New Physics prospects from $D^+ \to \pi^+ \ell^+ \ell^-$

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Based on "Disentangling QCD and New Physics in $D^+ \to \pi^+ \ell^+ \ell^-$" , by A. Bharucha, D. Boito and C. Méaux, arXiv:2011.12856 (hep-ph)

Talk at the LIO international conference on "Future colliders and the origin of mass", IP2I Lyon, 22 June 2021

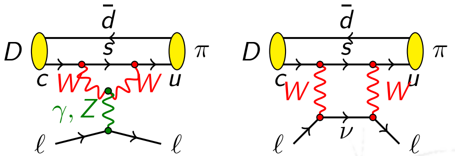
Why look at rare semi-leptonic charm decays?

Good question!

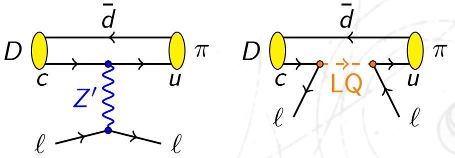
To test scales potentially beyond the direct searches at LHC

Also
- Anomalies in rare B decays, interesting to investigate charm
- Effort needed to improve theory for D decays in light of LHCb data

The rare decay $c \to u\ell\ell$ (BR $\sim 10^{-9}$) proceeds via loops in the SM



Beyond the SM, new particles can enter loops/generate new diagrams.

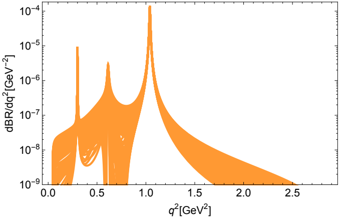


Most recent work on $D \to \pi\ell\ell$

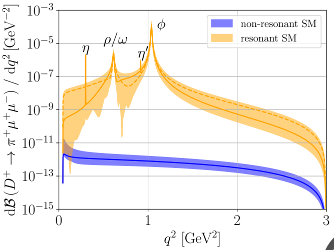
The latest references are:

- Boer and Hiller [arXiv:1510.00311]
- Fajfer and Kosnik [arXiv:1510.00965]
- Feldmann, Muller and Seidel [arXiv:1705.05891]
- Bause, Golz, Hiller and Tayduganov [1909.11108].
- Bause, Gisbert, Golz and Hiller [2004.01206].

[arXiv:1510.00965]



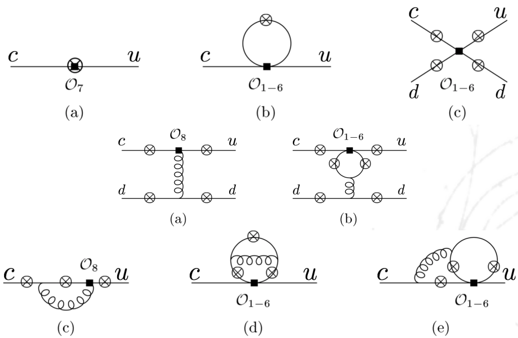
[arXiv:1909.11108]



Beyond the existing literature

The main aim of our work is to study the phenomenology including:

- Improved treatment of resonances, fit to e^+e^- data [Kruger, Sehgal [arXiv:hep-ph/9603237] and Lyons and Zwicky [arXiv:1406.0566]
- Implementation of non factorizable QCDf corrections, note large contribution from weak annihilation [Feldmann, Muller, Seidel, arXiv 1705.05891]



The Operator basis

$$\mathcal{H}_{\rm eff} = \mathcal{H}_{\rm eff}^{\rm SM} + \frac{4G_F}{\sqrt{2}} \sum_{i=7,9,10,S,P,T,T5} C_i\mathcal{O}_i + C_i\mathcal{O}_i' \qquad \text{where}$$

$$\mathcal{H}_{\rm eff}^{\rm SM}(m_b > \mu > m_c) = \frac{4\,G_F}{\sqrt{2}} \sum_{q=d,s} \lambda_q \left[C_1(\mu)\mathcal{O}_1^q + C_2(\mu)\mathcal{O}_2^q + \sum_{i=3}^{9} C_i(\mu)\mathcal{O}_i \right]$$

for $\lambda_q = V_{cq}^* V_{uq}$, $C_{10}^{(\prime),SM}$ and $C\prime, SM_{7,9}$, $C_{7/9} \to C_{7/9}^{\rm SM} + \Delta C_{7/9}^{\rm NP}$

$$\mathcal{O}_1^q = (\bar{u}_L\gamma_\mu T^a q_L)(\bar{q}_L\gamma_\mu T^a c_L), \quad \mathcal{O}_2^q = (\bar{u}_L\gamma_\mu q_L)(\bar{q}_L\gamma_\mu c_L),$$

$$\mathcal{O}_7 = -\frac{g_{\rm em} m_c}{16\pi^2}(\bar{u}_L\sigma^{\mu\nu}c_R)F_{\mu\nu}, \quad \mathcal{O}_8 = -\frac{g_s m_c}{16\pi^2}(\bar{u}_L\sigma^{\mu\nu}T^a c_R)G^a_{\mu\nu},$$

$$\mathcal{O}_9 = -\frac{\alpha_{\rm em}}{4\pi}(\bar{u}_L\gamma^\mu c_L)(\bar{\ell}\gamma^\mu\ell), \quad \mathcal{O}_{10}^{(\prime)} = -\frac{\alpha_{\rm em}}{4\pi}(\bar{u}\gamma^\mu c_{L(R)})(\bar{\ell}\gamma^\mu\gamma_5\ell),$$

$$\mathcal{O}_S^{(\prime)} = \frac{e^2}{(4\pi)^2}(\bar{u}P_{R(L)}c)(\bar{\ell}\ell), \quad \mathcal{O}_P^{(\prime)} = \frac{e^2}{(4\pi)^2}(\bar{u}P_{R(L)}c)(\bar{\ell}\gamma_5\ell),$$

$$\mathcal{O}_T = \frac{e^2}{(4\pi)^2}(\bar{u}\sigma_{\mu\nu}c)(\bar{\ell}\sigma^{\mu\nu}\ell), \quad \mathcal{O}_{T5} = \frac{e^2}{(4\pi)^2}(\bar{u}\sigma_{\mu\nu}c)(\bar{\ell}\sigma^{\mu\nu}\gamma_5\ell),$$

	C_1	C_2	C_3	C_4	C_5	C_6	$C_7^{\rm eff}$	$C_8^{\rm eff}$	C_9
LL	-0.890	1.072	-0.002	-0.041	0.000	0.000	0.057	-0.042	-0.095
NLL	-0.603	1.029	-0.003	-0.065	0.000	0.000	0.035	-0.045	-0.270
NNLL	-0.529	1.026	-0.004	-0.063	0.000	0.000	0.036	-0.048	-0.413

Boer, Müller, Seidel arXiv 1606.0552

Decay amplitude

The amplitude for the process can be expressed

$$A_{SD} = \frac{iG_F\alpha_e}{\sqrt{2}\pi}\left[V\bar{u}\,\not{p}v + A\bar{u}\,\not{p}\gamma_5 v + (S + T\cos\theta)\bar{u}v + (P + T_5\cos\theta)\bar{u}\gamma_5 v\right]$$

A, S, P, T, and T_5 are q^2-dependent functions of FFs and WCs:

$$V = f_+ C_9^{\mathrm{QCDf}} + \frac{8 f_T(q^2) m_l}{m_D + m_\pi} C_T, \qquad A = f_+(q^2)(C_{10} + C_{10}'),$$

$$S = \frac{m_D^2 - m_\pi^2}{2m_c} f_0(q^2)(C_S + C_S'),$$

$$P = \frac{m_D^2 - m_\pi^2}{2m_c} f_0(q^2)(C_P + C_P') - m_\ell\left[f_+(q^2) - \frac{m_D^2 - m_\pi^2}{q^2}\left(f_0(q^2) - f_+(q^2)\right)\right](C_{10} + C_{10}'),$$

$$T = \frac{2 f_T(q^2)\beta\lambda^{1/2}}{m_D + m_\pi} C_T, \qquad T_5 = \frac{2 f_T(q^2)\beta\lambda^{1/2}}{m_D + m_\pi} C_{T5}.$$

Incorporating QCDf corrections

The QCD factorization corrections have been absorbed into C_9^{QCDf}:

$$C_9^{\mathrm{QCDf}} = \lambda_b \left[(C_9 + C_9') + \frac{f_T}{f_+} \frac{2m_c}{m_D} (C_7 + C_7') \right] + \sum_{q=b,d} \lambda_q \left(\frac{2m_c}{m_D} a_s C_F C_{||}^{nf,q} \frac{f_T}{f_+} + Y^q \right.$$
$$\left. + \frac{2m_c}{m_D f_+} \sum_{\pm} \frac{\pi^2}{N_c} \frac{f_D f_\pi}{m_D} \sum_{\pm} \frac{d\omega}{\omega} \phi_{D,\pm}(\omega) \int_0^1 du \phi_\pi(u) T_{||,\pm}^{(q)}(u,\omega) \right)$$

where $T_{||,\pm}^{(q)} = T_{\pm}^{(0,q)} + a_s C_F T_{\pm}^{(nf,q)}$ and $C_{||}^{nf,q}$, $T_{-}^{(0/nf,q)}$ etc. are given in FMS17

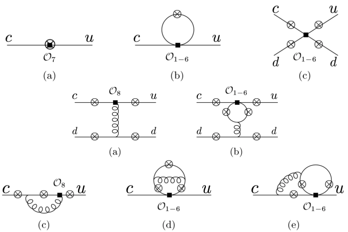
The quark loop functions are defined as follows (in terms of $h(s, m_q)$ parameterising the quark loop to be defined later):

$$\begin{aligned} Y^b = {} & [h(s,m_c) + h(s,m_u)] \left(7\, C_3 + \frac{4}{3}\, C_4 + 76\, C_5 + \frac{64}{3}\, C_6 \right) \\ & - h(s,m_s) \left(\frac{2}{3}\, C_1 + \frac{1}{2}\, C_2 + 3\, C_3 + 30\, C_5 \right) \\ & - h(s,m_d) \left(3\, C_3 + 30\, C_5 \right) + \frac{8}{9} \left(3\, C_3 + 16\, C_5 + \frac{16}{3}\, C_6 \right) , \\ Y^d = {} & - \left(\frac{2}{3}\, C_1 + \frac{1}{2}\, C_2 \right) [h(s,m_s) - h(s,m_d)] , \end{aligned}$$

Numerical comparison of different contributions

Contribution	$\propto \lambda_b$	$\propto \lambda_d$
C_9	-0.413	0
$Y^{(q)}$	$-1.303 + 0.034i$	$1.345 + 0.981i$
$C_{\rm FF}$	$-0.287 - 0.457i$	$-0.028 - 0.002i$
$C_{\rm Ann}$	$0.013 - 0.054i$	$0.503 - 2.100i$
$C_{\rm SS}$	$0.028 - 0.033i$	$0.005 + 0.002i$

Table: Individual contributions at NLO with no resonant contribution at $q^2 = 0.5 GeV^2$



Note:

- $\lambda^d \gg \lambda^b$
- Weak Annihilation dominates
- $Y^d = -\left(\frac{2}{3} C_1 + \frac{1}{2} C_2\right)[h(s, m_s) - h(s, m_d)]\,,$
- Y^d small if $h(m_s) \sim h(m_d)$

Weak Annihilation in QCDf and the OPE

- QCDf valid at small q^2, as here the pion is energetic, and in the heavy-quark limit $(E_\pi \gg \Lambda_{\mathrm{QCD}})$

$$C_9^{(d)}(s)\big|_{\mathrm{Ann}} = 8e_d \frac{\pi^2}{N_c} \frac{f_D f_\pi}{m_D} \frac{1}{f_+(s)} \frac{1}{\lambda_D^-(s)} \; 3C_2, \qquad \frac{1}{\lambda_D^-(s)} = \int_0^\infty d\omega \frac{\phi_D^-(\omega)}{\omega - s/m_D - i\epsilon}$$

- For low recoil, we can follow Beylich et al 2011, perform an operator product expansion (OPE) in $E_\pi/\sqrt{q^2}$ for $\sqrt{q^2} \gg E_\pi, \Lambda_{\mathrm{QCD}})$
- For the case of $D^+ \to \pi^+\ell^+\ell^-$, and we find

$$C_9^{(d)}(s)\big|_{\mathrm{Ann}}^{\mathrm{OPE}} = -1/3 \frac{8\pi^2 C_2 f_D f_\pi}{s f_+(s)}. \tag{1}$$

- WA for both QCDf and the OPE sizeable contribution as C_2 appears without any cancellation from other WCs $\Rightarrow$ very large contribution
- In differential BR for $q^2 < m_\phi$ use QCDf, for $q^2 > m_\phi$ OPE.

Note that the OPE result is not really valid down to m_ϕ nor up to the lowest recoil point, and therefore in phenomenological analysis we only consider q^2 in range [1.8, 2.3] GeV^2.

Modelling the resonances: The vacuum polarization and e^+e^- data

Relate $h_q(s) = \frac{12\pi^2}{N_c}\Pi^{(q)}(s)$ for $q = d, s$ to experimental data for $R(s)$ below charm threshold,

$$R \equiv R_{uds} = R_u + R_d + R_s = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi \sum_{q=u,d,s} \mathrm{Im}\,\Pi^{(q)}(s).$$

where $\Pi^{(q)}_{\mu\nu}(s) = i\int d^4x\, e^{ix\cdot q}\langle 0|T\{(\bar{q}\gamma_\mu q)(x)(\bar{q}\gamma_\nu q)(0)\}|0\rangle = (q_\mu q_\nu - g_{\mu\nu}s)\Pi^{(q)}(s).$, [1]

Relate R to $\mathrm{Im}\, h(s)$ (where exp. R is in terms of physical hadronic states $J^\mu_{1/0} = 1/\sqrt{2}(\bar{u}\gamma^\mu u \mp \bar{d}\gamma^\mu d)$.)

We therefore need a parameterization of $\mathrm{Im} h_{I/s}(s)$ to fit to the e^+e^- data, the simplest being a Breit-Wigner:

$$\mathrm{Im} h_{I/s}(s) = \mathrm{Im} f^{(R/\phi)}_{\mathrm{BW}}(s) = \mathrm{Im}\left(n_R e^{i\alpha_R}\frac{M_R^2}{M_R^2 - q^2 - i\sqrt{q^2}\,\Gamma_t}\right)$$

How can we improve on Breit-Wigner to have the correct analytic behaviour, i.e. a branch cut at $q^2 = 0$?

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[1] Here we follow the idea of Kruger and Sehgal [arXiv:hep-ph/9603237] and Lyons and Zwicky [arXiv:1406.0566]

The Shifman Model

$$f_{\rm mod}(q^2) = \hat{n}_V \left(1 + z_V \frac{\sigma_V^2}{M_V^2}\right)^{-1}, \quad z_V = \left(\frac{-q^2 - i\epsilon}{\sigma_V^2}\right)^{1-b_V/\pi}, \quad b_V = \frac{\Gamma_V}{M_V}.$$

For $\hat{\sigma}_V^2 = \sigma_V^2/M_V^2 = 1$, $b_V \ll 1$, $\mathrm{Im}\,[f_{\rm mod}]$ can then be approximated as,

$$\frac{1}{\pi}\,\mathrm{Im}\,[f_{\rm mod}] = \frac{\hat{n}_V\,\theta(q^2)}{\pi}\,\frac{|z_V|\,\hat{\sigma}_V^2\,\sin b_V}{1 - 2\,|z_V|\,\hat{\sigma}_V^2\,\cos b_V + |z_V|^2\,\hat{\sigma}_V^4} \simeq \frac{n_V\,\theta(q^2)}{\pi}\,\frac{q^2 M_V \Gamma_V}{(q^2 - M_V^2)^2 + q^2\,\Gamma_V^2},$$

Shifman model: build infinite tower of equidistant vector resonances with masses $M_n^2 = (n + a_0)\,\sigma^2$ and widths $\Gamma_n = bM_n$, for $n = \{0, 1, 2, \ldots\}$ and $a_0 = \mathrm{const}$ via

$$\pi(q^2) = \frac{1}{1 - b/\pi}\sum_{n=0}^{\infty}\frac{1}{n + a_0 + z} = -\frac{1}{1 - b/\pi}\,\Psi(z + a_0)\,, \qquad z = \left(\frac{-q^2 - i\epsilon}{\sigma^2}\right)^{1-b/\pi}.$$

Inf. sum over *all* resonances reproduces partonic result $\lim_{-q^2\to\infty}\pi(q^2) = -\ln\frac{-q^2}{\sigma^2} + \ldots$.
Reconstruct $h_q(s)$ from imaginary part using a once subtracted dispersion relation

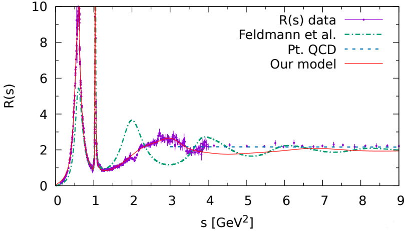
$$\tilde{h}_{l/s}(s) = \tilde{h}_{l/s}^{(\rm pt)}(-s_0) + \int_0^\infty ds'\,\frac{s_0 + s}{s_0 + (s')^2}\,\frac{\mathrm{Im}\,\tilde{h}_{l/s}(s')}{s' - s - i\epsilon} \quad \text{where} \quad \mathrm{Im}h_{l/s}(s) = \mathrm{Im}f_{\rm BW}^{(R/\phi)}(s) - \mathrm{Im}\left[\frac{\Psi(z_{l/s} + a_{l/s})}{1 - b_{l/s}/\pi}\right].$$

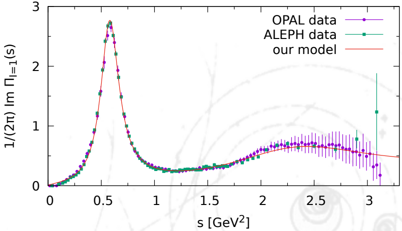
Subtraction constant is calculated from the perturbative result in the Euclidean Use $s_0 = 10$ GeV^2 and $\mu^2 = (1.5\ \mathrm{GeV})^2$.

Checked that results stable on varying subtraction pt and no. of subtractions.

Our Analysis

- Improve FMS17 description by fixing the parameters of our model from a comparison to $e^+e^- \to (\text{hadrons})$ and $\tau \to (\text{hadrons}) + \nu_\tau$ data.
- Simplicity and elegance of the model we use for the spectral functions appealing but cannot describe all features of data, model is not equally suitable for all the whole kinematic range, although improved wrt FMS17, doesn't precisely describe all experimental data available.
- Use publicly available Particle Data Group compilation of $R(s)$ data supplemented with $R(s)$ measurements from the BES and KEDR collaborations, also use the ALEPH and OPAL data for the vector isovector spectral function from $\tau \to (\text{hadrons}) + \nu_\tau$





Differential decay width following Bobeth, Hiller and Piranishvili

[arXiv:0709.4174]

wih respect to θ and q^2

$$\frac{d\Gamma(D \to \pi\ell\ell)}{dq^2 d\cos\theta} = N\,\lambda^{1/2}\,\beta\,\left(a_\ell(q^2) + b_\ell(q^2)\cos\theta + c_\ell(q^2)\cos^2\theta\right)$$

where $\beta = \sqrt{1 - 4m_l^2/q^2}$, $\lambda = (m_D^2 + m_\pi^2 + q^2)^2 - 4(m_D^2 m_\pi^2 + m_D^2 q^2 + m_\pi^2 q^2)$ and $N = \frac{G_F^2\alpha_e^2}{(4\pi)^5 m_D^3}$.

Angular coefficients can be written:

$$a_\ell(q^2) = \frac{\lambda}{2}(|V|^2 + |A|^2) + 8m_\ell^2 m_D^2 |A|^2 + 2q^2(\beta^2|S|^2 + |P|^2)$$

$$b_\ell(q^2) = 4Re\left[q^2(\beta^2 ST^* + PT_5^*) + m_\ell(\lambda^{1/2}\beta VS^* + (m_D^2 - m_\pi^2 + q^2)AT_5^*)\right]$$

$$c_\ell(q^2) = -\frac{\lambda\beta^2}{2}(|V|^2 + |A|^2) + 2q^2(\beta^2|T|^2 + |T_5|^2) + 4m_\ell\beta\lambda^{1/2}Re[VT^*].$$

Observables

Decay rate, Forward-backward asymmetry and the flat term

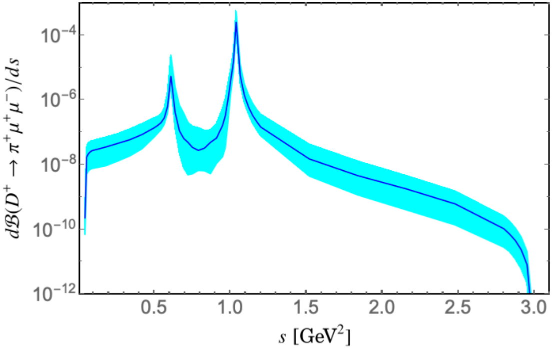
$$
\begin{aligned}
\frac{d\Gamma(D\to\pi\ell\ell)}{dq^2} &= 2N\lambda^{1/2}\beta\left[a_\ell(q^2)+\frac{c_\ell(q^2)}{3}\right]\\
&= N\lambda^{1/2}\beta\, d\hat{\Gamma}(q^2)\\
A_{\rm FB}(q^2) &= \frac{\int_0^1 d\cos\theta\,\frac{d\Gamma}{dq^2\,d\cos\theta}-\int_{-1}^0 d\cos\theta\,\frac{d\Gamma}{dq^2\,d\cos\theta}}{\int_{-1}^1 d\cos\theta\,\frac{d\Gamma}{dq^2\,d\cos\theta}}\equiv\frac{b_\ell(q^2)}{2(a_\ell(q^2)+c_\ell(q^2)/3)}\\
&= \frac{2q^2\sqrt{\lambda}}{m_c\, d\hat{\Gamma}(q^2)} f_0 f_T\left(Re(C_{T_5}C_P^*)+Re(C_T C_S^*)\right)\\
F_H(q^2) &= \frac{a_\ell(q^2)+c_\ell(q^2)}{a_\ell(q^2)+c_\ell(q^2)/3}\\
&= \frac{q^2}{d\hat{\Gamma}(q^2)}\left(\frac{m_D^2-m_\pi^2}{m_c^2}f_0^2(|C_P|^2+|C_S|^2)+16\lambda\frac{f_T^2\,(|C_T|^2+|C_{T_5}|^2)}{(m_D+m_\pi)^2}\right)
\end{aligned}
$$

Where $d\hat{\Gamma}(q^2) = (m_D^2-m_\pi^2)\frac{q^2}{m_c^2}f_0^2(|C_P|^2+|C_S|^2)+\frac{2}{3}\lambda f_+^2(|C_9^{\rm QCDf}|^2+|C_{10}|^2)+\frac{16}{3}\lambda q^2\frac{f_T^2(|C_T|^2+|C_{T_5}|^2)}{(m_D+m_\pi)^2}$.

Note that in SM only V is non-zero $\Rightarrow b_\ell(q^2)=0$ and $c_\ell(q^2)=-\beta^2 a_\ell(q^2)$

Differential branching ratio:

The full spectrum



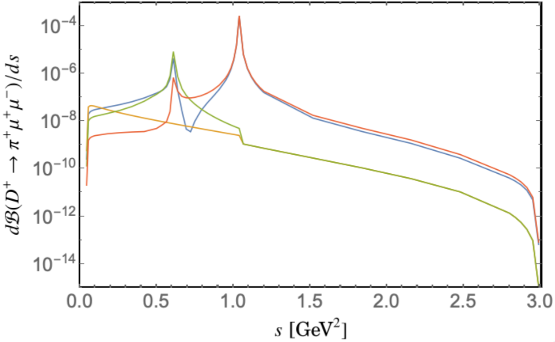
$$\frac{d\Gamma(D \to \pi\ell\ell)}{dq^2} = 2N\lambda^{1/2}\beta\left[a_\ell(q^2) + \frac{c_\ell(q^2)}{3}\right] = N\lambda^{1/2}\beta d\hat{\Gamma}(q^2)$$

Where $d\hat{\Gamma}(q^2) = (m_D^2 - m_\pi^2)\frac{q^2}{m_c^2} f_0^2(|C_P|^2 + |C_S|^2) + \frac{2}{3}\lambda f_+^2(|C_9^{\mathrm{QCDf}}|^2 + |C_{10}|^2) + \frac{16}{3}\lambda q^2 \frac{f_T^2(|C_T|^2 + |C_{T_5}|^2)}{(m_D + m_\pi)^2}$.

Integrated branching ratio measured by LHCb [LHCB-PAPER-2012-051]

Differential branching ratio:

The full spectrum



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Where $d\hat{\Gamma}(q^2) = (m_D^2 - m_\pi^2)\frac{q^2}{m_c^2} f_0^2(|C_P|^2 + |C_S|^2) + \frac{2}{3}\lambda f_+^2(|C_9^{\mathrm{QCDf}}|^2 + |C_{10}|^2) + \frac{16}{3}\lambda q^2 \frac{f_T^2(|C_T|^2 + |C_{T_5}|^2)}{(m_D + m_\pi)^2}$.

Integrated branching ratio measured by LHCb [LHCB-PAPER-2012-051]

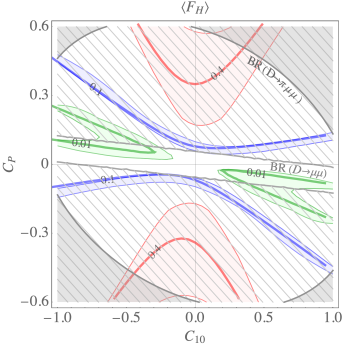
Implications for BSM models

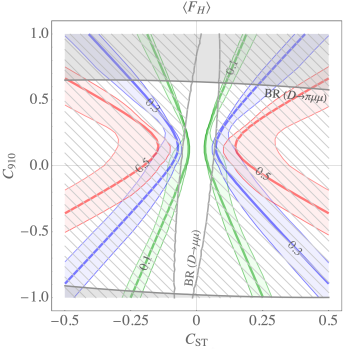
Focus on scalar/ vector leptoquarks (LQs), due both B anomalies (Crivellin 2017, Becirevic et al 2018, Angelescu et al 2018, Crivellin et al 2019) and the interesting effects in $c \to u\ell\ell$ transitions (de Boer et al 2015, Fajfer et al 2015)

- The Scalar LQs S_1 with quantum numbers $(3, 1, -1/3)$ and S_2 with $(3, 2, -7/6)$ are interesting as contribute to all WCs we consider
- For S_1, assume LH up-quark-muon coupling can be neglected, i.e. $\lambda_L^{u\mu} \sim 0$, such that the combination $\lambda_R^{u\mu}\lambda_R^{c\mu}$ controls $C_9' = C_{10}'$, and $\lambda_R^{u\mu}\lambda_L^{c\mu}$ controls $C_S' = C_P' = C_T/2 = C_{T_5}/2$.
- For S_2, assume RH up-quark-muon coupling can be neglected, i.e. $\lambda_R^{u\mu} \sim 0$, such that the combination $\lambda_L^{u\mu}\lambda_L^{c\mu}$ controls $C_9' = -C_{10}'$, and $\lambda_L^{u\mu}\lambda_R^{c\mu}$ controls $C_S' = -C_P' = C_T/2 = -C_{T_5}/2$.
- Vector LQs only contribute to $C_9^{(\prime)}$ and $C_{10}^{(\prime)}$. After taking into account the constraints from kaon decays, only vector LQs which can give rise to non-negligible Wilson coefficients are $\tilde{V}_1$ with quantum numbers $(3, 1, -5/3)$ with $C_9' = C_{10}'$ and $\tilde{V}_2$ with $(3, 2, 1/6)$ with $C_9' = -C_{10}'$, large values of un-primed C_9 and C_{10} cannot be generated

Flat term:

Possible effects of NP





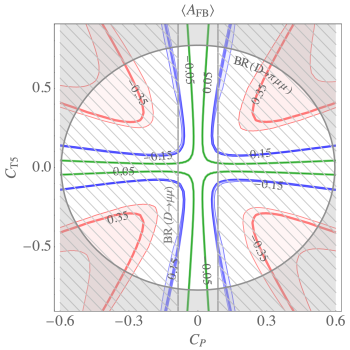
$$F_H(q^2) = \frac{\int_0^1 d\cos\theta \frac{d\Gamma}{dq^2 d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\Gamma}{dq^2 d\cos\theta}}{\int_{-1}^1 d\cos\theta \frac{d\Gamma}{dq^2 d\cos\theta}} \equiv \frac{a_\ell(q^2) + c_\ell(q^2)}{a_\ell(q^2) + c_\ell(q^2)/3}$$

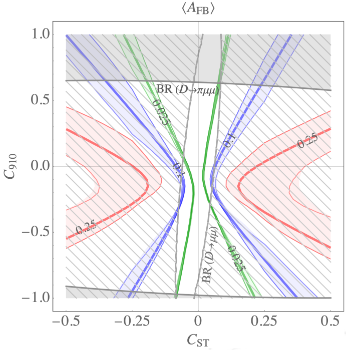
$$= \frac{q^2}{d\hat{\Gamma}(q^2)} \left(\frac{m_D^2 - m_\pi^2}{m_c^2} f_0^2(|C_P|^2 + |C_S|^2) + 16\lambda \frac{f_T^2\,(|C_T|^2 + |C_{T_5}|^2)}{(m_D + m_\pi)^2} \right)$$

Where $d\hat{\Gamma}(q^2) = (m_D^2 - m_\pi^2)\frac{q^2}{m_c^2} f_0^2(|C_P|^2 + |C_S|^2) + \frac{2}{3}\lambda f_+^2(|C_9^{\mathrm{QCDf}}|^2 + |C_{10}|^2) + \frac{16}{3}\lambda q^2 \frac{f_T^2(|C_T|^2 + |C_{T_5}|^2)}{(m_D + m_\pi)^2}$.

Forward-Backward/CP Asymmetry:

Possible effect of NP





$$A_{\rm FB}(q^2) \equiv \frac{b_\ell(q^2)}{2(a_\ell(q^2)+c_\ell(q^2)/3)} = \frac{2q^2\sqrt{\lambda}}{m_c\, d\hat{\Gamma}(q^2)} f_0 f_T \left(Re(C_{T_5}C_P^*) + Re(C_T C_S^*)\right)$$

Where $d\hat{\Gamma}(q^2) = (m_D^2 - m_\pi^2)\frac{q^2}{m_c^2} f_0^2(|C_P|^2 + |C_S|^2) + \frac{2}{3}\lambda f_+^2(|C_9^{\rm QCDf}|^2 + |C_{10}|^2) + \frac{16}{3}\lambda q^2 \frac{f_T^2(|C_T|^2+|C_{T_5}|^2)}{(m_D+m_\pi)^2}$.

Conclusions on the phenomenology

Uncertainties larger, particularly in high q^2 region. Include the weak annihilation from OPE at large q^2. This has a number of consequences for the pheno searches:

- Motivated as a combination of the OPE validity and the fact that uncertainties are still large above the ϕ, we propose to integrate observables in the range $q^2 = [1.8, 2.3]$ GeV^2
- We find that NP contributions to certain Wilson coefficients are subject to large theoretical uncertainties and it would be difficult to distinguish between different scenarios, however certain pairs (those shown) can be probed.
- We find that the uncertainty on A_{CP} is very large (an order of magnitude large than the central value), and therefore we find that distinguishing between different possible BSM phases would be difficult using this observable.
- Interestingly electrons in the final state can provide a cleaner relationship between the observable and the BSM contribution to the Wilson coefficients.

Experimental prospects for $D^+ \to \pi^+\ell^+\ell^-$

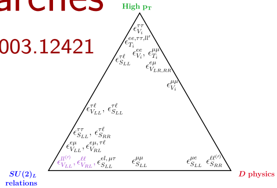
Experiment	Measurement	Sensitivity
LHCb	Angular observables	$\sim 0.2\%$ with $50\ \text{fb}^{-1}$, $\sim 0.08\%$ with $300\ \text{fb}^{-1}$
LHCb	Branching ratio	$\sim 10^{-8}$ with $50\ \text{fb}^{-1}$, $\sim 3 \times 10^{-9}$ with $300\ \text{fb}^{-1}$
Belle-II	Branching ratio	$\sim 10^{-8}$ (rescaling BaBar 2011)

Estimated projected exp. sensitivities from LHCb@Upgrade I ($50\ \text{fb}^{-1}$) and @Upgrade II ($300\ \text{fb}^{-1}$) for and Belle-II for $D^+ \to \pi^+\ell^+\ell^-$. LHCb projections from talks by Andrea Contu at Towards the Ultimate Precision in Flavour Physics, Durham, UK, April 2019 and Dominik Mitzel at RPF Town Hall Meeting, October 2020

- When LHCb has $50\ \text{fb}^{-1}$ of data, exp errors $\ll$ theory uncertainties $\Rightarrow$ contours easily distinguishable
- Even before $50\ \text{fb}^{-1}$, for F_H and $A_{\rm FB}$ sensitivity $\sim$10% could provide evidence for BSM physics
- Exp sensitivity$\sim$1% enough to perform precise fit to WCs (main problem theory uncertainties)
- For LQs, F_H, $A_{\rm FB}$ at most $\mathcal{O}(10\%)$, requiring sensitivities at the $\mathcal{O}(1\%)$. If Belle-II carries out angular analysis for electron case $\Rightarrow$ important complementary information.

Complementarity with collider searches

Fuentes-Martin, Greljo, Martin Camalich, Ruiz-Alvarez, arXiv:2003.12421



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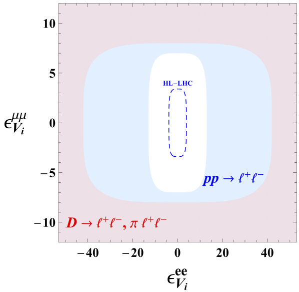


$$\mathcal{L}_{\rm NC} = \frac{4G_F}{\sqrt{2}}\frac{\alpha}{4\pi}\lambda_c \sum_{k,\alpha,\beta} \epsilon_k^{\alpha\beta}\mathcal{O}_k^{\alpha\beta} + \text{h.c.}$$

CC

α	$\lvert\epsilon_{V_i}^{\alpha\alpha}\rvert$	$\lvert\epsilon_{S_{LL,RR}}^{\alpha\alpha}(\mu)\rvert$ $\mu = 1$ TeV	$\lvert\epsilon_{S_{LL,RR}}^{\alpha\alpha}(\mu)\rvert$ $\mu = 2$ GeV	$\lvert\epsilon_{T_{L,R}}^{\alpha\alpha}(\mu)\rvert$ $\mu = 1$ TeV	$\lvert\epsilon_{T_{L,R}}^{\alpha\alpha}(\mu)\rvert$ $\mu = 2$ GeV
e	13 (3.9)	15 (4.5)	32 (9.5)	6.5 (2.0)	5.2 (1.6)
μ	7.0 (3.4)	8.1 (3.9)	17 (8.3)	3.5 (1.7)	2.8 (1.4)
τ	25 (12)	29 (13)	60 (28)	14 (6.6)	11 (5.2)

95% CL limits on NC WCs from $pp \to e^\alpha \bar{e}^\alpha$@LHC (HL-LHC (3 ab^{-1})), with $i = LL, RR, LR, RL$.

Summary

Improved predictions for $D \to \pi\ell\ell$

- Charm physics is gaining increased interest, due to the large numbers of charm mesons produced at LHCb and Belle-II
- Comprehensive analysis of decay conducted, tackling the increased complexity compared to $b \to s\ell\ell$ as expansion in $\Lambda_{\mathrm{QCD}}/m_c$ less effective, and larger resonance region
- Include weak annihilation within QCDf at low q^2 and also in the OPE at high q^2, due to the fact the strong hierarchy $\lambda_b \ll \lambda_d$ means that they are not suppressed as in the $b \to s$ case
- Our work benefits from recent Lattice QCD results for the form factors, as well as recent calculations of the Wilson coefficients at next-to leading order.
- Employed novel method, fitting Shifman model to $e^+e^- \to (\mathrm{hadrons})$ and $\tau \to (\mathrm{hadrons}) + \nu_\tau$ data, and further using the exp. value for the $D^+ \to \pi^+ R\,(R \to \ell^+\ell^-)$, with $R = \rho$, ω, and ϕ, BRs.
- Analysis of uncertainties involving Monte Carlo error propagation, taking into account dominant uncertainties from resonance model and renormalisation scales.

Summary

Conclusions and future prospects

- Our result for the differential BR can serve as a conservative prediction including uncertainties throughout the phase space, an important input for backgrounds in experimental searches
- Due to large weak-annihilation contribution+residual resonance contributions away from the resonance peaks, the integrated non-resonant branching ratios could be of the order of 10^{-9}, sensitivity of LHCb to the branching ratio will be $\sim 10^{-8}$ with 50 fb^{-1} and $\sim 10^{-9}$ with 200 fb^{-1}
- For our model-independent BSM analysis we focused on combinations C_{10}–C_P for F_H and C_P–C_{T5} for A_{FB}, where uncertainties small
- Of the LQ scenarios considered, for vector LQs A_{FB} vanishes and F_H suffers large theoretical uncertainties. In scalar leptoquark scenarios, particularly A_{FB}, can be precisely predicted.
- Look forward to the upcoming results for $D^+ \to \pi^+\ell^+\ell^-$ from LHCb, Belle-II and BES-III, from which we will obtain much-improved bounds on the Wilson coefficients and the models discussed. Urge exps. to measure $\langle F_H \rangle$ and $\langle A_{\mathrm{FB}} \rangle$ in the range q^2 from ~ 1.8 to 2.3 GeV^2, both for $D^+ \to \pi^+\mu^+\mu^-$ and $D^+ \to \pi^+e^+e^-$
- An experimental sensitivity of $\mathcal{O}(10\%)$ to these observables would already provide evidence for BSM physics in certain scenarios, and furthermore a sensitivity of $\mathcal{O}(1\%)$ would make a precise fit to the Wilson coefficients achievable.

Parameters and uncertainties

Parameters	Value	Reference
$m_s(m_s)$ [MeV]	95 ± 3	PDG 2018
m_c [GeV]	$1.67^{+0.07}_{-0.07}$	PDG 2018
$m_b(m_b)$ [GeV]	$4.18^{+0.04}_{-0.03}$	PDG 2018
$m_t(m_t)$ [GeV]	163.3 ± 2.7	Alekhin et al 2012
M_W [GeV]	80.385 ± 0.015	PDG 2018
ω_0[MeV]	450 ± 300	Feldmann et al 2017
f_{π^+} [MeV]	130.5 ± 16	PDG 2018
f_{D^+} [MeV]	212.15 ± 1.45	PDG 2018
a_2(1 GeV)	0.17 ± 0.08	Khodjamirian et al 2011
a_4(1 GeV)	0.06 ± 0.1	Khodjamirian et al 2011
$f(0)$	0.6117 ± 0.0354	ETM coll. 2017
$f_T(0)$	0.5063 ± 0.0786	ETM coll. 2018
c_+	-1.985 ± 0.347	ETM coll. 2017
c_0	-1.188 ± 0.256	ETM coll. 2017
c_T	-1.10 ± 1.03	ETM coll. 2018
P_V	0.1314 ± 0.0127	ETM coll. 2017
P_S	0.0342 ± 0.0122	ETM coll. 2017
P_T	0.1461 ± 0.0681	ETM coll. 2018
τ_{D^+} [ps]	1040 ± 7	PDG 2018
$\|V_{ud}\|$	0.97420 ± 0.0002	PDG 2018
$\|V_{cd}\|$	0.218 ± 0.004	PDG 2018
$\|V_{ub}\|$	$(4.09 \pm 0.39)10^{-3}$	PDG 2016
$\|V_{cb}\|$	$(40.5 \pm 1.5)10^{-3}$	PDG 2016
γ	$(73.2^{+6.3}_{-7.0})^{\circ}$	PDG 2016

Parameters and uncertainties

Parameter	Central value	Relative error
n_ρ	3.070	0.24%
m_ρ (GeV)	0.7653	0.034%
Γ_ρ (GeV)	0.1374	0.40%
$b_{I=1}$	0.323	1.2%
$\sigma^2_{I=1}$ (GeV^2)	2.476	fixed
$a_{I=1}$	0.974	fixed
n_ω	2.51	1.2%
m_ω (GeV)	0.78234	0.0072%
Γ_ω (GeV)	0.0088	1.4%
$b_{I=0}$	0.2	fixed
$\sigma^2_{I=0,1}$ (GeV^2)	2.476	fixed
$a_{I=0}$	1.5	22%
n_ϕ	1.9	0.3%
m_ϕ (GeV)	1.01921	0.0010%
Γ_ϕ (GeV)	0.00421	0.54%
σ^2_s (GeV^2)	3.6	24%
a_s	0.60	20%
b_s	0.20	12%

Implications for BSM models

- Following de Boer et al 2015, the contributions of LQs to WCs $C_9^{(\prime)}$, $C_{10}^{(\prime)}$, $C_S^{(\prime)}$, $C_P^{(\prime)}$, C_T and C_{T_5} in terms of couplings $\lambda_{L/R}^{u\ell/c\ell}$, for $\ell=\mu$, can be expressed as

$$C_{9,10}^{(\prime)} = \frac{\sqrt{2}\pi}{G_F\alpha_e} k_{9,10}^{(\prime)} \frac{\lambda^I_{i(j)}\left(\lambda^J_{i(j)}\right)^*}{M^2}\ , \quad C_T = \frac{\sqrt{2}\pi}{G_F\alpha_e} k_T \left(\frac{\lambda^I_i\left(\lambda^J_j\right)^*}{M^2} + \frac{\lambda^I_j\left(\lambda^J_i\right)^*}{M^2}\right),$$

$$C_{S,P}^{(\prime)} = \frac{\sqrt{2}\pi}{G_F\alpha_e} k_{S,P}^{(\prime)} \frac{\lambda^I_{j(i)}\left(\lambda^J_{i(j)}\right)^*}{M^2}\ , \quad C_{T_5} = \frac{\sqrt{2}\pi}{G_F\alpha_e} k_{T_5} \left(\frac{\lambda^I_i\left(\lambda^J_j\right)^*}{M^2} - \frac{\lambda^I_j\left(\lambda^J_i\right)^*}{M^2}\right),$$

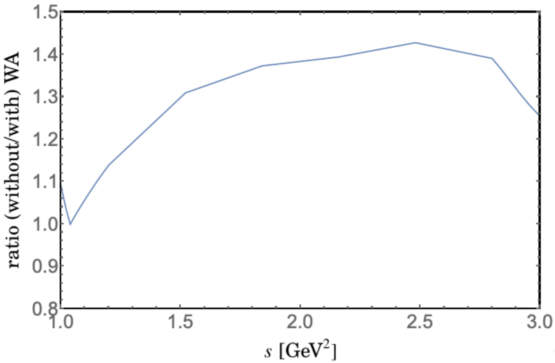
where $i,j = L, R$, and M is a generic scale of LQs

- $\mathcal{B}(K^+\to\pi^+\nu\bar\nu)$ for S_{1L} and $\mathcal{B}(K_L^0\to\mu\mu)$ for S_{2R}, V_2 and V_3 constraints very strong, affected WCs C_9, C_{10}, C_S and C_P can be neglected.

	I	J	i	j	k_9'	k_{10}'	$k_{S/P}'$	k_T	k_{T5}
$S_1\ (3,1,-1/3)$	(cl)	(ul)	L	R	$-\frac{1}{4}$	$-\frac{1}{4}$	$\mp\frac{1}{4}$	$-\frac{1}{8}$	$-\frac{1}{8}$
$S_2\ (3,2,-7/6)$	(ul)	(cl)	R	L	$-\frac{1}{4}$	$\frac{1}{4}$	$\mp\frac{1}{4}$	$-\frac{1}{8}$	$-\frac{1}{8}$
$\tilde V_1\ (3,1,-5/3)$	(ul)	(cl)	–	R	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0
$\tilde V_2\ (3,2,\ 1/6)$	(cl)	(ul)	–	L	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0

Differential branching ratio:

The full spectrum



$$\frac{d\Gamma(D\to\pi\ell\ell)}{dq^2} = 2N\lambda^{1/2}\beta\left[a_\ell(q^2)+\frac{c_\ell(q^2)}{3}\right] = N\lambda^{1/2}\beta d\hat{\Gamma}(q^2)$$

Where $d\hat{\Gamma}(q^2) = (m_D^2 - m_\pi^2)\frac{q^2}{m_c^2} f_0^2(|C_P|^2 + |C_S|^2) + \frac{2}{3}\lambda f_+^2(|C_9^{\mathrm{QCDf}}|^2 + |C_{10}|^2) + \frac{16}{3}\lambda q^2 \frac{f_T^2(|C_T|^2+|C_{T_5}|^2)}{(m_D+m_\pi)^2}$.

Integrated branching ratio measured by LHCb [LHCB-PAPER-2012-051]

Note on the calculation of uncertainties

- Parameters providing dominant contributions to uncertainties: phases of the resonances (ϕ_ω and ϕ_Φ), three parameters entering the ϕ resonance structure and the $s\bar{s}$ resonance excitations (n_Φ, σ^2_Φ and a_Φ) as well as two of the three renormalisation scales which enter the Wilson coefficients (μ_c and μ_W)
- Dependence highly non-linear, no reason to assume a Gaussian distribution for the scale variation $\Rightarrow$ Use a Monte Carlo method with $N = 1000$
- For BSM Physics scenarios impractical to apply this to a large number of points in plane: perform full MC error propagation ($N = 1000$) for a small subset of 9 points and extrapolate between these points

Weak Annihilation in QCDf

Contribution at leading order in α_s:

$$C_9^{(d)}(s)\big|_{\mathrm{Ann}} = 8e_d \frac{\pi^2}{N_c} \frac{f_D f_\pi}{m_D} \frac{1}{f_+(s)} \frac{1}{\lambda_D^-(s)} \; 3C_2,$$

$$C_9^{(b)}(s)\big|_{\mathrm{Ann}} = 8e_d \frac{\pi^2}{N_c} \frac{f_D f_\pi}{m_D} \frac{1}{f_+(s)} \frac{1}{\lambda_D^-(s)} \left[-C_3 - \frac{4}{3}(C_4 + 12C_5 + 16C_6)\right],$$

with the s-dependent moment

$$\frac{1}{\lambda_D^-(s)} = \int_0^\infty d\omega \frac{\phi_D^-(\omega)}{\omega - s/m_D - i\epsilon}$$

WA sizeable contribution as C_2 appears without any cancellation from other WCs $\Rightarrow$ very large contribution
Note in our final results we will modify $\lambda_D^-(s)$ to include those effects employing the ansatz of Feldmann et al 2017:

$$\frac{1}{\lambda_D^-(s)} = \int_0^\infty d\omega \frac{\phi_D^-(\omega) n_d j_d(s)}{\omega - s/m_D - i\epsilon}.$$

Weak annihilation in the OPE

- QCDf valid at small q^2, as here the pion is energetic, and in the heavy-quark limit $(E_\pi \gg \Lambda_{\rm QCD})$
- For low recoil, we can follow Beylich et al 2011, perform an operator product expansion (OPE) in $E_\pi/\sqrt{q^2}$ for $\sqrt{q^2} \gg E_\pi, \Lambda_{\rm QCD})$
- For the case of $D^+ \to \pi^+\ell^+\ell^-$, and we find

$$C_9^{(d)}(s)\Big|_{\rm Ann}^{\rm OPE} = -1/3\,\frac{8\pi^2 C_2 f_D f_\pi}{s f_+(s)}. \tag{2}$$

- $C_9^{(d)}(s)\big|_{\rm Ann}^{\rm OPE} \propto C_2$, $(C_4 + C_3/3$ for $B \to K\ell\ell)$, annihilation also imp. in the high-q^2 regime (other contributions Cabibbo suppressed)
- In differential BR for $q^2 < m_\phi$ use QCDf, for $q^2 > m_\phi$ OPE.

Note that the latter are not really valid down to m_ϕ nor up to the lowest recoil point, and therefore in phenomenological analysis we only consider q^2 in the range 1.8 to 2.3 GeV^2.

Form factors from the Lattice

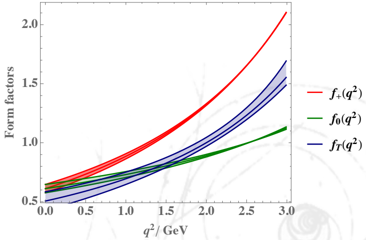
$$f_+^{D\to\pi}(q^2) = \frac{f^{D\to\pi}(0) + c_+^{D\to\pi}(z - z_0)\left(1 + \frac{z+z_0}{2}\right)}{1 - P_V\, q^2}\,, \tag{68}$$

$$f_0^{D\to\pi}(q^2) = \frac{f^{D\to\pi}(0) + c_0^{D\to\pi}(z - z_0)\left(1 + \frac{z+z_0}{2}\right)}{1 - P_S\, q^2}\,. \tag{69}$$

The values of the five parameters $f^{D\to\pi}(0)$, $c_+^{D\to\pi}$, P_V, $c_0^{D\to\pi}$ and P_S are collected in Table 6, with the corresponding covariance matrix given in Table 7.

$f^{D\to\pi}(0)$	$c_+^{D\to\pi}$	P_V (GeV^{-2})	$c_0^{D\to\pi}$	P_S (GeV^{-2})
0.6117 (354)	−1.985 (347)	0.1314 (127)	−1.188 (256)	0.0342 (122)

Scalar and vector (and Tensor) form factors of $D \to \pi(K)\ell\nu$ and $D \to \pi(K)\ell\ell$ decays with $N_f = 2+1+1$ twisted fermions - Phys.Rev. D96 (2017) no.5, 054514 arXiv:1706.03017 [hep-lat], Phys.Rev. D98 (2018) no.1, 014516, arXiv:1803.04807 [hep-lat]



Note that f_+ and f_0 in good agreement with Khodjamirian, Klein, Mannel, Offen arXiv 0907.2842.

Bounds on Wilson Coefficients

Constraints from $\mathcal{B}(D \to \ell\ell)$

$|C_S - C_S'|^2 + |C_P - C_P' + 0.1(C_{10} - C_{10}')|^2 \leq 0.008$, as $\mathcal{B}(D^0 \to \ell^+\ell^-) < 6.2\times 10^{-9}$ at 90 % C.L.

Assuming them to be real, $C_{10} - C_{10}' \leq 0.86$ and $C_{P(S)} - C_{P(S)}' \leq 0.087$

Constraints from $\mathcal{B}(D \to \pi\ell\ell)$

q^2-bin	90% C.L. Limit	$\mathcal{B}(D \to \pi\ell\ell)$ (SM)
full q^2	7.3×10^{-8}	4.2×10^{-7}
low q^2: $[0.250^2, 0.525^2]$ GeV^2	2.0×10^{-8}	6.3×10^{-9}
high q^2: $q^2 > 1.25^2$ GeV^2	2.6×10^{-8}	5.0×10^{-10}

	Reg. I	Reg. II
$\lvert C_7^{\mathrm{BSM}}\rvert$	≤ 1.58	≤ 0.67
$\lvert C_9^{\mathrm{BSM}}\rvert$	≤ 2.17	≤ 0.84
$\lvert C_{10} + C_{10}'\rvert$	≤ 0.938	≤ 1.1
$\lvert C_S + C_S'\rvert$	≤ 3.81	≤ 0.60
$\lvert C_P + C_P'\rvert$	≤ 3.28	≤ 0.60
$\lvert C_T\rvert$	≤ 3.50	≤ 0.68
$\lvert C_{T5}\rvert$	≤ 2.48	≤ 0.77