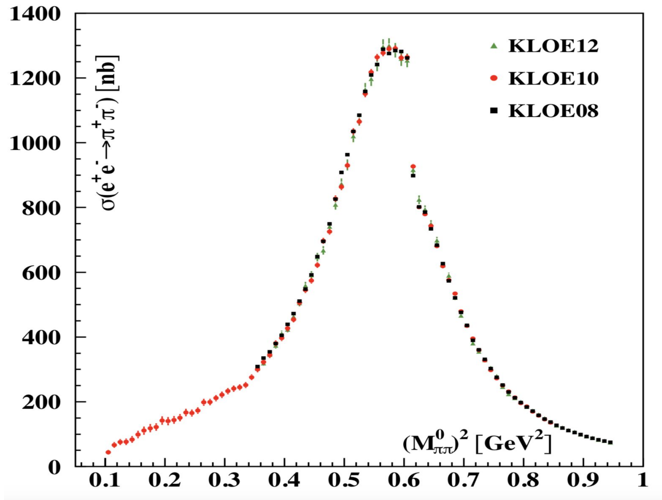
→ Precise $\sigma(e^+e^- \to$ hadrons) measurements at low energy are very important

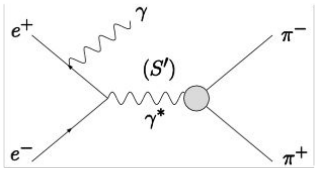
→ Do not use hadronic τ decays data anymore (less precise + theory uncertainties)

HVP: Data on $e^+e^- \rightarrow$ hadrons



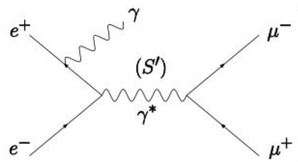


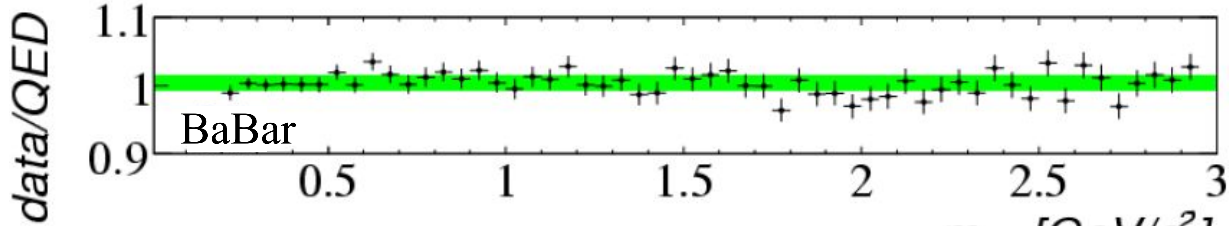




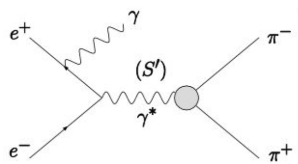
KLOE (08&10) + μμ (12) (ISR)

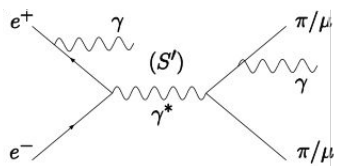
BaBar results (arXiv:0908.3589, PRL 103, 231801 (2009); arXiv:1205.2228(PRD)

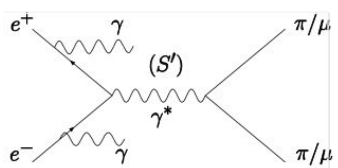


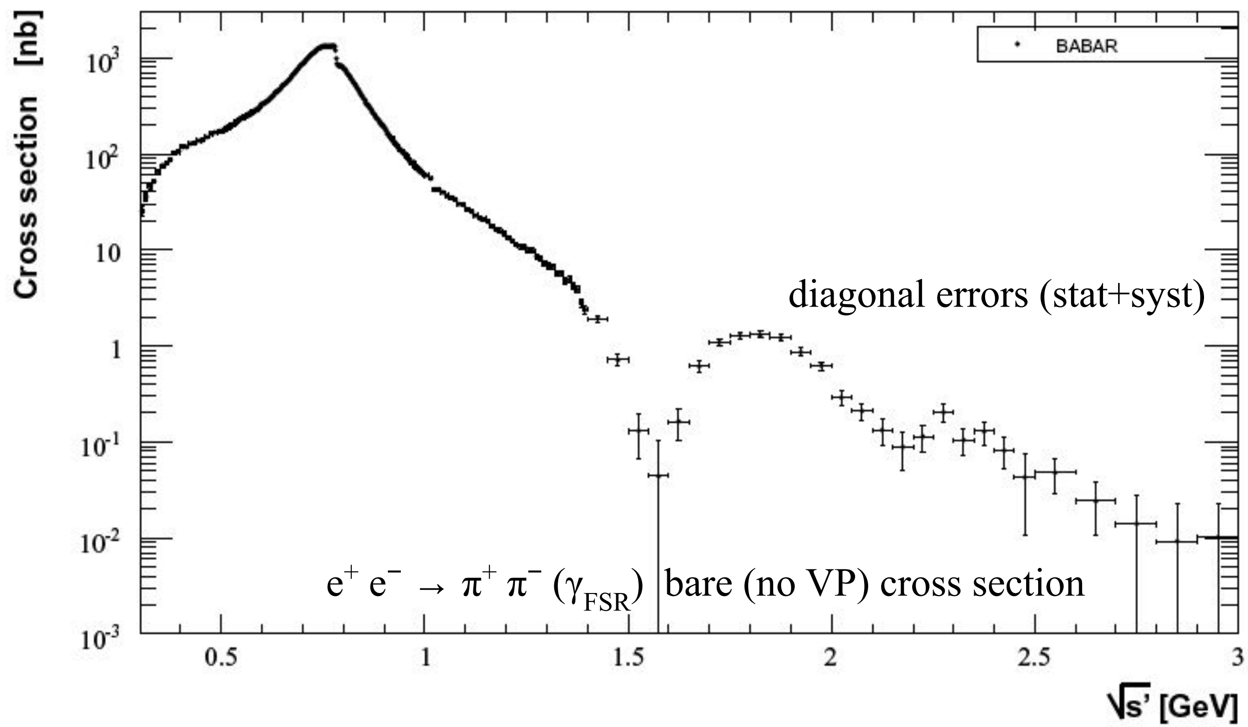


Absolute $\mu^+\mu^-$ cross section agrees with NLO QED within 1.1%









Combine cross section data: goal and requirements

→ Goal: combine experimental spectra with arbitrary point spacing / binning

→ Requirements:

- Properly propagate uncertainties and correlations
  (1st motivation for using DHMZ result as "baseline" in the TI White Paper)

- *Between measurements (data points/bins) of a given experiment*
  (covariance matrices and/or detailed split of uncertainties in sub-components)

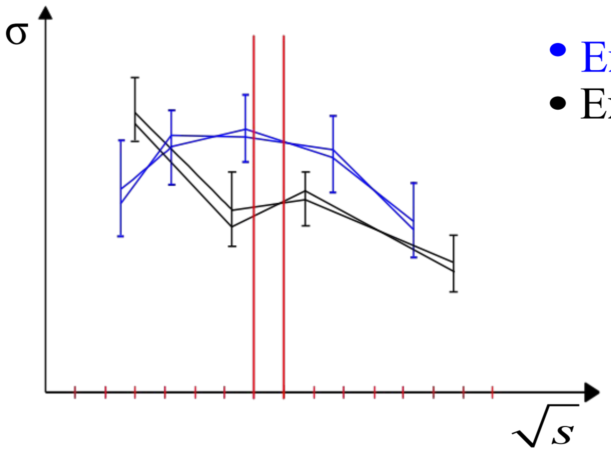
- *Between experiments* (common systematic uncertainties, e.g. VP)
  based on detailed information provided in publications

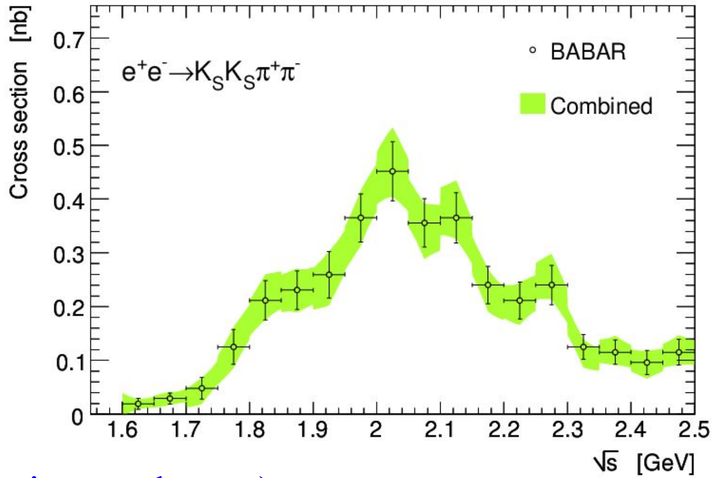
- *Between different channels* – motivated by understanding of the meaning of systematic uncertainties and identifying the common ones
  BABAR luminosity (ISR or BhaBha), efficiencies (photon, Ks, Kl, modeling);
  BABAR radiative corrections; $4\pi 2\pi^0 - \eta\omega$
  CMD2 $\eta\gamma - \pi^0\gamma$; CMD2/3 luminosity; SND luminosity;
  FSR; hadronic VP (old experiments)

- Minimize biases

- Optimize g-2 integral uncertainty
  (*without overestimating the precision with which the uncertainties of the measurements are known*)

Combination procedure implemented in HVPTools software





→ Define a (fine) final binning (to be filled and used for integrals etc.)

→ Linear/quadratic splines to interpolate between the points/bins of each experiment
- for binned measurements: preserve integral inside each bin
- closure test: replace nominal values of data points by Gounaris-Sakurai model and re-do the combination

  → *(non-)negligible bias for (linear)quadratic interpolation*

→ Fluctuate data points taking into account correlations & re-do the splines for each (pseudo-)experiment
- each uncertainty fluctuated coherently for all the points/bins that it impacts
- eigenvector decomposition for (statistical) covariance matrices

→ In each fine bin: minimize χ^2 and get average coefficients

Combination procedure implemented in HVPTools software

For each final bin:

→ Compute an average value for each measurement and its uncertainty

→ Compute correlation matrix between experiments

→ Minimize χ^2 and get average coefficients (weights)

→ Compute average between experiments and its uncertainty

Evaluation of integrals and propagation of uncertainties:

→ Integral(s) evaluated for nominal result and for each set of toy pseudo-experiments; uncertainty of integrals from RMS of results for all toys

→ The pseudo-experiments also used to derive (statistical & systematic) covariance matrices of combined cross sections → Integral evaluation

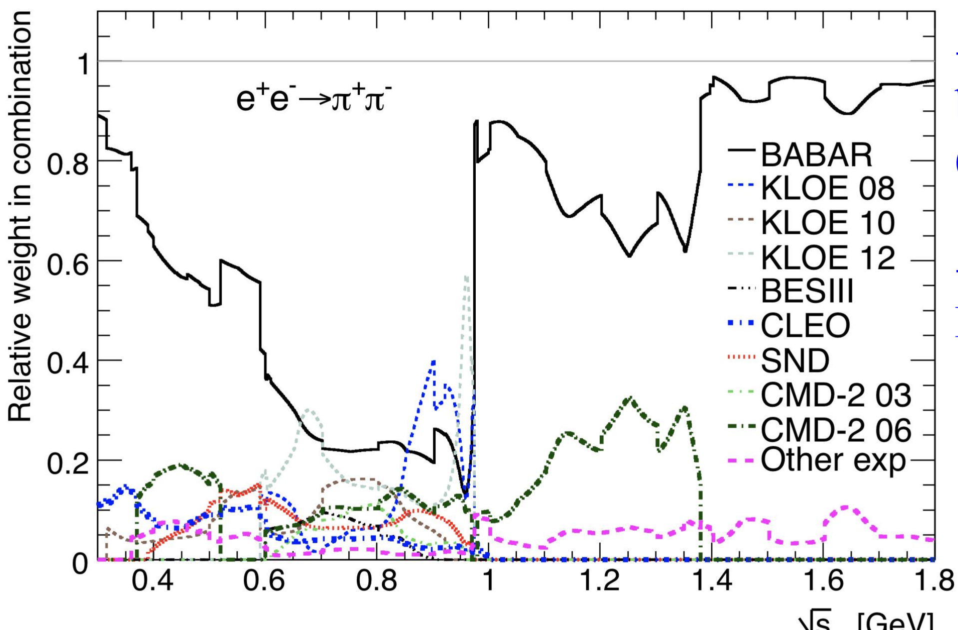
→ Uncertainties also propagated through $\pm 1\sigma$ shifts of each uncertainty:
- allows to account for correlations between different channels (for integrals and spectra)

→ *Checked consistency between the different approaches*

Combination procedure: weights of various measurements

For each final bin:
→ Minimize χ^2 and get average coefficients

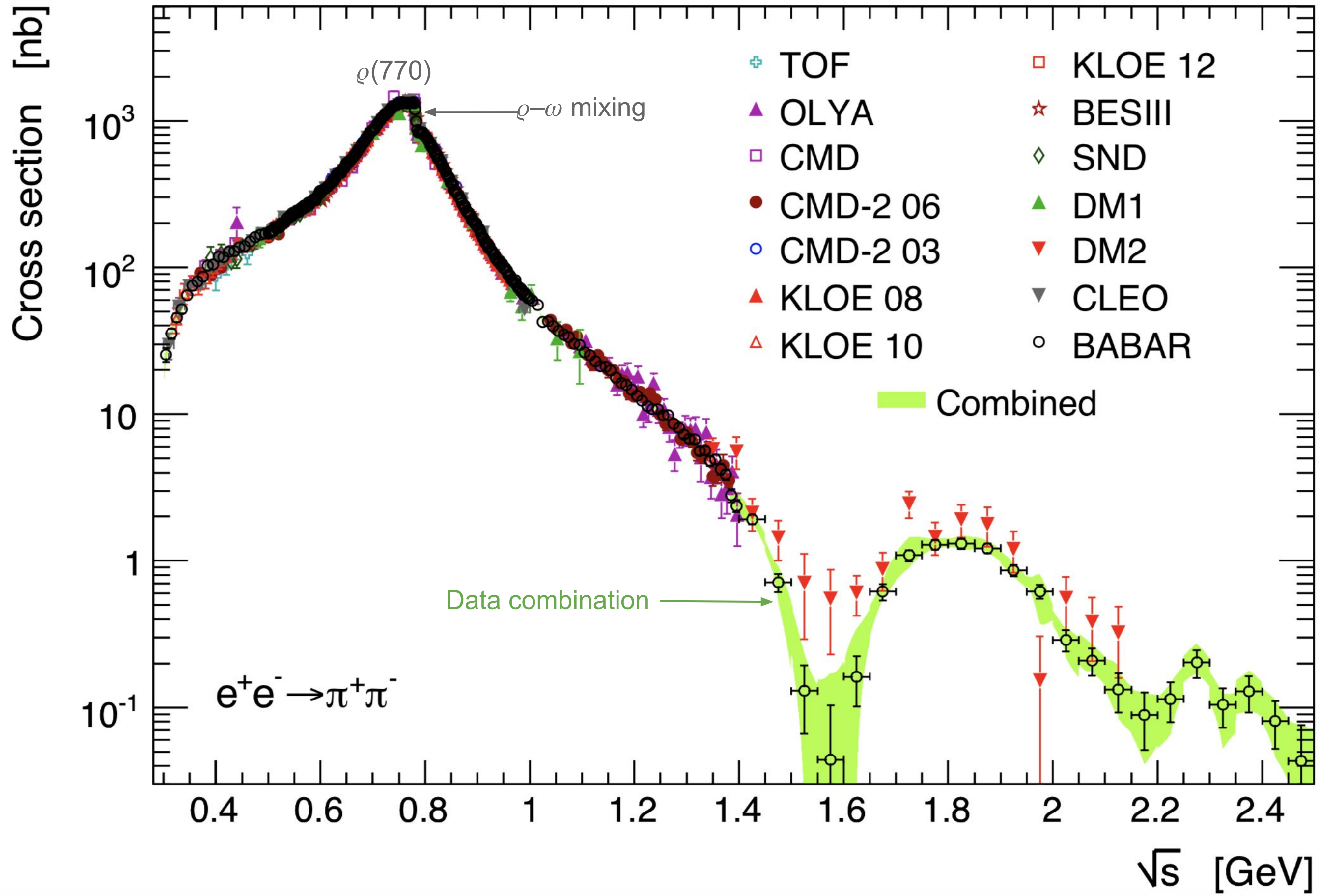
Note: average weights must account for bin sizes / point spacing of measurements
(do not over-estimate the weight of experiments with large bins)
→ weights in fine bins evaluated using a common (large) binning for measurements + interpolation
→ compare the precisions on the same footing



→ Bins used by KLOE larger than the ones by BABAR in ρ-ω interference region (factor ~3)

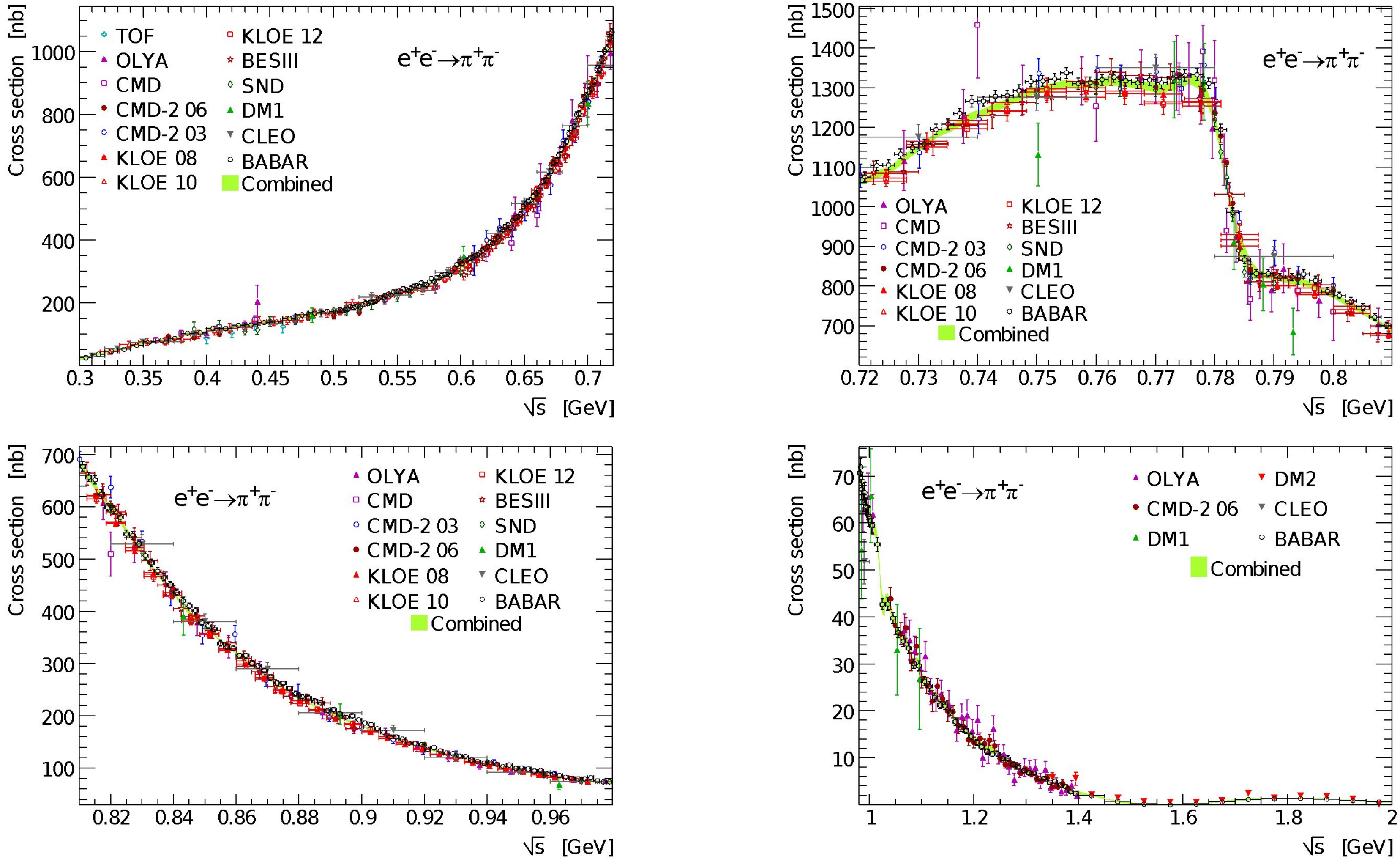
→ Average dominated by BaBar and KLOE, BaBar covering full range

Combination for the $e^+e^- \to \pi^+\pi^-$ channel

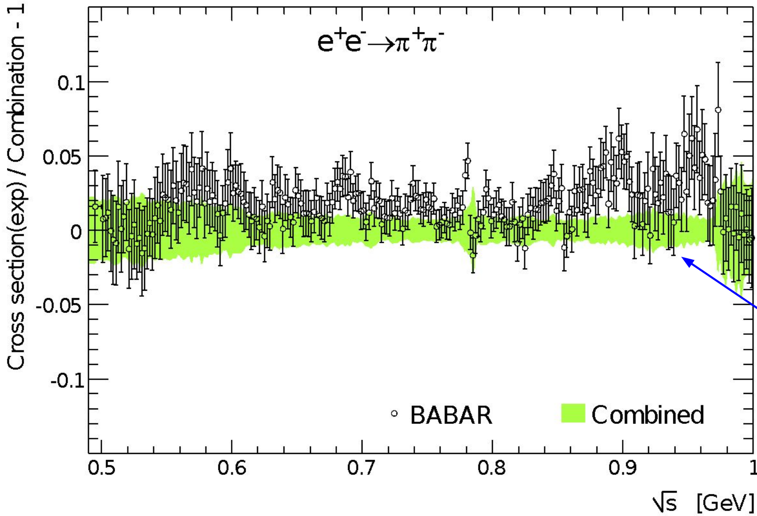


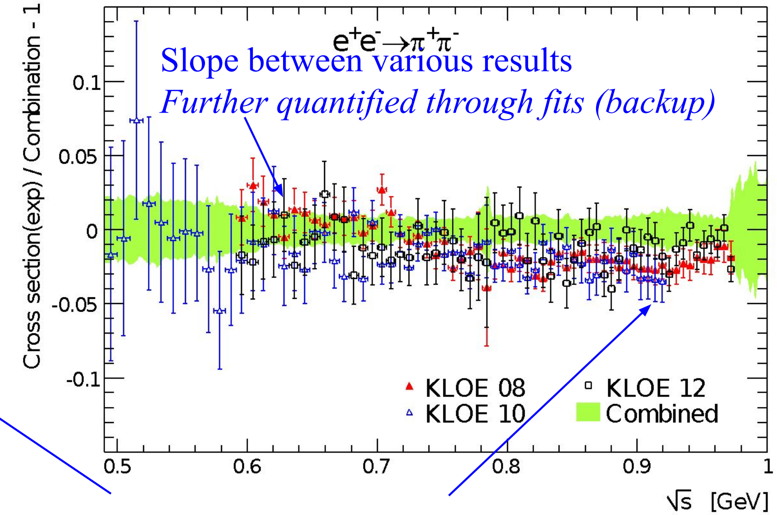
Procedure and software (HVPTools) for combining cross section data with arbitrary point spacing/binning

Combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel



Combination for the $e^+e^- \to \pi^+\pi^-$ channel





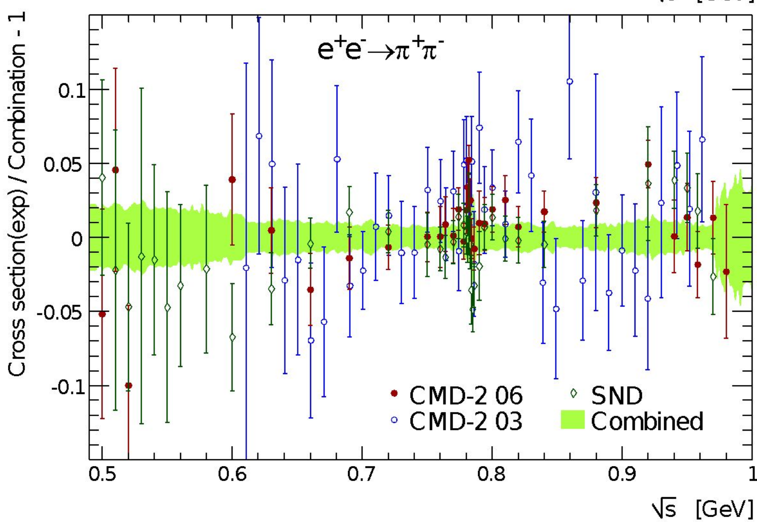
Slope between various results

Further quantified through fits (backup)

Local tension & systematic trends

Indication of "uncertainties on uncertainties"

(i.e. unaccounted biases)



Other experiments not yet precise enough to discriminate

(see however recent update from SND)

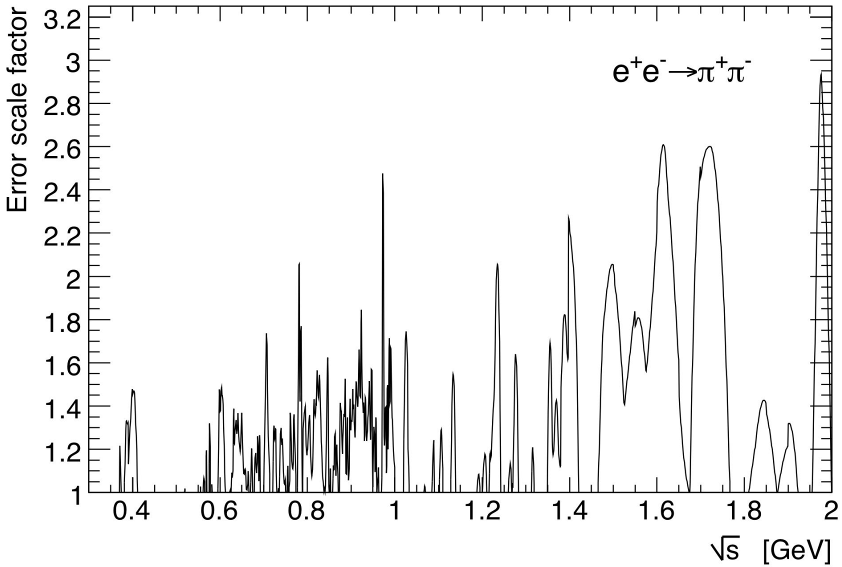
Combination procedure: compatibility between measurements

For each final bin:

→ χ^2 /ndof: test locally the level of agreement between input measurements, *taking into account the correlations*

→ Scale uncertainties in bins where χ^2 /ndof > 1 (PDG): *locally* conservative

→ Observed tension between BABAR and KLOE measurements



→ Tension between measurements:
indication of underestimated uncertainties
Motivates conservative uncertainty treatment in combination fit (evaluation of weights)

→ Included extra (dominant) uncertainty: difference between integrals without either BABAR or KLOE measurement to account for systematic deviations
(2nd motivation for using DHMZ result as "baseline" in White Paper)

Improving a_μ through fits for the $e^+e^- \to \pi^+\pi^-$ channel

→ Fit bare form-factor using 6 param. model based on *analyticity* and *unitarity*

$$|F_\pi^0|^2 = |R(s) \times J(s)|^2$$

$$R(s) = 1 + \alpha_V s + \frac{\kappa s}{m_\omega^2 - s - i m_\omega \Gamma_\omega}$$

(1611.09359, C. Hanhart et al.)

$$J(s) = e^{1-\frac{\delta_1(s_0)}{\pi}} \left(1 - \frac{s}{s_0}\right)^{\left[1-\frac{\delta_1(s_0)}{\pi}\right]\frac{s_0}{s}} \left(1 - \frac{s}{s_0}\right)^{-1} e^{\frac{s}{\pi}\int_{4m_\pi^2}^{s_0} dt \frac{\delta_1(t)}{t(t-s)}}$$

Omnès integral

(hep-ph/0402285, F.J. Yndurain et al.)

$$\cot \delta_1(s) = \frac{\sqrt{s}}{2k^3}\left(m_\rho^2 - s\right)\left[\frac{2m_\pi^3}{m_\rho^2\sqrt{s}} + B_0 + B_1\omega(s)\right]$$

$$k = \frac{\sqrt{s - 4m_\pi^2}}{2}$$

(1102.2183, F.J. Yndurain et al.)

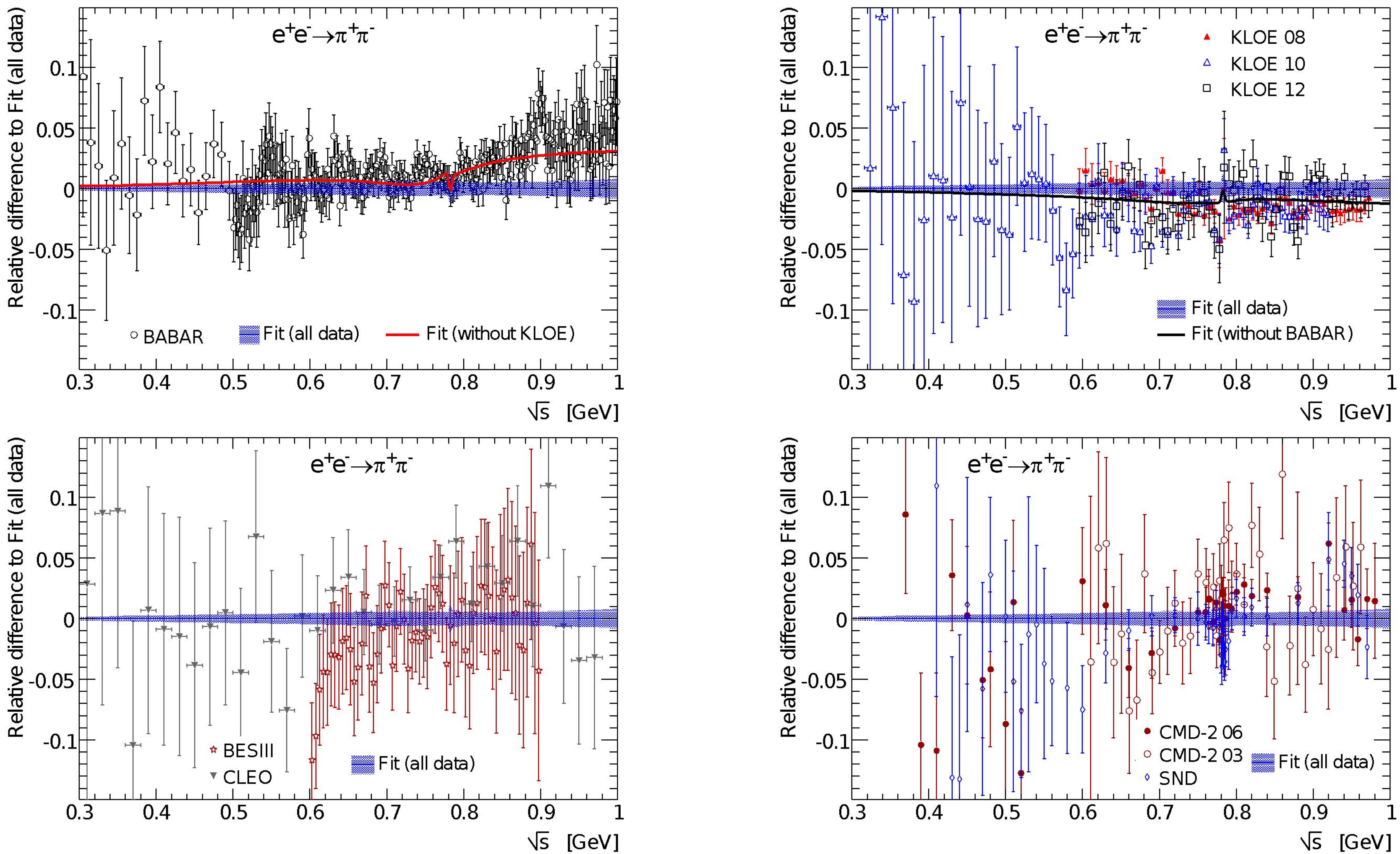
$$\omega(s) = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}} \qquad \sqrt{s_0} = 1.05 \text{ GeV}$$

→ Conservative χ^2 (diagonal matrix) & local rescaling of input uncertainties

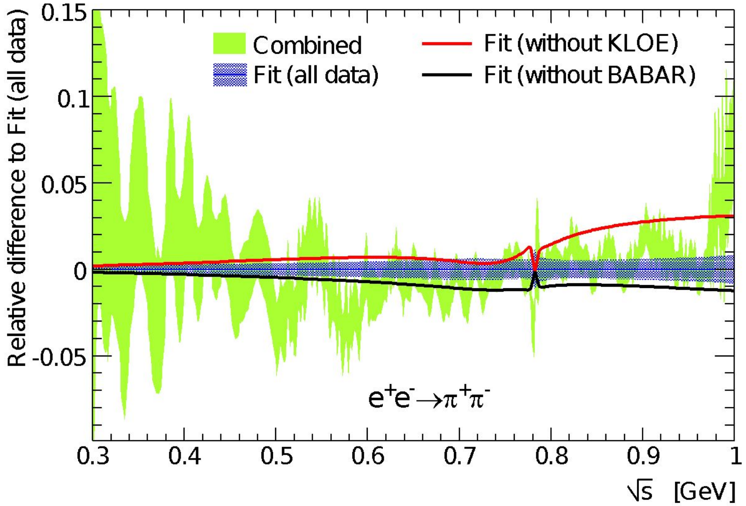
→ Full propagation of uncertainties & correlations using pseudo-experiments

DHMZ - 1908.00921

Fit performed up to 1 GeV: comparison with data



Fit performed up to 1 GeV, Result used up to 0.6 GeV



→ Use fit only below 0.6 GeV for a_μ integral:

- where data is less precise and scarce
- less impacted by potential uncertainties of inelastic effects

$\sqrt{s}$ range [GeV]	a_μ^{had} $[10^{-10}]$ Fit	a_μ^{had} $[10^{-10}]$ Data Integration
0.3 - 0.6	$109.80 \pm 0.37_{exp} \pm 0.36_{para*}$	$109.6 \pm 1.0_{exp}$

→ The difference 0.2 ± 0.9 (72% correlation accounted for)

→ The fit improves the precision by a factor ~2

(*) Parameter uncertainty corresponds to variations with/without the B_1 term in the phase shift formula and $\sqrt{s_0}$ varied from 1.05 GeV to 1.3 GeV (absolute values summed linearly), *checked to be statistically significant*

Combined results: Fit [<0.6GeV] + Data[0.6-1.8GeV]

→ Full uncertainty propagation using the same pseudo-experiments as for the spline-based combination: 62% correlation among the two contributions

$\sqrt{s}$ range [GeV]	a_μ^{had} $[10^{-10}]$ All data	a_μ^{had} $[10^{-10}]$ All but BABAR	a_μ^{had} $[10^{-10}]$ All but KLOE
threshold - 1.8	$506.9 \pm 1.9_{total}$	$505.0 \pm 2.1_{total}$	$510.6 \pm 2.2_{total}$

→ The difference "All but BABAR" and "All but KLOE" = 5.6
to be compared with 1.9 uncertainty with "All data"

- The local error inflation is not sufficient to amplify the uncertainty
- Global tension (normalisation/shape) not previously accounted for
- Potential underestimated uncertainty in at least one of the measurements?
- Other measurements not precise enough to discriminate BABAR / KLOE

→ Given the fact we do not know which dataset is problematic, we decide to:

- Add half of the discrepancy (2.8) as an additional uncertainty (correcting the local PDG inflation to avoid double counting)
- Take ("All but BABAR" + "All but KLOE") / 2 as central value

Uncertainties on uncertainties and on correlations

Topic of general interest, in other fields too (see backup)

1908.00921(DHMZ), 2006.04822(WP Theory Initiative)

Two different approaches for combining (e^+e^-) data

DHMZ:

→ χ^2 computed locally (in each fine bin), taking into account correlations between measurements (see previous slides)

→ used to determine the weights on the measurements in the combination and their level of agreement

→ uncertainties and correlations propagated using pseudo-experiments or ±1σ shifts of each uncertainty component

KNT:

→ χ^2 computed globally (for full mass range)

$$\chi^2_I = \sum_{i=1}^{N_{\rm tot}} \sum_{j=1}^{N_{\rm tot}} \left(R_i^{(m)} - \mathcal{R}_m^{i,I}\right) \mathbf{C}_I^{-1}\left(i^{(m)}, j^{(n)}\right) \left(R_j^{(n)} - \mathcal{R}_n^{j,I}\right)$$

KNT (1802.02995)

$$\chi^2 = \sum_{i=1}^{195} \sum_{j=1}^{195} \left(\sigma^0_{\pi\pi(\gamma)}(i) - \bar{\sigma}^0_{\pi\pi(\gamma)}(m)\right) \mathbf{C}^{-1}\left(i^{(m)}, j^{(n)}\right) \left(\sigma^0_{\pi\pi(\gamma)}(j) - \bar{\sigma}^0_{\pi\pi(\gamma)}(n)\right)$$

KLOE-KMT (1711.03085)

→ relies on description of correlations on long ranges

→ *One of the main sources of differences for the uncertainty on a_μ*

Evaluation of uncertainties and correlations (e^+e^-)

	$\sigma_{\pi\pi\gamma}$	$\sigma^0_{\pi\pi}$	F_π	$\Delta^{\pi\pi}a_\mu$
Reconstruction Filter	negligible			
Background subtraction	Tab. 1			0.3%
Trackmass	0.2%			
Pion cluster ID	negligible			
Tracking efficiency	0.3%			
Trigger efficiency	0.1%			
Acceptance	Tab. 2			0.2%
Unfolding	Tab. 3			negligible
L3 filter	0.1%			
$\sqrt{s}$ dependence of H	-	Tab. 4		0.2%
Luminosity	0.3%			
Experimental systematics				0.6%
FSR resummation	-	0.3%		
Radiator function H	-	0.5%		
Vacuum Polarization	-	0.1%	-	0.1%
Theory systematics				0.6%

$M^2_{\pi\pi}$ range (GeV^2)	Systematic error (%)
$0.35 \leq M^2_{\pi\pi} < 0.39$	0.6
$0.39 \leq M^2_{\pi\pi} < 0.43$	0.5
$0.43 \leq M^2_{\pi\pi} < 0.45$	0.4
$0.45 \leq M^2_{\pi\pi} < 0.49$	0.3
$0.49 \leq M^2_{\pi\pi} < 0.51$	0.2
$0.51 \leq M^2_{\pi\pi} < 0.64$	0.1
$0.64 \leq M^2_{\pi\pi} < 0.95$	-

KLOE 08 (0809.3950)

KLOE 10 (1006.5313)

	$\sigma_{\pi\pi\gamma}$	$\sigma^{\text{bare}}_{\pi\pi}$	$\lvert F_\pi\rvert^2$	$\Delta a^{\pi\pi}_\mu$
	threshold ; ρ-peak			(0.1 - 0.85 GeV^2)
Background Filter	0.5% ; 0.1%			negligible
Background subtraction	3.4% ; 0.1%			0.5%
$f_0 + \rho\pi$ bkg.	6.5% ; negl.			0.4%
Ω cut	1.4% ; negl.			0.2%
Trackmass cut	3.0% ; 0.2%			0.5%
π-e PID	0.3% ; negl.			negligible
Trigger	0.3% ; 0.2%			0.2%
Acceptance	1.9% ; 0.3%			0.5%
Unfolding	negl. ; 2.0%			negligible
Tracking	0.3%			
Software Trigger (L3)	0.1%			
Luminosity	0.3%			
Experimental syst.				1.0%
FSR treatment	-	7% ; negl.		0.8%
Radiator function H	-	0.5%		
Vacuum Polarization	-	Ref. [34]	-	0.1%
Theory syst.				0.9%

→ Systematics *evaluated* in ~wide mass ranges with sharp transitions

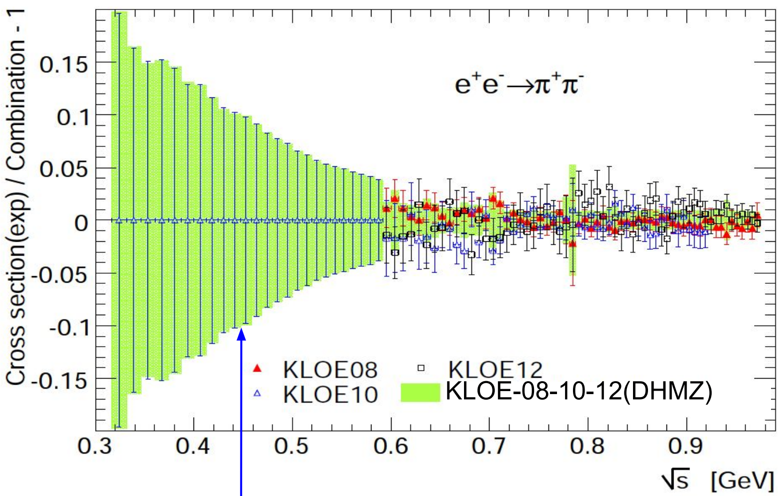
Evaluation of uncertainties and correlations (e^+e^-)

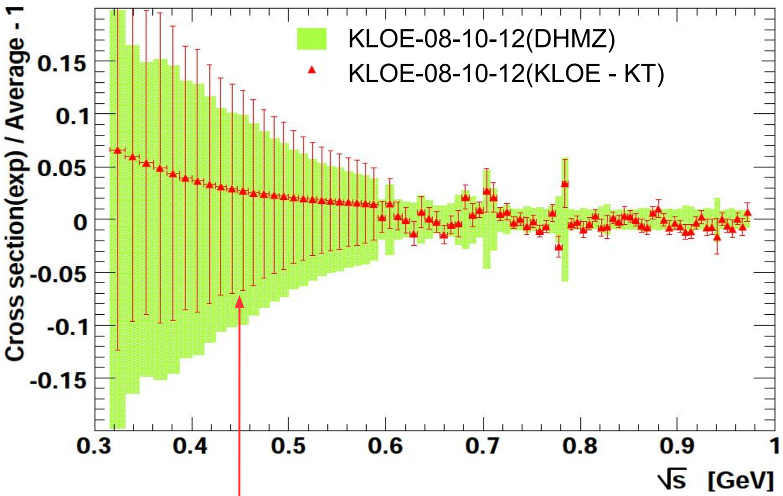
BABAR (1205.2228)

Sources	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.9	0.9-1.2	1.2-1.4	1.4-2.0	2.0-3.0
trigger/ filter	5.3	2.7	1.9	1.0	0.7	0.6	0.4	0.4
tracking	3.8	2.1	2.1	1.1	1.7	3.1	3.1	3.1
π-ID	10.1	2.5	6.2	2.4	4.2	10.1	10.1	10.1
background	3.5	4.3	5.2	1.0	3.0	7.0	12.0	50.0
acceptance	1.6	1.6	1.0	1.0	1.6	1.6	1.6	1.6
kinematic fit (χ^2)	0.9	0.9	0.3	0.3	0.9	0.9	0.9	0.9
correl $\mu\mu$ ID loss	3.0	2.0	3.0	1.3	2.0	3.0	10.0	10.0
$\pi\pi/\mu\mu$ non-cancel.	2.7	1.4	1.6	1.1	1.3	2.7	5.1	5.1
unfolding	1.0	2.7	2.7	1.0	1.3	1.0	1.0	1.0
ISR luminosity	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
sum (cross section)	13.8	8.1	10.2	5.0	6.5	13.9	19.8	52.4

→ Systematics *evaluated* in ~wide mass ranges with sharp transitions
(statistics limitations when going to narrow ranges)

Combining the 3 KLOE measurements





Local combination (DHMZ)

Information propagated between mass regions, through shifts of systematics - relying on correlations, amplitudes and shapes of systematics (KLOE-KT)

Combining the 3 KLOE measurements - $a_\mu^{\pi\pi}$ contribution

KLOE08 a_μ[0.6 ; 0.9] : 368.3 ± 3.2 [10^{-10}]
KLOE10 a_μ[0.6 ; 0.9] : 365.6 ± 3.3
KLOE12 a_μ[0.6 ; 0.9] : 366.8 ± 2.5

→ Correlation matrix:

	08	10	12
08	1	0.70	0.35
10	0.70	1	0.19
12	0.35	0.19	1

→ Amount of independent information provided by each measurement

→ KLOE-08-10-12(DHMZ) - a_μ[0.6 ; 0.9] : 366.5 ± 2.8 (Without χ^2 rescaling: ± 2.2)
→ Conservative treatment of uncertainties and correlations (*not perfectly known*) in weight determination

→ KLOE-08-10-12(KLOE-KT) - a_μ[0.6 ; 0.9]GeV : 366.9 ± 2.2 (Includes χ^2 rescaling)

→ Assuming perfect knowledge of the correlations to minimize average uncertainty

Uncertainties on uncertainties and correlations

Numerous indications of uncertainties on uncertainties and on correlations, with a direct impact on combination fits

→ Shapes of systematic uncertainties *evaluated* in ~wide mass ranges with sharp transitions

→ One standard deviation is statistically not well defined for systematic uncertainties

→ Systematic uncertainties like acceptance, tracking efficiency, background etc. not necessarily fully correlated between low and high mass

→ Are all systematic uncertainty components fully independent between each-other? (e.g. tracking and trigger)

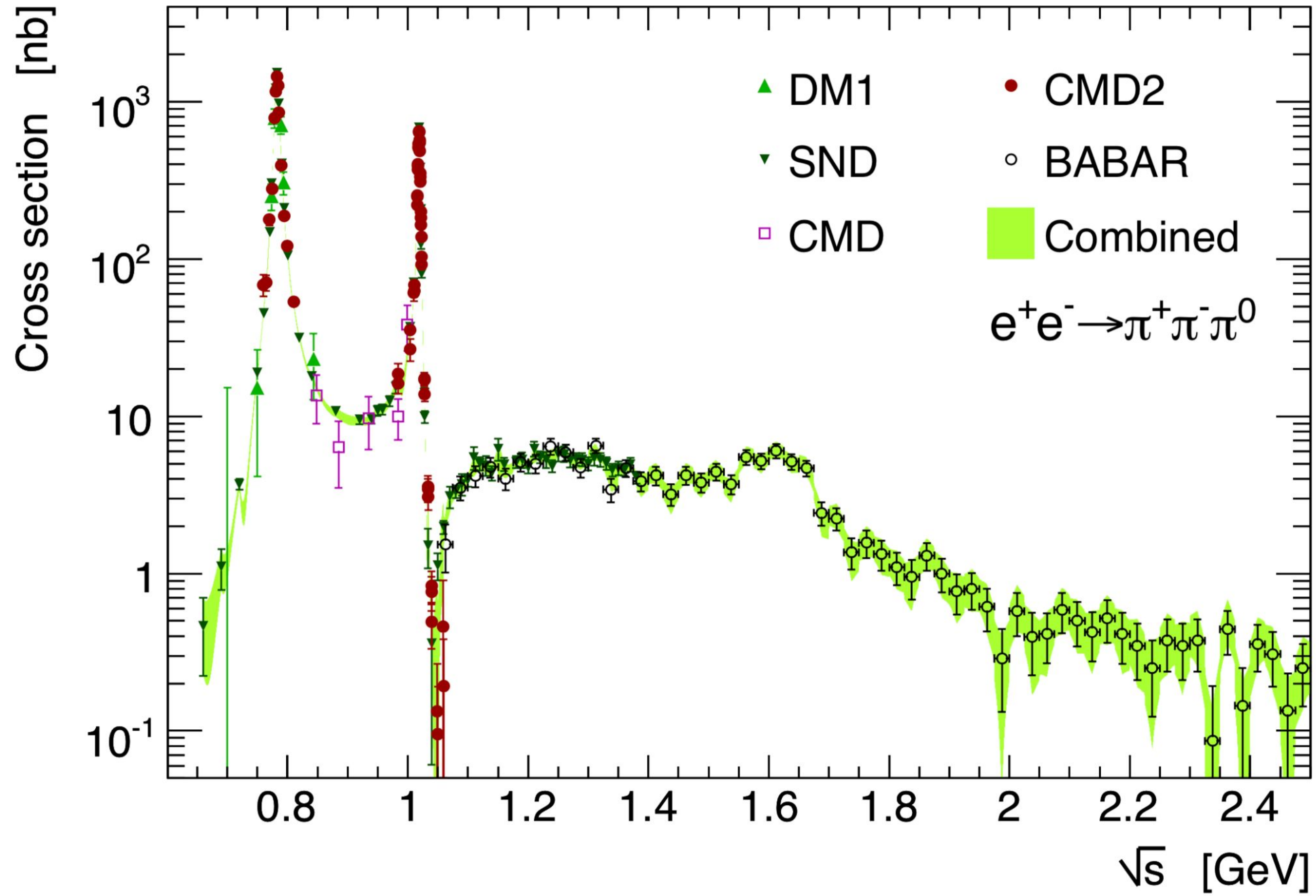
→ *Yield uncertainties on uncertainties and on correlations*

→ Tensions between measurements (BABAR/KLOE; 3 KLOE results etc.): *experimental indications of underestimated uncertainties*

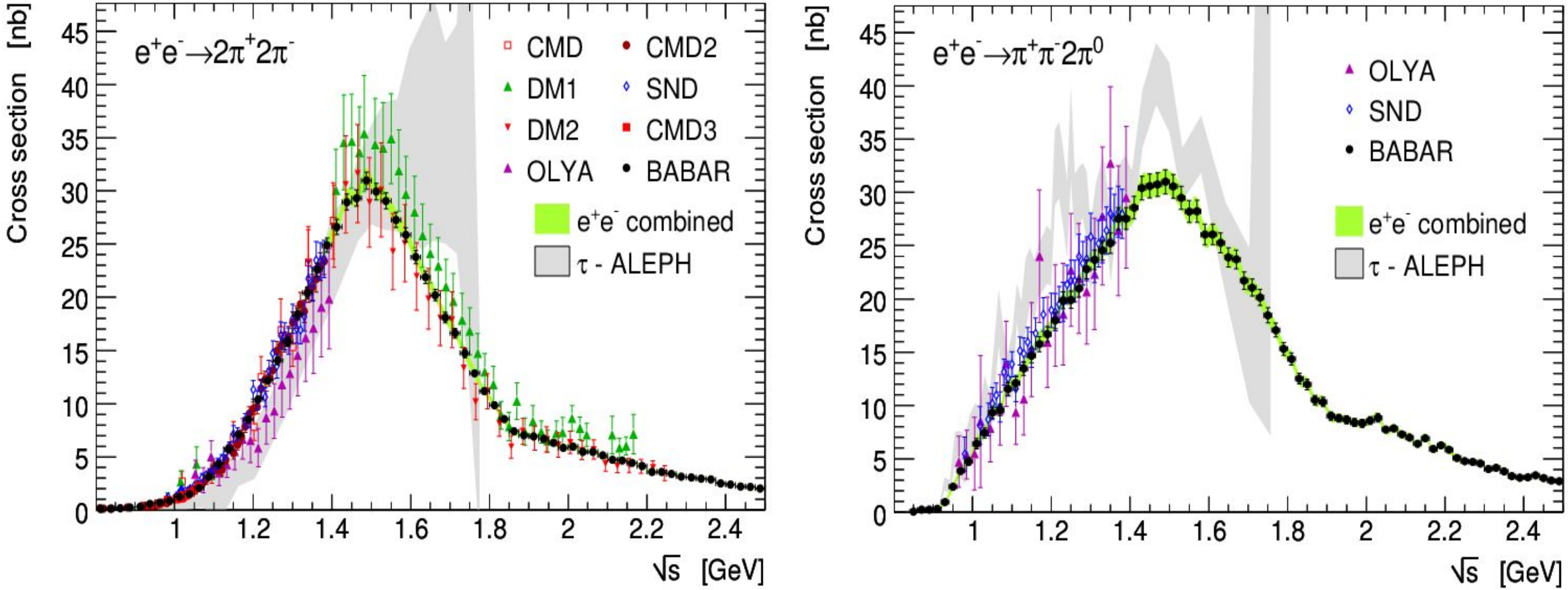
→ *Statistical methods* (χ^2 with correlations, likelihood fits, ratios of measured quantities etc.) *should not over-exploit the information on the amplitude and correlations of uncertainties*

Combination of measurements for various channels and total HVP contribution

Combination for the $e^+e^- \to \pi^+\pi^-\pi^0$ channel

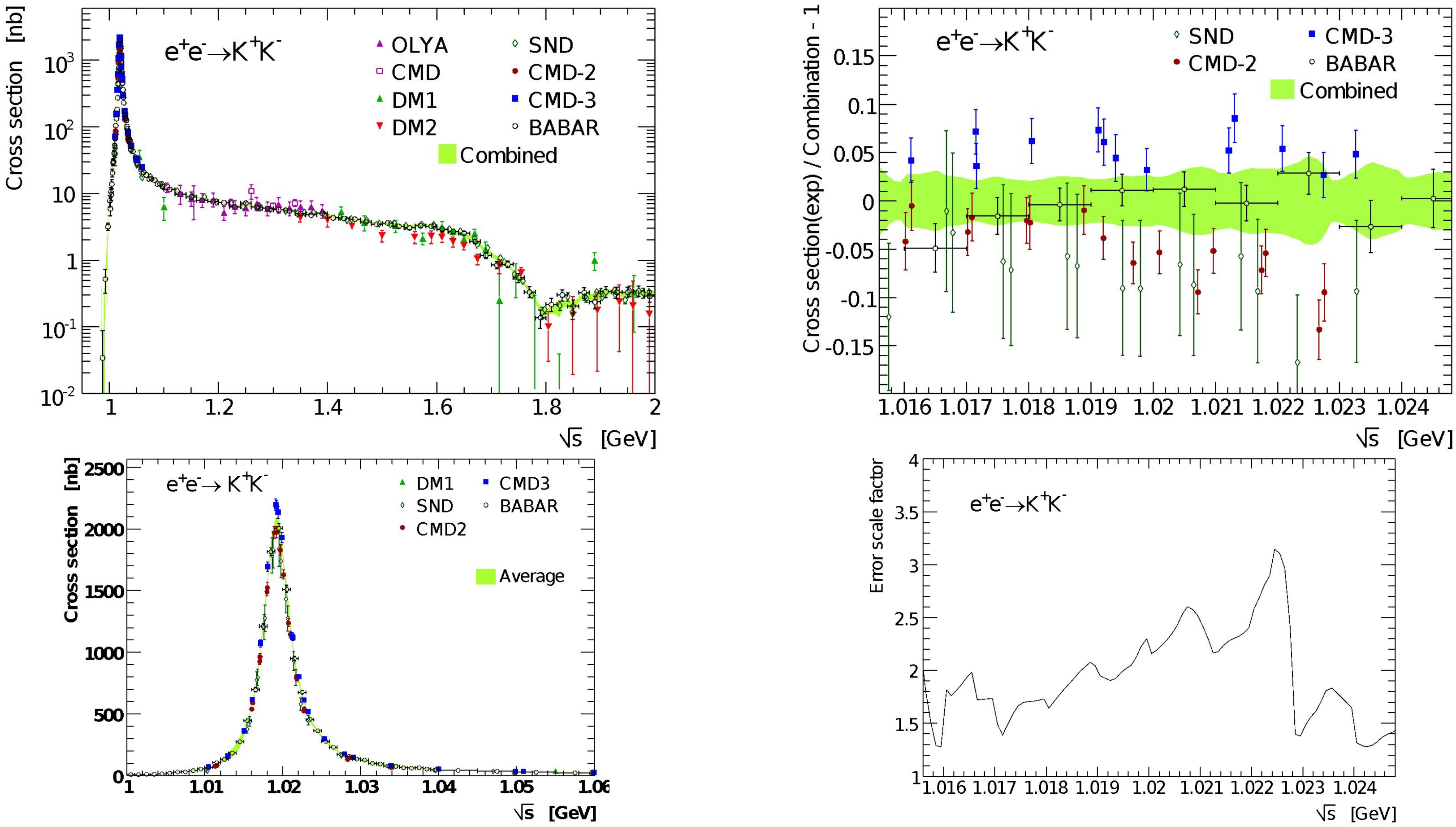


$e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$



→ Essentially normalization differences w.r.t. τ data: *cross-checks very desirable*

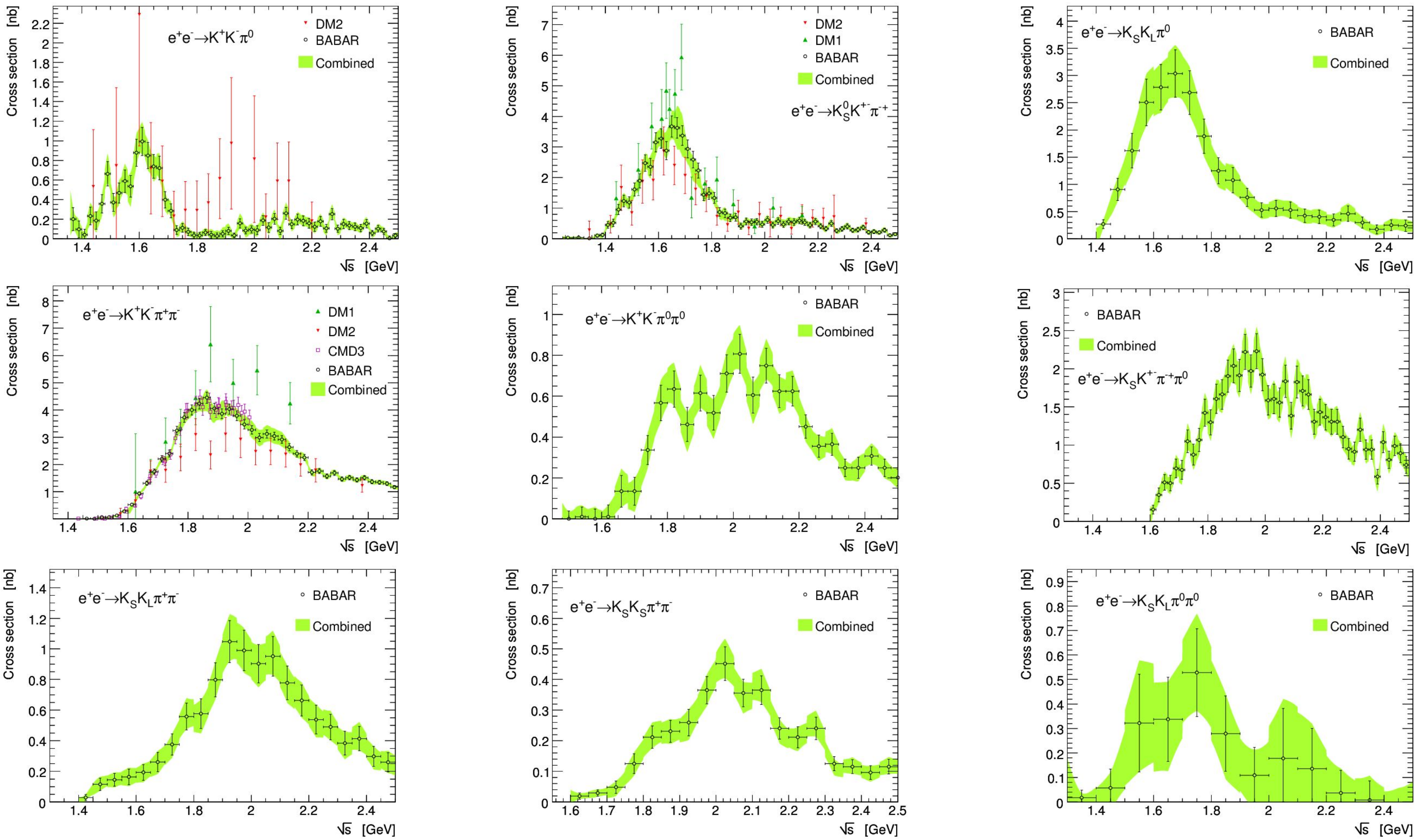
Combination for the $e^+e^- \to K^+K^-$ channel



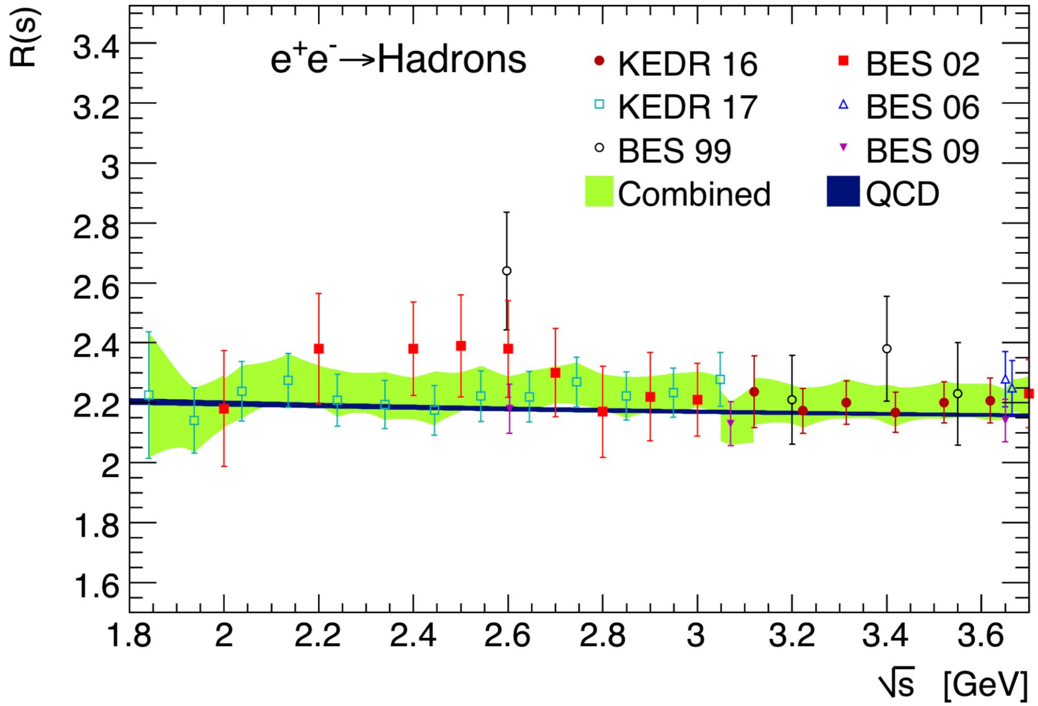
→ Tension between measurements

→ a_μ[→1.8GeV]: 23.08 ± 0.20 (stat.) ± 0.40 (syst.) [10^{-10}] (enhancement x 2.2)

Combination for the $e^+e^- \to KK\pi$ and $KK2\pi$ channels



Contributions from the 1.8 – 3.7 GeV region



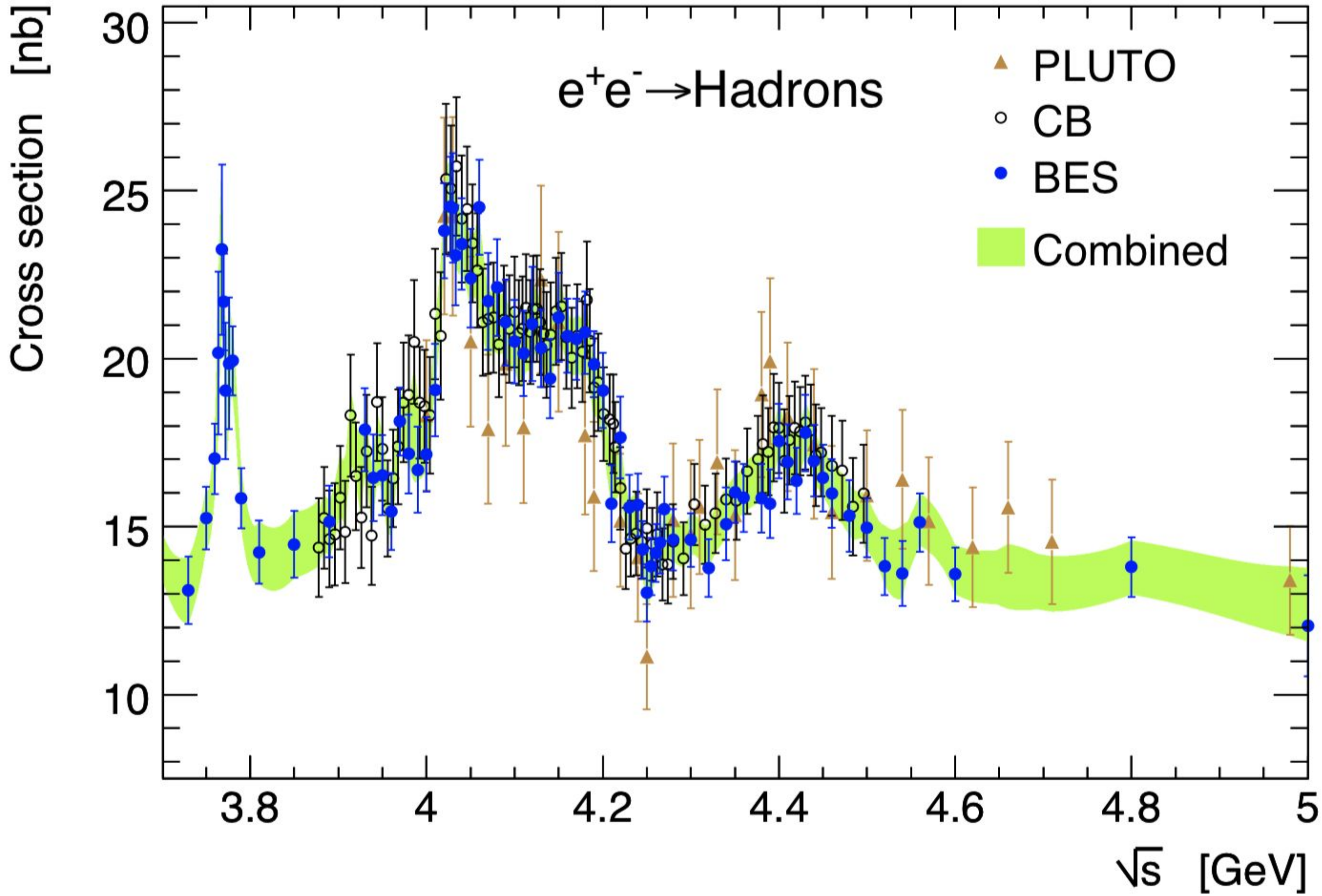
→ Contribution evaluated from pQCD (4 loops) + O(α_s^2) quark mass corrections

→ Uncertainties: α_s, truncation of perturbative series, CIPT/FOPT, m_q

→ 1.8-2.0 GeV: 7.65±0.31(data excl.); 8.30±0.09(QCD); added syst. 0.65 [10^{-10}]

→ 2.0-3.7 GeV: 25.82±0.61(data); 25.15 ± 0.19(QCD); agreement within 1σ

Contributions from the charm resonance region

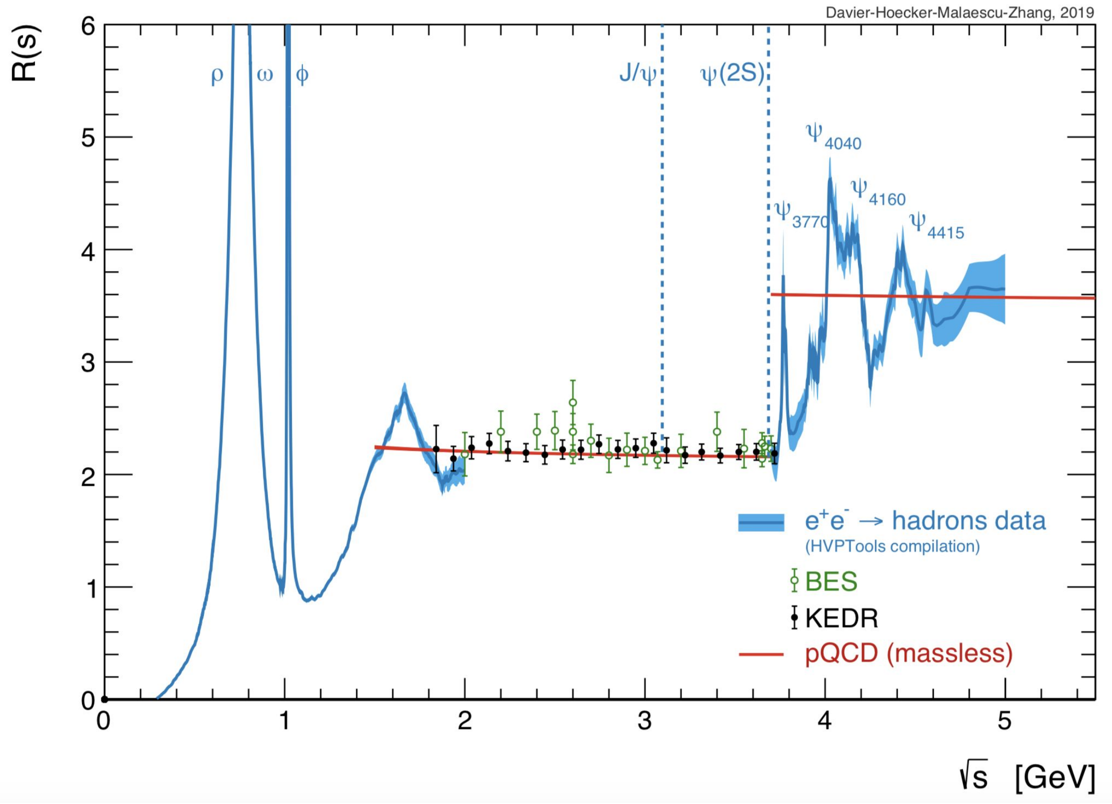


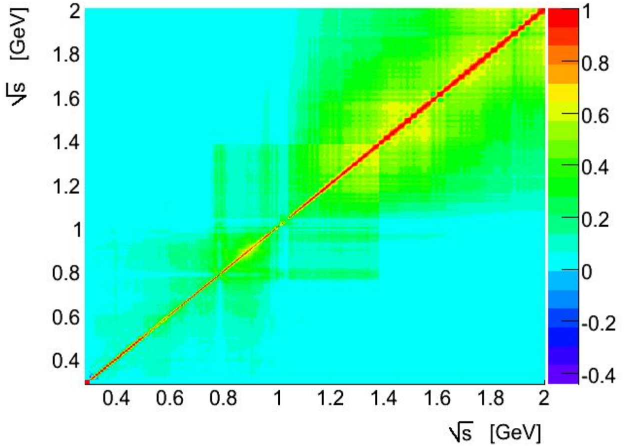
Situation in arXiv:1908.00921 (EPJC)

Channel	$a_\mu^{\rm had,LO}$ [10^{-10}]	$\Delta\alpha_{\rm had}(m_Z^2)$ [10^{-4}]
$\pi^0\gamma$	$4.41 \pm 0.06 \pm 0.04 \pm 0.07$	$0.35 \pm 0.00 \pm 0.00 \pm 0.01$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$507.85 \pm 0.83 \pm 3.23 \pm 0.55$	$34.50 \pm 0.06 \pm 0.20 \pm 0.04$
$\pi^+\pi^-\pi^0$	$46.21 \pm 0.40 \pm 1.10 \pm 0.86$	$4.60 \pm 0.04 \pm 0.11 \pm 0.08$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	$3.58 \pm 0.01 \pm 0.07 \pm 0.03$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	$4.45 \pm 0.02 \pm 0.12 \pm 0.07$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$	$0.21 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$	$0.15 \pm 0.01 \pm 0.03 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$	$0.25 \pm 0.02 \pm 0.02 \pm 0.05$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$	$0.03 \pm 0.00 \pm 0.03 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$	$0.35 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$	$0.12 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$	$0.20 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega 2\pi$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi,\pi\gamma,\eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$	$3.35 \pm 0.03 \pm 0.05 \pm 0.03$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	$1.74 \pm 0.01 \pm 0.03 \pm 0.02$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$	$0.78 \pm 0.02 \pm 0.03 \pm 0.02$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	$0.30 \pm 0.01 \pm 0.02 \pm 0.00$
$K\overline{K}\omega$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$	$0.11 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$	$0.00 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (BW integral)	6.20 ± 0.11	7.00 ± 0.13
$\psi(2S)$ (BW integral)	1.56 ± 0.05	2.48 ± 0.08
R data [3.7 − 5.0] GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
$R_{\rm QCD}$ [1.8 − 3.7 GeV]$_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\rm dual}$	$24.27 \pm 0.18 \pm 0.28_{\rm dual}$
$R_{\rm QCD}$ [5.0 − 9.3 GeV]$_{udsc}$	6.86 ± 0.04	34.89 ± 0.18
$R_{\rm QCD}$ [9.3 − 12.0 GeV]$_{udscb}$	1.20 ± 0.01	15.53 ± 0.04
$R_{\rm QCD}$ [12.0 − 40.0 GeV]$_{udscb}$	1.64 ± 0.00	77.94 ± 0.13
$R_{\rm QCD}$ [> 40.0 GeV]$_{udscb}$	0.16 ± 0.00	42.70 ± 0.05
$R_{\rm QCD}$ [> 40.0 GeV]$_t$	0.00 ± 0.00	-0.72 ± 0.01
Sum	$694.0 \pm 1.0 \pm 3.5 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\rm QCD}$	$275.29 \pm 0.15 \pm 0.72 \pm 0.23 \pm 0.15_\psi \pm 0.55_{\rm QCD}$

→ 32 exclusive channels are integrated up to 1.8 GeV

→ Only 0.016 ± 0.016% in missing (estimated) channels for a_μ





Sum of *32 exclusive channels* with *full propagation of correlations*

→ Performed non-trivial check:
a_μ from sum of individual channels and from Ree integral < 1.8 GeV

Theory initiative: prepare the Standard Model prediction for $(g-2)_\mu$

UPCOMING WORKSHOPS

Fourth Plenary Workshop of the Muon g-2 Theory Initiative
https://www-conf.kek.jp/muong-2theory/
A virtual workshop hosted by KEK (Tsukuba, Japan), to be held from 28 June - 02 July 2021.

PAST WORKSHOPS

The hadronic vacuum polarization from lattice QCD at high precision
https://indico.cern.ch/event/956699/
A virtual topical workshop of the Muon g-2 Theory Initiative, 16-20 Nov 2020.

Hadronic contributions to (g-2) μ
https://indico.fnal.gov/event/21626/
held at the Institute for Nuclear Theory, University of Washington, Seattle, WA, 9-13 September 2019

Second workshop of the Muon g-2 Theory Initiative
https://wwwth.kph.uni-mainz.de/g-2/
held at the Helmholtz Institute Mainz, University of Mainz, Mainz, Germany, 18-22 June 2018

Muon g-2 Theory Initiative Hadronic Light-by-Light working group workshop
https://indico.phys.uconn.edu/event/1/
held at the University of Connecticut, Storrs, CT, 12-14 March 2018

Workshop on Hadronic Vacuum Polarization Contributions to Muon g-2
https://www-conf.kek.jp/muonHVPws/
held at KEK, Tsukuba, Japan, 12-14 Feb 2018

First workshop of the Muon g-2 Theory Initiative
https://indico.fnal.gov/event/13795/
held in St. Charles, IL, USA, 3-6 June 2017

Put together in a *coherent & conservative* way the results of various groups, *before the Fermilab result*

https://muon-gm2-theory.illinois.edu

White Paper: arXiv:2006.04822 (Phys. Rept.)

Theory initiative white paper: executive summary

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

→ Dominant uncertainty: HVP LO → Based on *merging of model-independent methods*

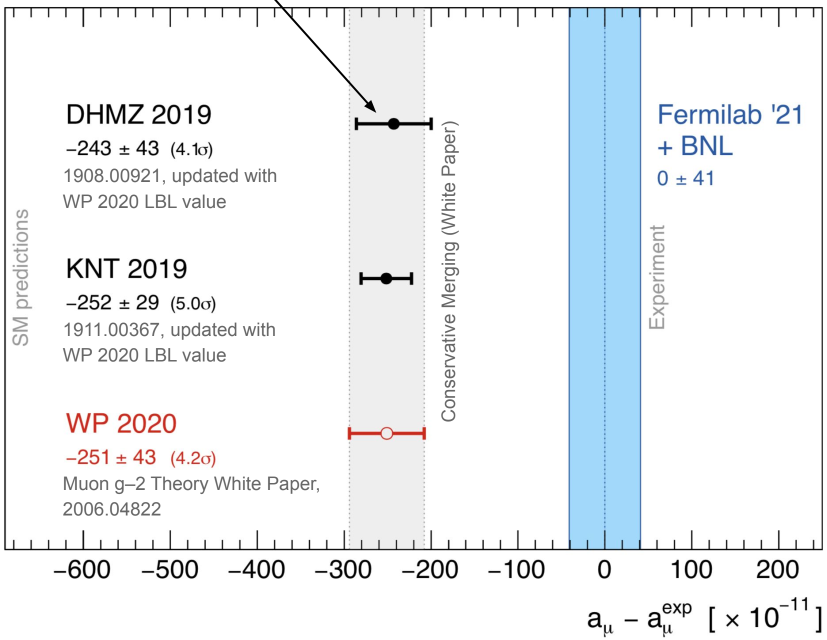
→ HLbL also has an important uncertainty

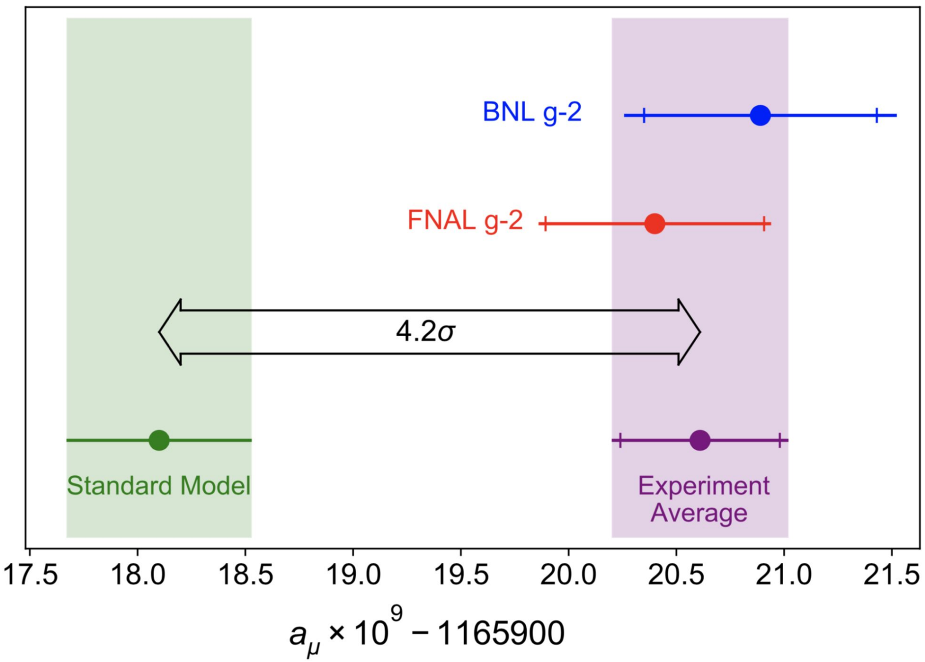
→ Lattice results become more and more interesting (*see next talk*)

→ A tension between the BNL measurement and SM prediction: ~ 3.7 σ

Status of a_μ

Important to account for BABAR-KLOE diff. & inter-channel correlations





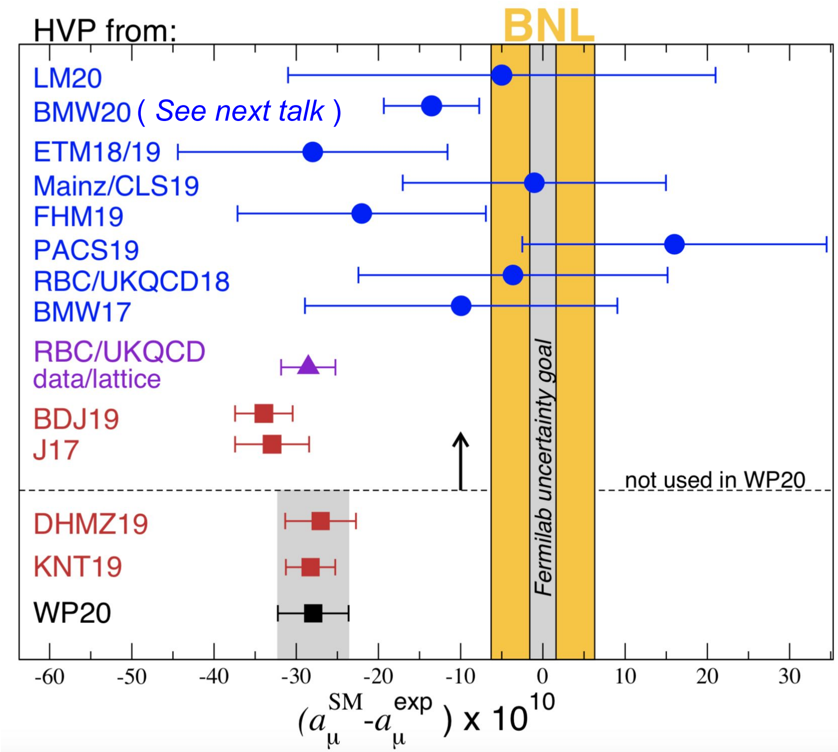
→ Caution about significance:
- statistics-dominated measurement
- prediction uncertainty limited by non-Gaussian systematic effects

→ Nevertheless, large discrepancy between measurement and SM

Impact of correlations between a_μ and α_{QED} on the EW fit

2008.08107(BM, Matthias Schott)

See also: Crivellin et al, 2003.04886; Keshavarzi et al., 2006.12666 ;de Rafael, 2006.13880; Colangelo et al, 2010.07943



Approaches considered for treating the a_μ - α_{QED} correlations

Studied approaches probing different hypotheses concerning the possible source(s) of the a_μ tension(s) :

(0) Scaling factor applied to the HVP contribution from some energy range of the hadronic spectrum

→ Approaches taking into account (*for the first time*) the full correlations between the uncertainties of the HVP contributions to a_μ and α_{QED} , based on input from DHMZ 19 (arXiv:1908.00921):
correlations between points/bins of a measurement in a given channel, between different measurements in the same channel, between different channels; full treatment of the BABAR-KLOE tension in the $\pi^+\pi^-$ channel

Computation (Energy range)	$a_\mu^{\text{HVP, LO}}$ $[10^{-10}]$	$\Delta\alpha_{\text{had}}(M_Z^2)$ $[10^{-4}]$	ρ
Phenomenology (Full HVP)	694.0 ± 4.0	275.3 ± 1.0	44%
Phenomenology ([Th.; 1.8 GeV])	635.5 ± 3.9	55.4 ± 0.4	86%
Phenomenology ([Th.; 1 GeV])	539.8 ± 3.8	36.3 ± 0.3	99.5%
Lattice (Full HVP)BMW 20 (v1)	712.4 ± 4.5	-	-

(1) Cov. matrix of a_μ and α_{QED} (Pheno) described by a nuisance parameter (NP_1) impacting both quantities (used to shift a_μ to some "target" value - coherent shift applied to α_{QED}) and another one (NP_2) impacting only α_{QED} (used in the EW fit)
Note: "target" values chosen in order to reach agreement with the BMW 20 prediction / Experimental a_μ ($\pm 1\sigma$)

Uncertainty components	$a_\mu^{\text{HVP, LO}}$	$\Delta\alpha_{\text{had}}(M_Z^2)$
NP_1	$\sigma(a_\mu^{\text{HVP, LO}})$	$\sigma(\Delta\alpha_{\text{had}}(M_Z^2)) \cdot \rho$
NP_2	0	$\sigma(\Delta\alpha_{\text{had}}(M_Z^2)) \cdot \sqrt{1-\rho^2}$

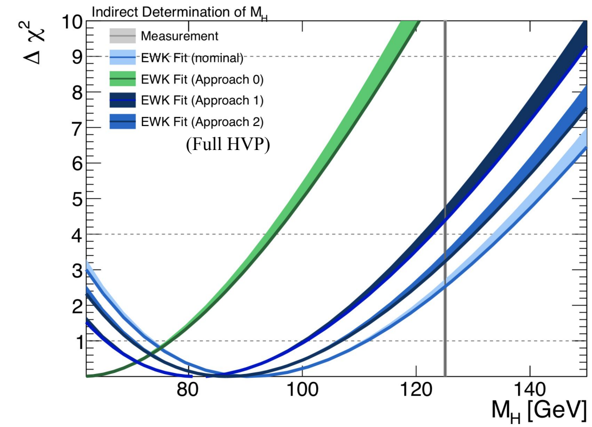
(2) Include the HVP contribution to a_μ as extra parameter in the EW fit, constrained by the Pheno & BMW 20 values
Note: Also accounted for the coherent impact of α_S on the HVP contribution and on the EW fit

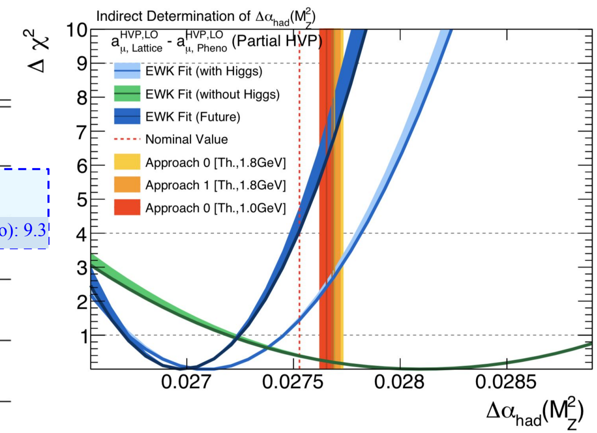
Results: comparing the Phenomenology & BMW 20 values

$a_\mu^{\text{HVP, LO}}$ shift (Energy range)	*Approach 0* Scaling factor	*Approach 0* $\Delta'\alpha_{\text{had}}(M_Z^2)$	*Approach 1* Shift NP$_1$	*Approach 1* $\sigma'\left(\Delta\alpha_{\text{had}}(M_Z^2)\right)$	*Approach 1* $\Delta'\alpha_{\text{had}}(M_Z^2)$
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ (Full HVP)	1.027	0.02826	4.6	$9.0 \cdot 10^{-5}$	0.02774
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ (Full HVP)	1.020	0.02808	3.5	$9.0 \cdot 10^{-5}$	0.02769
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.8 GeV])	1.029	0.02769	4.7	$9.5 \cdot 10^{-5}$	0.02768
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.8 GeV])	1.022	0.02765	3.5	$9.5 \cdot 10^{-5}$	0.02764
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1 GeV])	1.034	0.02765	-	-	-
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1 GeV])	1.026	0.02762	-	-	-

→ Large scaling factors (w.r.t. exp. uncertainties) & significant shifts of NP$_1$

$a_\mu^{\text{HVP, LO}}$ shift (Energy range)	Nominal $\Delta'\alpha_{\text{had}}(M_Z^2)$	Nominal χ^2/ndf	*Approach 0* $\Delta'\alpha_{\text{had}}(M_Z^2)$	*Approach 0* χ^2/ndf	*Approach 1* $\Delta'\alpha_{\text{had}}(M_Z^2)$	*Approach 1* χ^2/ndf	*Approach 2* $\Delta'\alpha_{\text{had}}(M_Z^2)$	*Approach 2* χ^2/ndf
	0.02753	18.6/16 (p=0.29)	-	-	-	-	0.02753	28.1/17 (p=0.04) χ^2(BMW20-Pheno): 9.3
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ (Full HVP)	-	-	0.02826	27.6/16 (p=0.04)	0.02774	20.3/16 (p=0.21)	-	
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.8 GeV])	-	-	0.02769	19.9/16 (p=0.22)	0.02768	19.8/16 (p=0.23)	-	-
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.0 GeV])	-	-	0.02765	19.6/16 (p=0.24)	-	-	-	-

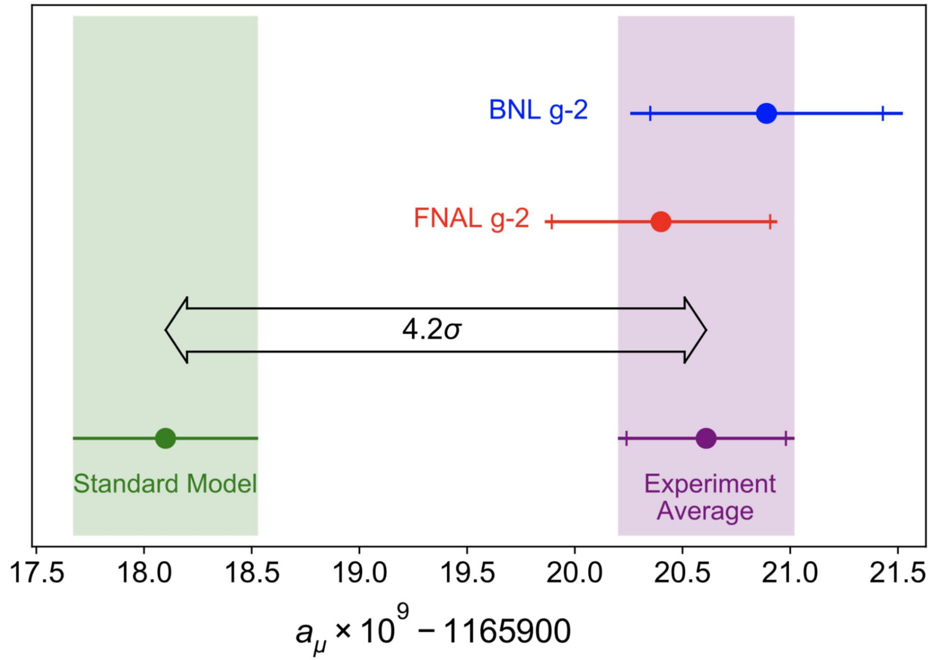




→ Addressing the BMW 20 - Pheno difference for a_μ has little impact on the EW fit, except for the unrealistic scenario rescaling the full HVP contribution

Note: Similar conclusions for the comparison with the Experimental a_μ value (see backup)

We have an interesting, long standing, multifaceted problem to solve...



???

Backup

Lepton Magnetic Anomaly: from Dirac to QED

$$\vec{\mu} = g\frac{e}{2m}\vec{s} \qquad\qquad a = \frac{g-2}{2}$$

Dirac (1928) g_e=2 a_e=0

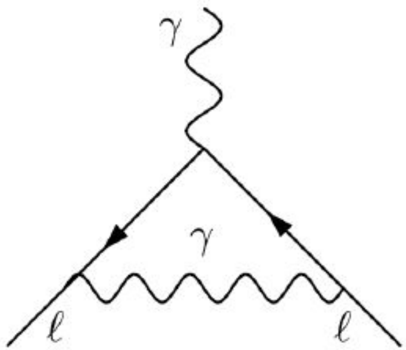
anomaly discovered:

Kusch-Foley (1948) $a_e = (1.19 \pm 0.05)\ 10^{-3}$

and explained by O(α) QED contribution:

Schwinger (1948) $a_e = \alpha/2\pi = 1.16\ 10^{-3}$

first triumph of QED



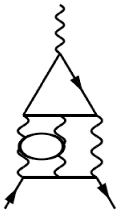
⇒ a_e sensitive to quantum fluctuations of fields

More Quantum Fluctuations

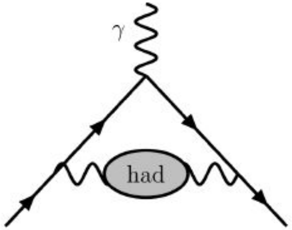
$$a = a^{\text{QED}} + a^{\text{had}} + a^{\text{weak}} \qquad + \,?\; a^{\text{new physics}}\, ?$$

typical contributions:

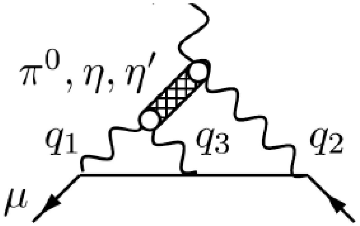
QED up to O(α^5) (Kinoshita et al.)



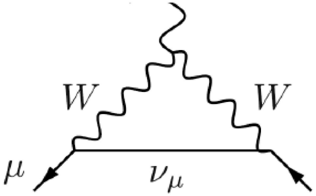
Hadrons vacuum polarization



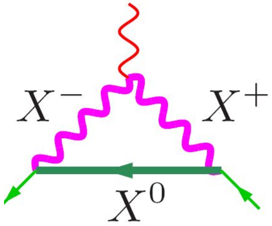
light-by-light (dispersive & lattice QCD)



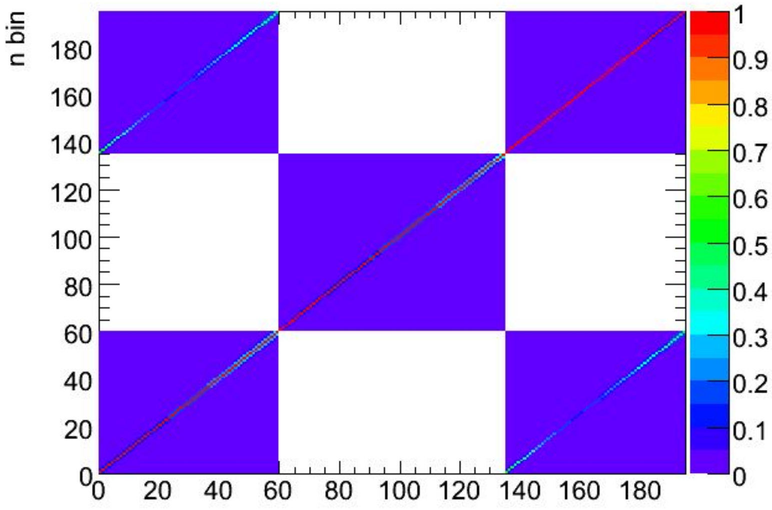
Electroweak

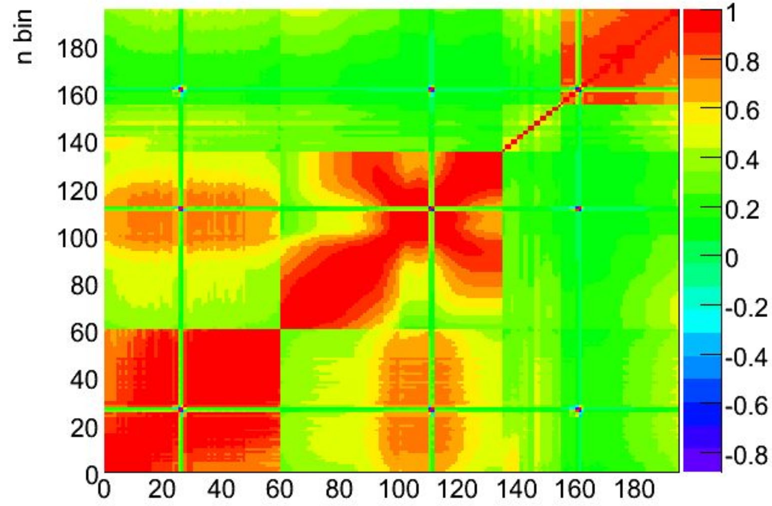


new physics at high mass scale



$\delta a_l \propto \frac{m_l^2}{M^2}$ $\Rightarrow$ a_μ much more sensitive to high scales



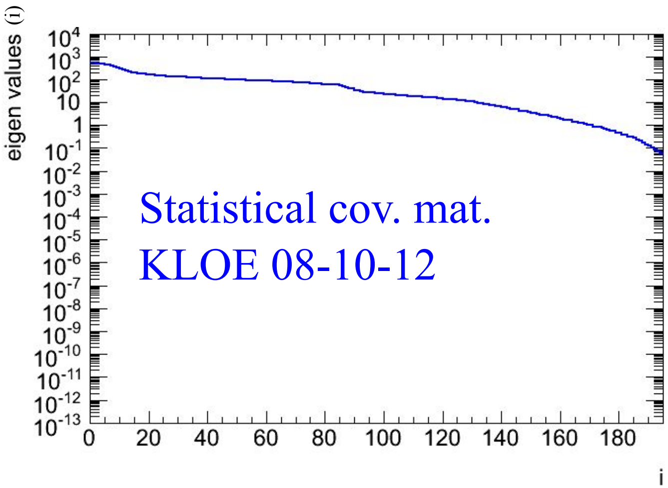


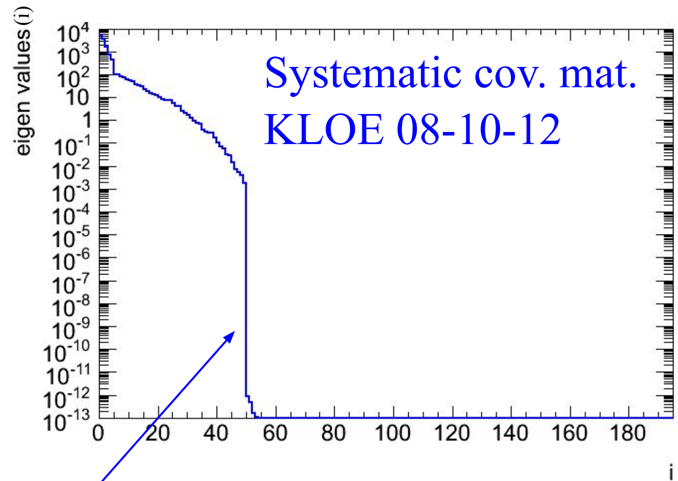
KLOE: *08* *10* *12* n bin

KLOE: *08* *10* *12* n bin

→ Statistical and systematic correlation matrices among the 3 measurements

Treatment of the KLOE data – eigenvector decomposition





$$C = S \cdot D \cdot S^T$$

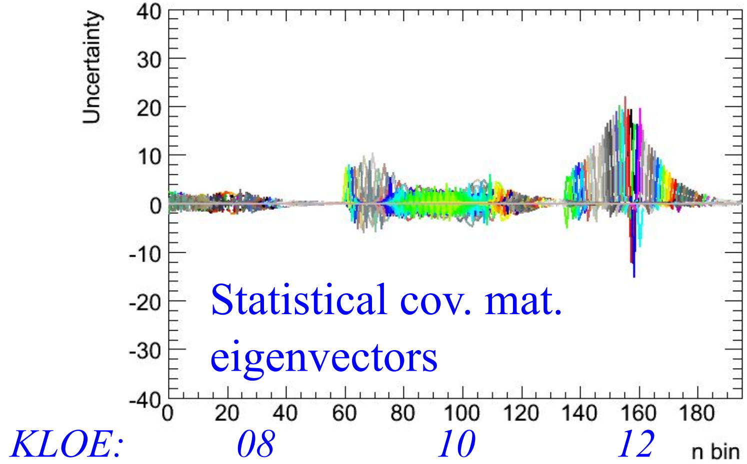
$$D = \begin{pmatrix} \ddots & 0 & 0 \\ 0 & \sigma_i^2 & 0 \\ 0 & 0 & \ddots \end{pmatrix}$$

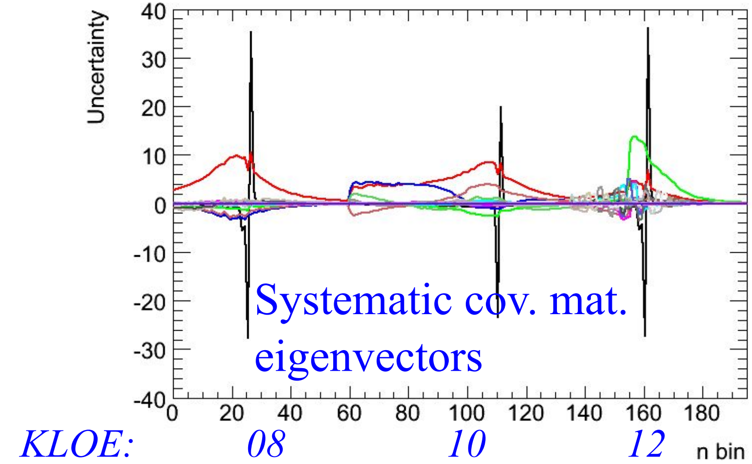
$$S = (V_1 \quad \dots \quad V_n)$$

→ "counting" the number of independent components (50) used to build the covariance matrix

→ Problem of negative eigenvalues for previous systematic covariance matrix solved
(informed KLOE collaboration about the problem in summer 2016)

Treatment of the KLOE data – eigenvector decomposition





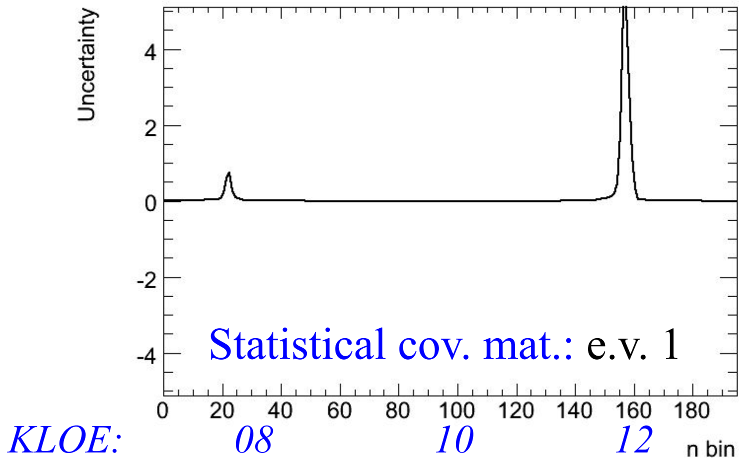
→ Each normalized eigenvector (σ_i*V_i) treated as an uncertainty fully correlated between the bins

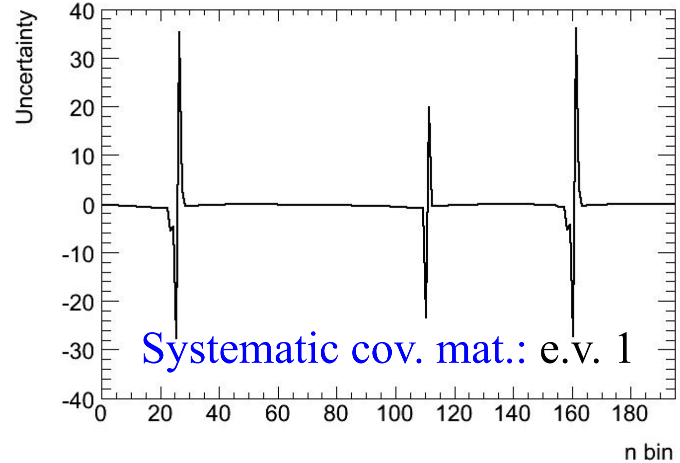
→ All these uncertainties are independent between each-other

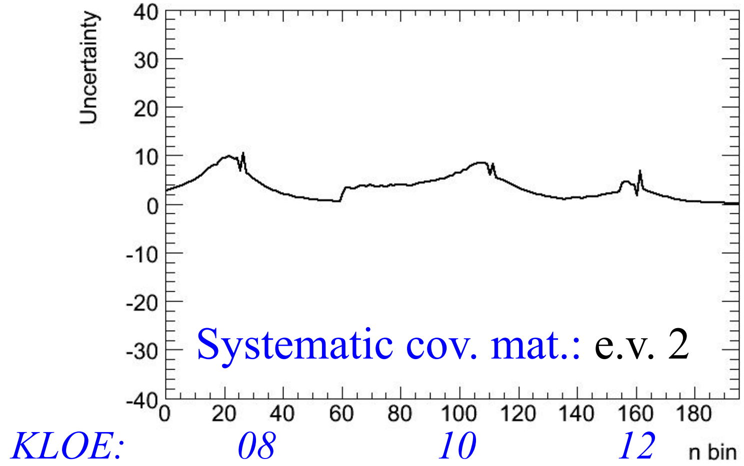
$$C = \sum_{i=1}^{N_{bins}} \sigma_i^2 \cdot C(V_i)$$

→ Checked exact matching with the original matrices + with all a_μ integrals and uncertainties published by KLOE

Treatment of the KLOE data – eigenvector decomposition



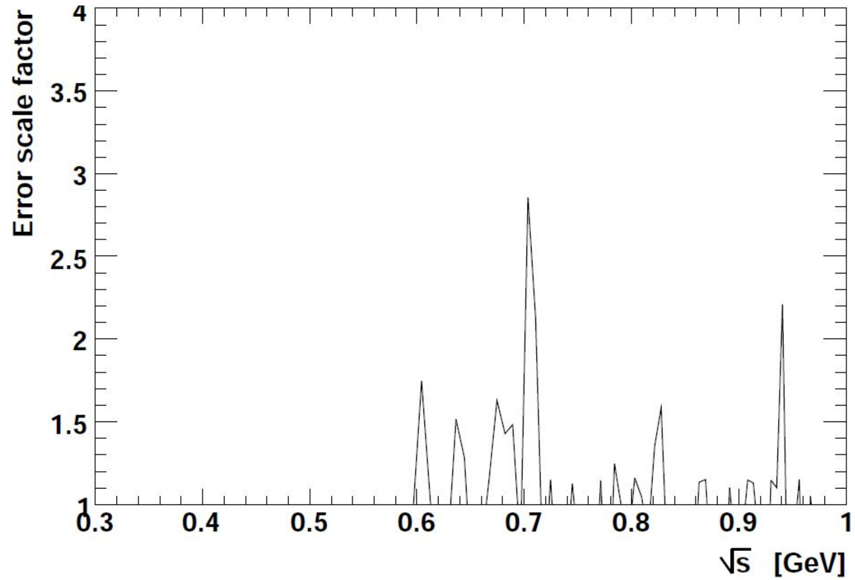




→ Eigenvectors carry the general features of the correlations:

- long-range for systematics
- ~short-range for statistical uncertainties + correlations between KLOE 08 & 12

Local comparison of the 3 KLOE measurements



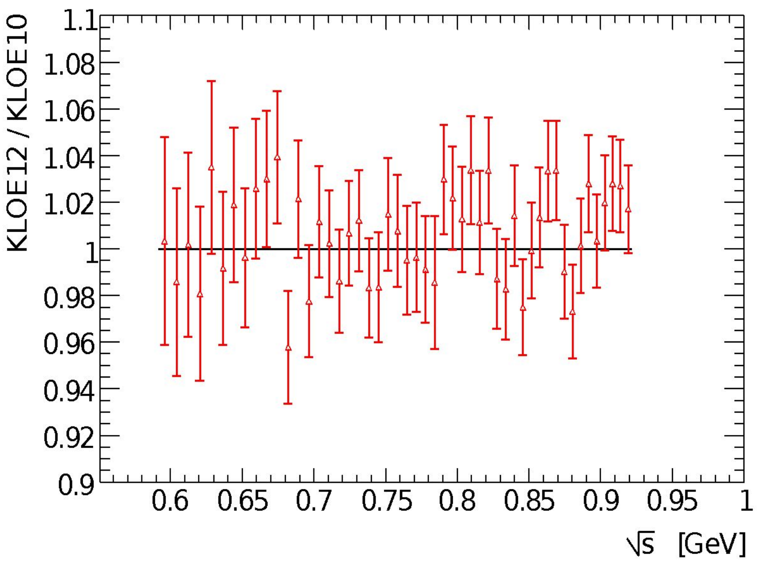
→ Local χ^2 /ndof test of the local compatibility between KLOE 08 & 10 & 12, taking into account the correlations: some tensions observed

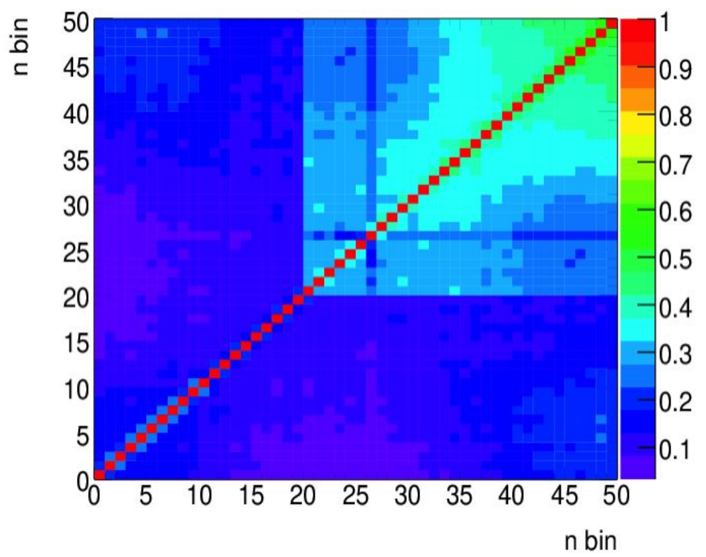
→ Does not probe general trends of the difference between the measurements (e.g. slopes in the ratio)

Ratios between measurements

→ Compute ratio between pairs of KLOE measurements

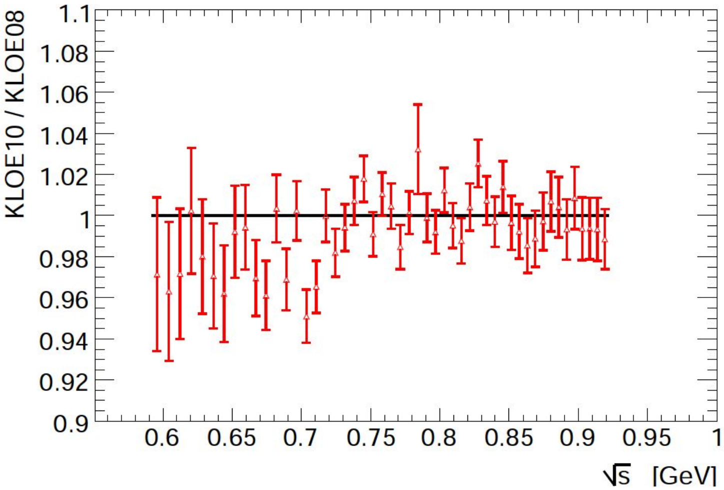
→ Full propagation of uncertainties and correlations using pseudo-experiments (agreement with analytical linear uncertainty propagation)

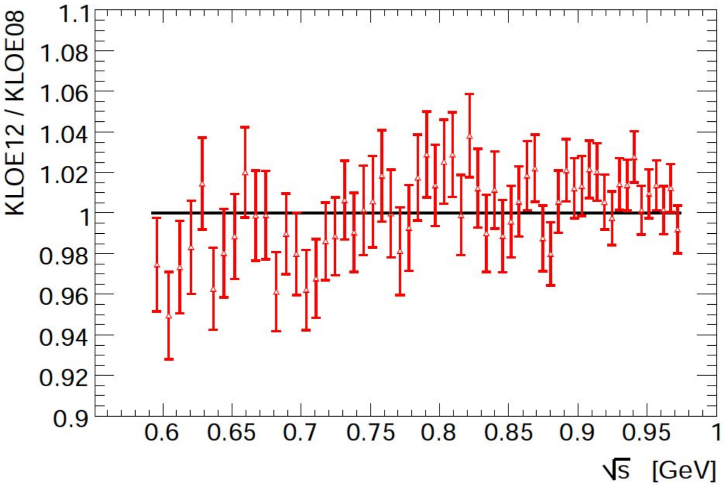


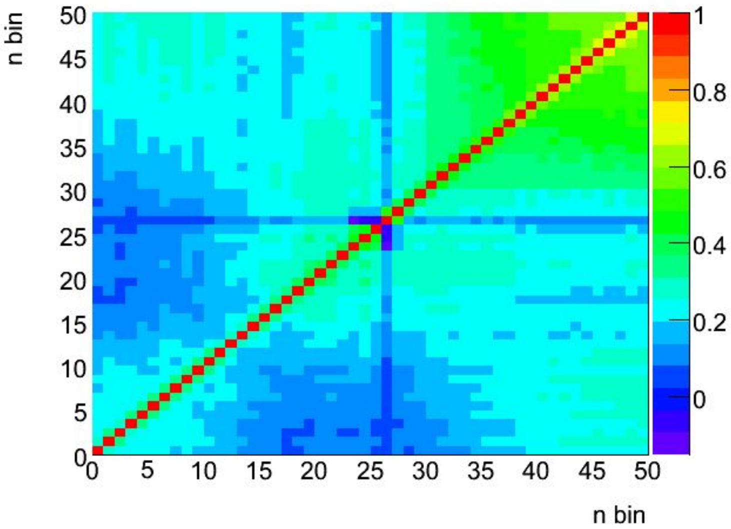


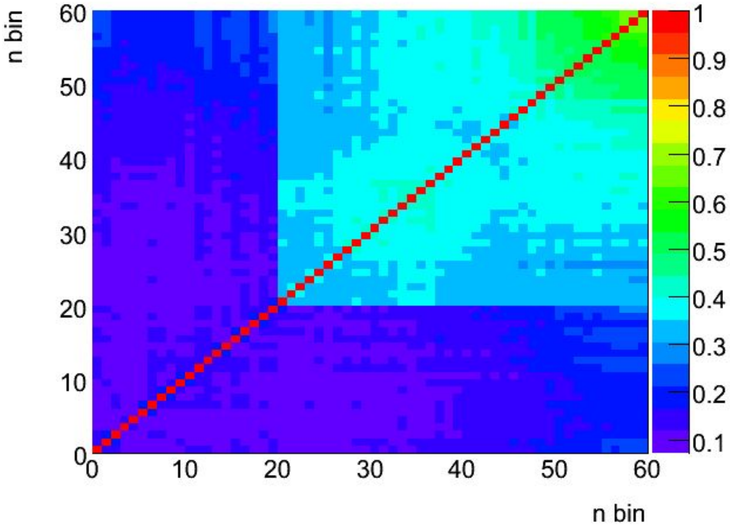
→ Good agreement between KLOE 10 and KLOE 12

Ratios between measurements









Direct comparison of the 3 KLOE measurements

→ Quantitative comparison between the ratios and unity, taking into account correlations

KLOE 10 / KLOE 08

χ^2 [0.35;0.85] GeV^2 : 79.0 / 50(DOF)
p-value= 0.0056

χ^2 [0.35;0.58] GeV^2 : 46.2 / 23(DOF)
p-value= 0.0028

χ^2 [0.58;0.85] GeV^2 : 29.7 / 27(DOF)
p-value= 0.33

χ^2 [0.64;0.85] GeV^2 : 20.7 / 21(DOF)
p-value= 0.47

KLOE 12 / KLOE 08

χ^2 [0.35;0.95] GeV^2 : 73.7 / 60(DOF)
p-value= 0.11

χ^2 [0.35;0.58] GeV^2 : 21.8 / 23(DOF)
p-value= 0.53

χ^2 [0.35;0.64] GeV^2 : 27.5 / 29(DOF)
p-value= 0.55

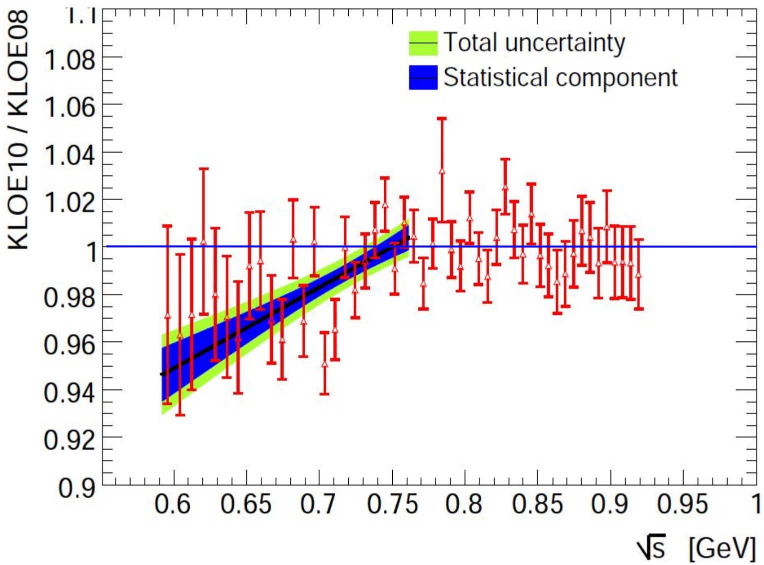
χ^2 [0.64;0.95] GeV^2 : 39.4 / 31(DOF)
p-value= 0.14

Quantitative comparisons of the KLOE measurements

→ Quantitative comparison between the ratios and unity, taking into account correlations

→ Fitting the ratio taking into account correlations

→ Full propagation of uncertainties and correlations – 3 methods yielding consistent results: ±1σ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit



Comparison with Unity:

χ^2 [0.35;0.85] GeV^2 : 79.0 / 50(DOF)

p-value= 0.0056

χ^2 [0.35;0.58] GeV^2 : 46.2 / 23(DOF)

p-value= 0.0028

χ^2 [p0 + p1√s]: 36.1 / 21(DOF)

p-value= 0.02

p0 : 0.745 ± 0.085

p1 : 0.341 ± 0.117

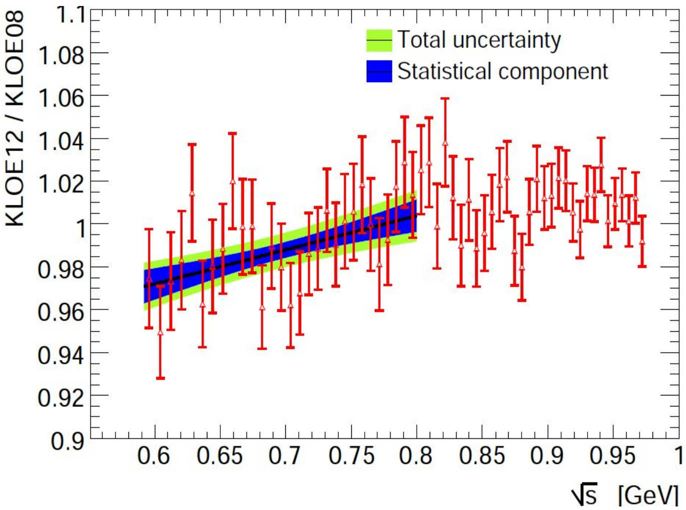
→ Significant shift & slope (~2.5-3σ) at low √s, no significant shift at high √s
Similar shift & slope for KLOE 12 / KLOE 08 (*see below*)

→ Should motivate conservative treatment of uncertainties and correlations in combination

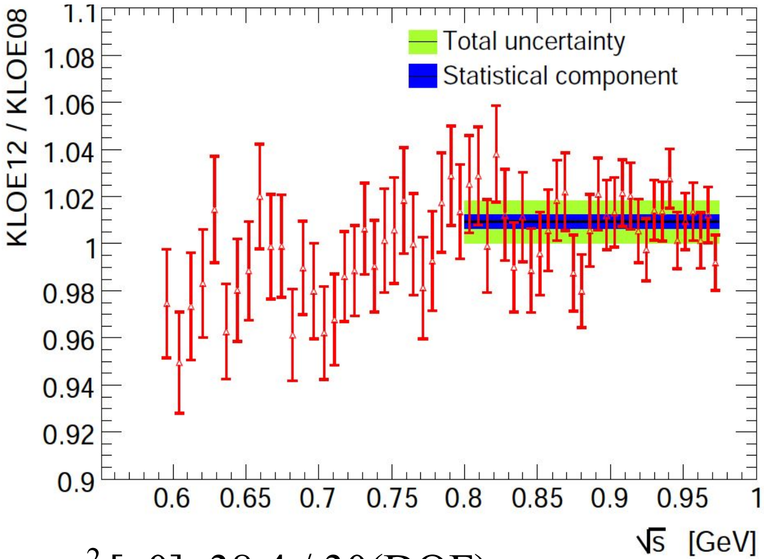
Direct comparison of the 3 KLOE measurements

→ Fitting the ratio taking into account correlations

→ Full propagation of uncertainties and correlations – 3 methods yielding consistent results: ±1σ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit



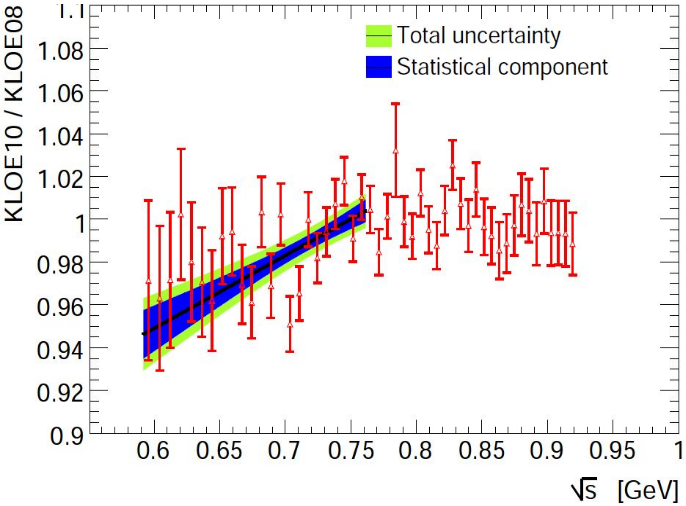
χ^2 [p0 + p1√s]: 20.7 / 27(DOF)
p-value= 0.80
p0 : 0.876 ± 0.056
p1 : 0.159 ± 0.081



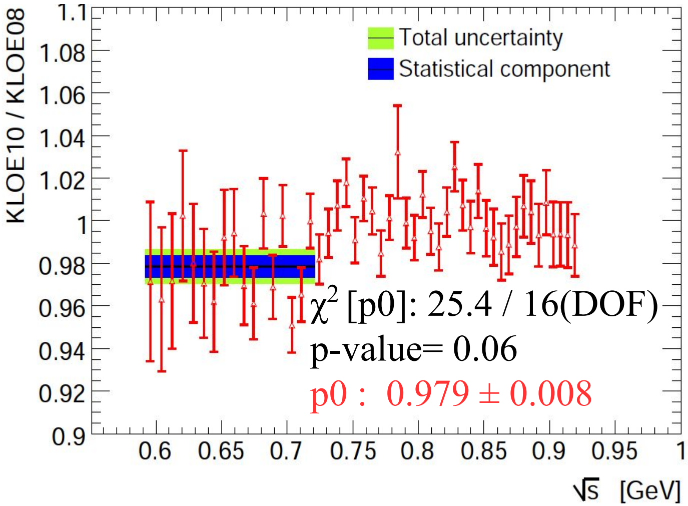
χ^2 [p0]: 38.4 / 30(DOF)
p-value= 0.14
p0 : 1.009 ± 0.009

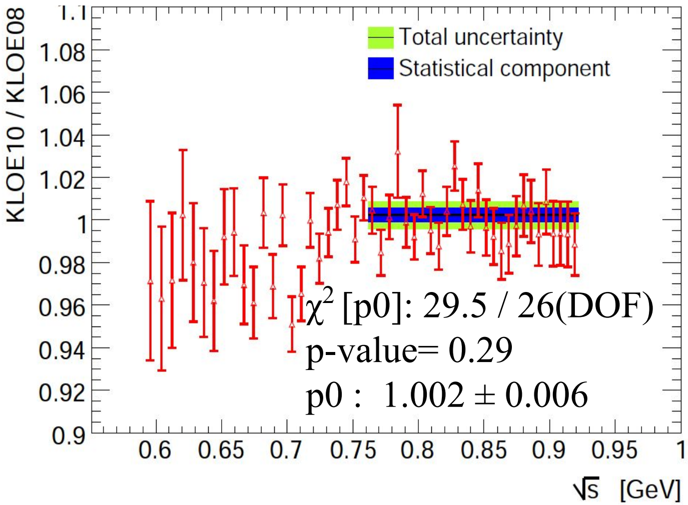
→ Significant shift and slope (~2σ) at low √s, no significant shift at high √s

Direct comparison of the 3 KLOE measurements

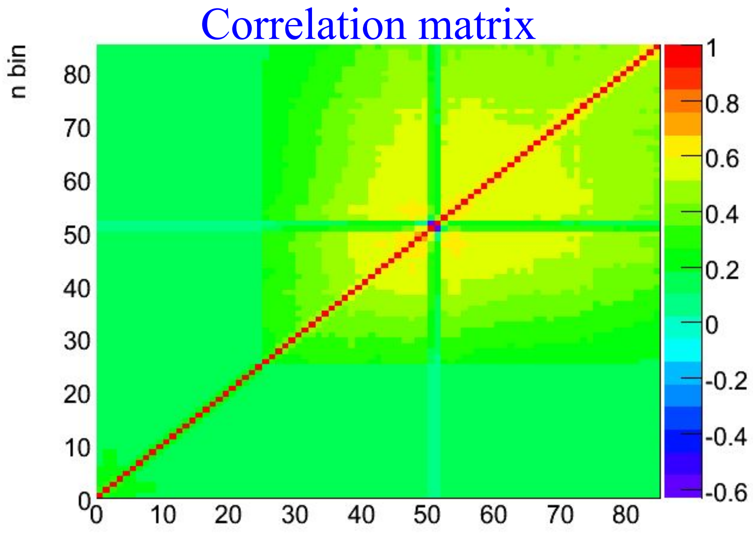


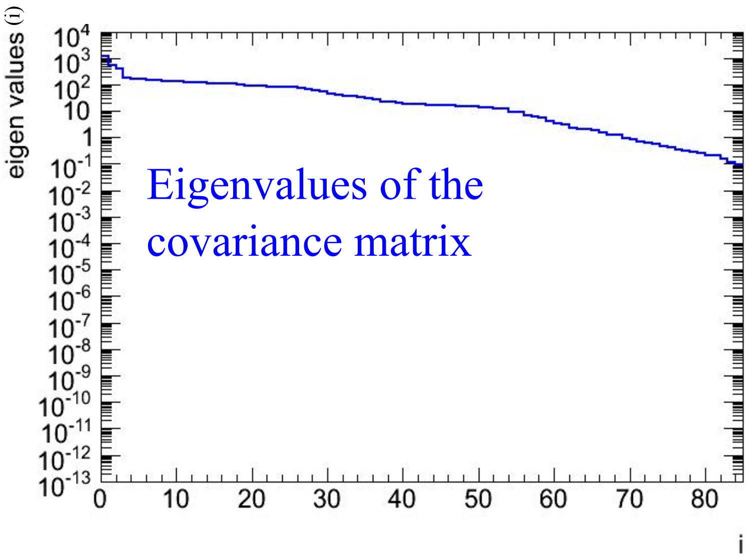
χ^2 [p0 + p1√s]: 36.1 / 21(DOF)
p-value= 0.02
p0 : 0.745 ± 0.085
p1 : 0.341 ± 0.117

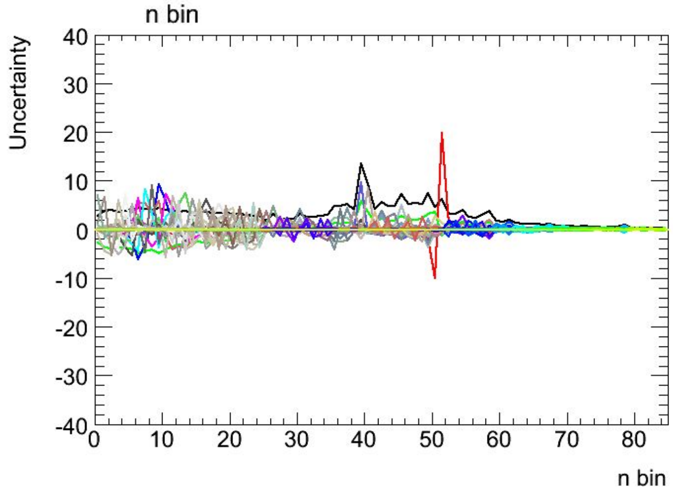




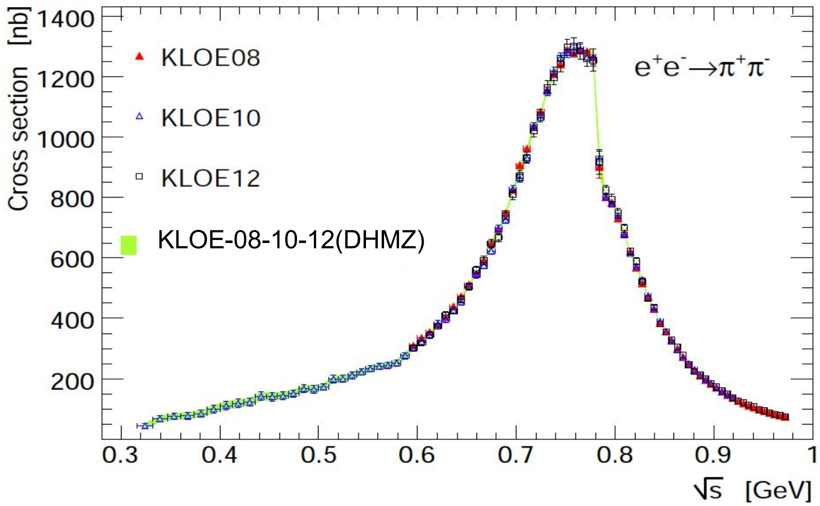
→ Significant shift and slope (~2.5-3σ) at low √s, no significant shift at high √s

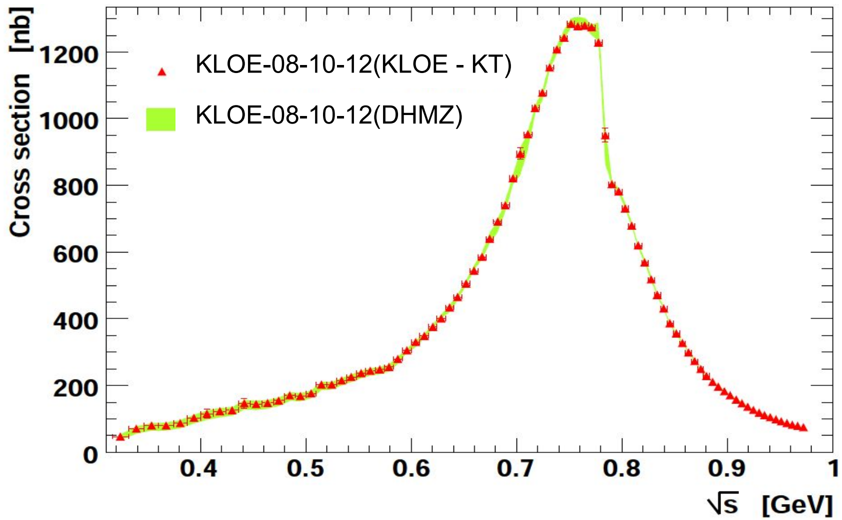

Correlation matrix
n bin
n bin
0
10
20
30
40
50
60
70
80
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6

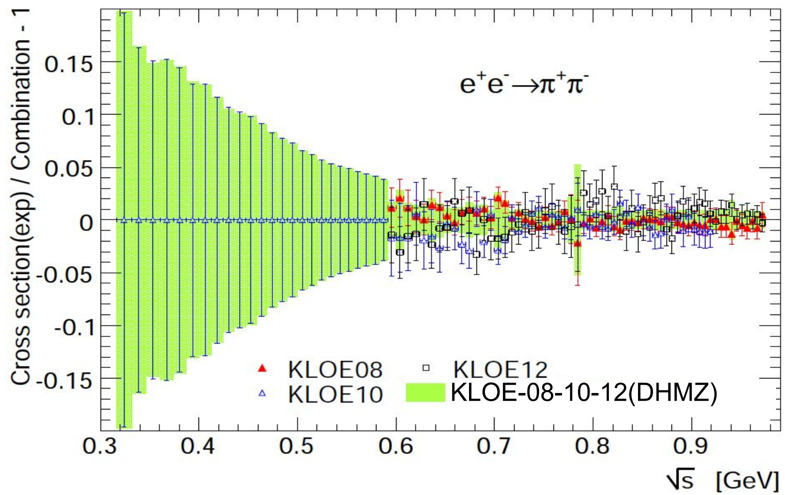

eigen values (i)
Eigenvalues of the
covariance matrix
i

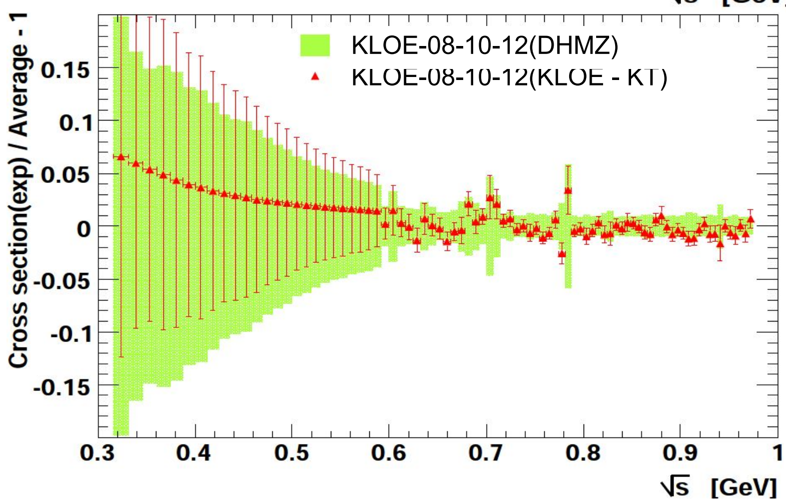

Uncertainty
n bin
40
30
20
10
0
-10
-20
-30
-40

Combining the 3 KLOE measurements









$a_\mu^{\pi\pi}$ contribution [0.28; 1.8] GeV – spline-based (2018)

→ Updated result:

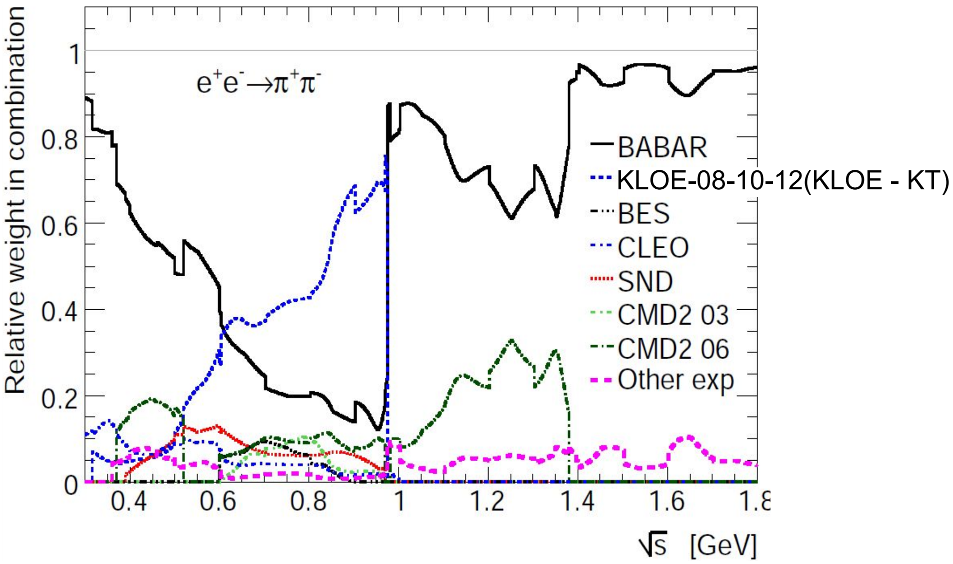
506.70 ± 2.32 (± 1.01 (stat.) ± 2.08 (syst.)) [10^{-10}]

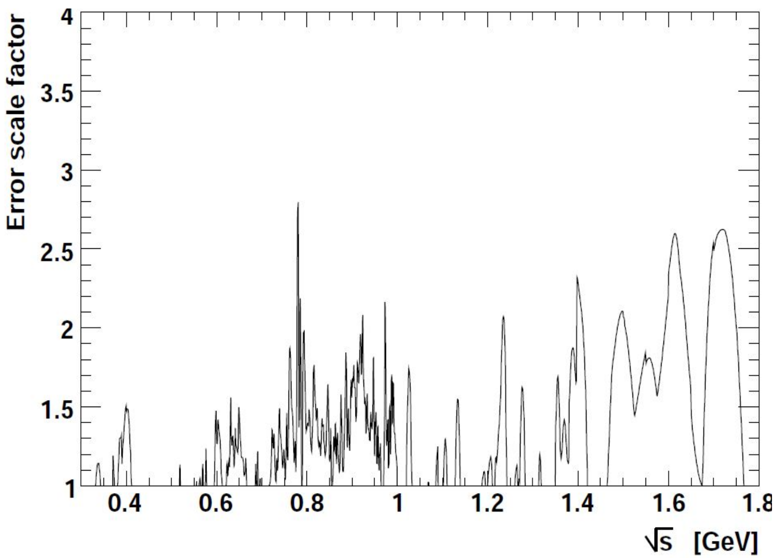
(after uncertainty enhancement by ~14% caused by the tension between inputs, taken into account through a local rescaling)

Total uncertainty: 5.9 (2003) → 2.8 (2011) → 2.6 (2017) → 2.3 (2018)

$a_\mu^{\pi\pi}$ contribution [0.28; 1.8] GeV – spline-based (2018)

→ with KLOE-08-10-12 (KLOE-KT) used as input: 506.55 ± 2.38 [10^{-10}]
(after uncertainty enhancement by 18% caused by the tension between inputs, taken into account through a local rescaling)
→ Compensation between uncertainty reduction for KLOE-08-10-12 (KLOE-KT), inducing a change of weights in DHMZ combination, and tension enhancement





Fit parameters, uncertainties and correlations $e^+e^- \to \pi^+\pi^-$

	α_V	$\kappa[10^{-4}]$	B_0	B_1	m_ρ [MeV]	m_ω [MeV]
α_V	0.133 ± 0.020	0.52	-0.45	-0.97	0.90	-0.25
$\kappa[10^{-4}]$		21.6 ± 0.5	-0.33	-0.57	0.64	-0.08
B_0			1.040 ± 0.003	0.40	-0.40	0.29
B_1				-0.13 ± 0.11	-0.96	0.20
m_ρ [MeV]					774.5 ± 0.8	-0.17
m_ω [MeV]						782.0 ± 0.1

→ κ corresponds to a Br $(\omega \to \pi^+\pi^-)$ of $(2.09 \pm 0.09) \cdot 10^{-2}$, in agreement with the result extracted from the fit of arXiv:1810.00007, $(1.95 \pm 0.08) \cdot 10^{-2}$. Both values disagree with the PDG average $(1.51 \pm 0.12) \cdot 10^{-2}$, dominated by the result of arXiv:1611.09359 which uses fits to essentially the same data.

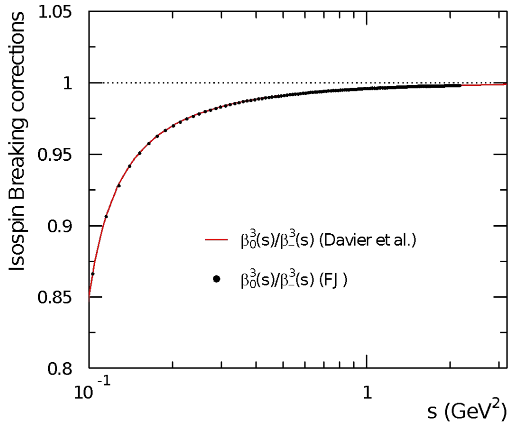
→ The fitted ω mass is found to be lower than the PDG average obtained from 3π decays by $(0.65 \pm 0.12 \pm 0.12_{PDG})$ MeV, in agreement with previous fits of the $\rho - \omega$ interference in the 2π spectrum (see e.g. arXiv:1205.2228 and arXiv:1810.00007).

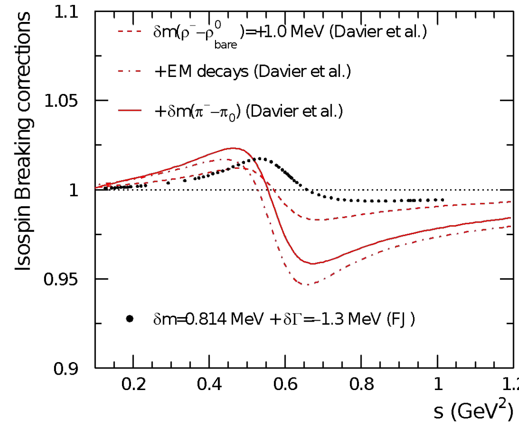
Comparison with IB-corrected τ data

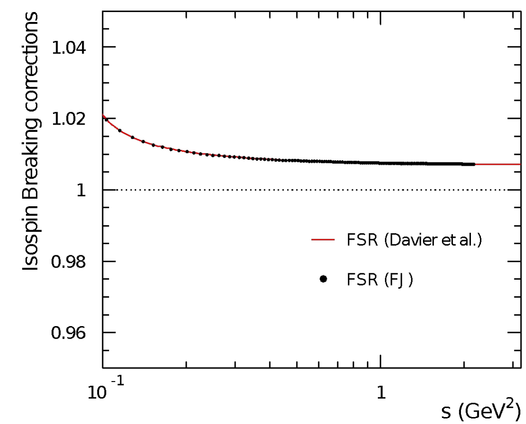
$$v_{1,X^-}(s) = \frac{m_\tau^2}{6|V_{ud}|^2} \frac{\mathcal{B}_{X^-}}{\mathcal{B}_e} \frac{1}{N_X} \frac{dN_X}{ds} \times \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)^{-1} \frac{R_{\rm IB}(s)}{S_{\rm EW}}$$

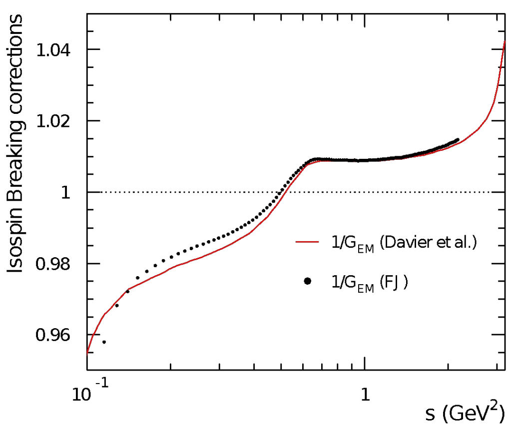
$$R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left|\frac{F_0(s)}{F_-(s)}\right|^2$$

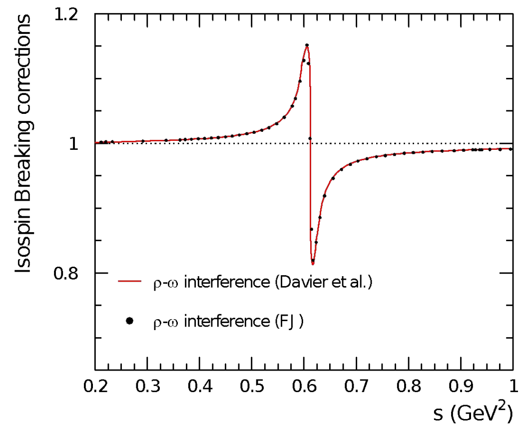
→ Comparing corrections used by Davier et al. with the ones by F. Jegerlehner









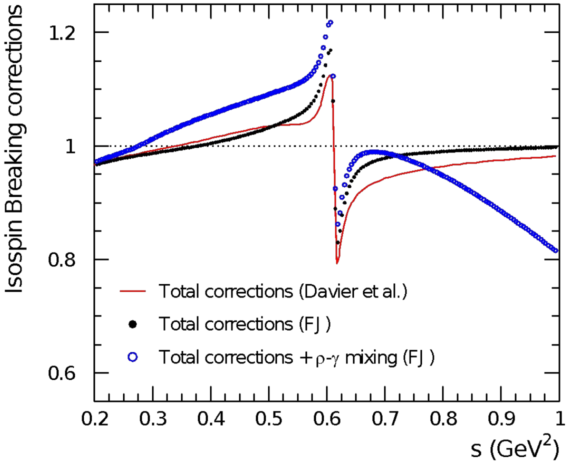


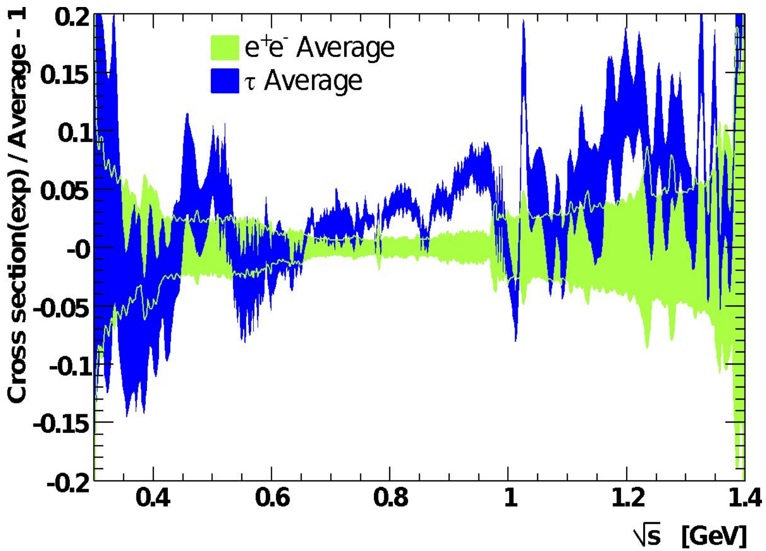
Plots by Z. Zhang, based on private communication with F. Jegerlehner

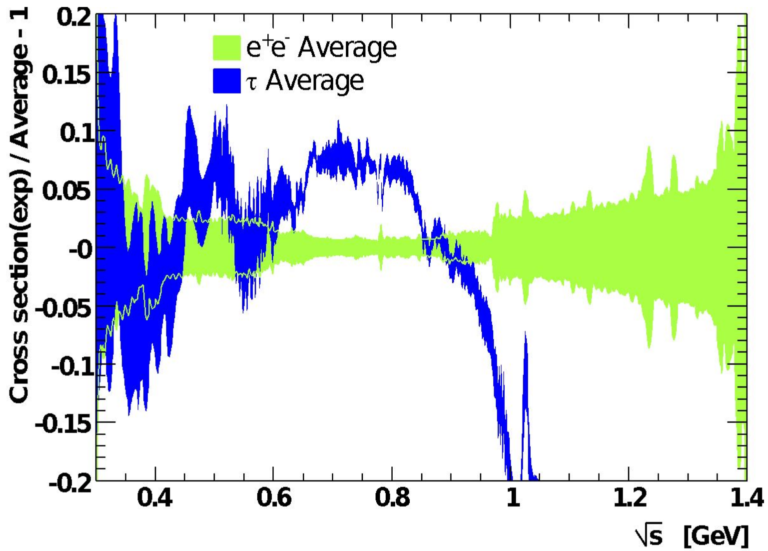
Comparison with IB-corrected τ data

→ for a_μ, $e^+e^- - \tau$ difference of 2.2 σ (Davier et al.)

→ the ρ−γ mixing correction proposed in arXiv:1101.2872 (FJ) seems to over-estimate the $e^+e^- - \tau$ difference







χ^2 definitions and properties

$$\chi^2(\mathbf{d};\mathbf{t}) = \sum_{i,j} (d_i - t_i) \cdot \left[C^{-1}(\mathbf{t})\right]_{ij} \cdot (d_j - t_j) \qquad C_{ij} = C^{stat}_{ij} + \sum_k s^k_i \cdot s^k_j$$

$$\chi^2(\mathbf{d};\mathbf{t}) = \min_{\beta_a} \left\{ \sum_{i,j} \left[d_i - \left(1 + \sum_a \beta_a \cdot \left(\boldsymbol{\epsilon}^{\pm}_a(\beta_a) \right)_i \right) t_i \right] \cdot \left[C^{-1}_{\rm su}(\mathbf{t}) \right]_{ij} \cdot \left[d_j - \left(1 + \sum_a \beta_a \cdot \left(\boldsymbol{\epsilon}^{\pm}_a(\beta_a) \right)_j \right) t_j \right] + \sum_a \beta_a^2 \right\},$$

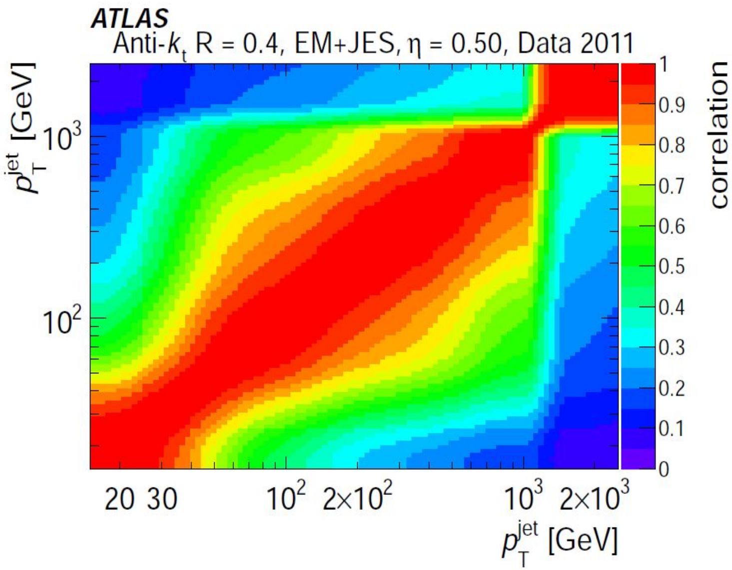
→ Two χ^2 definitions, with systematic uncertainties included in covariance matrix or treated as fitted "nuisance parameters"

→ Equivalent for symmetric Gaussian uncertainties
(1312.3524 - ATLAS)

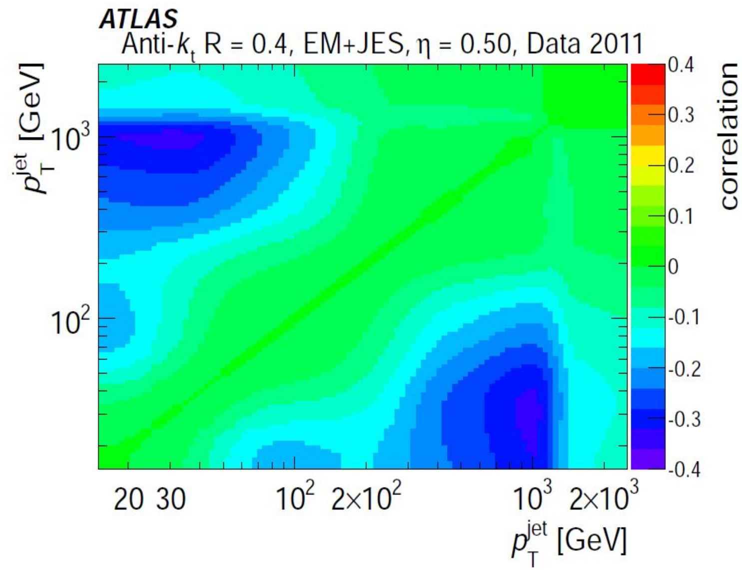
→ *Both approaches assume the knowledge of the amplitude, shape (phase-space dependence) and correlations of systematic uncertainties*

Example: published uncertainties on correlations

1406.0076 – ATLAS jet energy scale uncertainties



Nominal correlation scenario



Weaker - stronger correlation scenarios

Scaling factors and NP shifts

$a_\mu^{\text{HVP, LO}}$ shift (Energy range)	*Approach* 0		*Approach* 1		
	Scaling factor	$\Delta'\alpha_{\text{had}}(M_Z^2)$	Shift NP_1	$\sigma'\left(\Delta\alpha_{\text{had}}(M_Z^2)\right)$	$\Delta'\alpha_{\text{had}}(M_Z^2)$
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ (Full HVP)	1.027	0.02826	4.6	$9.0 \cdot 10^{-5}$	0.02774
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ (Full HVP)	1.020	0.02808	3.5	$9.0 \cdot 10^{-5}$	0.02769
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.8 GeV])	1.029	0.02769	4.7	$9.5 \cdot 10^{-5}$	0.02768
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.8 GeV])	1.022	0.02765	3.5	$9.5 \cdot 10^{-5}$	0.02764
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1 GeV])	1.034	0.02765	-	-	-
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1 GeV])	1.026	0.02762	-	-	-
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM (Pheno)}}$ (Full HVP)	1.037	0.02856	6.6	$9.0 \cdot 10^{-5}$	0.02782
$(a_\mu^{\text{Exp}} - 1\sigma) - a_\mu^{\text{SM (Pheno)}}$ (Full HVP)	1.028	0.02831	5.0	$9.0 \cdot 10^{-5}$	0.02775
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM (Pheno)}}$ ([Th.; 1.8 GeV])	1.041	0.02776	6.6	$9.5 \cdot 10^{-5}$	0.02774
$(a_\mu^{\text{Exp}} - 1\sigma) - a_\mu^{\text{SM (Pheno)}}$ ([Th.; 1.8 GeV])	1.031	0.02770	5.0	$9.5 \cdot 10^{-5}$	0.02769
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM (Pheno)}}$ ([Th.; 1 GeV])	1.048	0.02771	-	-	-
$(a_\mu^{\text{Exp}} - 1\sigma) - a_\mu^{\text{SM (Pheno)}}$ ([Th.; 1 GeV])	1.036	0.02766	-	-	-

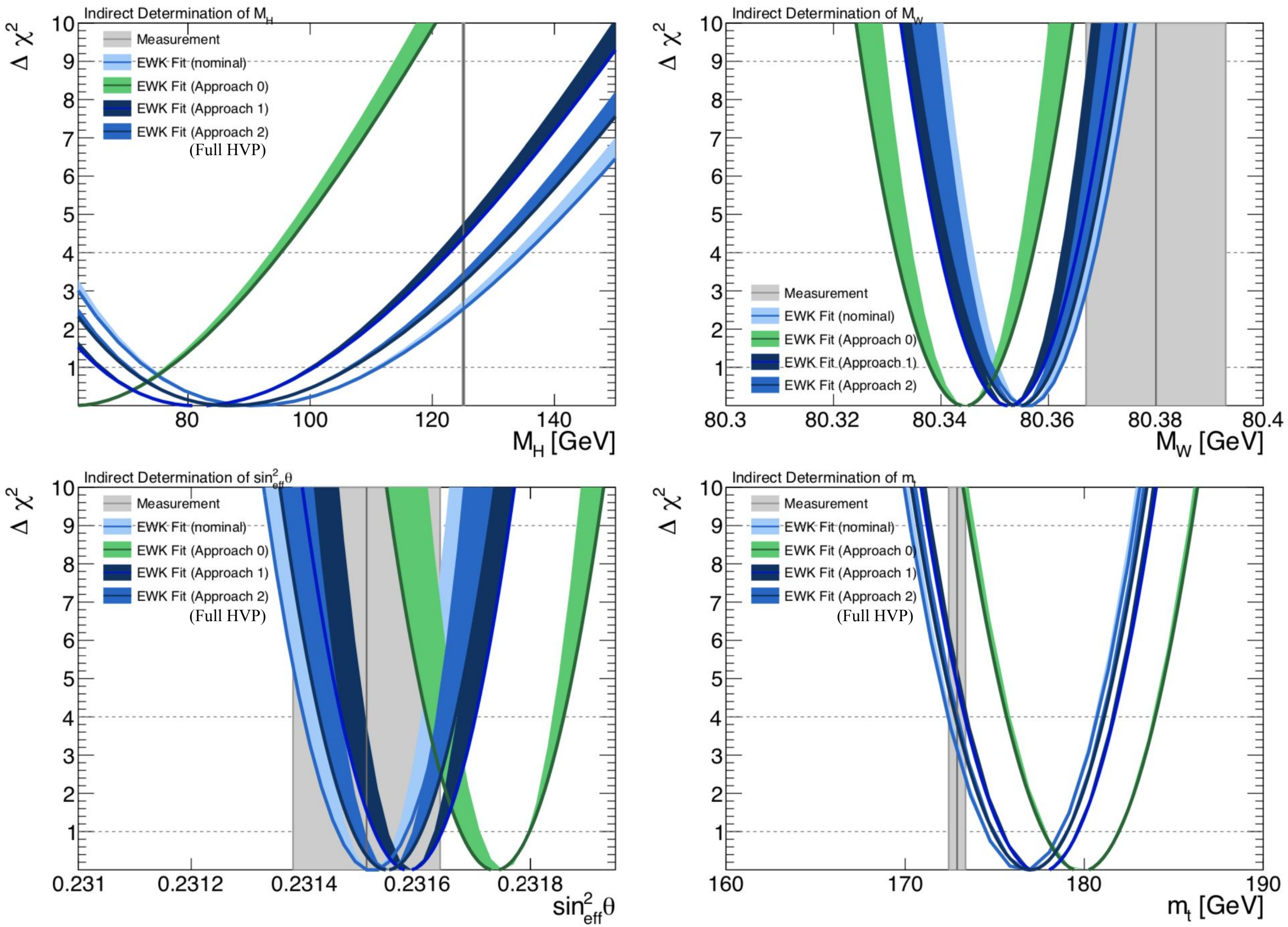
→ Large scaling factors (w.r.t. uncertainties) & significant shifts of NP_1

EW fit inputs and χ^2 results

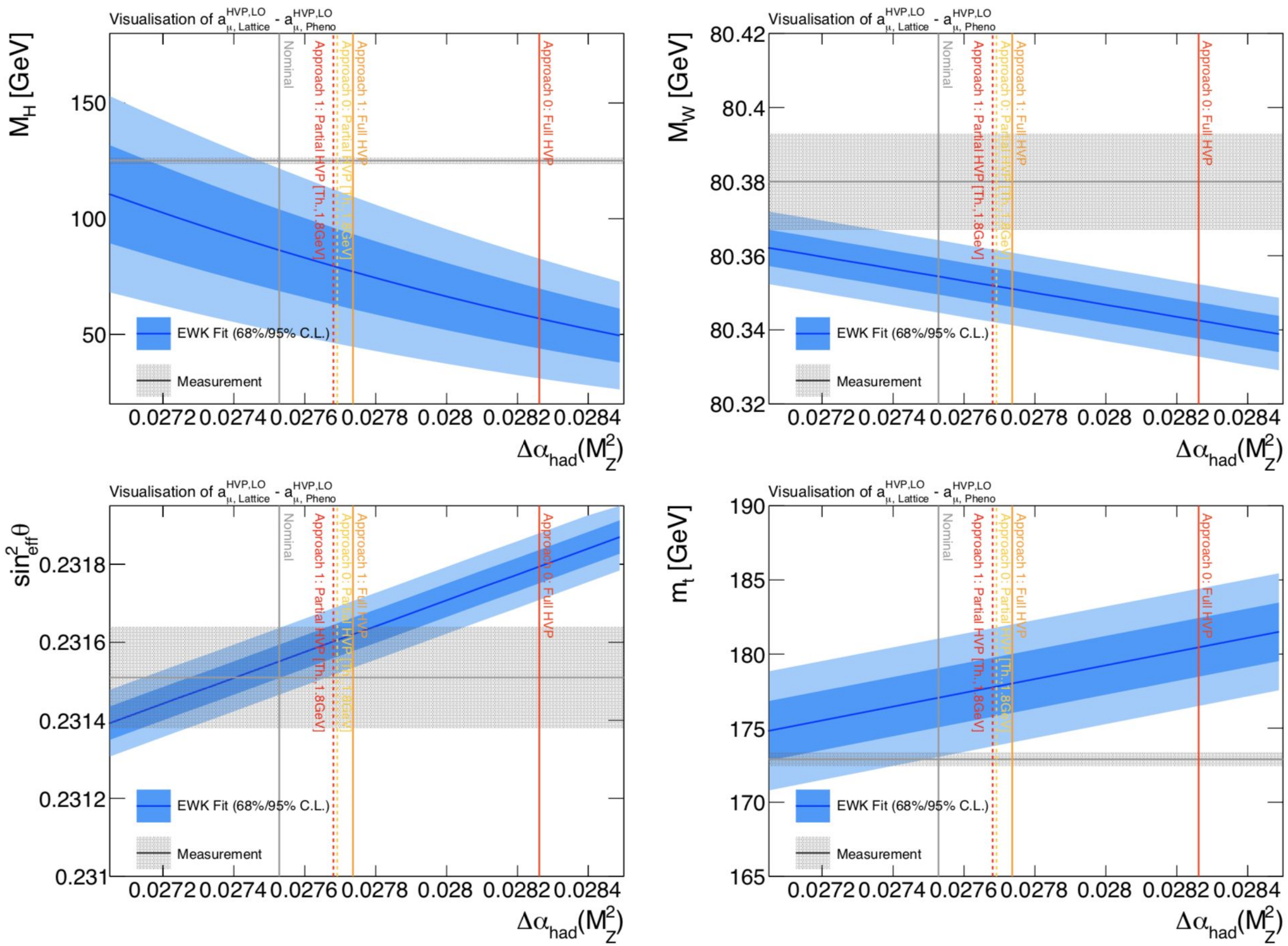
LEP/LHC/Tevatron					
M_Z [GeV]	91.188 ± 0.002	R_c^0	0.1721 ± 0.003	M_H [GeV]	125.09 ± 0.15
$\sigma^0_{\rm had}$ [nb]	41.54 ± 0.037	R_b^0	0.21629 ± 0.00066	M_W [GeV]	80.380 ± 0.013
Γ_Z [GeV]	2.495 ± 0.002	A_c	0.67 ± 0.027	m_t [GeV]	172.9 ± 0.5
A_l (SLD)	0.1513 ± 0.00207	A_l (LEP)	0.1465 ± 0.0033	$\sin^2\theta^l_{\rm eff}$	0.2314 ± 0.00023
$A^l_{\rm FB}$	0.0171 ± 0.001	m_c [GeV]	$1.27^{+0.07}_{-0.11}$ GeV	**After HL-LHC**	
$A^c_{\rm FB}$	0.0707 ± 0.0035	m_b [GeV]	$4.20^{+0.17}_{-0.07}$ GeV	M_W [GeV]	80.380 ± 0.008
$A^b_{\rm FB}$	0.0992 ± 0.0016	$\alpha_s(M_Z)$	0.1198 ± 0.003	$\sin^2\theta^l_{\rm eff}$	0.2314 ± 0.00012
R_l^0	20.767 ± 0.025	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ $[10^{-5}]$	2760 ± 9	m_t [GeV]	172.9 ± 0.3

$a_\mu^{\rm HVP,\,LO}$ shift (Energy range)	Nominal		*Approach* 0		*Approach* 1		*Approach* 2	
	$\Delta'\alpha_{\rm had}(M_Z^2)$	χ^2/ndf	$\Delta'\alpha_{\rm had}(M_Z^2)$	χ^2/ndf	$\Delta'\alpha_{\rm had}(M_Z^2)$	χ^2/ndf	$\Delta'\alpha_{\rm had}(M_Z^2)$	χ^2/ndf
	0.02753	18.6/16 (p=0.29)	-	-	-	-	0.02753	28.1/17 (p=0.04) χ^2(BMW20-Pheno): 9.3
$a_\mu^{\rm HVP,\,LO}$(Lattice) $- a_\mu^{\rm HVP,\,LO}$(Pheno) (Full HVP)	-	-	0.02826	27.6/16 (p=0.04)	0.02774	20.3/16 (p=0.21)	-	
$a_\mu^{\rm HVP,\,LO}$(Lattice) $- a_\mu^{\rm HVP,\,LO}$(Pheno) ([Th.; 1.8 GeV])	-	-	0.02769	19.9/16 (p=0.22)	0.02768	19.8/16 (p=0.23)	-	-
$a_\mu^{\rm HVP,\,LO}$(Lattice) $- a_\mu^{\rm HVP,\,LO}$(Pheno) ([Th.; 1.0 GeV])	-	-	0.02765	19.6/16 (p=0.24)	-	-	-	-
$a_\mu^{\rm Exp} - a_\mu^{\rm SM}$ (Pheno) (Full HVP)	-	-	0.02856	33.6/16 (p=0.01)	0.02782	21.2/16 (p=0.17)	-	-
$a_\mu^{\rm Exp} - a_\mu^{\rm SM}$ (Pheno) ([Th.; 1.8 GeV])	-	-	0.02776	20.6/16 (p=0.19)	0.02774	20.4/16 (p=0.20)	-	-
$a_\mu^{\rm Exp} - a_\mu^{\rm SM}$ (Pheno) ([Th.; 1.0 GeV])	-	-	0.02771	20.1/16 (p=0.22)	-	-	-	-

EW fit results: χ^2 scans



EW fit results: parameter scans for varying $\Delta\alpha_{had}(M_Z^2)$



EW fit results: indirect determination of $\Delta\alpha_{had}(M_Z^2)$