On gluon polarization at small x

Cyrille Marquet

Centre de Physique Théorique
Ecole Polytechnique & CNRS

Spin physics and TMDs

TMDs are crucial to describe hard processes in polarized collisions
(e.g. Drell-Yan and semi-inclusive DIS)

8 leading-twist TMDs

Sivers function

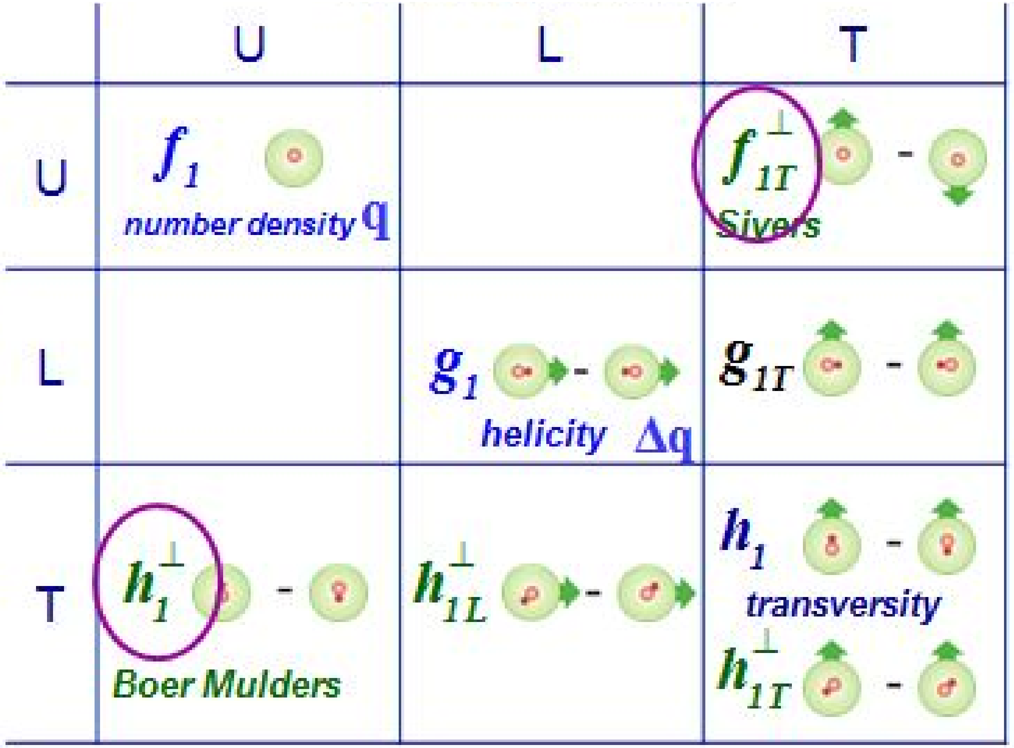
correlation between transverse spin of the nucleon and transverse momentum of the quark

Boer-Mulders function

correlation between transverse spin and transverse momentum of the quark in unpolarized nucleon

nucleon polarization

quark polarization



	U	L	T
U	f_1 number density q		$f_{1T}^{\perp}$ - Sivers
L		g_1 - helicity Δq	g_{1T} -
T	$h_1^{\perp}$ - Boer Mulders	$h_{1L}^{\perp}$ -	h_1 - transversity $h_{1T}^{\perp}$ -

Spin physics and TMDs

TMDs are crucial to describe hard processes in polarized collisions

(e.g. Drell-Yan and semi-inclusive DIS)

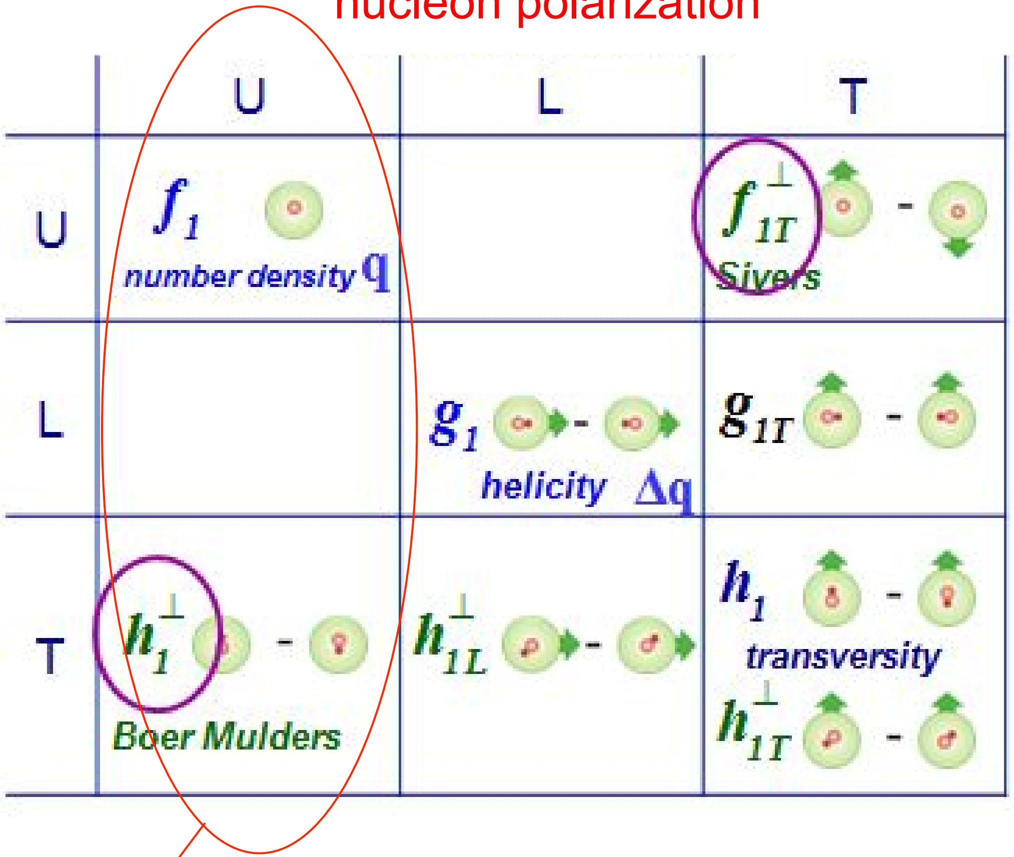
8 leading-twist TMDs

Sivers function

correlation between transverse spin of the nucleon and transverse momentum of the quark

Boer-Mulders function

correlation between transverse spin and transverse momentum of the quark in unpolarized nucleon



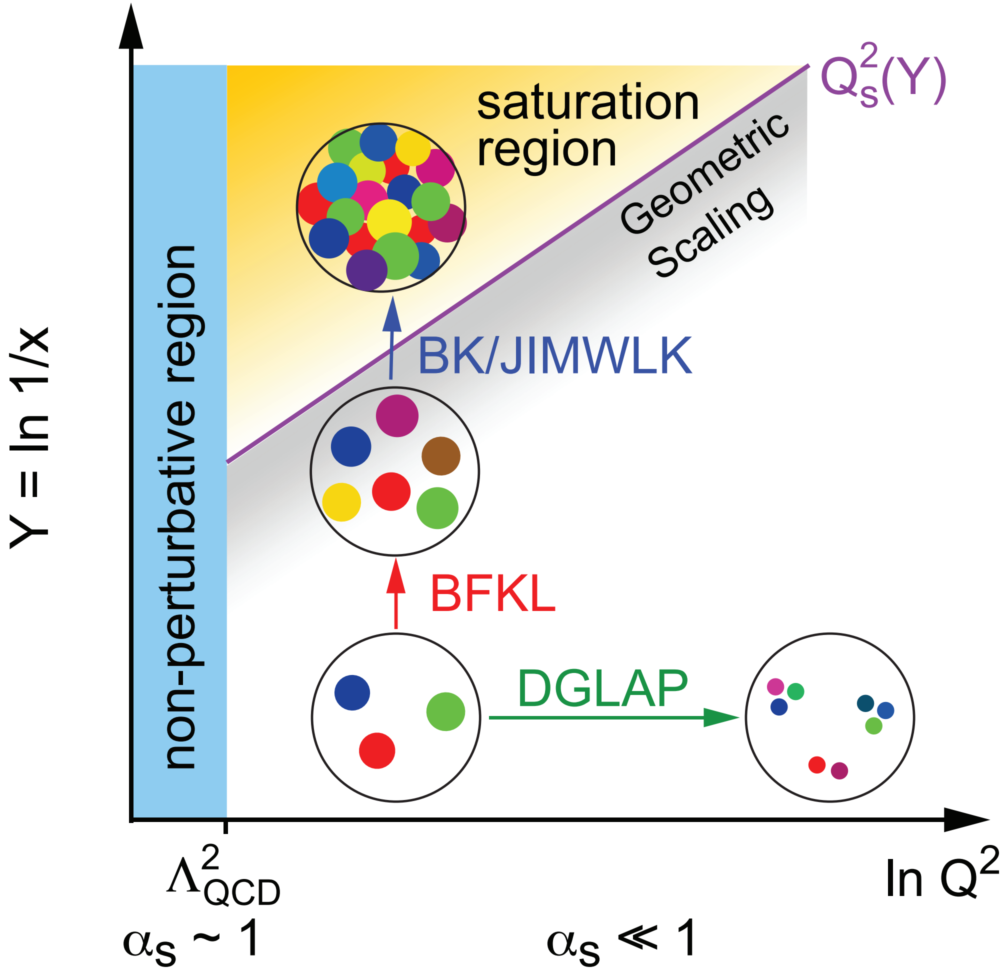
nucleon polarization

quark polarization

	U	L	T
U	f_1 number density q		$f_{1T}^{\perp}$ Sivers
L		g_1 helicity Δq	g_{1T}
T	$h_1^{\perp}$ Boer Mulders	$h_{1L}^{\perp}$	h_1 transversity $h_{1T}^{\perp}$

I will discuss those for gluons

Small x and saturation



what does small-x physics have to say about gluon TMDs ?

Unpolarized & linearly-polarized gluon TMDs

Generic definitions of gluon TMDs

I consider only hadronic/nuclear states that are *unpolarized*

$$2\int\frac{d\xi^+d^2\boldsymbol{\xi}_t}{(2\pi)^3p_A^-}e^{ixp_A^-\xi^+-ik_t\cdot\boldsymbol{\xi}_t}\left\langle A\right|\mathrm{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}_t\right)F^{j-}\left(0\right)\right]\left|A\right\rangle$$

$$=\frac{\delta_{ij}}{2}\mathcal{F}(x,k_t)+\left(\frac{k_ik_j}{k_t^2}-\frac{\delta_{ij}}{2}\right)\mathcal{H}(x,k_t)$$

unpolarized gluon TMD

linearly-polarized gluon TMD

Generic definitions of gluon TMDs

I consider only hadronic/nuclear states that are *unpolarized*

$$2\int\frac{d\xi^+d^2\boldsymbol{\xi}_t}{(2\pi)^3p_A^-}e^{ixp_A^-\xi^+-ik_t\cdot\boldsymbol{\xi}_t}\left\langle A\right|\mathrm{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}_t\right)F^{j-}\left(0\right)\right]\left|\,A\right\rangle$$

$$=\underbrace{\frac{\delta_{ij}}{2}\mathcal{F}(x,k_t)}_{\text{unpolarized gluon TMD}}+\left(\frac{k_ik_j}{k_t^2}-\frac{\delta_{ij}}{2}\right)\underbrace{\mathcal{H}(x,k_t)}_{\text{linearly-polarized gluon TMD}}$$

- at small x, $\mathcal{F}=\mathcal{H}$ in the BFKL regime:

$$\mathcal{F}(x,k_t)=UGD(x,k_t)+\mathcal{O}(Q_s^2/k_t^2)$$

$$\mathcal{H}(x,k_t)=\underbrace{UGD(x,k_t)}_{\text{so-called unintegrated gluon distribution}}+\mathcal{O}(Q_s^2/k_t^2)$$

Gluon TMDs and gauge links

- the naive operator definition is not gauge-invariant

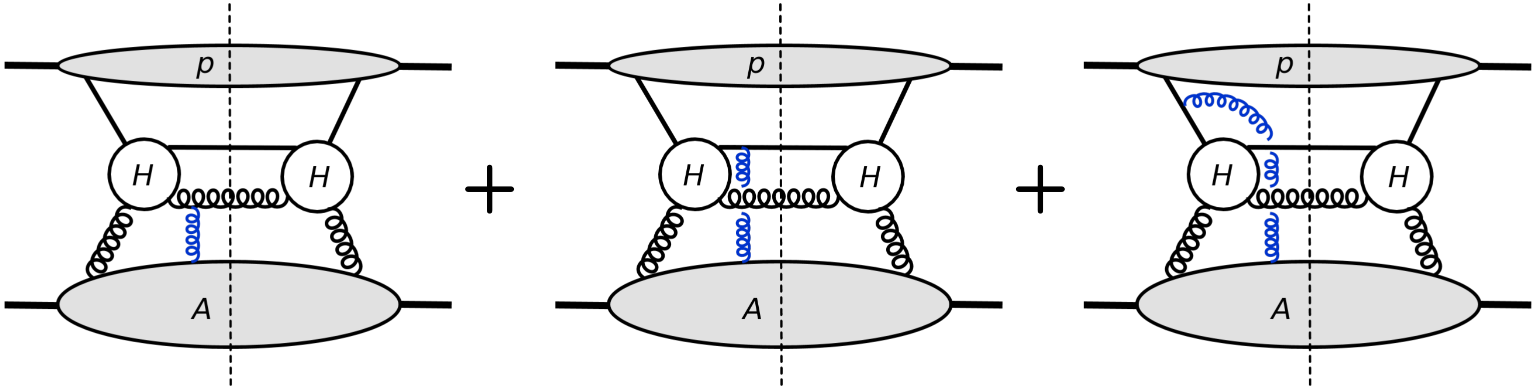
$$\mathcal{F}_{g/A}(x_2, k_t) \overset{\text{naive}}{=} 2 \int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \boldsymbol{\xi}_t} \left\langle A \right| \mathrm{Tr} \left[F^{i-} \left(\xi^+, \boldsymbol{\xi}_t\right) F^{i-} (0) \right] \left| A \right\rangle$$

Gluon TMDs and gauge links

- the naive operator definition is not gauge-invariant

$$\mathcal{F}_{g/A}(x_2, k_t) \overset{\text{naive}}{=} 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}_t} \left\langle A\right|\mathrm{Tr}\left[F^{i-}\left(\xi^+, \boldsymbol{\xi}_t\right) F^{i-}(0)\right]\left|A\right\rangle$$

- a theoretically consistent definition requires to include more diagrams



$+$ similar diagrams with $2, 3, \ldots$ gluon exchanges

They all contribute at leading power and need to be resummed.

this is done by including gauge links in the operator definition

Process-dependent TMDs

- the proper operator definition(s)

some gauge link $\mathcal{P}\exp\left[-ig\int_{\alpha}^{\beta} d\eta^{\mu} A^{a}(\eta)\,T^{a}\right]$

$$\mathcal{F}_{g/A}(x_2, k_t) = 2\int \frac{d\xi^{+} d^{2}\boldsymbol{\xi}_t}{(2\pi)^{3} p_A^{-}} e^{ix_2 p_A^{-}\xi^{+} - ik_t\cdot\boldsymbol{\xi}_t} \left\langle A\right|\mathrm{Tr}\left[F^{i-}\left(\xi^{+},\boldsymbol{\xi}_t\right) U_{[\xi,0]} F^{i-}(0)\right]\left|A\right\rangle$$

  - ▶ $U_{[\alpha,\beta]}$ renders gluon distribution gauge invariant

different processes require a different gauge-link structure,
implying in turn different gluon TMDs

Process-dependent TMDs

- the proper operator definition(s)

$$\mathcal{F}_{g/A}(x_2, k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}_t} \left\langle A\right|\mathrm{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}_t\right) U_{[\xi,0]} F^{i-}(0)\right]\left|A\right\rangle$$

several paths are possible for the gauge links

examples :

- in the large k_t limit, the process dependence of the gauge links disappears (like for the integrated gluon distribution), and a single gluon distribution is sufficient

$$\mathcal{F}_{g/A}(x_2, k_t) = UGD(x_2, k_t) + \mathcal{O}(Q_s^2/k_t^2)$$

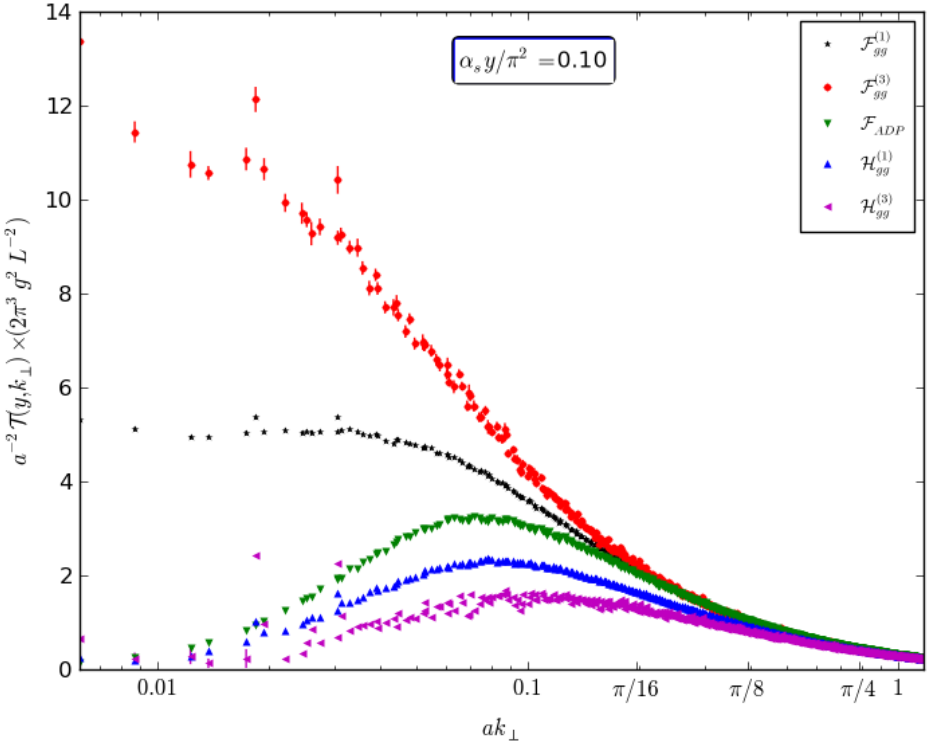
$$\mathcal{F}(x_2, k_t) - \mathcal{H}(x_2, k_t) = \mathcal{O}(Q_s^2/k_t^2)$$

JIMWLK numerical results

using a code written by Claude Roiesnel

initial condition at y=0 : MV model
evolution: JIMWLK at leading log

CM, Petreska, Roiesnel (2016)
CM, Roiesnel, Taels (2017)

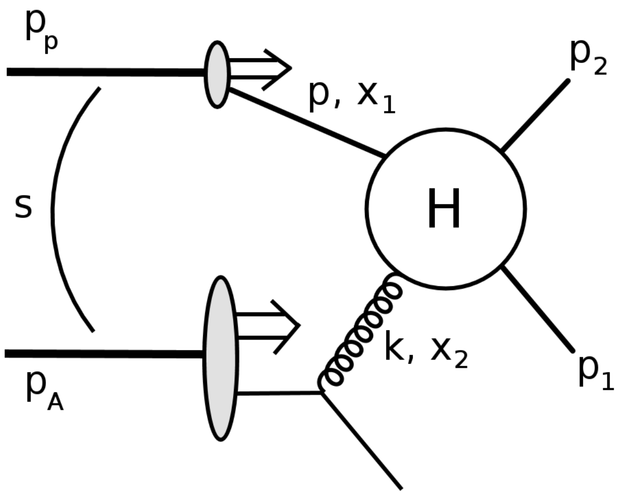


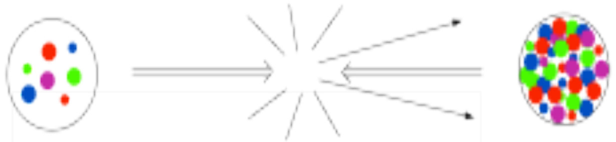
saturation effects impact the various gluon TMDs in very different ways

Probing the gluon TMDs
at collider

The context: forward di-jets

- large-x projectile (proton) on small-x target (proton or nucleus)





$$\langle k_{1t}\rangle \sim \Lambda_{QCD} \qquad \langle k_{2t}\rangle \sim Q_s(x_2)$$

$$Q_s(x_2) \gg \Lambda_{QCD}$$

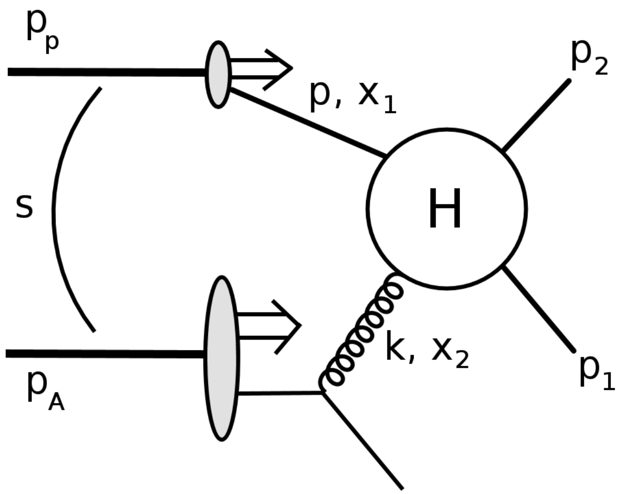
so-called “dilute-dense” kinematics

Incoming partons’ energy fractions:

$$\begin{aligned} x_1 &= \tfrac{1}{\sqrt{s}}\left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \\ x_2 &= \tfrac{1}{\sqrt{s}}\left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \end{aligned} \quad \xrightarrow{y_1, y_2 \gg 0} \quad \begin{aligned} x_1 &\sim 1 \\ x_2 &\ll 1 \end{aligned}$$

The context: forward di-jets

- large-x projectile (proton) on small-x target (proton or nucleus)



$$\langle k_{1t}\rangle \sim \Lambda_{QCD} \qquad \langle k_{2t}\rangle \sim Q_s(x_2)$$

$$Q_s(x_2) \gg \Lambda_{QCD}$$

so-called “dilute-dense” kinematics

Incoming partons’ energy fractions:

$$\begin{aligned} x_1 &= \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \\ x_2 &= \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \end{aligned} \quad \xrightarrow{y_1,y_2\gg 0} \quad \begin{aligned} x_1 &\sim 1 \\ x_2 &\ll 1 \end{aligned}$$

Gluon’s transverse momentum (p_{1t}, p_{2t} imbalance):

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}|\cos\Delta\phi$$

relevant regime here : $|p_{1t}|,\ |p_{2t}| \sim \mathbf{P} \gg Q_s$

TMD factorization

- TMDs appear in cross-sections by pairs:

several pairs may be involved depending on the process

$$\mathrm{d}\sigma \propto f(x_1) \sum_c H^{ij}_{(c)}(\mathbf{P}, k_t) \Big[\frac{1}{2}\delta^{ij} \mathcal{F}_{(\mathrm{c})}(x_2, k_t) + \Big(\frac{k^i k^j}{k_t^2} - \frac{1}{2}\delta^{ij} \Big) \mathcal{H}_{(\mathrm{c})}(x_2, k_t) \Big]$$

hard factors

standard pdf

TMD factorization

- TMDs appear in cross-sections by pairs:

several pairs may be involved depending on the process

$$\mathrm{d}\sigma \propto f(x_1) \sum_c H^{ij}_{(c)}(\mathbf{P}, k_t) \Big[\frac{1}{2} \delta^{ij} \mathcal{F}_{(\mathrm{c})}(x_2, k_t) + \Big(\frac{k^i k^j}{k_t^2} - \frac{1}{2} \delta^{ij} \Big) \mathcal{H}_{(\mathrm{c})}(x_2, k_t) \Big]$$

hard factors

standard pdf

- TMD factorization involves $H^{ij}_{(c)}(\mathbf{P}, k_t = 0)$

need $\mathbf{P} \gg k_t, \; Q_s$

- improved TMD (ITMD) factorization involves

$$H^{ij}_{(c)}(\mathbf{P}, k_t) = H^{ij}_{(c)}(\mathbf{P}, k_t = 0) + \sum_n c_n (k_t/\mathbf{P})^n$$

all-order resummation of higher
“kinematic” twists

Processes sensitive to $\mathcal{H}$

- ITMD factorization may be rewritten

$$d\sigma \propto f(x_1) \sum_c \Big[H^{ns}_{(c)}(\mathbf{P},k_t)\mathcal{F}_{(\mathrm{c})}(x_2,k_t) + H^{h}_{(c)}(\mathbf{P},k_t)\Big(\underbrace{\mathcal{H}_{(\mathrm{c})}(x_2,k_t) - \mathcal{F}_{(\mathrm{c})}(x_2,k_t)}_{= 0 \text{ in BFKL regime}}\Big)\Big]$$

projections onto
“non-sense” polarization

$$H^{ns}_{(c)} = H^{ij}_{(c)} k^i k^j / k_t^2$$

projections onto linear polarization

$$H^{h}_{(c)} = H^{ij}_{(c)} (k^i k^j / k_t^2 - \delta^{ij}/2)$$

Processes sensitive to $\mathcal{H}$

- ITMD factorization may be rewritten

$$d\sigma \propto f(x_1) \sum_c \left[H^{ns}_{(c)}(\mathbf{P}, k_t)\mathcal{F}_{(\mathrm{c})}(x_2, k_t) + H^{h}_{(c)}(\mathbf{P}, k_t)\underbrace{\Big(\mathcal{H}_{(\mathrm{c})}(x_2, k_t) - \mathcal{F}_{(\mathrm{c})}(x_2, k_t) \Big)}_{\text{= 0 in BFKL regime}} \right]$$

projections onto
"non-sense" polarization

$$H^{ns}_{(c)} = H^{ij}_{(c)} k^i k^j / k_t^2$$

projections onto linear polarization

$$H^{h}_{(c)} = H^{ij}_{(c)} (k^i k^j / k_t^2 - \delta^{ij}/2)$$

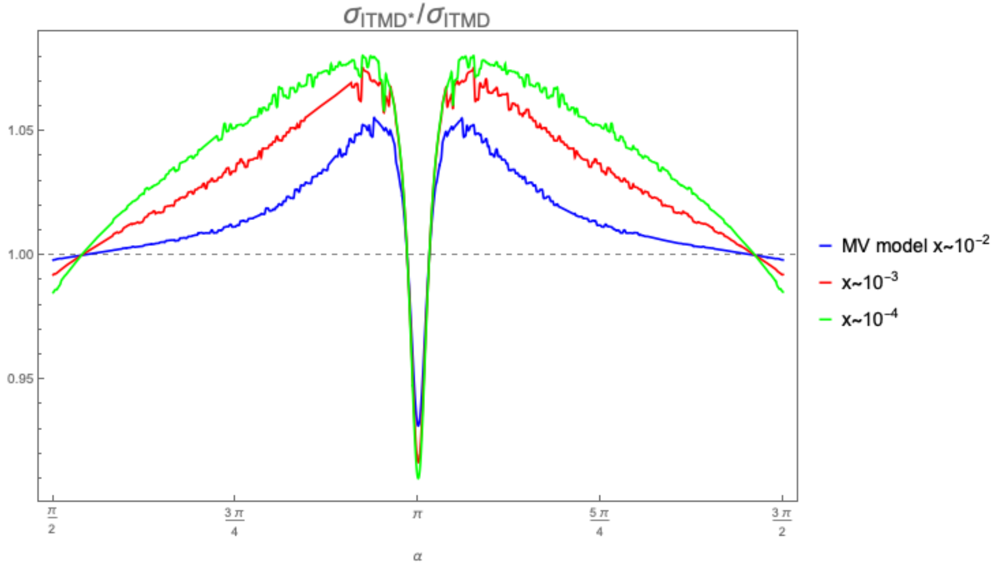
- processes for which the $H^{h}_{(c)}$ hard factors are non-zero:
  - dijets in deep inelastic scattering (e+p or e+A)
  - heavy-quark pair production (in photo-production or p+A collisions)
  - trijets or more

linearly-polarized gluons come with a cos(2ϕ) modulation
(at small $\mathrm{k_t}$ / **P**) where ϕ is the angle between k_t and **P**

Dijets in deep inelatic scattering

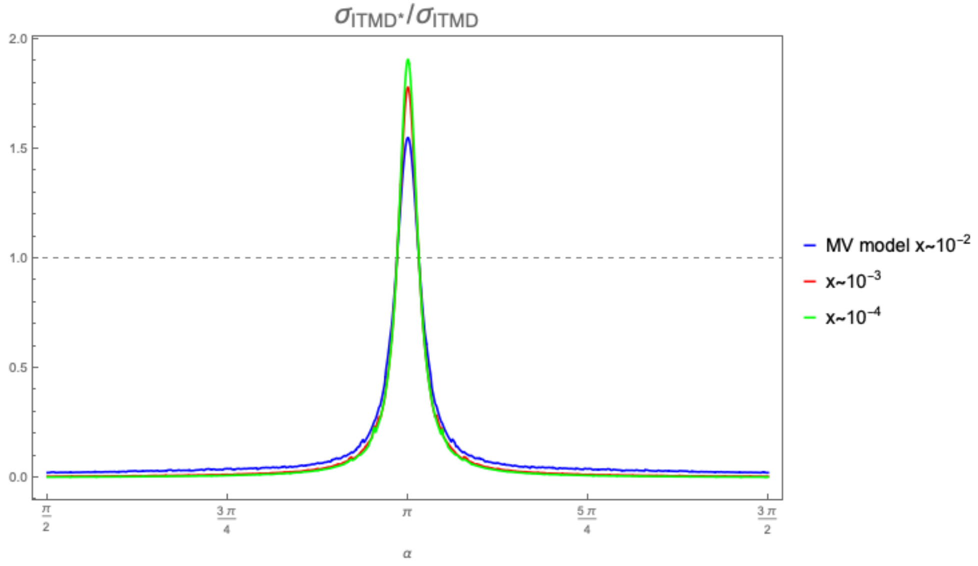
- looking at $\Delta\Phi$ distributions:

Altinoluk, CM, Taels (2021)



$$\frac{\text{BFKL}}{\text{BFKL + saturation}}$$

for transverse photon



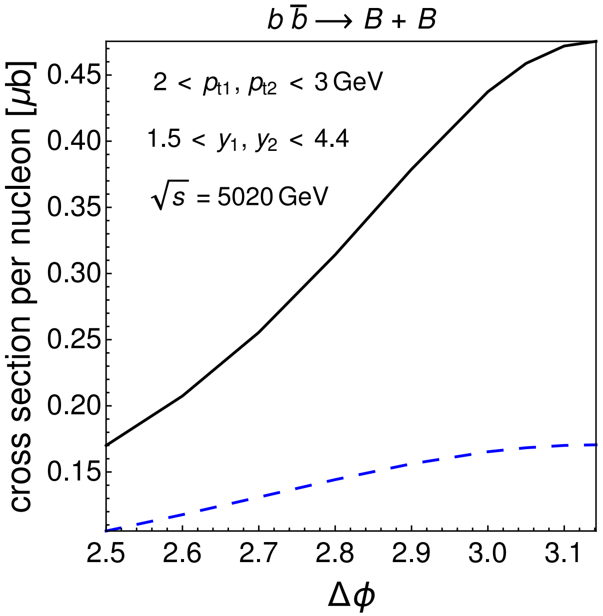
$$\frac{\text{BFKL}}{\text{BFKL + saturation}}$$

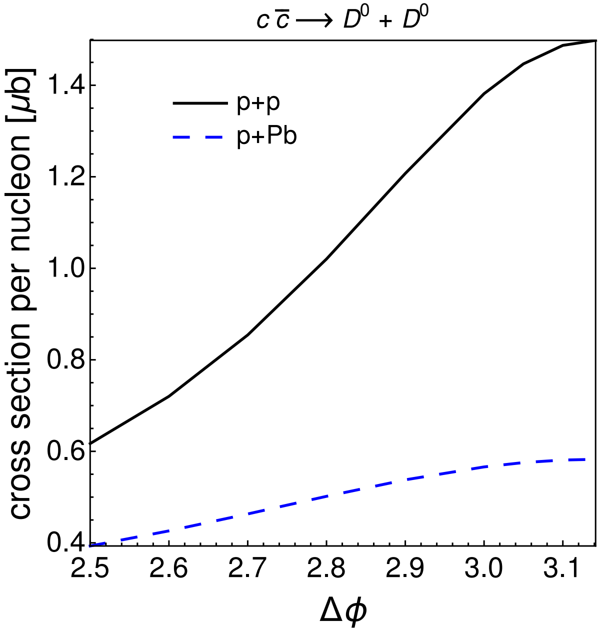
for longitudinal photon

Forward $Q\bar{Q}$ pair in p+A collisions

- preliminary study performed for HL-LHC yellow report

CM, Giacalone (2018)





ITMD hard factors to be implemented

important away from $\Delta\Phi = \pi$, when $k_t \sim \mathbf{P}$

soft-gluon resummation to be added as well

important near $\Delta\Phi = \pi$, when $\log(\mathbf{P}/k_t)$ becomes large

Conclusions

- different processes involve different gluon TMDs, with different operator definitions
- each operator definition provides an unpolarized gluon TMD and a linearly-polarized one
- the various gluon TMDs coincide at large transverse momentum, in the linear regime
- however, they differ significantly from one another at low transverse momentum, in the non-linear saturation regime
- one could use small-x gluons which are not fully linearly polarized to look for saturation effects