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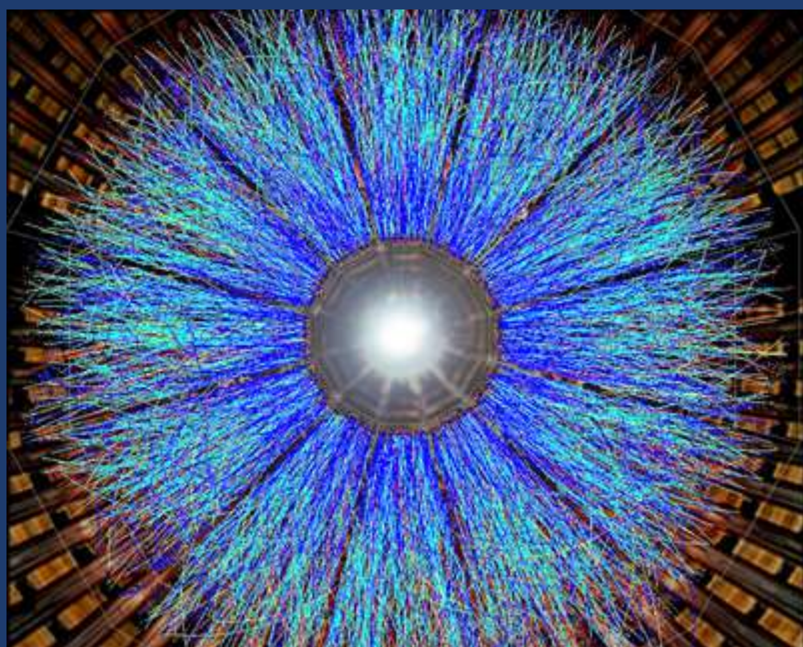
HYDRODYNAMISATION AT STRONG COUPLING

SEWM, 2.6.2021

‘SPECTRUM’ OF QUANTUM FIELD THEORIES

microscopic quantum field theory (QCD, …) gives rise to collective ‘late-time’ dynamics



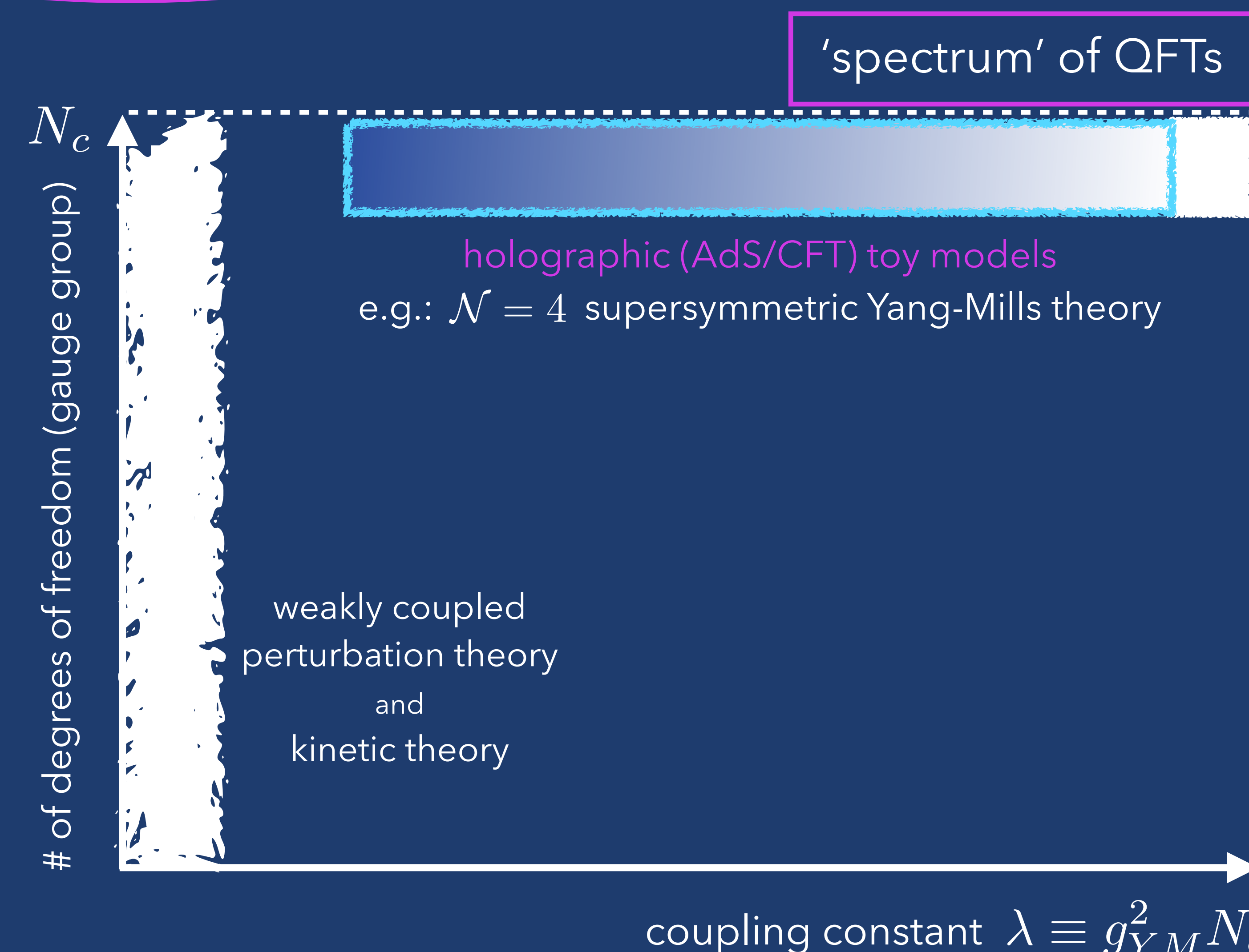
states of interest such as quark-gluon plasma that evolve in real time are strongly coupled

$t \sim 10^{-23}$ seconds

$T \sim 3 \times 10^{12}$ Kelvin

$B \lesssim 10^{15}$ Tesla

hydrodynamics works ‘unreasonably’ well at time scales of *fm/c*



HOLOGRAPHIC (ADS/CFT) TOY MODELS

- $\mathcal{N} = 4$ (maximally) supersymmetric Yang-Mills theory
  - conformal theory with gluons, fermions and scalars in the adjoint *SU(N$_c$)*
  - various deformations make it look more like QCD
  - at the centre of dualities and mathematical structures

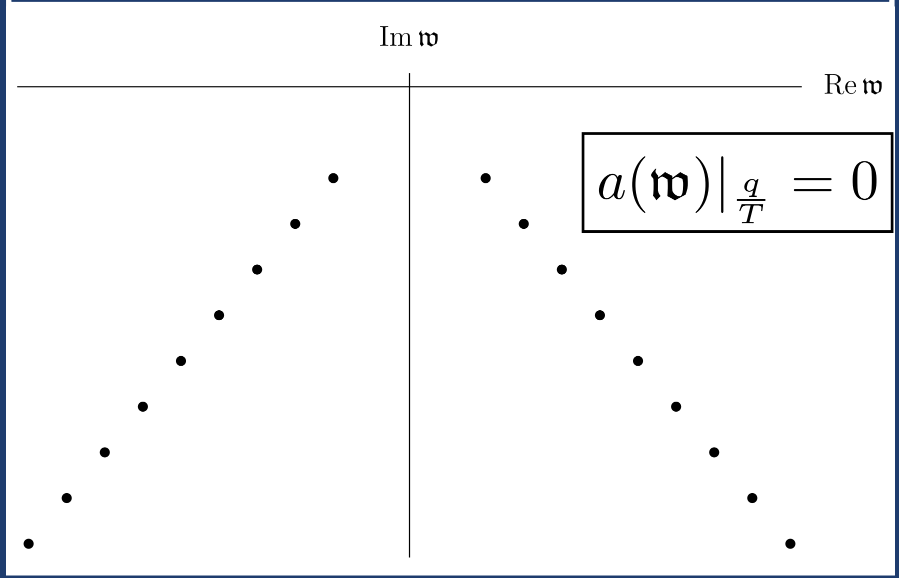
- duality: theory *A* = theory *B*

- holography is a result of string theory – quantum theory of gravity [Maldacena (1997)]

QFT in 4d		$=$	*quantum gravity in 5d (really 10d)*	
strongly coupled	$\lambda \gg 1$	$=$	*weakly coupled gravity*	$\lambda \ll 1$
large # of d.o.f.'s	$N_c \to \infty$	$=$	classical gravity	$G_N \ll 1$

- gravity (black holes) allows to analyse (thermal, dense) spectra at strong coupling through classical perturbations in $1/\lambda^{3/2} \ll 1$

$$\langle T_{\mu\nu}(-\omega,-q), T_{\rho\sigma}(\omega,q)\rangle_R \sim \frac{b(\omega,q)}{a(\omega,q)}$$



HOLOGRAPHIC (ADS/CFT) TOY MODELS

- exact Wilson loop for all couplings at $T = 0$ and $N_c \to \infty$
  [Erickson, Semenoff, Zarembo (2000)]

$$\langle W_C \rangle = \frac{2}{\sqrt{\lambda}} I_1\left(\sqrt{\lambda}\right)$$

QFT-type result

holography-type result

$$\lambda \ll 1: \qquad \langle W_C \rangle = 1 + \frac{\lambda}{8} + \frac{\lambda^2}{192} + \cdots$$

$$\lambda \gg 1: \qquad \langle W_C \rangle \sim \sqrt{\frac{2}{\pi}} \frac{e^{\sqrt{\lambda}}}{\lambda^{3/4}} \left(1 - \frac{3}{8\sqrt{\lambda}} - \frac{15}{128\lambda} + \cdots\right)$$

- thermal entropy density

``3/4 problem"

SEE LAST TALK TODAY

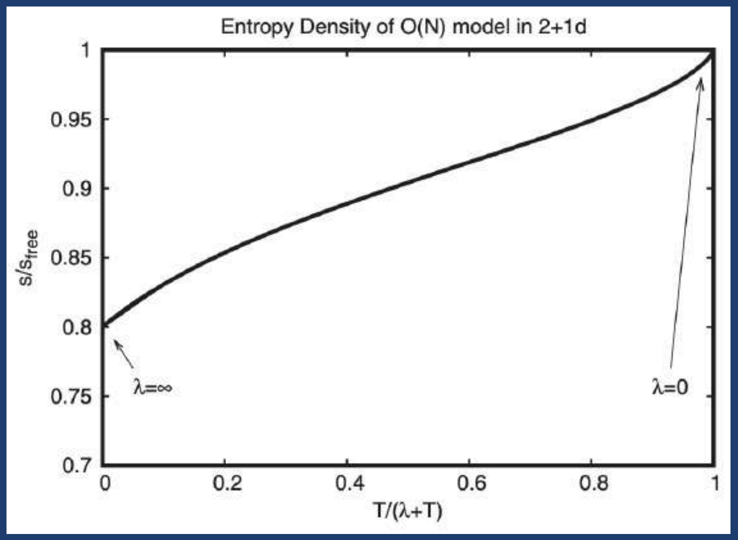
QFT: $\lambda \ll 1: \qquad s/s_0 = 1 - \frac{3}{2\pi^2}\lambda + \frac{\sqrt{2}+3}{\pi^3}\lambda^{3/2} + \#\lambda^2 + \#\lambda^2 \ln\lambda \cdots$ [**Du**, Strickland, Tantary (2021)]

holography: $\lambda \gg 1: \qquad s/s_0 = \frac{3}{4} + \frac{45\zeta(3)}{32}\frac{1}{\lambda^{3/2}} + \cdots$ [Gubser, Klebanov, Tseytlin (1998)]

Padé interpolation between two results as in Blaizot, Iancu, Kraemmer, Rebhan (2007)

- analytic QFT result in the large-N vector model [Romatschke, PRL (2019)]

4/5 instead of 3/4 →



OUTLINE

- hydrodynamics
- applicability of hydrodynamics at strong coupling
- hydrodynamisation at strong coupling
- summary

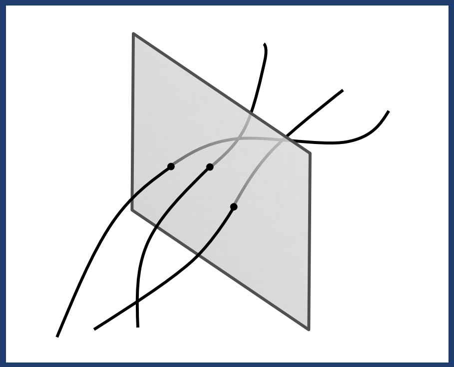
HYDRODYNAMICS

- low-energy limit of QFTs – a Schwinger-Keldysh effective field theory
  [Grozdanov, Polonyi (2013); Crossley, Glorioso, Liu (2015); Haehl, Loganayagam, Rangamani (2015); …]

- conservation laws (equations of motion) of globally conserved operators

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu J^{\mu} = 0 \qquad \ldots \nabla_\mu J^{\mu\nu} = 0$$

higher-form currents in MHD
[Grozdanov, Hofman, Iqbal,
PRD (2017)]



- tensor structures (symmetries, gradient expansions) and transport coefficients (QFT)

$$T^{\mu\nu} = (\varepsilon + p)\, u^\mu u^\nu + p g^{\mu\nu} - \eta \sigma^{\mu\nu}[\partial u] - \ldots\,, \qquad \partial u^\mu \sim \partial T \ll 1$$

- small ∂ = small frequency-momentum $u^\mu, T \sim e^{-i\omega t + i\mathbf{q}\cdot\mathbf{x}}$: $\omega/T \sim q/T \ll 1$

- dispersion relations:

shear diffusion	sound
$\omega = -iDq^2$	$\omega = \pm v_s q - i\Gamma q^2$

equilibrium temperature

$q = \sqrt{\mathbf{q}^2}$

HYDRODYNAMICS

- infinite, all-order hydrodynamic expansion

$$T^{\mu\nu} = \sum_{n=0}^{\infty} \left[\sum_{i}^{N} \lambda_i^{(n)} \mathcal{T}_{(n)}^{\mu\nu} \right] \xrightarrow[u^\mu \sim T \sim e^{-i\omega t + iqz}]{\nabla_\mu T^{\mu\nu} = 0} \boxed{\omega(q) = \sum_{n=1}^{\infty} \alpha_n q^n}$$

- independent terms at each order:
  1st order [Navier-Stokes (1821)] = 2; 2nd order [BRSSS (2007)] = 15; etc. [Grozdanov, Kaplis, PRD (2016); Diles, et. al., JHEP (2020) and A. Jaiswal, PRC (2013) for kinetic theory]

- 1st order hydrodynamics can be made stable by a frame choice [Bemfica, Disconzi, Noronha (2019); Kovtun (2019)]

- non-analytic corrections (long-time tails) should become important for QGP

$$\langle T_{\mu\nu}(-\omega, -q), T_{\rho\sigma}(\omega, q) \rangle_R \sim \frac{b(\omega, q)}{a(\omega, q)} + \frac{\#}{N_c^2} |\omega|^{1/2}$$

- what is hydrodynamics at finite *N*?

HYDRODYNAMICS

- $\mathcal{N}=4$ supersymmetric Yang-Mills theory at $N_c \to \infty$:

first order (1/1):	$\eta = \lambda_1^{(1)} = \# + \#/\lambda^{3/2} + \cdots$	Buchel, Liu, Starinets (2004)
second order (5/5):	$\lambda_i^{(2)} = \#_i + \#_i/\lambda^{3/2} + \cdots, \quad i = \{1,\ldots,5\}$	Grozdanov, Starinets (2014)
third order (5/20):	$\lambda_i^{(3)} = \#_i + \cdots, \quad i = \{1,\ldots,5\}$	Grozdanov, Kaplis (2016)

- universality of *1st*-order hydro (*2*-pt functions)

  [Kovtun, Son, Starinets (2004)]

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \cdots\right)$$

weak coupling:

$$\frac{\eta}{s} \sim \frac{T^3}{\lambda^4 \ln(1/\lambda)}$$

[Huot, Jeon, Moore (2006)]

- *2nd*-order hydro and its, more general, universality (*3*-pt functions)

$$\tau_\Pi = \frac{2-\ln 2}{2\pi T} + \frac{375\zeta(3)}{32\pi T \lambda^{3/2}} + \cdots$$

$$\lambda_1 = \frac{N_c^2 T^2}{16}\left(1 + \frac{175\zeta(3)}{4\lambda^{3/2}} + \cdots\right)$$

weak coupling:

$$\lambda_1 \sim \frac{T^2}{\lambda^4 \ln^2(1/\lambda)}$$

[York, Moore (2009)]

$$2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0 + \frac{0}{\lambda^{3/2}} + O(1/\lambda^2)$$

[Haack, Yarom (2008); Grozdanov, Starinets (2014)]

- bounds e.g. KSS: $\eta/s \geq 1/(4\pi)$
  (long history…)
  some more mathematical recent work [Grozdanov, PRL (2021)]

HYDRODYNAMICS

- dispersion relations [Grozdanov, Kovtun, Starinets, Tadić, JHEP (2019)]

$$\omega(q) = \sum_{n=1}^{\infty} \alpha_n q^n$$

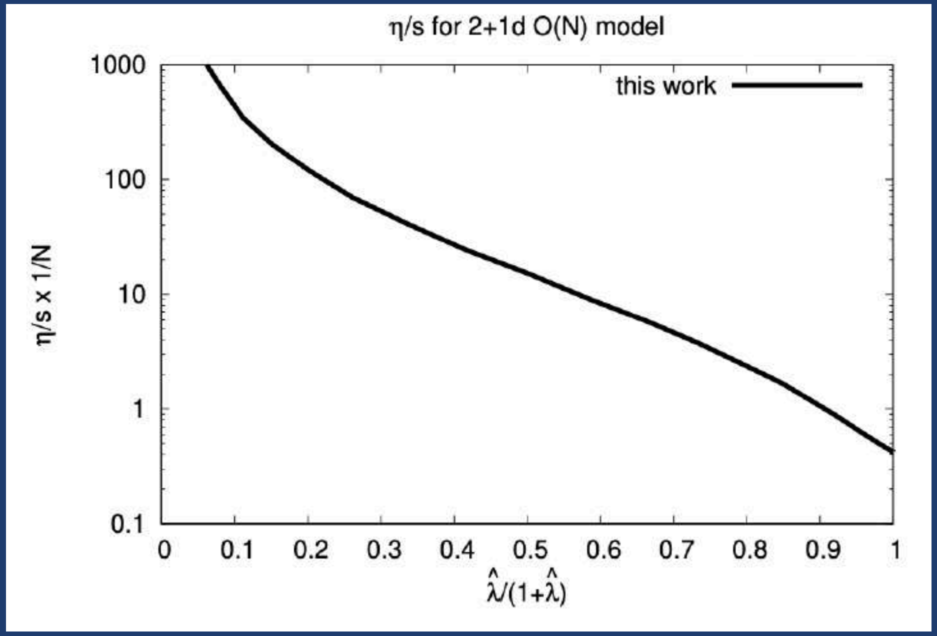
shear diffusion:

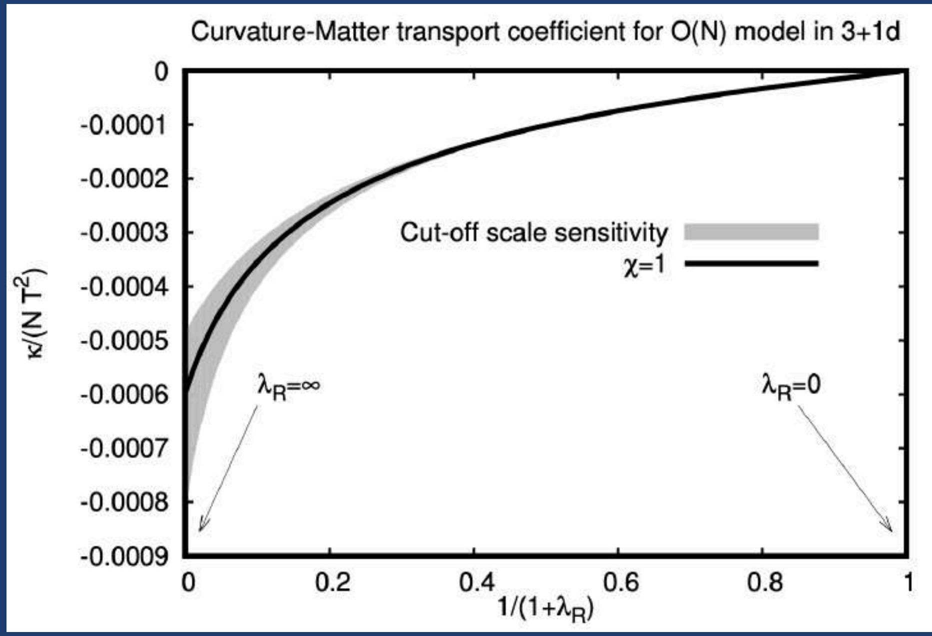
$$\omega = \pm\frac{1}{\sqrt{3}}q - \frac{i}{6\pi T}q^2 \pm \frac{3-2\ln 2}{24\sqrt{3}\pi^2 T^2}q^3 - \frac{i\left(\pi^2 - 24 + 24\ln 2 - 12\ln^2 2\right)}{864\pi^3 T^3}q^4 \pm \cdots$$

sound:

$$\omega = -\frac{i}{4\pi T}q^2 - \frac{i(1-\ln 2)}{32\pi^3 T^3}q^4 - \frac{i(24\ln^2 2 - \pi^2)}{96\,(2\pi T)^5}q^6 - \frac{i\left[2\pi^2(\ln 32 - 1) - 21\zeta(3) - 24\ln 2(1 + \ln 2(\ln 32 - 3))\right]}{384\,(2\pi T)^7}q^8 + \cdots$$

- QFT methods used for large-*N* vector models can compute certain transport coefficients for all couplings [… Aarts, Martinez Resco, JHEP (2004); … Romatschke, PRD (2019); Romatschke (2021)]





ASIDE: HYDRODYNAMICS AND QUANTUM CHAOS

- precise analytic connection between hydrodynamics and quantum chaos [Grozdanov, Schalm, Scopelliti, PRL (2017); Blake, Lee, Liu, JHEP (2018); Blake, Davison, Grozdanov, Liu, JHEP (2018); Grozdanov, JHEP (2019)]

- resumed hydrodynamic series (e.g. sound)

$$\omega(q) = \sum_{n=1}^{\infty} \alpha_n \left(T, \mu_i, \langle \mathcal{O}_j \rangle, \lambda\right) q^n$$

passes through a "chaos point" at imaginary momentum

$$\omega(q = i\lambda_L / v_B) = i\lambda_L = 2\pi T i$$

'quantum' chaos spreads $\sim e^{\lambda_L (t - |\mathbf{x}|/v_B)}$

'quantum' Lyapunov exponent

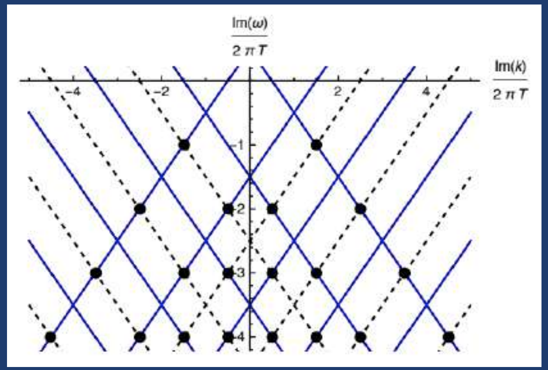
where the associated *2*-pt function is "0/0":

'quantum' butterfly velocity

$$\mathrm{Res}\, G_R^{\varepsilon\varepsilon} \left(\omega = i\lambda_L, q = i\lambda_L / v_B\right) = 0$$

- triviality of Einstein's equations at the horizon [Blake, Davison, Grozdanov, Liu, JHEP (2018)]

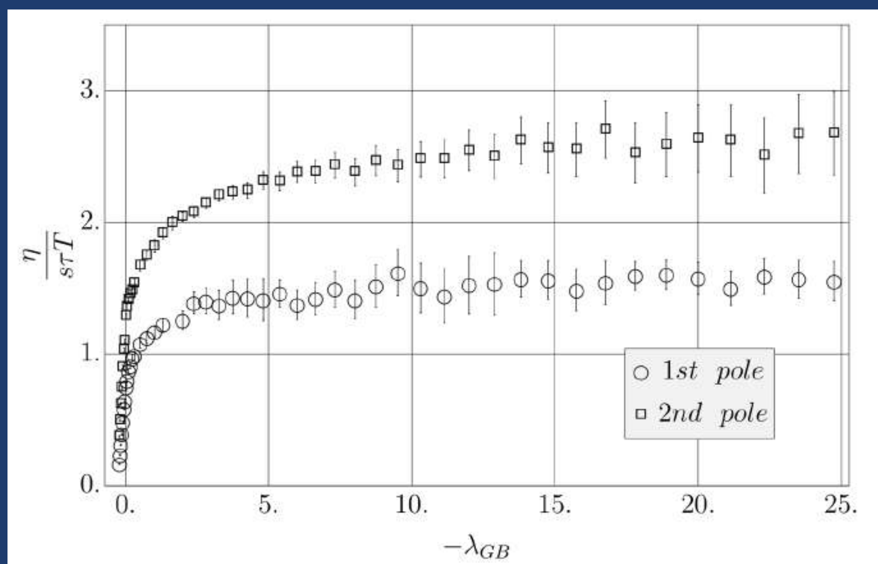
- infinite constraints on correlators at multiples of Matsubara frequencies [Grozdanov, Kovtun, Starinets, Tadić, JHEP (2019); Blake, Davison, Vegh, JHEP (2019)]



APPLICABILITY OF HYDRODYNAMICS

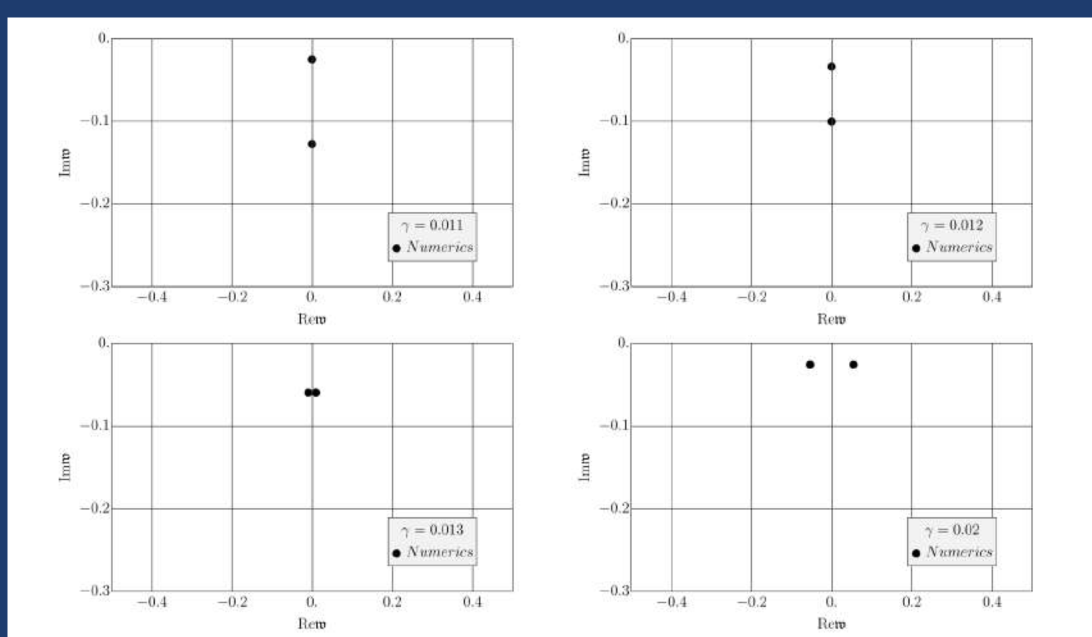
- tune from strong to weak coupling: quasiparticles, breakdown of hydrodynamics

[Grozdanov, Kaplis, Starinets, JHEP (2016)]



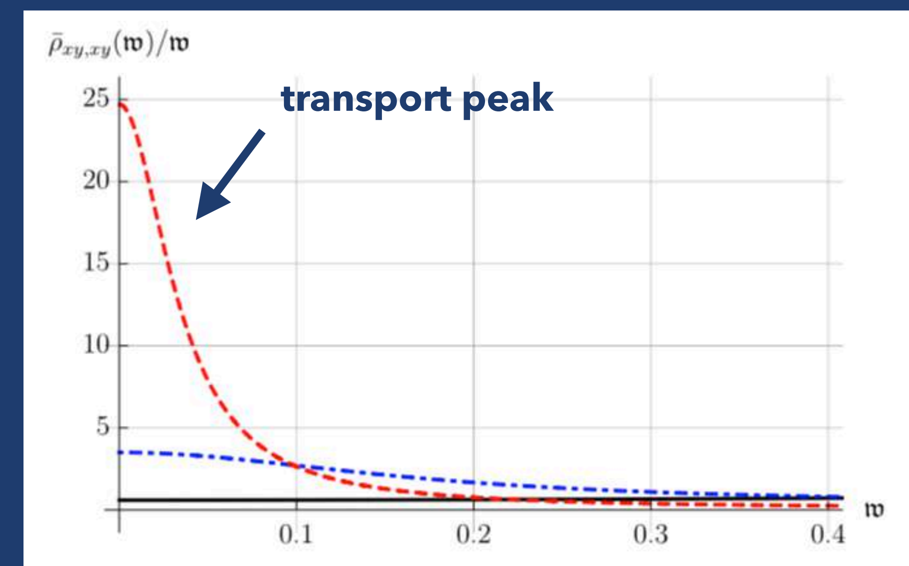
$$\tau_R^{(n)} = -\frac{1}{\operatorname{Im}\omega_n}$$

$$\eta/s \sim C \times \tau_R T$$



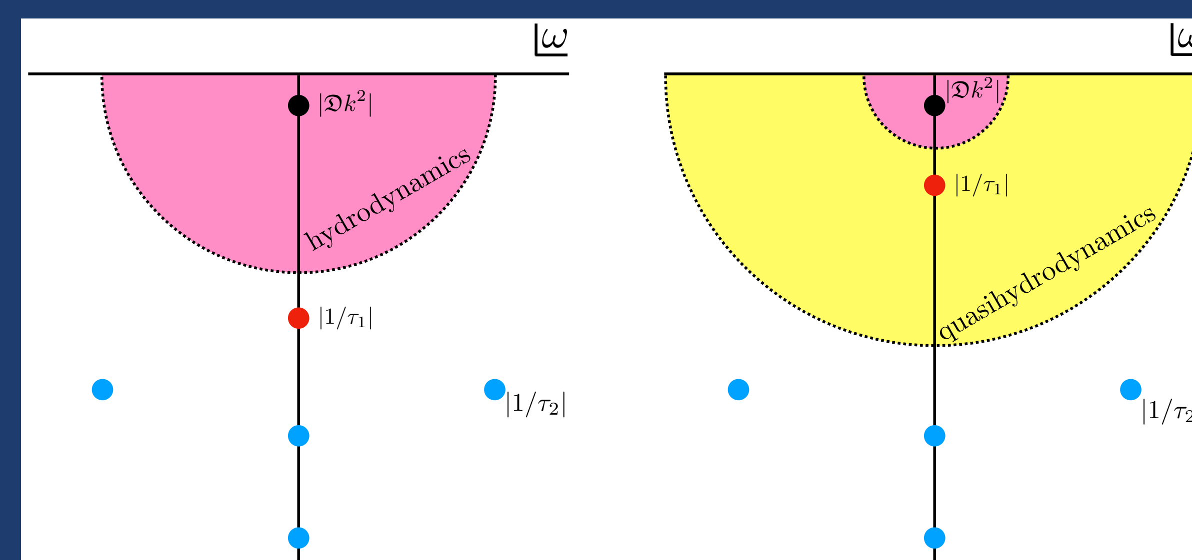
$$q_c/T \sim \lambda^{3/2}$$

[Casalderey-Solana, Grozdanov, Starinets, PRL (2018)]



$$\rho = -2\operatorname{Im}\langle T_{\mu\nu}(\omega, q), T_{\rho\sigma}(-\omega, -q)\rangle_R$$

[Grozdanov, Lucas, Poovuttikul, PRD (2019)]



$$\tau_1 \approx \frac{373\zeta(3)}{32\pi T\lambda^{3/2}} \approx \frac{4.5}{T\lambda^{3/2}}$$

quasihydrodynamics consistent with MIS

APPLICABILITY OF HYDRODYNAMICS

- a precise way of defining regime of applicability: radius of convergence
- hydrodynamic modes as complex spectral curves
  [Grozdanov, Kovtun, Starinets, Tadić, PRL (2019) and JHEP (2019)] [**see also SERANTES's TALK**]

hydro: $\det \mathcal{L}(\mathbf{q}^2,\omega)=0$

QNM: $a(\mathbf{q}^2,\omega)=0$

$\longrightarrow$ $P(\mathbf{q}^2,\omega)=0$ $\Longrightarrow$ $\omega_i(\mathbf{q}^2)$

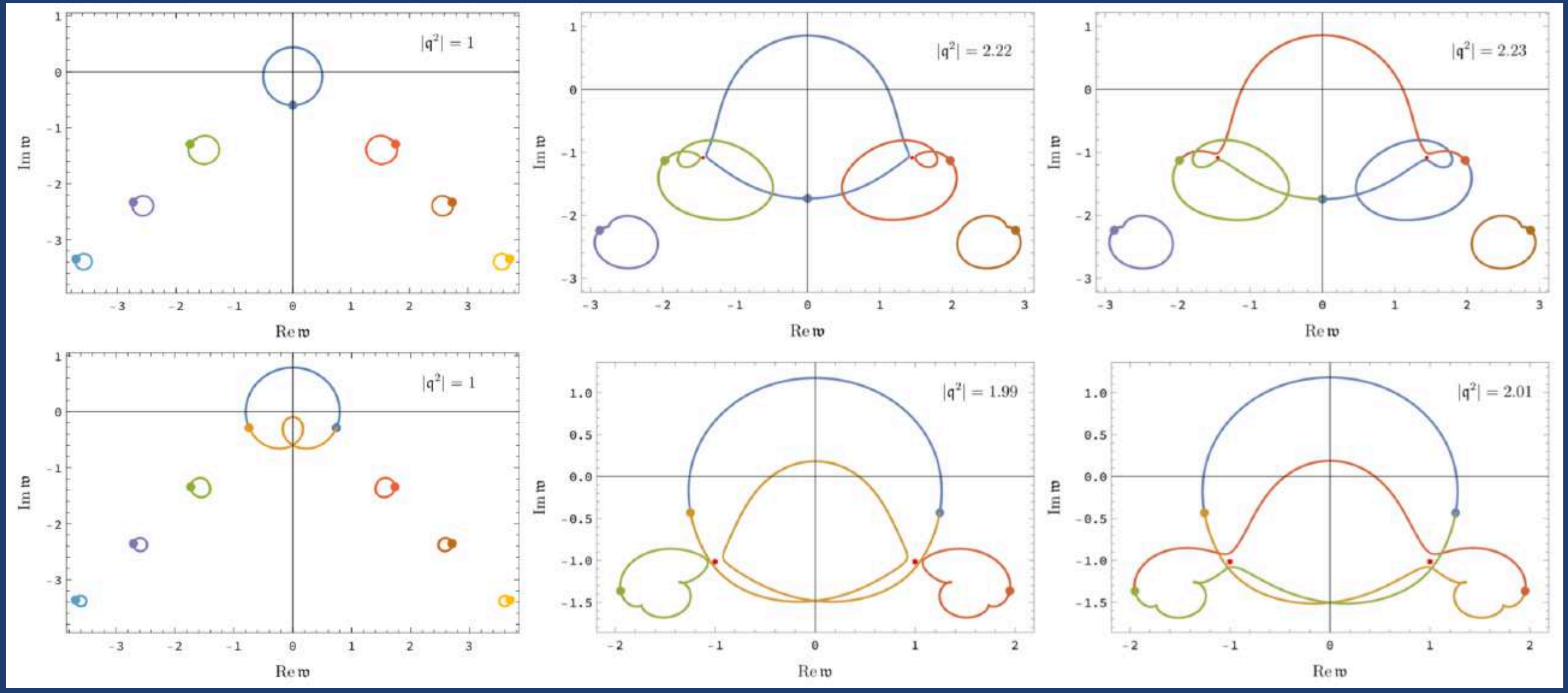
$$\mathfrak{w}=\frac{\omega}{2\pi T},\ \mathfrak{q}=\frac{|\mathbf{q}|}{2\pi T}\in\mathbb{C}$$

- dispersion relations have a finite radius of convergence

$$\mathfrak{q}^2=\left|\mathfrak{q}^2\right|e^{i\theta}$$

$$R=\min\left|\mathfrak{q}_{\text{collision}}\right|$$

level-crossing



- convergence guaranteed up to the nearest (non-trivial) critical point (branch point)
  cf., Newton polygon or Darboux theorem [see Grozdanov, Starinets, Tadić, JHEP (2021)]

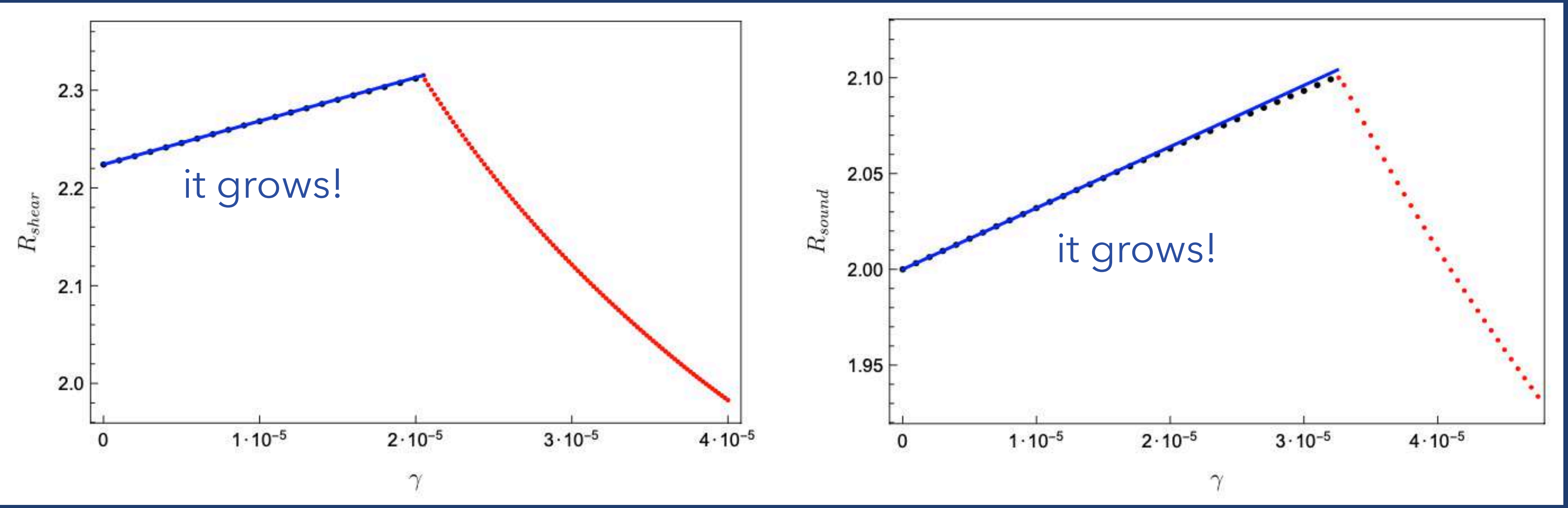
APPLICABILITY OF HYDRODYNAMICS

- hydrodynamic series are convergent (Puiseux) series

$$\mathfrak{w}_{\text{shear}} = -i \sum_{n=1}^{\infty} c_n \left(\mathfrak{q}^2\right)^n = -i\mathfrak{D}\mathfrak{q}^2 + \ldots$$

$$\mathfrak{w}_{\text{sound}} = -i \sum_{n=1}^{\infty} a_n e^{\pm \frac{i\pi n}{2}} \left(\mathfrak{q}^2\right)^{n/2} = \pm v_s \mathfrak{q} - \frac{i}{2}\mathfrak{G}\mathfrak{q}^2 + \ldots$$

- finite coupling radius of convergence in $\mathcal{N} = 4$ [Grozdanov, Starinets, Tadić, JHEP (2021)]



plot from $\lambda = \infty$ to $\lambda \approx 200$

$$R_{\text{shear}}(\lambda) = 2.22 \left(1 + 674.15\, \lambda^{-3/2} + \cdots\right)$$
$$R_{\text{sound}}(\lambda) = 2 \left(1 + 481.68\, \lambda^{-3/2} + \cdots\right)$$

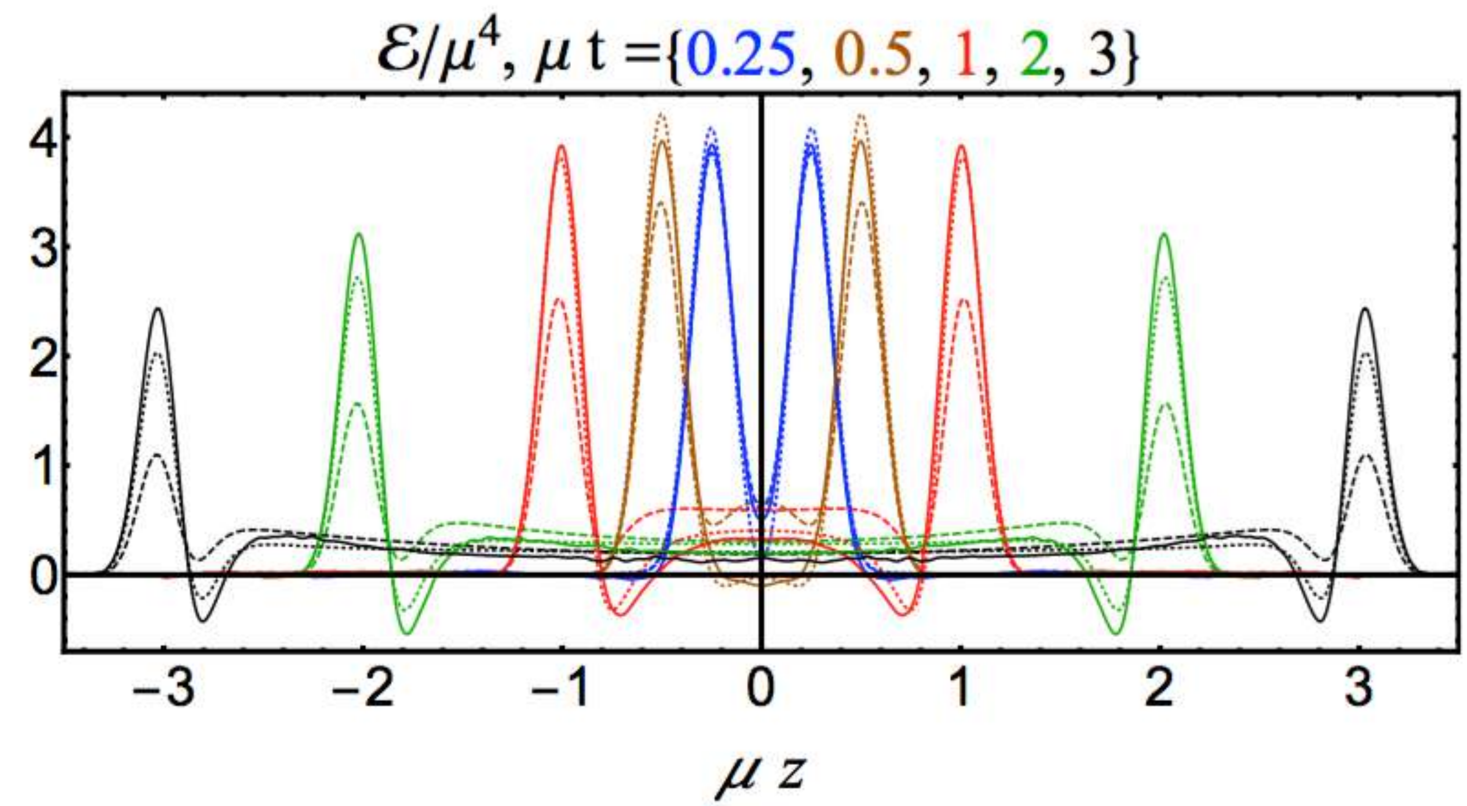
→ $q/T \sim O(10)$

↓

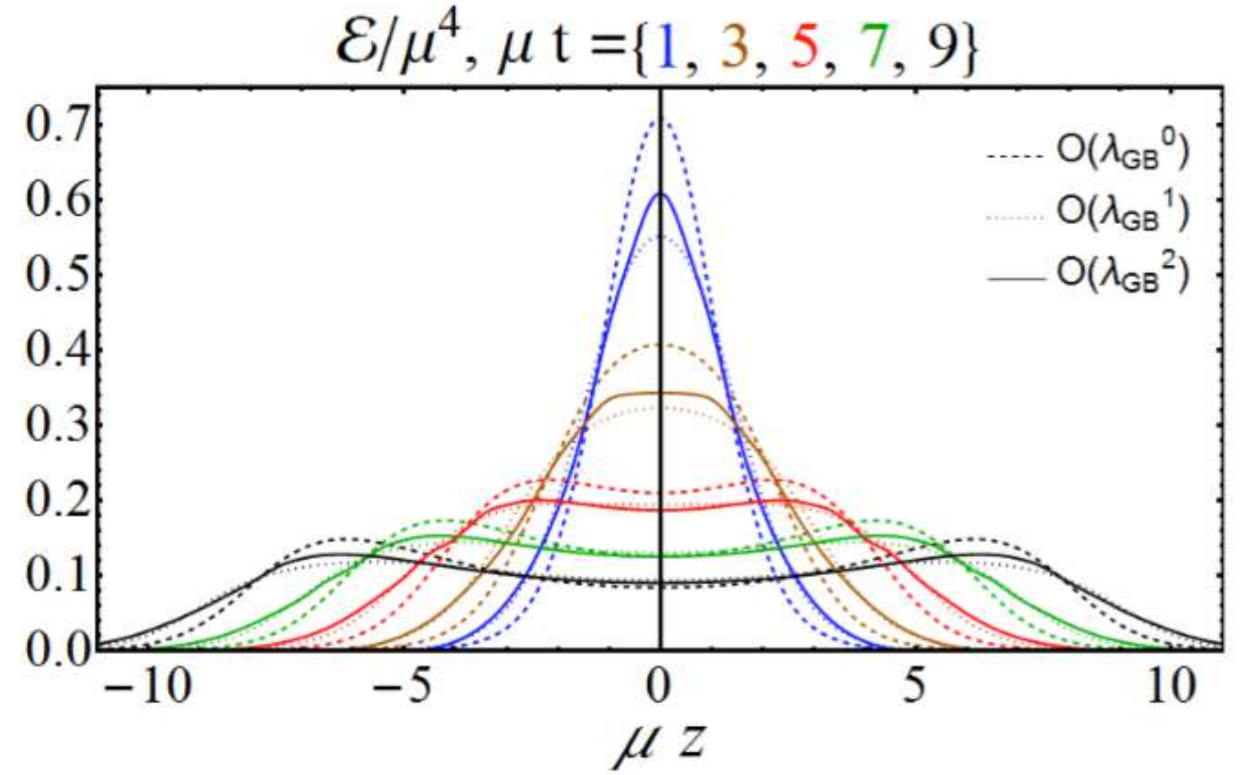
orders of magnitude larger radius of convergence than naive $q/T \ll 1$ – this is a precise incarnation of the "unreasonable effectiveness of hydrodynamics"

HYDRODYNAMISATION AT STRONG COUPLING

- initial study of holographic heavy-ion collisions Chesler and Yaffe in 2011
- numerical studies of gravity and black hole formation (thermalisation)
- holographic model with coupling dependence [Grozdanov, van der Schee, PRL (2016)]
- at intermediate coupling, ``nuclei” experience less stopping and have more energy deposited near the light cone

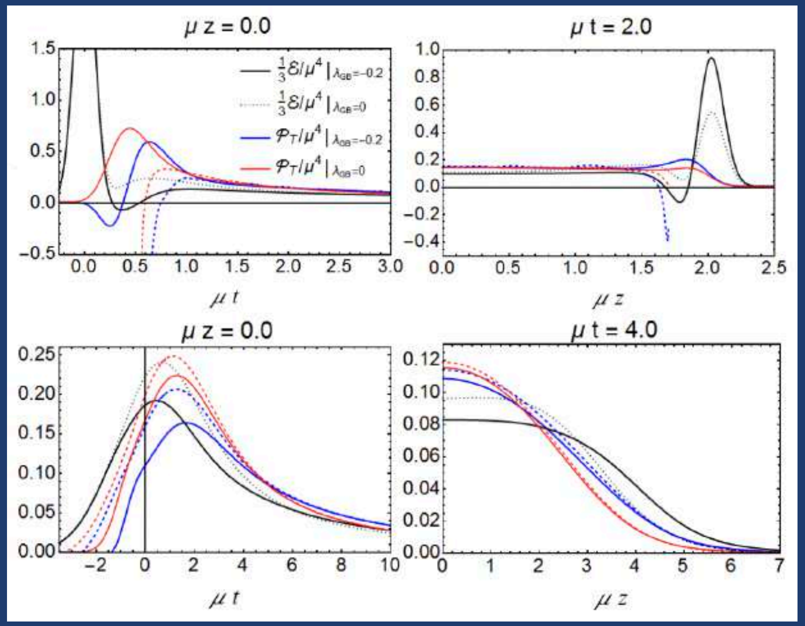


narrow
sheets



wide
sheets

HYDRODYNAMISATION AT STRONG COUPLING



- what is hydrodynamisation (isotropisation, thermalisation)?

see also BREWER's TALK from Monday

- model shows worse applicability of hydrodynamics at (lower) finite coupling

- delayed hydrodynamisation is a quantifiable prediction

viscosity measures the coupling (monotonicity)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 4\lambda_{GB}\right)$$

$$t_{\mathrm{hyd}} T_{\mathrm{hyd}} = \{0.41 - 0.52\lambda_{GB},\ 0.43 - 6.3\lambda_{GB}\}$$

→

at $\lambda_{GB} = -0.2$:

η/s increases by 80%

⇓

$t_{\mathrm{hyd}} T_{\mathrm{hyd}}$ increases by

$\{25\%, 290\%\}$

↑ narrow sheets

↑ wide sheets

- other predictions: entropy production, rapidity distributions

HYDRODYNAMISATION AT STRONG COUPLING

- coupling dependence in the presence of finite baryon number density
  [Folkestad, Grozdanov, Rajagopal, van der Schee, JHEP (2019)]

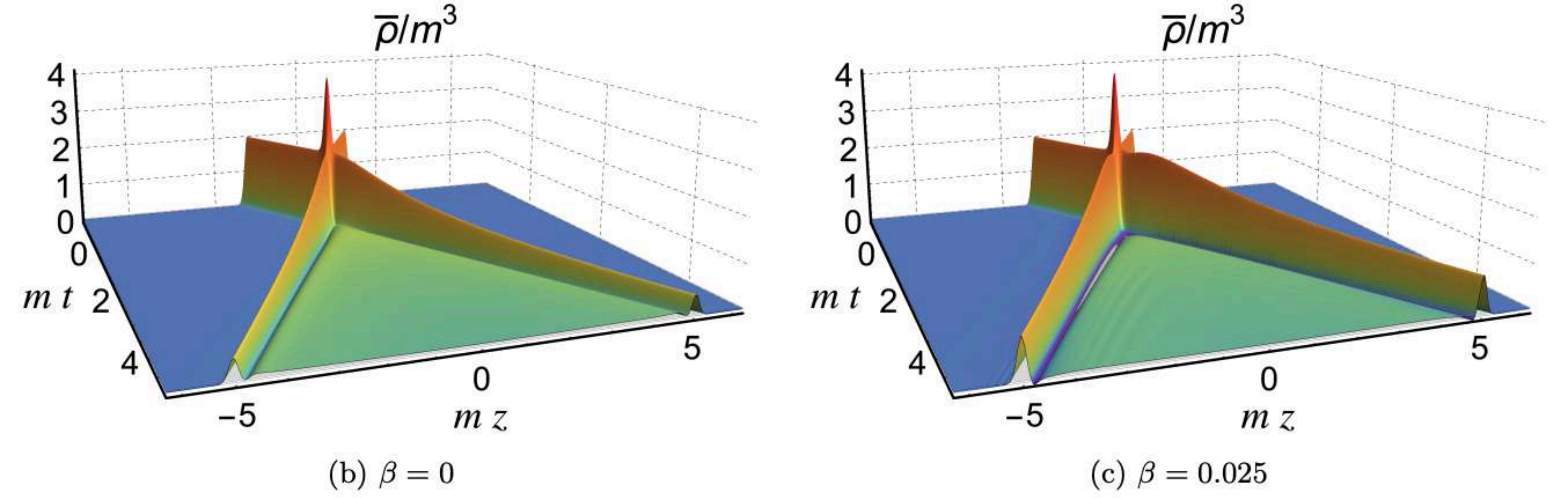
- fix gravitational couplings through phenomenological considerations

viscosity: $\lambda_{GB} = -0.2 \quad \left[\Longrightarrow \frac{\eta}{s} = \frac{1.8}{4\pi}\right]$

charge susceptibility: $\left.\frac{\chi}{\chi_0}\right|_{\text{QCD}} \in [0.8, 0.9] \wedge \frac{\eta}{s} \in \left[\frac{1}{4\pi}, \frac{3}{4\pi}\right] \quad \Longrightarrow \quad \beta \in [0.04, 0.15]$

[lattice QCD data: *T* in *250 – 400 MeV*]

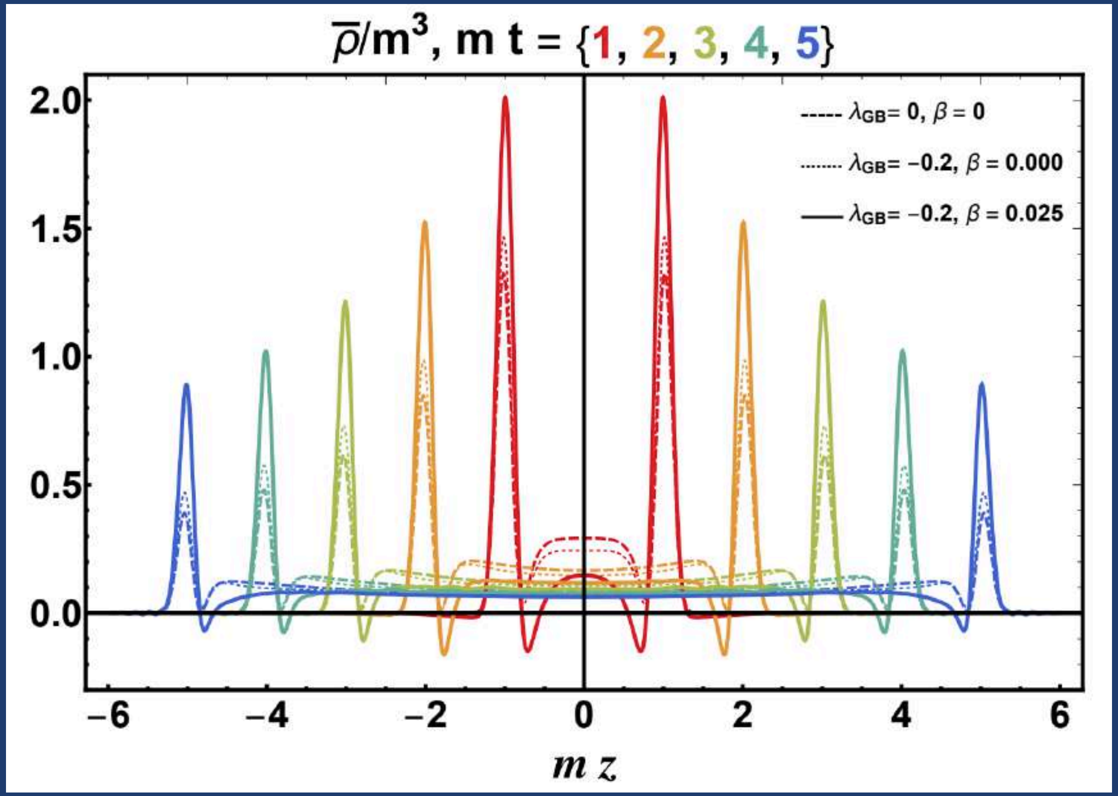
- at finite coupling significantly less stopping of the baryon charge
  [resolves a puzzle from Casalderrey-Solana, Mateos, van der Schee, Triana, JHEP (2016)]



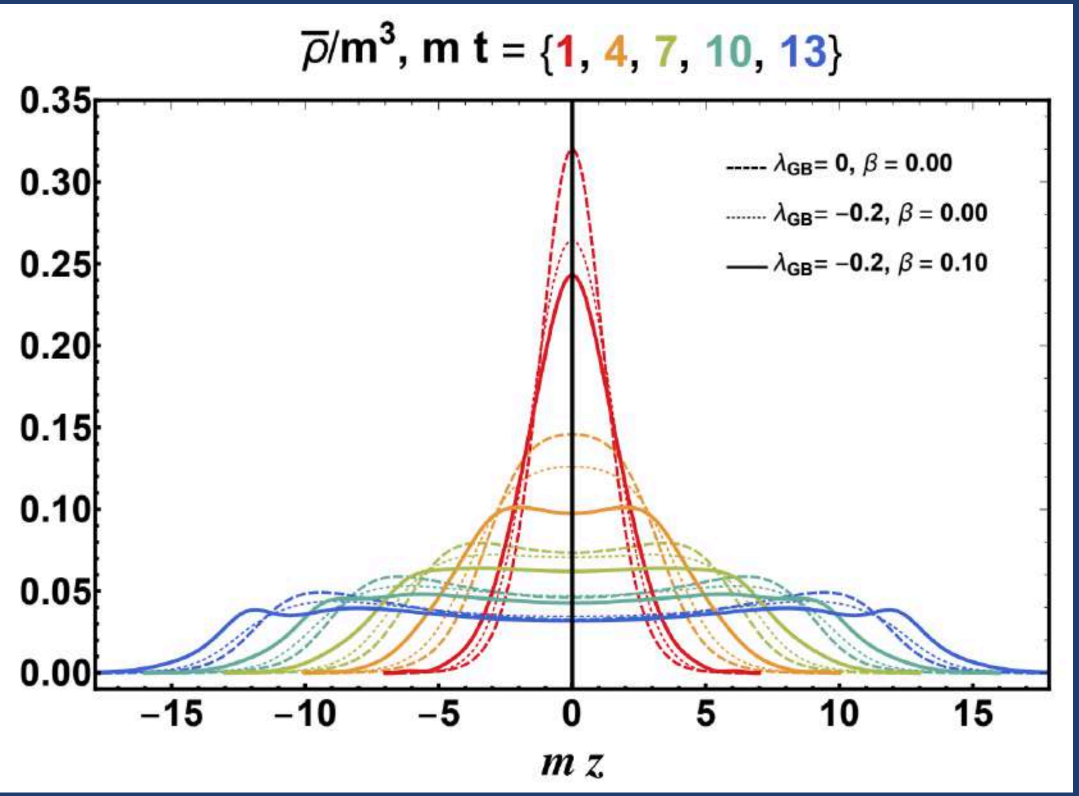
(b) $\beta = 0$ (c) $\beta = 0.025$

HYDRODYNAMISATION AT STRONG COUPLING

- thin sheets are significantly more sensitive to coupling corrections

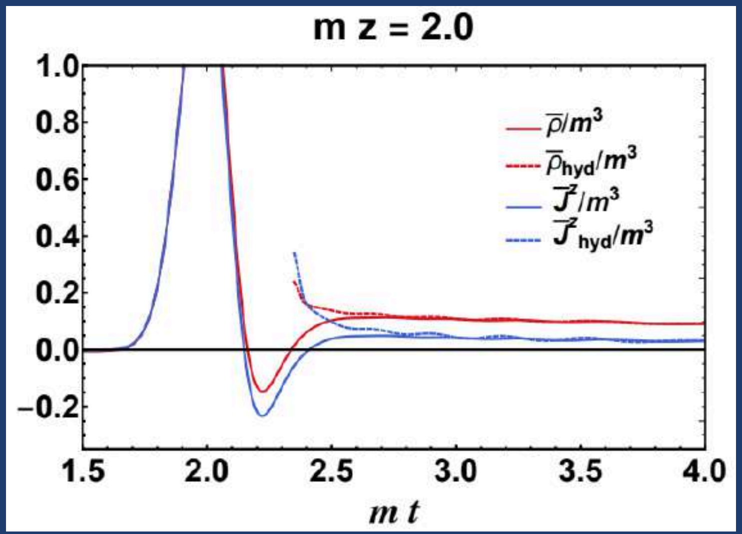


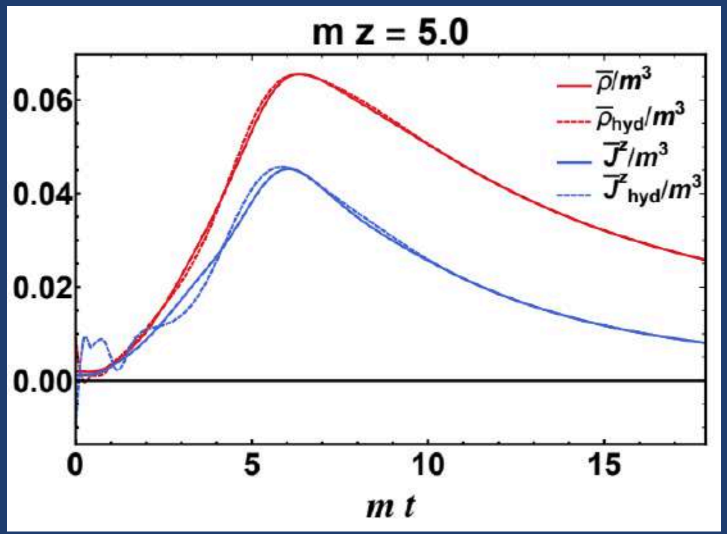
thin sheets



thick sheets

- hydrodynamisation at strong coupling with 'baryon' charge

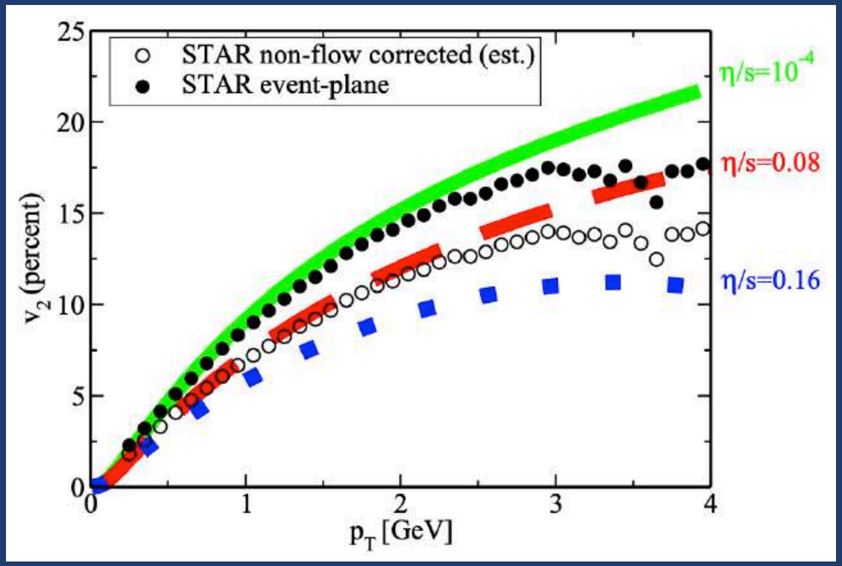




SUMMARY

SUMMARY

- holographic duality is an invaluable tool for exploring QFTs at strong (finite) coupling
- available methods enable concrete calculations in- and far-from-equilibrium
- compare with measurements (higher-order hydrodynamics, relaxation times, hydrodynamisation, …)



[from Luzum, Romatschke, PRC (2018)]

- do higher-order hydrodynamics or long-time tails play a measurable role in QGP [work in progress]?
- hydrodynamics has exceptional convergence properties at strong coupling; how about non-linear versions of this statement? [**see also SERANTES's TALK**]
- holography at finite number of colours N_c
- is quantum chaos important for QGP, jet quenching, …?

THANK YOU!