Bottom-up thermalization and hydrodynamisation on a hybrid hydrodynamic attractor surface

Toshali Mitra

Ayan Mukhopadhyay,Sukrut Mondkar
Anton Rebhan, Alexander Soloviev

based on arxiv: 2006.09383

The Institute of Mathematical Sciences, HBNI

SEWM, 2021

2 july 2021

Introduction

- The puzzle of applicability of hydrodynamics at far from equilibrium is solved by **Hydrodynamic attractor**.
  [M P.Heller and M Spalinski, arXiv:1503.07514]

- Hydrodynamic attractors are evolution obtained by resumming the derivative expansion to all orders and imposing regular initial conditions.

Atttractors from different microscopic theory:

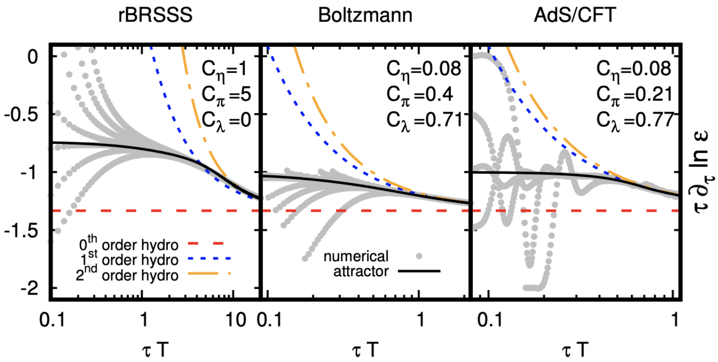

Figure: Numerical results for energy density evolution as a function of τT [P Romatschke, arXiv:1704.08699]

For approach to the attractor at early and late time at weak and strong coupling see- [A Kurkela, Wilke van der Schee, Urs Achim Wiedemann and B Wu, arXiv: 1907.08101]

Why Semi-holographic approach?

- Coexistence of Weak and strong coupling simultaneously in the QGP state calls for the need of an effective theory to describe both the degrees of freedom.
- Semi-holographic approach incorporates both perturbative and non-perturbative (modelled by holography as a classical gravity theory in 5-D) descriptions consistently in one framework.
  [T Faulkner and J Polchinski arXiv: 1001.5049,
  A Mukhopadhyay and G Policastro, arXiv:1306.3941,
  E Iancu and A Mukhopadhyay, arXiv: 1410.6448]
- Democratic coupling between weakly and strongly self interacting subsystems allows us to construct the low energy dynamics of the full system from effective low energy descriptions of the subsystems.

Coupling Scheme:

[S Banerjee, N Gaddam, and A Mukhopadhyay, arXiv: 1701.01229]

- Principles of democratic coupling is based on the following scheme-
  - The two sector interacts by deforming the marginal/relevant coupling of the respective theories.The marginal/relevant couplings and background metric of each sub-sector becomes local algebraic functions of the operators of the other sector.
  - The full dynamics can be solved iteratively. Convergence is quick and leads to conserved energy-momentum tensor of the full system in the physical background metric.
    [E Iancu and A Mukhopadhyay, arXiv: 1410.6448,
    A Mukhopadhyay, F Preis, A Rebhan, and Stefan A Stricker, arXiv:1512.06445,
    C Ecker, A Mukhopadhyay, F Preis, A Rebhan, and A Soloviev, arXiv:1806.01850]

Metric coupling

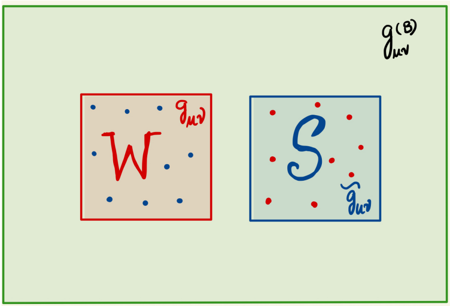

Figure: Schematic diagram of two coupled system

Metric coupling

[A Kurkela, A Mukhopadhyay, F Preis, A Rebhan and A Soloviev, arXiv: 1805.05213]

- Action formalism, $g^{(B)}_{\mu\nu}$ **is the background metric**

$$\mathcal{S} = \mathcal{S}_1[g_{\mu\nu}] + \mathcal{S}_2[\tilde{g}_{\mu\nu}] + \frac{1}{2\gamma}\int d^d x \sqrt{-g^{(B)}}(g_{\mu\alpha} - g^{(B)}_{\mu\alpha})g^{(B)\alpha\beta}(\tilde{g}_{\beta\nu} - g^{(B)}_{\beta\nu})g^{(B)\nu\mu} + \frac{1}{2\gamma}\frac{\gamma'}{d\gamma' - \gamma}\int d^d x \sqrt{-g^{(B)}}(g_{\mu\nu}g^{(B)\mu\nu} - d)(\tilde{g}_{\alpha\beta}g^{(B)\alpha\beta} - d)$$

- Lowest order metric coupling:

$$\mathbf{W} \leftarrow g_{\mu\nu} = g^{(B)}_{\mu\nu} + \left(\gamma\, g^{(B)}_{\mu\alpha}\tilde{t}^{\alpha\beta}g^{(B)}_{\beta\nu} + \tilde{\gamma}\; \tilde{t}^{\alpha\beta}g^{(B)}_{\alpha\beta}\; g^{(B)}_{\mu\nu}\right)\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}}$$

$$\mathbf{S} \leftarrow \tilde{g}_{\mu\nu} = g^{(B)}_{\mu\nu} + \left(\gamma\, g^{(B)}_{\mu\alpha}t^{\alpha\beta}g^{(B)}_{\beta\nu} + \tilde{\gamma}\; t^{\alpha\beta}g^{(B)}_{\alpha\beta}\; g^{(B)}_{\mu\nu}\right)\frac{\sqrt{-g}}{\sqrt{-g^{(B)}}}$$

- Full EM-Tensor

$$T^{\mu}{}_{\nu} = \frac{1}{2}[(t^{\mu}{}_{\nu} + t_{\nu}{}^{\mu})\frac{\sqrt{-g}}{\sqrt{-g^{(\mathrm{B})}}} + (\tilde{t}^{\mu}{}_{\nu} + \tilde{t}_{\nu}{}^{\mu})\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(\mathrm{B})}}}] + \Delta K \delta^{\mu}_{\nu}$$

$$\Delta K = -\frac{\gamma}{2}\left(t^{\rho\alpha}\frac{\sqrt{-g}}{\sqrt{-g^{(\mathrm{B})}}}\right)g^{(\mathrm{B})}_{\alpha\beta}\left(\tilde{t}^{\beta\sigma}\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(\mathrm{B})}}}\right)g^{(\mathrm{B})}_{\sigma\rho} - \frac{\tilde{\gamma}}{2}\left(t^{\alpha\beta}g^{(\mathrm{B})}_{\alpha\beta}\frac{\sqrt{-g}}{\sqrt{-g^{(\mathrm{B})}}}\right)\left(\tilde{t}^{\sigma\rho}g^{(\mathrm{B})}_{\sigma\rho}\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(\mathrm{B})}}}\right)$$

Points to remember:

- $g_{\mu\nu}[\tilde{t}^{\gamma\delta}]$ and $\tilde{g}_{\mu\nu}[t^{\alpha\beta}]$
- $\nabla_{\mu} t^{\mu}_{\nu} = 0$ and $\tilde{\nabla}_{\mu}\tilde{t}^{\mu}_{\nu} = 0$, together $\nabla^{(B)}_{\mu} T^{\mu}_{\nu} = 0$

Hybrid Attractor

- Study of non-equilibrium evolution of viscous hybrid fluid model in a boost-invariant Bjorken-flow.

- **Set up:**
  - Background metric: $g^{(B)}_{\mu\nu} = \text{diag}(-1, 1, 1, \tau^2)$
    W: $g_{\mu\nu} = \text{diag}(-a^2, b^2, b^2, c^2)$ **S**: $\tilde{g}_{\mu\nu} = \text{diag}(-\tilde{a}^2, \tilde{b}^2, \tilde{b}^2, \tilde{c}^2)$
  - The sub-systems are parametrized with different transport coefficient

$$C_\eta = \frac{\eta}{S}, \quad C_\tau = \tau_\pi T$$

    W: $C_\eta = 10\tilde{C}_\eta$ and $C_\tau = 5C_\eta$ **S**: $\tilde{C}_\eta = 1/4\pi$ and $\tilde{C}_\tau = (2 - log2)/2\pi$

  - $t^{\mu\nu} = diag(\frac{\epsilon}{a^2}, \frac{\epsilon/3+\phi/2}{b^2}, \frac{\epsilon/3+\phi/2}{b^2}, \frac{\epsilon/3-\phi}{c^2})$
    $\tilde{t}^{\mu\nu} = diag(\frac{\tilde{\epsilon}}{\tilde{a}^2}, \frac{\tilde{\epsilon}/3+\tilde{\phi}/2}{\tilde{b}^2}, \frac{\tilde{\epsilon}/3+\tilde{\phi}/2}{\tilde{b}^2}, \frac{\tilde{\epsilon}/3-\tilde{\phi}}{\tilde{c}^2})$

- Four ODEs + six algebraic equations,

  - $\nabla_\mu t^{\mu\nu} = 0$ and $\tilde{\nabla}_\mu \tilde{t}^{\mu\nu} = 0$

$$\partial_\tau \epsilon + \epsilon \partial_\tau \log(b^{8/3} c^{4/3}) + \phi \partial_\tau \log(b/c) = 0$$

$$\tilde{\partial}_\tau \tilde{\epsilon} + \tilde{\epsilon} \tilde{\partial}_\tau \log(\tilde{b}^{8/3} \tilde{c}^{4/3}) + \tilde{\phi} \tilde{\partial}_\tau \log(\tilde{b}/\tilde{c}) = 0$$

  - $\tau_\pi (u.\nabla + 1)\pi^{\mu\nu} = -\eta \sigma^{\mu\nu}$ and $\tilde{\tau}_\pi (\tilde{u}.\tilde{\nabla} + 1)\tilde{\pi}^{\mu\nu} = -\tilde{\eta}\tilde{\sigma}^{\mu\nu}$,

$$\tau_\pi \partial_\tau \phi + \frac{4}{3} \eta \partial_\tau \log(b/c) + [a + \frac{4}{3} \tau_\pi \partial_\tau \log(b^2 c)]\phi = 0$$

$$\tilde{\tau}_\pi \tilde{\partial}_\tau \tilde{\phi} + \frac{4}{3} \tilde{\eta} \tilde{\partial}_\tau \log(\tilde{b}/\tilde{c}) + [\tilde{a} + \frac{4}{3} \tilde{\tau}_\pi \tilde{\partial}_\tau \log(\tilde{b}^2 \tilde{c})]\tilde{\phi} = 0$$

  - Two metric coupling equations- $\tilde{a}, \tilde{b}, \tilde{c}$ and a, b, c

- There exist a four dimensional phase space spanned by the dynamical variable ϵ , ϕ, $\tilde{\epsilon}$ and $\tilde{\phi}$

Attractor curve

- Introducing dimensionless variable (Measure of anisotropy)

$$\chi := \frac{\phi}{\epsilon + P}$$

- The set of attractor solution is a 2-D manifold parametrized by dimensionless energy densities $\alpha := \lim_{\tau\to\infty} \epsilon\tau^{4/3}$ and $\beta := \lim_{\tau\to\infty} \tilde{\epsilon}\tau^{4/3}$.

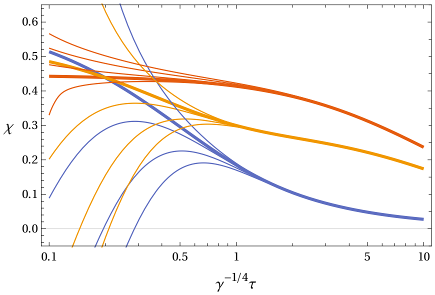

Figure: **Attractor solutions** , thin lines are neighbouring trajectories.
Strong system, *Weak system* and *Full system*

Attractor curve

- $\epsilon_0 = \epsilon(\tau_0)$ and $\tilde{\epsilon_0} = \tilde{\epsilon}(\tau_0)$, τ_0 is some non-zero reference time.
- Fine tune χ and $\tilde{\chi}$, run the solution backward and forward in time.

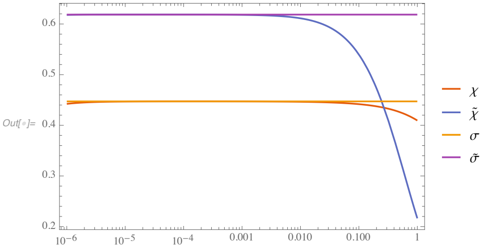

Figure: Numerically determined attractor solution at early time

at $\tau \to 0$,

$$\chi \to \sqrt{\frac{C_\eta}{C_\tau}} := \sigma$$

$$\tilde{\chi} \to \sqrt{\frac{\tilde{C}_\eta}{\tilde{C}_\tau}} := \tilde{\sigma}$$

Bottom-Up thermalization!!

- At initial time total energy is concentrated to hard sector which gets distributed to the soft sector over a stretch of time.
- At late time the weak system re-dominates as if in the hadron gas crossover.

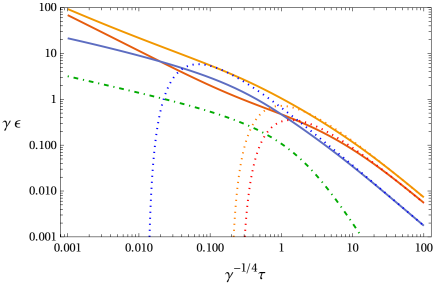

Figure: *Strong system*, *Full system* and *Weak system*
⋯ *First-order hydrodynamic*
· — · *Interaction energy of the subsystem*

For $\tilde{\sigma} > \sigma$

at $\tau \to 0$

$$\mathcal{E}_2/\mathcal{E}_1 \sim \tau^{8(\tilde{\sigma}-\sigma)/3}$$

Single fluid:

- The two sectors do not equilibrate locally with each other but the full system at late time behaves as a Single fluid with shear viscosity.
- The shear viscosity and EOS is determined by the attractor curve followed by the system.

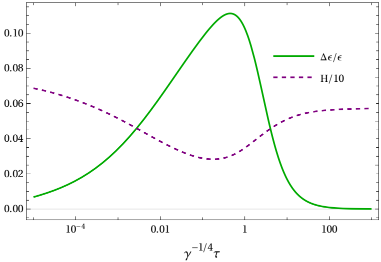

Figure: *Interaction energy of the subsystems over total energy and Averaged effective shear viscosity*

$$C_\eta^{eff} = \lim_{\tau\to\infty} \mathbf{H}(\tau) = \frac{C_\eta \alpha^{4/3} + \tilde{C}_\eta \beta^{4/3}}{\alpha^{4/3} + \beta^{4/3}}$$

Hydrodynamization time

- Weakly coupled sector is strongly anisotropic compared to the soft sector.
- Hydrodynamization time is longer for hard sector than for soft.

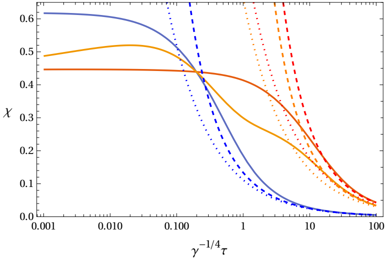

Figure: *Strong System*, *Weak System* and *Full system*
$\cdots$ *first-order hydro*
– – *Second-order hydro*

Hydrodynamization time :

$$\frac{|\Delta P|}{P} := \frac{|\phi - \phi_{1st}|}{P} < 0.1, \quad \tau > \tau_{hd}$$

$$\tau_{hd} > \tilde{\tau}_{hd}$$

Table

Table: Subsystem hydrodynamization times χ, and $\tilde{\chi}$ for three scenarios with different values of $\mathcal{E}_1(1) = \mathcal{E}_2(1) =: \mathcal{E}(1)$, where all dimensionful quantities are given in units of γ. The last column gives the ratio $R_{\rm hd} := \tau_{\rm hd}/\tilde{\tau}_{\rm hd}$.

$\mathcal{E}(1)$	$\tau_{\rm hd}$	$\chi_{\rm hd}$	$\tilde{\tau}_{\rm hd}$	$\tilde{\chi}_{\rm hd}$	$R_{\rm hd}$
0.32	10.2	0.203	3.90	0.0525	2.62
0.26	12.0	0.215	2.08	0.101	5.76
0.052	25.5	0.210	1.39	0.211	18.4

- Ratio of hydrodynamization time depends on the initial condition
- Anisotropy, $A = \frac{P_\perp - P_L}{P} \sim 6\chi$

Essence:

- **Coupling of two fluid in democratic manner.**
- Emergence of **hydrodynamic attractor** in a composite system which is well behaved at early time.
- **Bottom-up thermalization is universal in hybrid system** involving weakly coupled and strongly coupled degrees of freedom.
- **Full system behaves as a single fluid** with transport coefficients determined by which attractor curve on the two dimensional attractor surface the system goes to.
- **Insight into small and large system.** - Smaller densities lead to delayed hydrodynamization of the weakly coupled system while larger densities delay that of the strongly coupled system. The stong system hydrodynamizes faster typically.

Ongoing...

- More specific to semi-holography is the coupling of kinetic theory to black hole to study the late time behaviour of the system.

Thank you