Renormalization group optimized perturbation at finite temperatures in QCD

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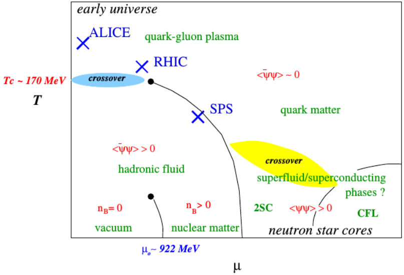
Overview

Phase diagram of QCD



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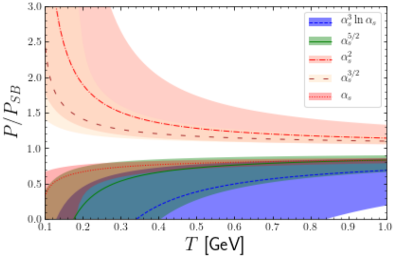
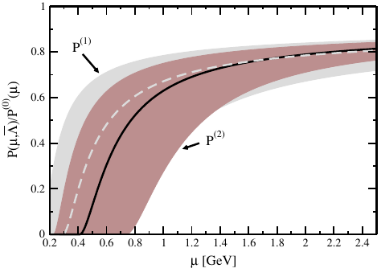
- LQCD works at finite T and low μ (ALICE).
- At intermediate T and μ (RHIC) LQCD fails (sign problem) (see Patztor and Ziegler talks for improvements in LQCD at finite μ).
- Low T and high μ important to describe neutron star cores (Tyler Gorda talk).

[1] Picture from arXiv:physics/0105022

The renormalization scale M varies (by convention) between πT and $4\pi T$ (μ and 4μ)

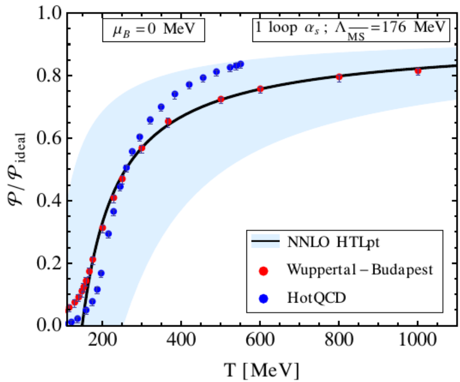




Kajantie et al, Phys.Rev.D 67,105008(2003)   Vuorinen, Phys.Rev.D 68,054017(2003); Kurkela at al, Phys.Rev.D 81,105021(2010)

- Poor convergence (oscillations).
- Scale dependence is quite significant.
- Scale dependence increases at higher perturbative orders.

Hard Thermal Loops Perturbation Theory (HTLpt at NNLO)



Haque et al, JHEP 1405, 027 (2014)

- Results close to LQCD when $M = 2\pi T$.
- High temperature approximation $m/T \ll 1$.
- Also presents a quite significant scale dependence.

Alternative

RGOPT

Alternative resummation technique built to satisfy renormalization group properties.
Already used to calculate:
The QCD coupling constant, α_s. [Kneur and Neveu (2013)]
The quark condensate from the spectral function. [Kneur and Neveu (2015)]
Applied in scalar models at finite temperature. [Kneur and Pinto (2015), Ferrari et al (2017)]
Applied in QCD at finite densities. [Kneur, Pinto and Restrepo (2019)]

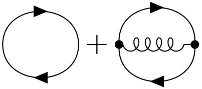
Important

Here we apply the RGOPT only to the quark sector of QCD, while the gluon sector is given by its perturbative expression
The gluonic sector has several technical difficulties to apply the RGOPT (under investigation, Kneur and Pinto)

Steps to apply the RGOPT

1. Restore RG invariance in the perturbative massive theory (explicitly non RG invariant even at 1L order)

$$P_q^{\rm RGPT} = P_q^{PT} - m^4 \sum_{k \geq 0} s_k g^{k-1},$$

where $g = 4\pi\alpha_s$ and $P_q^{\rm PT} =$ 

2. Deform the RG invariant perturbative pressure by rescaling the coupling and adding an *arbitrary* variational mass term.

$$P^{\rm RGOPT} = P^{\rm RGPT}(g \to \delta g, m \to m(1-\delta)^a)$$

Here $m_c = 0$ ($N_f = 3$) (chiral limit), δ is a dummy expansion parameter (interpolates between the free ($\delta = 0$) and the interacting theory ($\delta = 1$))

Steps to apply the RGOPT

3. Expand in powers of δ up to the order considered, set $\delta \to 1$ (original theory) and fix the arbitrary mass parameter m with the variational criterion called mass optimization prescription (MOP)

$$\partial_m P^{\mathrm{RGOPT}}(\mathcal{O}(\delta^k), \delta \to 1)\Big|_{\overline{m}} \equiv 0$$

4. Fix the exponent a at order δ^0 (one loop) using the reduced ($\mathrm{m}_c = 0$) RG equation, $[M\partial_M + \beta(g)\partial_g]P^{\mathrm{RGOPT}}_{\delta\to 1} = 0$, which leads to $a = \gamma_0/(2b_0)$.
   At NLO (two loops) the RG equation can provide an alternative determination of $\overline{m}$.

Solutions are chosen so that they match asymptotic freedom behavior for $g \to 0$ (at NLO are unique) .

Solution up to order δ

- At LO (1L) the RGOPT produces non trivial scale invariant results where a is fixed by the RG equation and $\overline{m}$ by the MOP equation
- At NLO (order δ), solving the MOP or the RG equation with g given by the exact perturbative 2L expression one finds complex solutions in the $\overline{\text{MS}}$ scheme. What to do?

How to recover real solutions? Renormalization scheme change (RSC)

$m \to m(1 + B_2 g^2)$ (done in P^{RGPT})
This adds an extra term $-4gm^4 s_0 B_2$ to the pressure.

B_2 is a free parameter. However $B_2 g^2$ must be small so that the deviation from the original scheme remains moderate.
B_2 must be fixed by an extra, sensible prescription that allows real solutions.

MOP prescription

$$\partial_m P_q^{\mathrm{RGOPT}}\Big|_{\overline{m}} \equiv 0 \Rightarrow -\ln \frac{m^2}{M^2} + B_{mop} - \frac{2\pi}{3g}\sqrt{D_{mop}} = 0$$

$$B_2 = -\frac{329}{1728\pi^2\, g}$$

RG prescription

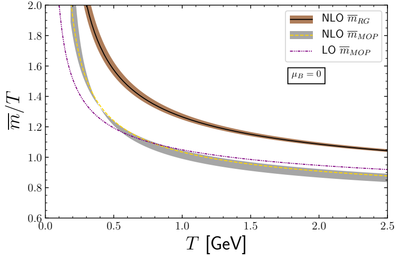
$$[M\partial_M + \beta(g)\partial_g] P_q^{\mathrm{RGOPT}} = 0 \Rightarrow -\ln \frac{m^2}{M^2} + B_{rg} \mp \frac{8\pi^2}{g}\sqrt{\frac{2}{3} D_{rg}} = 0$$

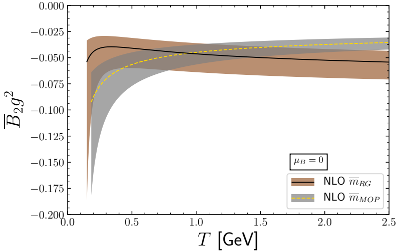
We choose B_2 such that $D_{rg} = 0$

Results

$$\Lambda_{\overline{\text{MS}}} = 335 \text{ MeV}, \ \ln \frac{M}{\Lambda_{\overline{\text{MS}}}} = \frac{1}{2b_0\, g} + \frac{b_1}{2b_0^2} \ln \left(\frac{b_0 g}{1+\frac{b_1}{b_0} g} \right) \text{ (exact 2L running)},$$

$$\pi T \leq M \leq 4\pi T$$

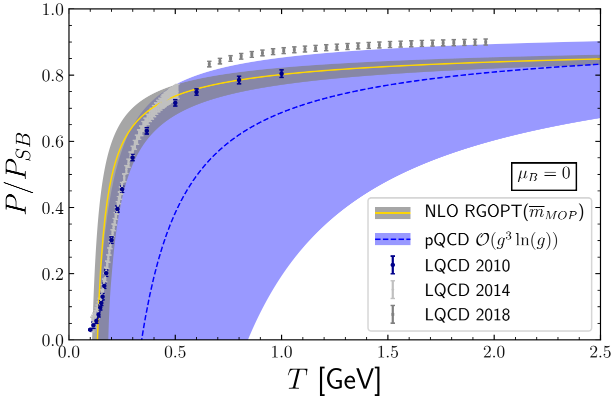




- The variational thermal RGOPT masses behave very similar to a screened thermal mass.
- The RSC deviation from the $\overline{\text{MS}}$ scheme remains moderate.

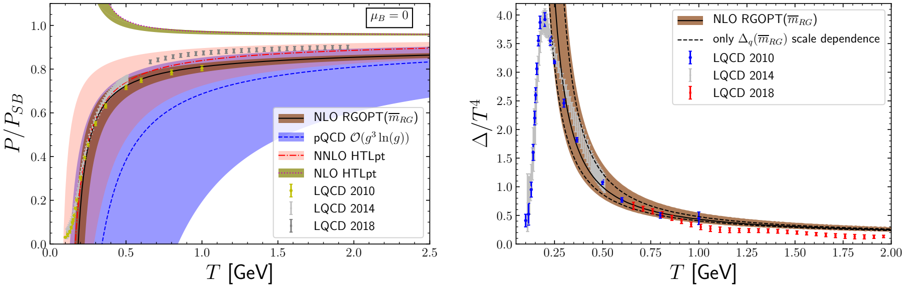
Results (for MOP prescription)

$P = P_g^{\rm PT} + P_q^{\rm RGOPT}$, with $P_g^{\rm PT} = \left(N_c^2 - 1\right) \frac{\pi^2 T^4}{45} \left(1 - \frac{15}{4}\frac{g}{4\pi^2}\right)$ (2L order).



Kajantie et al, Phys.Rev. D 67, 105008 (2003); Borsanyi et al, JHEP 11, 077 (2010); Phys. Lett. B 730, 99 (2014); Bazavov et al, Phys. Rev. D 97, 014510 (2018)

Results (for RG prescription)



- NLO RGOPT results very close to LQCD 2010 (HTLpt NNLO and pQCD N^3LO)
- Considerable reduction of scale dependence.
- RGOPT also fails (at this stage) to describe the phase transitions.

Conclusions

- As a first approximation we considered the perturbative gluonic expression plus the RGOPT quark pressure. This approximation seems appropriate to describe QCD matter above $T \sim 0.25$ GeV $\sim 1.5T_c$.
- Our results at NLO are very close to LQCD data, especially the RG prescription.
- When compared with pQCD and HTLpt, the RGOPT drastically reduces the uncertainty implied by the scale dependence.

For more details
arXiv:1908.08363 and arXiv:2101.02124

THANKS!!

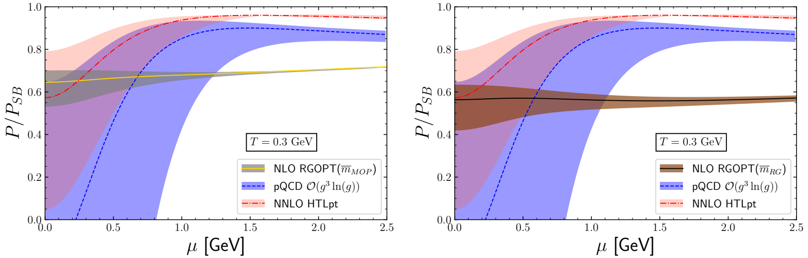
Backup

$$B_{mop} = -\frac{7\pi^2}{9g} + \frac{5}{6} + 4\pi^2 \left(J_2' + \frac{T^2}{m^2} J_2 \right), \tag{1}$$

$$\begin{aligned} D_{mop} \quad = \quad & 9\frac{\pi^2}{4} - \frac{47}{6} g - g^2 \left(\frac{35}{16\pi^2} + 288\frac{\pi^2}{7} B_2 \right) \\ & + 36\pi^2 g^2 \left(J_2'^2 + \frac{T^4}{m^4} J_2^2 \right) + 9g(g - 2\pi^2) \left(\frac{T^2}{m^2} J_2 - J_2' \right) \\ & + 8\pi^2 g^2 \frac{T^2}{m^2} (3J_2 - 1) J_2' - 48\pi^2 g^2 \left(J_3' + \frac{T^2}{m^2} J_3 \right) \quad , \end{aligned} \tag{2}$$

$$B_{rg} = -\frac{1}{b_0\, g} + \frac{172}{81} - \frac{64}{81} \left(\frac{4g}{9\pi^2} \right) \frac{1}{1 + \frac{4g}{9\pi^2}} + 8\pi^2 \frac{T^2}{m^2} J_2, \tag{3}$$

$$D_{rg} = -\left(\frac{3}{7} B_2 + \frac{11}{384\pi^4} \right) g^2 - \frac{g}{27} \frac{(4g + 81\pi^2)}{(4g + 9\pi^2)^2} + g^2 \frac{T^4}{m^4} J_2 \left(J_2 - \frac{1}{6} \right) - g^2 \frac{T^2}{m^2} J \tag{4}$$



P/P_{SB}
μ [GeV]
$T = 0.3$ GeV
NLO RGOPT($\overline{m}_{MOP}$)
pQCD $\mathcal{O}(g^3 \ln(g))$
NNLO HTLpt
NLO RGOPT($\overline{m}_{RG}$)